

sec. 15-3 The Force Law for Simple Harmonic Motion•1

- 1 An object undergoing simple harmonic motion takes 0.25 s to travel from one point of zero velocity to the next such point. The distance between those points is 36 cm. Calculate the (a) period, (b) frequency, and (c) amplitude of the motion.

Answer:

(a) 0.50 s; (b) 2.0 Hz; (c) 18 cm

- 2 A 0.12 kg body undergoes simple harmonic motion of amplitude 8.5 cm and period 0.20 s. (a) What is the magnitude of the maximum force acting on it? (b) If the oscillations are produced by a spring, what is the spring constant?
- 3 What is the maximum acceleration of a platform that oscillates at amplitude 2.20 cm and frequency 6.60 Hz?

Answer:

37.8 m/s²

- 4 An automobile can be considered to be mounted on four identical springs as far as vertical oscillations are concerned. The springs of a certain car are adjusted so that the oscillations have a frequency of 3.00 Hz. (a) What is the spring constant of each spring if the mass of the car is 1450 kg and the mass is evenly distributed over the springs? (b) What will be the oscillation frequency if five passengers, averaging 73.0 kg each, ride in the car with an even distribution of mass?
- 5 **SSM** In an electric shaver, the blade moves back and forth over a distance of 2.0 mm in simple harmonic motion, with frequency 120 Hz. Find (a) the amplitude, (b) the maximum blade speed, and (c) the magnitude of the maximum blade acceleration.

Answer:

(a) 1.0 mm; (b) 0.75 m/s; (c) 5.7×10^2 m/s²

- 6 A particle with a mass of 1.00×10^{-20} kg is oscillating with simple harmonic motion with a period of 1.00×10^{-5} s and a maximum speed of 1.00×10^3 m/s. Calculate (a) the angular frequency and (b) the maximum displacement of the particle.
- 7 **SSM** A loudspeaker produces a musical sound by means of the oscillation of a diaphragm whose amplitude is limited to $1.00 \mu\text{m}$. (a) At what frequency is the magnitude a of the diaphragm's acceleration equal to g ? (b) For greater frequencies, is a greater than or less than g ?

Answer:

(a) 498 Hz; (b) greater

- 8 What is the phase constant for the harmonic oscillator with the position function $x(t)$ given in Fig. 15-28 if the position function has the form $x = x_m \cos(\omega t + \phi)$? The vertical axis scale is set by $x_s = 6.0$ cm.

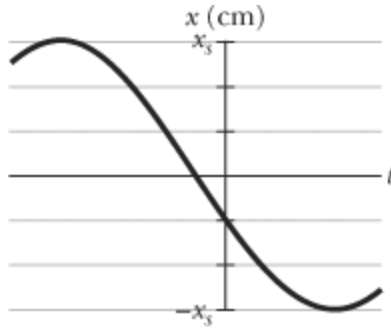


Figure 15-28 Problem 8.

- 9 The function $x = (6.0 \text{ m}) \cos[(3\pi \text{ rad/s})t + \pi/3 \text{ rad}]$ gives the simple harmonic motion of a body. At $t = 2.0 \text{ s}$, what are the (a) displacement, (b) velocity, (c) acceleration, and (d) phase of the motion? Also, what are the (e) frequency and (f) period of the motion?

Answer:

- (a) 3.0 m; (b) -49 m/s; (c) $-2.7 \times 10^2 \text{ m/s}^2$; (d) 20 rad; (e) 1.5 Hz; (f) 0.67 s
- 10 An oscillating block–spring system takes 0.75 s to begin repeating its motion. Find (a) the period, (b) the frequency in hertz, and (c) the angular frequency in radians per second.
- 11 In Fig. 15-29, two identical springs of spring constant 7580 N/m are attached to a block of mass 0.245 kg. What is the frequency of oscillation on the frictionless floor?

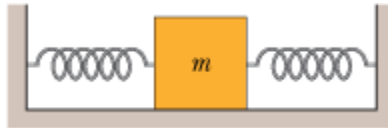


Figure 15-29 Problems 11 and 21.

Answer:

39.6 Hz

- 12 What is the phase constant for the harmonic oscillator with the velocity function $v(t)$ given in Fig. 15-30 if the position function $x(t)$ has the form $x = x_m \cos(\omega t + \phi)$? The vertical axis scale is set by $v_s = 4.0 \text{ cm/s}$.

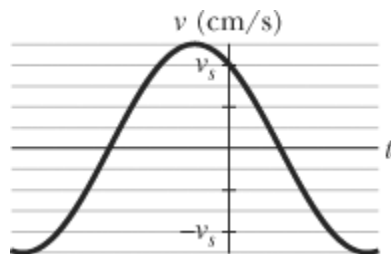


Figure 15-30 Problem 12.

- 13 **SSM** An oscillator consists of a block of mass 0.500 kg connected to a spring. When set into oscillation with amplitude 35.0 cm, the oscillator repeats its motion every 0.500 s. Find the (a) period, (b) frequency, (c) angular frequency, (d) spring constant, (e) maximum speed, and (f)

magnitude of the maximum force on the block from the spring.

Answer:

(a) 0.500 s; (b) 2.00 Hz; (c) 12.6 rad/s; (d) 79.0 N/m; (e) 4.40 m/s; (f) 27.6 N

••14 A simple harmonic oscillator consists of a block of mass 2.00 kg attached to a spring of spring constant 100 N/m. When $t = 1.00$ s, the position and velocity of the block are $x = 0.129$ m and $v = 3.415$ m/s. (a) What is the amplitude of the oscillations? What were the (b) position and (c) velocity of the block at $t = 0$ s?

••15 **SSM** Two particles oscillate in simple harmonic motion along a common straight-line segment of length A . Each particle has a period of 1.5 s, but they differ in phase by $\pi/6$ rad. (a) How far apart are they (in terms of A) 0.50 s after the lagging particle leaves one end of the path? (b) Are they then moving in the same direction, toward each other, or away from each other?

Answer:

(a) $0.18A$; (b) same direction

••16 Two particles execute simple harmonic motion of the same amplitude and frequency along close parallel lines. They pass each other moving in opposite directions each time their displacement is half their amplitude. What is their phase difference?

••17 **ILW** An oscillator consists of a block attached to a spring ($k = 400$ N/m). At some time t , the position (measured from the system's equilibrium location), velocity, and acceleration of the block are $x = 0.100$ m, $v = -13.6$ m/s, and $a = -123$ m/s². Calculate (a) the frequency of oscillation, (b) the mass of the block, and (c) the amplitude of the motion.

Answer:

(a) 5.58 Hz; (b) 0.325 kg; (c) 0.400 m

••18 **GO** At a certain harbor, the tides cause the ocean surface to rise and fall a distance d (from highest level to lowest level) in simple harmonic motion, with a period of 12.5 h. How long does it take for the water to fall a distance $0.250d$ from its highest level?

••19 A block rides on a piston that is moving vertically with simple harmonic motion. (a) If the SHM has period 1.0 s, at what amplitude of motion will the block and piston separate? (b) If the piston has an amplitude of 5.0 cm, what is the maximum frequency for which the block and piston will be in contact continuously?

Answer:

(a) 25 cm; (b) 2.2 Hz

••20 **GO** Figure 15-31a is a partial graph of the position function $x(t)$ for a simple harmonic oscillator with an angular frequency of 1.20 rad/s; Fig. 15-31b is a partial graph of the corresponding velocity function $v(t)$. The vertical axis scales are set by $x_s = 5.0$ cm and $v_s = 5.0$ cm/s. What is the phase constant of the SHM if the position function $x(t)$ is in the general form $x = x_m \cos(\omega t + \phi)$?

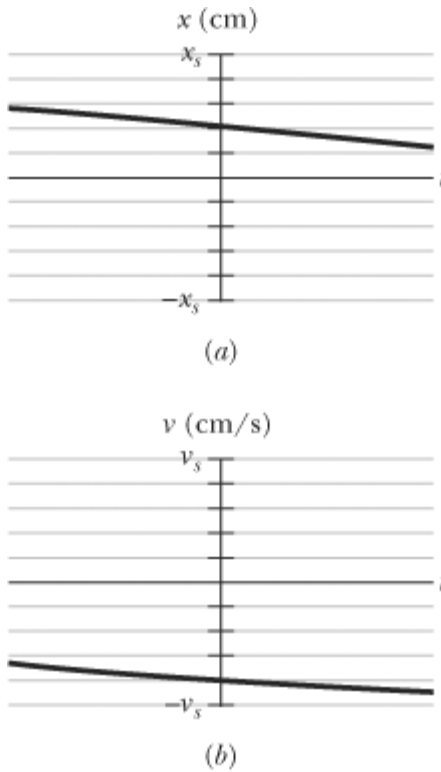


Figure 15-31 Problem 20.

- 21 **ILW** In Fig. 15-29, two springs are attached to a block that can oscillate over a frictionless floor. If the left spring is removed, the block oscillates at a frequency of 30 Hz. If, instead, the spring on the right is removed, the block oscillates at a frequency of 45 Hz. At what frequency does the block oscillate with both springs attached?

Answer:

54 Hz

- 22 **GO** Figure 15-32 shows block 1 of mass 0.200 kg sliding to the right over a frictionless elevated surface at a speed of 8.00 m/s. The block undergoes an elastic collision with stationary block 2, which is attached to a spring of spring constant 1208.5 N/m. (Assume that the spring does not affect the collision.) After the collision, block 2 oscillates in SHM with a period of 0.140 s, and block 1 slides off the opposite end of the elevated surface, landing a distance d from the base of that surface after falling height $h = 4.90$ m. What is the value of d ?

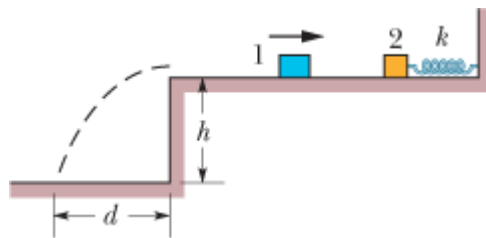


Figure 15-32 Problem 22.

- 23 **SSM WWW** A block is on a horizontal surface (a shake table) that is moving back and forth

horizontally with simple harmonic motion of frequency 2.0 Hz. The coefficient of static friction between block and surface is 0.50. How great can the amplitude of the SHM be if the block is not to slip along the surface?

Answer:

3.1 cm

- 24 In Fig. 15-33, two springs are joined and connected to a block of mass 0.245 kg that is set oscillating over a frictionless floor. The springs each have spring constant $k = 6430 \text{ N/m}$. What is the frequency of the oscillations?

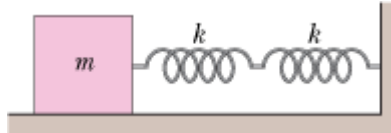


Figure 15-33 Problem 24.

- 25 **GO** In Fig. 15-34, a block weighing 14.0 N, which can slide without friction on an incline at angle $\theta = 40.0^\circ$, is connected to the top of the incline by a massless spring of unstretched length 0.450 m and spring constant 120 N/m. (a) How far from the top of the incline is the block's equilibrium point? (b) If the block is pulled slightly down the incline and released, what is the period of the resulting oscillations?

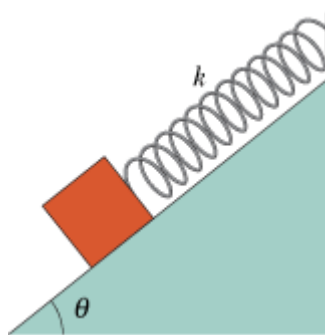


Figure 15-34 Problem 25.

Answer:

(a) 0.525 m; (b) 0.686 s

- 26 **GO** In Fig. 15-35, two blocks ($m = 1.8 \text{ kg}$ and $M = 10 \text{ kg}$) and a spring ($k = 200 \text{ N/m}$) are arranged on a horizontal, frictionless surface. The coefficient of static friction between the two blocks is 0.40. What amplitude of simple harmonic motion of the spring-blocks system puts the smaller block on the verge of slipping over the larger block?

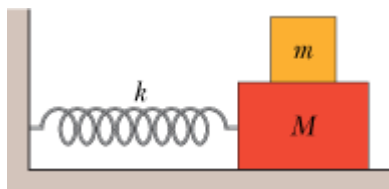


Figure 15-35 Problem 26.

sec. 15-4 Energy in Simple Harmonic Motion

- 27 **SSM** When the displacement in SHM is one-half the amplitude x_m , what fraction of the total energy is (a) kinetic energy and (b) potential energy? (c) At what displacement, in terms of the amplitude, is the energy of the system half kinetic energy and half potential energy?

Answer:

(a) 0.75; (b) 0.25; (c) $2^{-0.5}x_m$

- 28 Figure 15-36 gives the one-dimensional potential energy well for a 2.0 kg particle (the function $U(x)$ has the form bx^2 and the vertical axis scale is set by $U_s = 2.0$ J). (a) If the particle passes through the equilibrium position with a velocity of 85 cm/s, will it be turned back before it reaches $x = 15$ cm? (b) If yes, at what position, and if no, what is the speed of the particle at $x = 15$ cm?

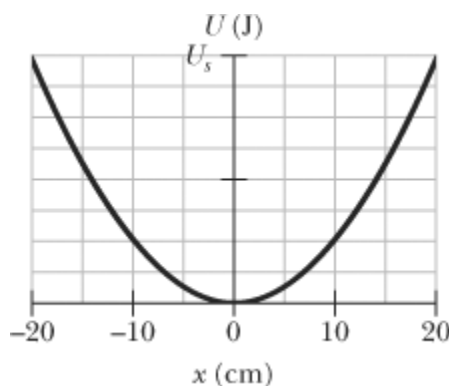


Figure 15-36 Problem 28.

- 29 **SSM** Find the mechanical energy of a block–spring system having a spring constant of 1.3 N/cm and an oscillation amplitude of 2.4 cm.

Answer:

37 mJ

- 30 An oscillating block–spring system has a mechanical energy of 1.00 J, an amplitude of 10.0 cm, and a maximum speed of 1.20 m/s. Find (a) the spring constant, (b) the mass of the block, and (c) the frequency of oscillation.
- 31 **ILW** A 5.00 kg object on a horizontal frictionless surface is attached to a spring with $k = 1000$ N/m. The object is displaced from equilibrium 50.0 cm horizontally and given an initial velocity of 10.0 m/s back toward the equilibrium position. What are (a) the motion's frequency, (b) the initial potential energy of the block–spring system, (c) the initial kinetic energy, and (d) the motion's amplitude?

Answer:

(a) 2.25 Hz; (b) 125 J; (c) 250 J; (d) 86.6 cm

- 32 Figure 15-37 shows the kinetic energy K of a simple harmonic oscillator versus its position x . The vertical axis scale is set by $K_s = 4.0$ J. What is the spring constant?

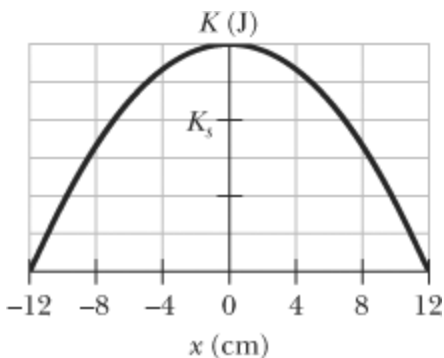


Figure 15-37 Problem 32.

- 33 GO A block of mass $M = 5.4$ kg, at rest on a horizontal frictionless table, is attached to a rigid support by a spring of constant $k = 6000$ N/m. A bullet of mass $m = 9.5$ g and velocity \vec{v} of magnitude 630 m/s strikes and is embedded in the block (Fig. 15-38). Assuming the compression of the spring is negligible until the bullet is embedded, determine (a) the speed of the block immediately after the collision and (b) the amplitude of the resulting simple harmonic motion.

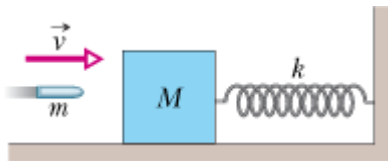


Figure 15-38 Problem 33.

Answer:

(a) 1.1 m/s; (b) 3.3 cm

- 34 GO In Fig. 15-39, block 2 of mass 2.0 kg oscillates on the end of a spring in SHM with a period of 20 ms. The block's position is given by $x = (1.0 \text{ cm}) \cos(\omega t + \pi/2)$. Block 1 of mass 4.0 kg slides toward block 2 with a velocity of magnitude 6.0 m/s, directed along the spring's length. The two blocks undergo a completely inelastic collision at time $t = 5.0$ ms. (The duration of the collision is much less than the period of motion.) What is the amplitude of the SHM after the collision?

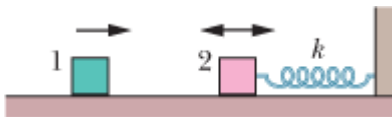


Figure 15-39 Problem 34.

- 35 A 10 g particle undergoes SHM with an amplitude of 2.0 mm, a maximum acceleration of magnitude 8.0×10^3 m/s², and an unknown phase constant. What are (a) the period of the motion, (b) the maximum speed of the particle, and (c) the total mechanical energy of the oscillator? What is the magnitude of the force on the particle when the particle is at (d) its maximum displacement and (e) half its maximum displacement?

Answer:

(a) 3.1 ms; (b) 4.0 m/s; (c) 0.080 J; (d) 80 N; (e) 40 N

- 36 If the phase angle for a block–spring system in SHM is $\pi/6$ rad and the block's position is given by $x = x_m \cos(\omega t + \phi)$, what is the ratio of the kinetic energy to the potential energy at time $t = 0$?
- 37 A massless spring hangs from the ceiling with a small object attached to its lower end. The object is initially held at rest in a position y_i such that the spring is at its rest length. The object is then released from y_i and oscillates up and down, with its lowest position being 10 cm below y_i . (a) What is the frequency of the oscillation? (b) What is the speed of the object when it is 8.0 cm below the initial position? (c) An object of mass 300 g is attached to the first object, after which the system oscillates with half the original frequency. What is the mass of the first object? (d) How far below y_i is the new equilibrium (rest) position with both objects attached to the spring?

Answer:

(a) 2.2 Hz; (b) 56 cm/s; (c) 0.10 kg; (d) 20.0 cm

sec. 15-5 An Angular Simple Harmonic Oscillator

- 38 A 95 kg solid sphere with a 15 cm radius is suspended by a vertical wire. A torque of 0.20 N·m is required to rotate the sphere through an angle of 0.85 rad and then maintain that orientation. What is the period of the oscillations that result when the sphere is then released?
- 39 **SSM WWW** The balance wheel of an old-fashioned watch oscillates with angular amplitude π rad and period 0.500 s. Find (a) the maximum angular speed of the wheel, (b) the angular speed at displacement $\pi/2$ rad, and (c) the magnitude of the angular acceleration at displacement $\pi/4$ rad.

Answer:

(a) 39.5 rad/s; (b) 34.2 rad/s; (c) 124 rad/s²

sec. 15-6 Pendulums

- 40 **ILW** A physical pendulum consists of a meter stick that is pivoted at a small hole drilled through the stick a distance d from the 50 cm mark. The period of oscillation is 2.5 s. Find d .
- 41 **SSM** In Fig. 15-40, the pendulum consists of a uniform disk with radius $r = 10.0$ cm and mass 500 g attached to a uniform rod with length $L = 500$ mm and mass 270 g. (a) Calculate the rotational inertia of the pendulum about the pivot point. (b) What is the distance between the pivot point and the center of mass of the pendulum? (c) Calculate the period of oscillation.

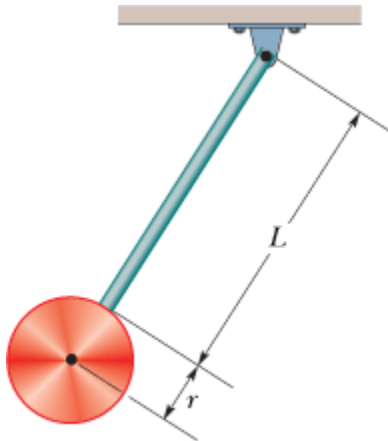


Figure 15-40 Problem 41.

Answer:

(a) $0.205 \text{ kg}\cdot\text{m}^2$; (b) 47.7 cm ; (c) 1.50 s

- 42 Suppose that a simple pendulum consists of a small 60.0 g bob at the end of a cord of negligible mass. If the angle between the cord and the vertical is given by

$$\theta = (0.0800 \text{ rad}) \cos [(4.43 \text{ rad / s})t + \phi],$$

what are (a) the pendulum's length and (b) its maximum kinetic energy?

- 43 (a) If the physical pendulum of Fig. 15-11 and the associated sample problem is inverted and suspended at point P , what is its period of oscillation? (b) Is the period now greater than, less than, or equal to its previous value?

Answer:

(a) 1.64 s ; (b) equal

- 44 A physical pendulum consists of two meter-long sticks joined together as shown in Fig. 15-41. What is the pendulum's period of oscillation about a pin inserted through point A at the center of the horizontal stick?

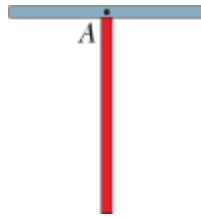


Figure 15-41 Problem 44.

- 45 A performer seated on a trapeze is swinging back and forth with a period of 8.85 s . If she stands up, thus raising the center of mass of the *trapeze + performer* system by 35.0 cm , what will be the new period of the system? Treat *trapeze + performer* as a simple pendulum.

Answer:

8.77 s

- 46 A physical pendulum has a center of oscillation at distance $2L/3$ from its point of suspension. Show that the distance between the point of suspension and the center of oscillation for a physical pendulum of any form is I/mh , where I and h have the meanings assigned to them in Eq. 15-29 and m is the mass of the pendulum.
- 47 In Fig. 15-42, a physical pendulum consists of a uniform solid disk (of radius $R = 2.35 \text{ cm}$) supported in a vertical plane by a pivot located a distance $d = 1.75 \text{ cm}$ from the center of the disk. The disk is displaced by a small angle and released. What is the period of the resulting simple harmonic motion?

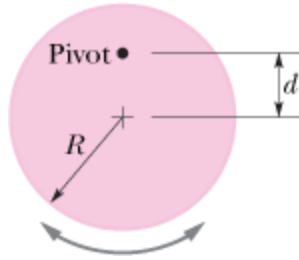


Figure 15-42 Problem 47.

Answer:

0.366 s

- 48 A rectangular block, with face lengths $a = 35$ cm and $b = 45$ cm, is to be suspended on a thin horizontal rod running through a narrow hole in the block. The block is then to be set swinging about the rod like a pendulum, through small angles so that it is in SHM. Figure 15-43 shows one possible position of the hole, at distance r from the block's center, along a line connecting the center with a corner. (a) Plot the period of the pendulum versus distance r along that line such that the minimum in the curve is apparent. (b) For what value of r does that minimum occur? There is actually a line of points around the block's center for which the period of swinging has the same minimum value. (c) What shape does that line make?

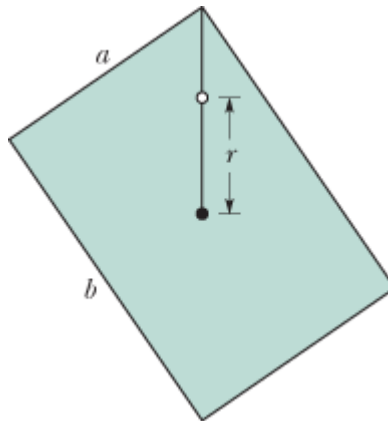


Figure 15-43 Problem 48.

- 49 GO The angle of the pendulum of Fig. 15-9b is given by $\theta = \theta_m \cos[(4.44 \text{ rad/s})t + \phi]$. If at $t = 0$, $\theta = 0.040$ rad and $d\theta/dt = -0.200$ rad/s, what are (a) the phase constant and (b) the maximum angle θ_m ? (Hint: Don't confuse the rate $d\theta/dt$ at which θ changes with the ω of the SHM.)

Answer:

(a) 0.845 rad; (b) 0.0602 rad

- 50 A thin uniform rod (mass = 0.50 kg) swings about an axis that passes through one end of the rod and is perpendicular to the plane of the swing. The rod swings with a period of 1.5 s and an angular amplitude of 10° . (a) What is the length of the rod? (b) What is the maximum kinetic energy of the rod as it swings?
- 51 In Fig. 15-44, a stick of length $L = 1.85$ m oscillates as a physical pendulum. (a) What value of distance x between the stick's center of mass and its pivot point O gives the least period? (b) What

is that least period?

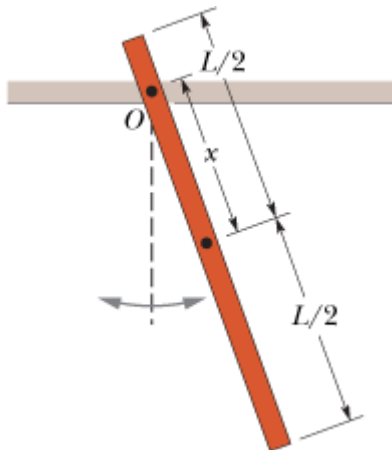


Figure 15-44 Problem 51.

Answer:

(a) 0.53 m; (b) 2.1 s

52 GO The 3.00 kg cube in Fig. 15-45 has edge lengths $d = 6.00$ cm and is mounted on an axle through its center. A spring ($k = 1200$ N/m) connects the cube's upper corner to a rigid wall. Initially the spring is at its rest length. If the cube is rotated 3° and released, what is the period of the resulting SHM?

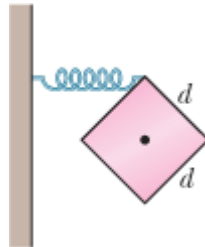


Figure 15-45 Problem 52.

53 SSM ILW In the overhead view of Fig. 15-46, a long uniform rod of mass 0.600 kg is free to rotate in a horizontal plane about a vertical axis through its center. A spring with force constant $k = 1850$ N/m is connected horizontally between one end of the rod and a fixed wall. When the rod is in equilibrium, it is parallel to the wall. What is the period of the small oscillations that result when the rod is rotated slightly and released?

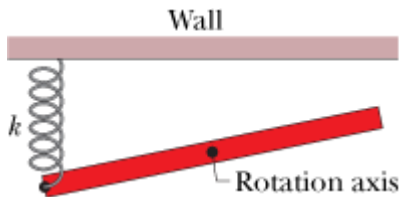


Figure 15-46 Problem 53.

Answer:

0.0653 s

- 54 GO In Fig. 15-47a, a metal plate is mounted on an axle through its center of mass. A spring with $k = 2000 \text{ N/m}$ connects a wall with a point on the rim a distance $r = 2.5 \text{ cm}$ from the center of mass. Initially the spring is at its rest length. If the plate is rotated by 7° and released, it rotates about the axle in SHM, with its angular position given by Fig. 15-47b. The horizontal axis scale is set by $t_s = 20 \text{ ms}$. What is the rotational inertia of the plate about its center of mass?

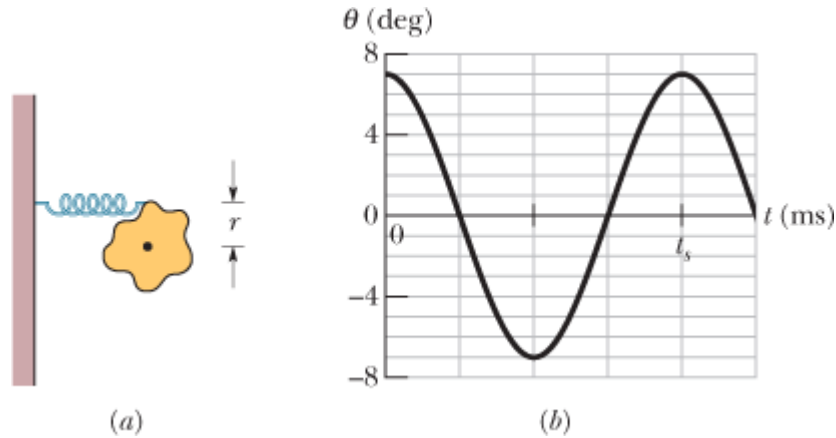


Figure 15-47 Problem 54.

- 55 A pendulum is formed by pivoting a long thin rod about a point on the rod. In a series of experiments, the period is measured as a function of the distance x between the pivot point and the rod's center. (a) If the rod's length is $L = 2.20 \text{ m}$ and its mass is $m = 22.1 \text{ g}$, what is the minimum period? (b) If x is chosen to minimize the period and then L is increased, does the period increase, decrease, or remain the same? (c) If, instead, m is increased without L increasing, does the period increase, decrease, or remain the same?

Answer:

(a) 2.26 s; (b) increases; (c) same

- 56 In Fig. 15-48, a 2.50 kg disk of diameter $D = 42.0 \text{ cm}$ is supported by a rod of length $L = 76.0 \text{ cm}$ and negligible mass that is pivoted at its end. (a) With the massless torsion spring unconnected, what is the period of oscillation? (b) With the torsion spring connected, the rod is vertical at equilibrium. What is the torsion constant of the spring if the period of oscillation has been decreased by 0.500 s?

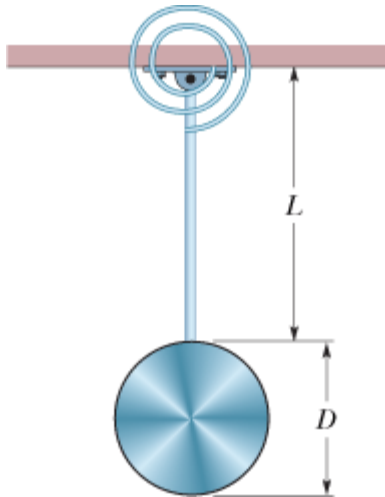


Figure 15-48 Problem 56.

sec. 15-8 Damped Simple Harmonic Motion

- 57 The amplitude of a lightly damped oscillator decreases by 3.0% during each cycle. What percentage of the mechanical energy of the oscillator is lost in each cycle?

Answer:

6.0%

- 58 In a damped oscillator with $m = 250$ g, $k = 85$ N/m, and $b = 70$ g/s, what is the ratio of the amplitude of the damped oscillations to the initial amplitude at the end of 20 cycles?
- 59 **SSM WWW** In Fig. 15-14, the block has a mass of 1.50 kg and the spring constant is 8.00 N/m. The damping force is given by $-b(dx/dt)$, where $b = 230$ g/s. The block is pulled down 12.0 cm and released. (a) Calculate the time required for the amplitude of the resulting oscillations to fall to one-third of its initial value. (b) How many oscillations are made by the block in this time?

Answer:

(a) 14.3 s; (b) 5.27

- 60 The suspension system of a 2000 kg automobile “sags” 10 cm when the chassis is placed on it. Also, the oscillation amplitude decreases by 50% each cycle. Estimate the values of (a) the spring constant k and (b) the damping constant b for the spring and shock absorber system of one wheel, assuming each wheel supports 500 kg.

sec. 15-9 Forced Oscillations and Resonance

- 61 For Eq. 15-45, suppose the amplitude x_m is given by

$$x_m = \frac{F_m}{\left[m^2 (\omega_d^2 - \omega^2)^2 + b^2 \omega_d^2 \right]^{1/2}}$$

where F_m is the (constant) amplitude of the external oscillating force exerted on the spring by the rigid support in Fig. 15-14. At resonance, what are the (a) amplitude and (b) velocity amplitude of the oscillating object?

Answer:



(a) $F_m/b\omega$; (b) F_m/b

- 62 Hanging from a horizontal beam are nine simple pendulums of the following lengths: (a) 0.10, (b) 0.30, (c) 0.40, (d) 0.80, (e) 1.2, (f) 2.8, (g) 3.5, (h) 5.0, and (i) 6.2 m. Suppose the beam undergoes horizontal oscillations with angular frequencies in the range from 2.00 rad/s to 4.00 rad/s. Which of the pendulums will be (strongly) set in motion?
- 63 A 1000 kg car carrying four 82 kg people travels over a “washboard” dirt road with corrugations 4.0 m apart. The car bounces with maximum amplitude when its speed is 16 km/h. When the car stops, and the people get out, by how much does the car body rise on its suspension?

Answer:

5.0 cm

Additional Problems

- 64  Although California is known for earthquakes, it has large regions dotted with precariously balanced rocks that would be easily toppled by even a mild earthquake. The rocks have stood this way for thousands of years, suggesting that major earthquakes have not occurred in those regions during that time. If an earthquake were to put such a rock into sinusoidal oscillation (parallel to the ground) with a frequency of 2.2 Hz, an oscillation amplitude of 1.0 cm would cause the rock to topple. What would be the magnitude of the maximum acceleration of the oscillation, in terms of g ?
- 65 A loudspeaker diaphragm is oscillating in simple harmonic motion with a frequency of 440 Hz and a maximum displacement of 0.75 mm. What are the (a) angular frequency, (b) maximum speed, and (c) magnitude of the maximum acceleration?
- Answer:**
- (a) 2.8×10^3 rad/s; (b) 2.1 m/s; (c) 5.7 km/s^2
- 66 A uniform spring with $k = 8600 \text{ N/m}$ is cut into pieces 1 and 2 of unstretched lengths $L_1 = 7.0 \text{ cm}$ and $L_2 = 10 \text{ cm}$. What are (a) k_1 and (b) k_2 ? A block attached to the original spring as in Fig. 15-5 oscillates at 200 Hz. What is the oscillation frequency of the block attached to (c) piece 1 and (d) piece 2?
- 67  In Fig. 15-49, three 10 000 kg ore cars are held at rest on a mine railway using a cable that is parallel to the rails, which are inclined at angle $\theta = 30^\circ$. The cable stretches 15 cm just before the coupling between the two lower cars breaks, detaching the lowest car. Assuming that the cable obeys Hooke's law, find the (a) frequency and (b) amplitude of the resulting oscillations of the remaining two cars.

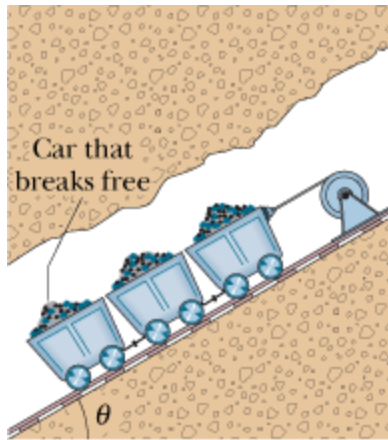


Figure 15-49 Problem 67.

Answer:

(a) 1.1 Hz; (b) 5.0 cm

68 A 2.00 kg block hangs from a spring. A 300 g body hung below the block stretches the spring 2.00 cm farther. (a) What is the spring constant? (b) If the 300 g body is removed and the block is set into oscillation, find the period of the motion.

69 **SSM** The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 0.76 m. If the piston moves with simple harmonic motion with an angular frequency of 180 rev/min, what is its maximum speed?

Answer:

7.2 m/s

70 A wheel is free to rotate about its fixed axle. A spring is attached to one of its spokes a distance r from the axle, as shown in Fig. 15-50. (a) Assuming that the wheel is a hoop of mass m and radius R , what is the angular frequency ω of small oscillations of this system in terms of m , R , r , and the spring constant k ? What is ω if (b) $r = R$ and (c) $r = 0$?

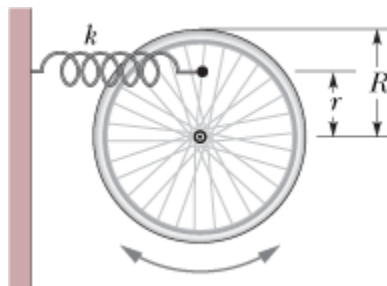


Figure 15-50 Problem 70.

71 A 50.0 g stone is attached to the bottom of a vertical spring and set vibrating. If the maximum speed of the stone is 15.0 cm/s and the period is 0.500 s, find the (a) spring constant of the spring, (b) amplitude of the motion, and (c) frequency of oscillation.

Answer:

(a) 7.90 N/m; (b) 1.19 cm; (c) 2.00 Hz

72A uniform circular disk whose radius R is 12.6 cm is suspended as a physical pendulum from a point on its rim. (a) What is its period? (b) At what radial distance $r < R$ is there a pivot point that gives the same period?

73 SSM A vertical spring stretches 9.6 cm when a 1.3 kg block is hung from its end. (a) Calculate the spring constant. This block is then displaced an additional 5.0 cm downward and released from rest. Find the (b) period, (c) frequency, (d) amplitude, and (e) maximum speed of the resulting SHM.

Answer:

(a) 1.3×10^2 N/m; (b) 0.62 s; (c) 1.6 Hz; (d) 5.0 cm; (e) 0.51 m/s

74A massless spring with spring constant 19 N/m hangs vertically. A body of mass 0.20 kg is attached to its free end and then released. Assume that the spring was unstretched before the body was released. Find (a) how far below the initial position the body descends, and the (b) frequency and (c) amplitude of the resulting SHM.

75A 4.00 kg block is suspended from a spring with $k = 500$ N/m. A 50.0 g bullet is fired into the block from directly below with a speed of 150 m/s and becomes embedded in the block. (a) Find the amplitude of the resulting SHM. (b) What percentage of the original kinetic energy of the bullet is transferred to mechanical energy of the oscillator?

Answer:

(a) 16.6 cm; (b) 1.23%

76A 55.0 g block oscillates in SHM on the end of a spring with $k = 1500$ N/m according to $x = x_m \cos(\omega t + \phi)$. How long does the block take to move from position $+0.800x_m$ to (a) position $+0.600x_m$ and (b) position $-0.800x_m$?

77Figure 15-51 gives the position of a 20 g block oscillating in SHM on the end of a spring. The horizontal axis scale is set by $t_s = 40.0$ ms. What are (a) the maximum kinetic energy of the block and (b) the number of times per second that maximum is reached? (*Hint:* Measuring a slope will probably not be very accurate. Find another approach.)

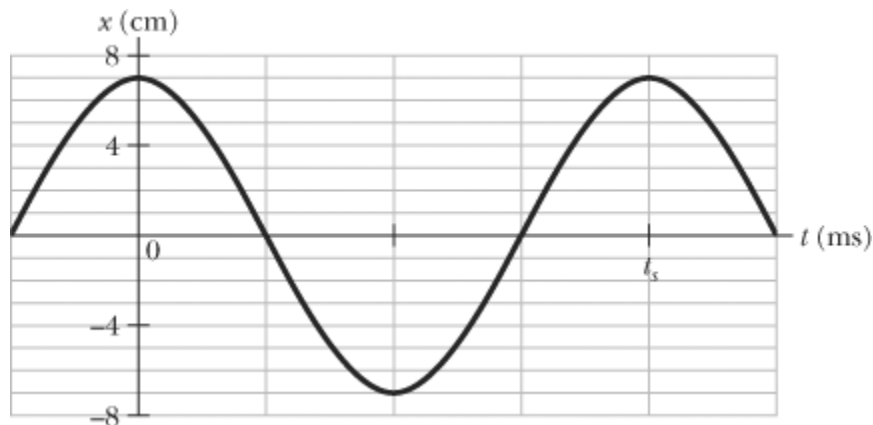


Figure 15-51 Problems 77 and 78.

Answer:

(a) 1.2 J; (b) 50

78 Figure 15-51 gives the position $x(t)$ of a block oscillating in SHM on the end of a spring ($t_s = 40.0$ ms). What are (a) the speed and (b) the magnitude of the radial acceleration of a particle in the corresponding uniform circular motion?

79 Figure 15-52 shows the kinetic energy K of a simple pendulum versus its angle θ from the vertical. The vertical axis scale is set by $K_s = 10.0$ mJ. The pendulum bob has mass 0.200 kg. What is the length of the pendulum?

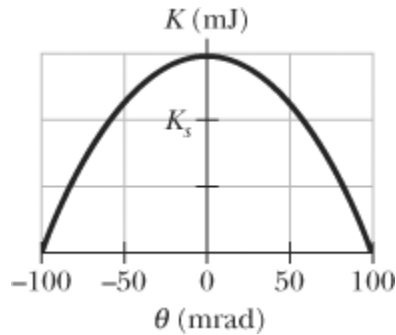


Figure 15-52 Problem 79.

Answer:

1.53 m

80 A block is in SHM on the end of a spring, with position given by $x = x_m \cos(\omega t + \phi)$. If $\phi = \pi/5$ rad, then at $t = 0$ what percentage of the total mechanical energy is potential energy?

81 A simple harmonic oscillator consists of a 0.50 kg block attached to a spring. The block slides back and forth along a straight line on a frictionless surface with equilibrium point $x = 0$. At $t = 0$ the block is at $x = 0$ and moving in the positive x direction. A graph of the magnitude of the net force \vec{F} on the block as a function of its position is shown in Fig. 15-53. The vertical scale is set by $F_s = 75.0$ N. What are (a) the amplitude and (b) the period of the motion, (c) the magnitude of the maximum acceleration, and (d) the maximum kinetic energy?

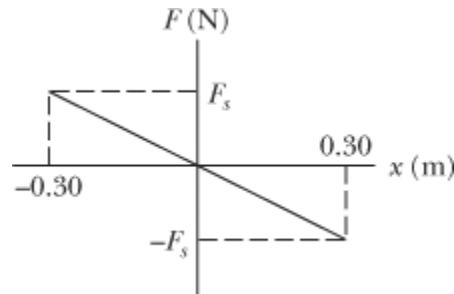


Figure 15-53 Problem 81.

Answer:

(a) 0.30 m; (b) 0.28 s; (c) $1.5 \times 10^2 \text{ m/s}^2$; (d) 11 J

82A simple pendulum of length 20 cm and mass 5.0 g is suspended in a race car traveling with constant speed 70 m/s around a circle of radius 50 m. If the pendulum undergoes small oscillations in a radial direction about its equilibrium position, what is the frequency of oscillation?

83The scale of a spring balance that reads from 0 to 15.0 kg is 12.0 cm long. A package suspended from the balance is found to oscillate vertically with a frequency of 2.00 Hz. (a) What is the spring constant? (b) How much does the package weigh?

Answer:

(a) 1.23 kN/m; (b) 76.0 N

84A 0.10 kg block oscillates back and forth along a straight line on a frictionless horizontal surface. Its displacement from the origin is given by

$$x = (10 \text{ cm}) \cos[(10 \text{ rad/s})t + \pi/2 \text{ rad}].$$

(a) What is the oscillation frequency? (b) What is the maximum speed acquired by the block? (c) At what value of x does this occur? (d) What is the magnitude of the maximum acceleration of the block? (e) At what value of x does this occur? (f) What force, applied to the block by the spring, results in the given oscillation?

85The end point of a spring oscillates with a period of 2.0 s when a block with mass m is attached to it. When this mass is increased by 2.0 kg, the period is found to be 3.0 s. Find m .

Answer:

1.6 kg

86The tip of one prong of a tuning fork undergoes SHM of frequency 1000 Hz and amplitude 0.40 mm. For this tip, what is the magnitude of the (a) maximum acceleration, (b) maximum velocity, (c) acceleration at tip displacement 0.20 mm, and (d) velocity at tip displacement 0.20 mm?

87A flat uniform circular disk has a mass of 3.00 kg and a radius of 70.0 cm. It is suspended in a horizontal plane by a vertical wire attached to its center. If the disk is rotated 2.50 rad about the wire, a torque of 0.0600 N · m is required to maintain that orientation. Calculate (a) the rotational inertia of the disk about the wire, (b) the torsion constant, and (c) the angular frequency of this torsion pendulum when it is set oscillating.

Answer:

(a) $0.735 \text{ kg} \cdot \text{m}^2$; (b) 0.0240 N · m; (c) 0.181 rad/s

88A block weighing 20 N oscillates at one end of a vertical spring for which $k = 100 \text{ N/m}$; the other end of the spring is attached to a ceiling. At a certain instant the spring is stretched 0.30 m beyond its relaxed length (the length when no object is attached) and the block has zero velocity. (a) What is the net force on the block at this instant? What are the (b) amplitude and (c) period of the resulting simple harmonic motion? (d) What is the maximum kinetic energy of the block as it oscillates?

89A 3.0 kg particle is in simple harmonic motion in one dimension and moves according to the equation

$$x = (5.0 \text{ m}) \cos[(\pi/3 \text{ rad/s})t - \pi/4 \text{ rad}],$$

with t in seconds. (a) At what value of x is the potential energy of the particle equal to half the total

energy? (b) How long does the particle take to move to this position x from the equilibrium position?

Answer:

(a) 3.5 m; (b) 0.75 s

90 A particle executes linear SHM with frequency 0.25 Hz about the point $x = 0$. At $t = 0$, it has displacement $x = 0.37$ cm and zero velocity. For the motion, determine the (a) period, (b) angular frequency, (c) amplitude, (d) displacement $x(t)$, (e) velocity $v(t)$, (f) maximum speed, (g) magnitude of the maximum acceleration, (h) displacement at $t = 3.0$ s, and (i) speed at $t = 3.0$ s.

91 SSM What is the frequency of a simple pendulum 2.0 m long (a) in a room, (b) in an elevator accelerating upward at a rate of 2.0 m/s^2 , and (c) in free fall?

Answer:

(a) 0.35 Hz; (b) 0.39 Hz; (c) 0 (no oscillation)

92 A grandfather clock has a pendulum that consists of a thin brass disk of radius $r = 15.00$ cm and mass 1.000 kg that is attached to a long thin rod of negligible mass. The pendulum swings freely about an axis perpendicular to the rod and through the end of the rod opposite the disk, as shown in Fig. 15-54. If the pendulum is to have a period of 2.000 s for small oscillations at a place where $g = 9.800 \text{ m/s}^2$, what must be the rod length L to the nearest tenth of a millimeter?

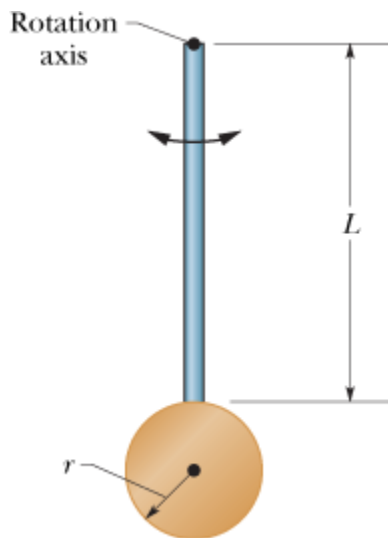


Figure 15-54 Problem 92.

93 A 4.00 kg block hangs from a spring, extending it 16.0 cm from its unstretched position. (a) What is the spring constant? (b) The block is removed, and a 0.500 kg body is hung from the same spring. If the spring is then stretched and released, what is its period of oscillation?

Answer:

(a) 245 N/m; (b) 0.284 s

94 What is the phase constant for SMH with $a(t)$ given in Fig. 15-55 if the position function $x(t)$ has the form $x = x_m \cos(\omega t + \phi)$ and $a_s = 4.0 \text{ m/s}^2$?

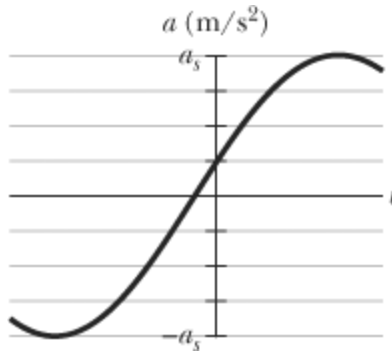



Figure 15-55 Problem 94.

- 95 An engineer has an odd-shaped 10 kg object and needs to find its rotational inertia about an axis through its center of mass. The object is supported on a wire stretched along the desired axis. The wire has a torsion constant $\kappa = 0.50 \text{ N} \cdot \text{m}$. If this torsion pendulum oscillates through 20 cycles in 50 s, what is the rotational inertia of the object?

Answer:

$$0.079 \text{ kg} \cdot \text{m}^2$$

- 96  A spider can tell when its web has captured, say, a fly because the fly's thrashing causes the web threads to oscillate. A spider can even determine the size of the fly by the frequency of the oscillations. Assume that a fly oscillates on the *capture thread* on which it is caught like a block on a spring. What is the ratio of oscillation frequency for a fly with mass m to a fly with mass $2.5m$?
- 97 A torsion pendulum consists of a metal disk with a wire running through its center and soldered in place. The wire is mounted vertically on clamps and pulled taut. Figure 15-56a gives the magnitude τ of the torque needed to rotate the disk about its center (and thus twist the wire) versus the rotation angle θ . The vertical axis scale is set by $\tau_s = 4.0 \times 10^{-3} \text{ N} \cdot \text{m}$. The disk is rotated to $\theta = 0.200 \text{ rad}$ and then released. Figure 15-56b shows the resulting oscillation in terms of angular position versus time t . The horizontal axis scale is set by $t_s 0.40 \text{ s}$. (a) What is the rotational inertia of the disk about its center? (b) What is the maximum angular speed $d\theta/dt$ of the disk? (*Caution:* Do not confuse the (constant) angular frequency of the SHM with the (varying) angular speed of the rotating disk, even though they usually have the same symbol ω . *Hint:* The potential energy U of a torsion pendulum is equal to $\frac{1}{2} \kappa \theta^2$, analogous to $U = \frac{1}{2} \kappa x^2$ for a spring.)

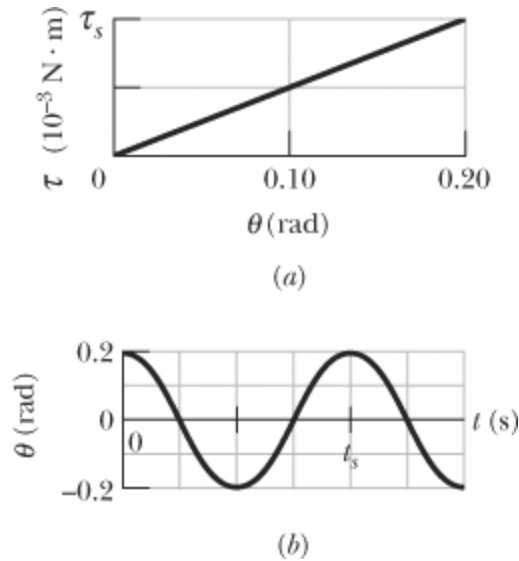


Figure 15-56 Problem 97.

Answer:

(a) $8.11 \times 10^{-5} \text{ kg} \cdot \text{m}^2$; (b) 3.14 rad/s

- 98** When a 20 N can is hung from the bottom of a vertical spring, it causes the spring to stretch 20 cm. (a) What is the spring constant? (b) This spring is now placed horizontally on a frictionless table. One end of it is held fixed, and the other end is attached to a 5.0 N can. The can is then moved (stretching the spring) and released from rest. What is the period of the resulting oscillation?
- 99** For a simple pendulum, find the angular amplitude θ_m at which the restoring torque required for simple harmonic motion deviates from the actual restoring torque by 1.0%. (See “Trigonometric Expansions” in Appendix E.)

Answer:

14.0°

- 100** In Fig. 15-57, a solid cylinder attached to a horizontal spring ($k = 3.00 \text{ N/m}$) rolls without slipping along a horizontal surface. If the system is released from rest when the spring is stretched by 0.250 m, find (a) the translational kinetic energy and (b) the rotational kinetic energy of the cylinder as it passes through the equilibrium position. (c) Show that under these conditions the cylinder's center of mass executes simple harmonic motion with period

$$T = 2\pi \sqrt{\frac{3M}{2k}},$$

where M is the cylinder mass. (*Hint:* Find the time derivative of the total mechanical energy.)

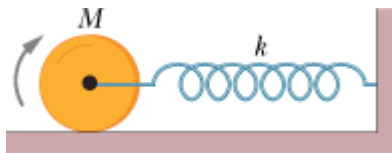


Figure 15-57 Problem 100.

101 SSM A 1.2 kg block sliding on a horizontal frictionless surface is attached to a horizontal spring with $k = 480 \text{ N/m}$. Let x be the displacement of the block from the position at which the spring is unstretched. At $t = 0$ the block passes through $x = 0$ with a speed of 5.2 m/s in the positive x direction. What are the (a) frequency and (b) amplitude of the block's motion? (c) Write an expression for x as a function of time.

Answer:

(a) 3.2 Hz ; (b) 0.26 m ; (c) $x = (0.26 \text{ m}) \cos(20t - \pi/2)$, with t in seconds

102 A simple harmonic oscillator consists of an 0.80 kg block attached to a spring ($k = 200 \text{ N/m}$). The block slides on a horizontal frictionless surface about the equilibrium point $x = 0$ with a total mechanical energy of 4.0 J . (a) What is the amplitude of the oscillation? (b) How many oscillations does the block complete in 10 s ? (c) What is the maximum kinetic energy attained by the block? (d) What is the speed of the block at $x = 0.15 \text{ m}$?

103 A block sliding on a horizontal frictionless surface is attached to a horizontal spring with a spring constant of 600 N/m . The block executes SHM about its equilibrium position with a period of 0.40 s and an amplitude of 0.20 m . As the block slides through its equilibrium position, a 0.50 kg putty wad is dropped vertically onto the block. If the putty wad sticks to the block, determine (a) the new period of the motion and (b) the new amplitude of the motion.

Answer:

(a) 0.44 s ; (b) 0.18 m

104 A damped harmonic oscillator consists of a block ($m = 2.00 \text{ kg}$), a spring ($k = 10.0 \text{ N/m}$), and a damping force ($F = -bv$). Initially, it oscillates with an amplitude of 25.0 cm ; because of the damping, the amplitude falls to three-fourths of this initial value at the completion of four oscillations. (a) What is the value of b ? (b) How much energy has been "lost" during these four oscillations?

105 A block weighing 10.0 N is attached to the lower end of a vertical spring ($k = 200.0 \text{ N/m}$), the other end of which is attached to a ceiling. The block oscillates vertically and has a kinetic energy of 2.00 J as it passes through the point at which the spring is un-stretched. (a) What is the period of the oscillation? (b) Use the law of conservation of energy to determine the maximum distance the block moves both above and below the point at which the spring is unstretched. (These are not necessarily the same.) (c) What is the amplitude of the oscillation? (d) What is the maximum kinetic energy of the block as it oscillates?

Answer:

(a) 0.45 s ; (b) 0.10 m above and 0.20 m below; (c) 0.15 m ; (d) 2.3 J

106 A simple harmonic oscillator consists of a block attached to a spring with $k = 200 \text{ N/m}$. The block slides on a frictionless surface, with equilibrium point $x = 0$ and amplitude 0.20 m . A graph of the block's velocity v as a function of time t is shown in Fig. 15-58. The horizontal scale is set by $t_s = 0.20 \text{ s}$. What are (a) the period of the SHM, (b) the block's mass, (c) its displacement at $t = 0$, (d) its acceleration at $t = 0.10 \text{ s}$, and (e) its maximum kinetic energy?

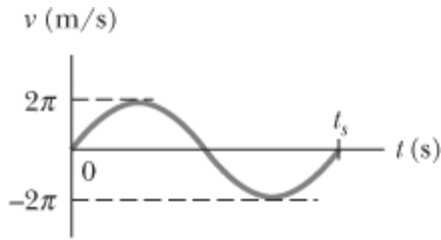


Figure 15-58 Problem 106.

- 107** The vibration frequencies of atoms in solids at normal temperatures are of the order of 10^{13} Hz. Imagine the atoms to be connected to one another by springs. Suppose that a single silver atom in a solid vibrates with this frequency and that all the other atoms are at rest. Compute the effective spring constant. One mole of silver (6.02×10^{23} atoms) has a mass of 108 g.

Answer:

$$7 \times 10^2 \text{ N/m}$$