




## sec. 10-2 The Rotational Variables

- 1 A good baseball pitcher can throw a baseball toward home plate at 85 mi/h with a spin of 1800 rev/min. How many revolutions does the baseball make on its way to home plate? For simplicity, assume that the 60 ft path is a straight line.

**Answer:**

14 rev

- 2 What is the angular speed of (a) the second hand, (b) the minute hand, and (c) the hour hand of a smoothly running analog watch? Answer in radians per second.
- 3  When a slice of buttered toast is accidentally pushed over the edge of a counter, it rotates as it falls. If the distance to the floor is 76 cm and for rotation less than 1 rev, what are the (a) smallest and (b) largest angular speeds that cause the toast to hit and then topple to be butter-side down?

**Answer:**

(a) 4.0 rad/s; (b) 11.9 rad/s

- 4 The angular position of a point on a rotating wheel is given by  $\theta = 2.0 + 4.0t^2 + 2.0t^3$ , where  $\theta$  is in radians and  $t$  is in seconds. At  $t = 0$ , what are (a) the point's angular position and (b) its angular velocity? (c) What is its angular velocity at  $t = 4.0$  s? (d) Calculate its angular acceleration at  $t = 2.0$  s. (e) Is its angular acceleration constant?
- 5 **ILW** A diver makes 2.5 revolutions on the way from a 10-m-high platform to the water. Assuming zero initial vertical velocity, find the average angular velocity during the dive.

**Answer:**

11 rad/s

- 6 The angular position of a point on the rim of a rotating wheel is given by  $\theta = 4.0t - 3.0t^2 + t^3$ , where  $\theta$  is in radians and  $t$  is in seconds. What are the angular velocities at (a)  $t = 2.0$  s and (b)  $t = 4.0$  s? (c) What is the average angular acceleration for the time interval that begins at  $t = 2.0$  s and ends at  $t = 4.0$  s? What are the instantaneous angular accelerations at (d) the beginning and (e) the end of this time interval?
- 7 The wheel in Fig. 10-27 has eight equally spaced spokes and a radius of 30 cm. It is mounted on a fixed axle and is spinning at 2.5 rev/s. You want to shoot a 20 cm-long arrow parallel to this axle and through the wheel without hitting any of the spokes. Assume that the arrow and the spokes are very thin. (a) What minimum speed must the arrow have? (b) Does it matter where between the axle and rim of the wheel you aim? If so, what is the best location?

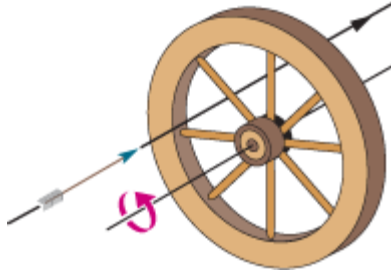


Figure 10-27 Problem 7.

**Answer:**

(a) 4.0 m/s; (b) no

- 8 The angular acceleration of a wheel is  $\alpha = 6.0t^4 - 4.0t^2$ , with  $\alpha$  in radians per second-squared and  $t$  in seconds. At time  $t = 0$ , the wheel has an angular velocity of +2.0 rad/s and an angular position of +1.0 rad. Write expressions for (a) the angular velocity (rad/s) and (b) the angular position (rad) as functions of time (s).

### sec. 10-4 Rotation with Constant Angular Acceleration

- 9 A drum rotates around its central axis at an angular velocity of 12.60 rad/s. If the drum then slows at a constant rate of  $4.20 \text{ rad/s}^2$ , (a) how much time does it take and (b) through what angle does it rotate in coming to rest?

**Answer:**

(a) 3.00 s; (b) 18.9 rad

- 10 Starting from rest, a disk rotates about its central axis with constant angular acceleration. In 5.0 s, it rotates 25 rad. During that time, what are the magnitudes of (a) the angular acceleration and (b) the average angular velocity? (c) What is the instantaneous angular velocity of the disk at the end of the 5.0 s? (d) With the angular acceleration unchanged, through what additional angle will the disk turn during the next 5.0 s?
- 11 A disk, initially rotating at 120 rad/s, is slowed down with a constant angular acceleration of magnitude  $4.0 \text{ rad/s}^2$ . (a) How much time does the disk take to stop? (b) Through what angle does the disk rotate during that time?


**Answer:**


(a) 30 s; (b)  $1.8 \times 10^3 \text{ rad}$

- 12 The angular speed of an automobile engine is increased at a constant rate from 1200 rev/min to 3000 rev/min in 12 s. (a) What is its angular acceleration in revolutions per minute-squared? (b) How many revolutions does the engine make during this 12 s interval?
- 13 **ILW** A flywheel turns through 40 rev as it slows from an angular speed of 1.5 rad/s to a stop. (a) Assuming a constant angular acceleration, find the time for it to come to rest. (b) What is its angular acceleration? (c) How much time is required for it to complete the first 20 of the 40 revolutions?

**Answer:**

(a)  $3.4 \times 10^2$  s; (b)  $-4.5 \times 10^{-3}$  rad/s<sup>2</sup>; (c) 98 s

••14  A disk rotates about its central axis starting from rest and accelerates with constant angular acceleration. At one time it is rotating at 10 rev/s; 60 revolutions later, its angular speed is 15 rev/s. Calculate (a) the angular acceleration, (b) the time required to complete the 60 revolutions, (c) the time required to reach the 10 rev/s angular speed, and (d) the number of revolutions from rest until the time the disk reaches the 10 rev/s angular speed.

••15  A wheel has a constant angular acceleration of  $3.0 \text{ rad/s}^2$ . During a certain 4.0 s interval, it turns through an angle of 120 rad. Assuming that the wheel started from rest, how long has it been in motion at the start of this 4.0 s interval?

**Answer:**

8.0 s

••16 A merry-go-round rotates from rest with an angular acceleration of  $1.50 \text{ rad/s}^2$ . How long does it take to rotate through (a) the first 2.00 rev and (b) the next 2.00 rev?

••17 At  $t = 0$ , a flywheel has an angular velocity of  $4.7 \text{ rad/s}$ , a constant angular acceleration of  $-0.25 \text{ rad/s}^2$ , and a reference line at  $\theta_0 = 0$ . (a) Through what maximum angle  $\theta_{\text{max}}$  will the reference line turn in the positive direction? What are the (b) first and (c) second times the reference line will be at  $\theta = \frac{1}{2}\theta_{\text{max}}$ ? At what (d) negative time and (e) positive time will the reference line be at  $\theta = -10.5 \text{ rad}$ ? (f) Graph  $\theta$  versus  $t$ , and indicate the answers to (a) through (e) on the graph.

**Answer:**

(a) 44 rad; (b) 5.5 s; (c) 32 s; (d) -2.1 s; (e) 40 s

### sec. 10-5 Relating the Linear and Angular Variables


•18 If an airplane propeller rotates at 2000 rev/min while the airplane flies at a speed of 480 km/h relative to the ground, what is the linear speed of a point on the tip of the propeller, at radius 1.5 m, as seen by (a) the pilot and (b) an observer on the ground? The plane's velocity is parallel to the propeller's axis of rotation.

•19 What are the magnitudes of (a) the angular velocity, (b) the radial acceleration, and (c) the tangential acceleration of a spaceship taking a circular turn of radius 3220 km at a speed of 29 000 km/h?

**Answer:**

(a)  $2.50 \times 10^{-3} \text{ rad/s}$ ; (b)  $20.2 \text{ m/s}^2$ ; (c) 0

•20 An object rotates about a fixed axis, and the angular position of a reference line on the object is given by  $\theta = 0.40e^{2t}$ , where  $\theta$  is in radians and  $t$  is in seconds. Consider a point on the object that is 4.0 cm from the axis of rotation. At  $t = 0$ , what are the magnitudes of the point's (a) tangential component of acceleration and (b) radial component of acceleration?

•21  Between 1911 and 1990, the top of the leaning bell tower at Pisa, Italy, moved toward the south at an average rate of 1.2 mm/y. The tower is 55 m tall. In radians per second, what is the average angular speed of the tower's top about its base?

**Answer:**

$$6.9 \times 10^{-13} \text{ rad/s}$$

- 22 An astronaut is being tested in a centrifuge. The centrifuge has a radius of 10 m and, in starting, rotates according to  $\theta = 0.30t^2$ , where  $t$  is in seconds and  $\theta$  is in radians. When  $t = 5.0$  s, what are the magnitudes of the astronaut's (a) angular velocity, (b) linear velocity, (c) tangential acceleration, and (d) radial acceleration?
- 23 **SSM WWW** A flywheel with a diameter of 1.20 m is rotating at an angular speed of 200 rev/min. (a) What is the angular speed of the flywheel in radians per second? (b) What is the linear speed of a point on the rim of the flywheel? (c) What constant angular acceleration (in revolutions per minute-squared) will increase the wheel's angular speed to 1000 rev/min in 60.0 s? (d) How many revolutions does the wheel make during that 60.0 s?

**Answer:**

(a) 20.9 rad/s; (b) 12.5 m/s; (c) 800 rev/min<sup>2</sup>; (d) 600 rev

- 24 A vinyl record is played by rotating the record so that an approximately circular groove in the vinyl slides under a stylus. Bumps in the groove run into the stylus, causing it to oscillate. The equipment converts those oscillations to electrical signals and then to sound. Suppose that a record turns at the rate of  $33\frac{1}{3}$  rev/min, the groove being played is at a radius of 10.0 cm, and the bumps in the groove are uniformly separated by 1.75 mm. At what rate (hits per second) do the bumps hit the stylus?
- 25 **SSM** (a) What is the angular speed  $\omega$  about the polar axis of a point on Earth's surface at latitude 40° N? (Earth rotates about that axis.) (b) What is the linear speed  $v$  of the point? What are (c)  $\omega$  and (d)  $v$  for a point at the equator?

**Answer:**

(a)  $7.3 \times 10^{-5}$  rad/s; (b)  $3.5 \times 10^2$  m/s; (c)  $7.3 \times 10^{-5}$  rad/s; (d)  $4.6 \times 10^2$  m/s

- 26 The flywheel of a steam engine runs with a constant angular velocity of 150 rev/min. When steam is shut off, the friction of the bearings and of the air stops the wheel in 2.2 h. (a) What is the constant angular acceleration, in revolutions per minute-squared, of the wheel during the slowdown? (b) How many revolutions does the wheel make before stopping? (c) At the instant the flywheel is turning at 75 rev/min, what is the tangential component of the linear acceleration of a flywheel particle that is 50 cm from the axis of rotation? (d) What is the magnitude of the net linear acceleration of the particle in (c)?
- 27 A record turntable is rotating at  $33\frac{1}{3}$  rev/min. A watermelon seed is on the turntable 6.0 cm from the axis of rotation. (a) Calculate the acceleration of the seed, assuming that it does not slip. (b) What is the minimum value of the coefficient of static friction between the seed and the turntable if the seed is not to slip? (c) Suppose that the turntable achieves its angular speed by starting from rest and undergoing a constant angular acceleration for 0.25 s. Calculate the minimum coefficient of static friction required for the seed not to slip during the acceleration period.

**Answer:**

(a) 73 cm/s<sup>2</sup>; (b) 0.075; (c) 0.11

- 28 In Fig. 10-28, wheel A of radius  $r_A = 10$  cm is coupled by belt B to wheel C of radius  $r_C = 25$  cm. The angular speed of wheel A is increased from rest at a constant rate of 1.6 rad/s<sup>2</sup>. Find the time needed for wheel C to reach an angular speed of 100 rev/min, assuming the belt does not slip.

(Hint: If the belt does not slip, the linear speeds at the two rims must be equal.)

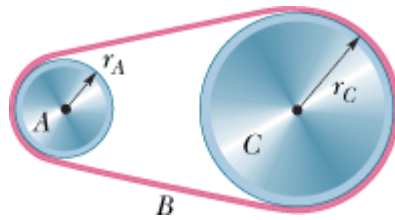


Figure 10-28 Problem 28.

- 29 An early method of measuring the speed of light makes use of a rotating slotted wheel. A beam of light passes through one of the slots at the outside edge of the wheel, as in Fig. 10-29, travels to a distant mirror, and returns to the wheel just in time to pass through the next slot in the wheel. One such slotted wheel has a radius of 5.0 cm and 500 slots around its edge. Measurements taken when the mirror is  $L = 500$  m from the wheel indicate a speed of light of  $3.0 \times 10^5$  km/s. (a) What is the (constant) angular speed of the wheel? (b) What is the linear speed of a point on the edge of the wheel?

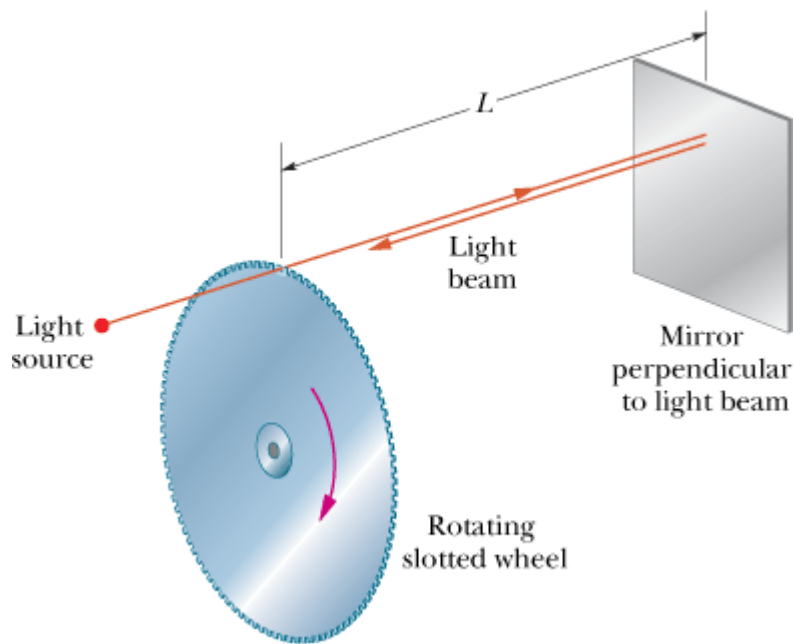



Figure 10-29 Problem 29.

**Answer:**

(a)  $3.8 \times 10^3$  rad/s; (b)  $1.9 \times 10^2$  m/s

- 30 A gyroscope flywheel of radius 2.83 cm is accelerated from rest at  $14.2 \text{ rad/s}^2$  until its angular speed is 2760 rev/min. (a) What is the tangential acceleration of a point on the rim of the flywheel during this spin-up process? (b) What is the radial acceleration of this point when the flywheel is spinning at full speed? (c) Through what distance does a point on the rim move during the spin-up?
- 31  A disk, with a radius of 0.25 m, is to be rotated like a merry-go-round through 800 rad, starting from rest, gaining angular speed at the constant rate  $\alpha_1$  through the first 400 rad and then

losing angular speed at the constant rate  $-\alpha_1$  until it is again at rest. The magnitude of the centripetal acceleration of any portion of the disk is not to exceed  $400 \text{ m/s}^2$ . (a) What is the least time required for the rotation? (b) What is the corresponding value of  $\alpha_1$ ?

**Answer:**

(a) 40 s; (b)  $2.0 \text{ rad/s}^2$

- 32 A pulsar is a rapidly rotating neutron star that emits a radio beam the way a lighthouse emits a light beam. We receive a radio pulse for each rotation of the star. The period  $T$  of rotation is found by measuring the time between pulses. The pulsar in the Crab nebula has a period of rotation of  $T = 0.033 \text{ s}$  that is increasing at the rate of  $1.26 \times 10^{-5} \text{ s/y}$ . (a) What is the pulsar's angular acceleration  $\alpha$ ? (b) If  $\alpha$  is constant, how many years from now will the pulsar stop rotating? (c) The pulsar originated in a supernova explosion seen in the year 1054. Assuming constant  $\alpha$ , find the initial  $T$ .

### sec. 10-6 Kinetic Energy of Rotation

- 33 **SSM** Calculate the rotational inertia of a wheel that has a kinetic energy of 24 400 J when rotating at 602 rev/min.

**Answer:**

$12.3 \text{ kg} \cdot \text{m}^2$

- 34 Figure 10-30 gives angular speed versus time for a thin rod that rotates around one end. The scale on the  $\omega$  axis is set by  $\omega_s = 6.0 \text{ rad/s}$ . (a) What is the magnitude of the rod's angular acceleration? (b) At  $t = 4.0 \text{ s}$ , the rod has a rotational kinetic energy of 1.60 J. What is its kinetic energy at  $t = 0$ ?

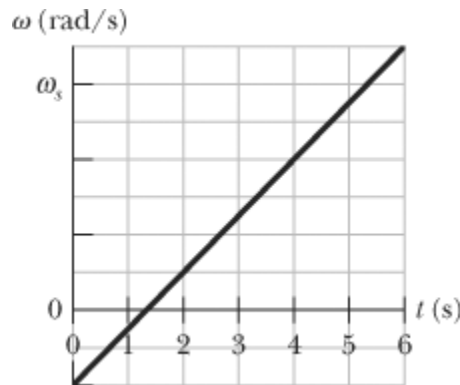


Figure 10-30 Problem 34.

### sec. 10-7 Calculating the Rotational Inertia

- 35 **SSM** Two uniform solid cylinders, each rotating about its central (longitudinal) axis at 235 rad/s, have the same mass of 1.25 kg but differ in radius. What is the rotational kinetic energy of (a) the smaller cylinder, of radius 0.25 m, and (b) the larger cylinder, of radius 0.75 m?

**Answer:**

(a) 1.1 kJ; (b) 9.7 kJ

- 36 Figure 10-31a shows a disk that can rotate about an axis at a radial distance  $h$  from the center of the disk. Figure 10-31b gives the rotational inertia  $I$  of the disk about the axis as a function of that

distance  $h$ , from the center out to the edge of the disk. The scale on the  $I$  axis is set by  $I_A = 0.050 \text{ kg} \cdot \text{m}^2$  and  $I_B = 0.150 \text{ kg} \cdot \text{m}^2$ . What is the mass of the disk?

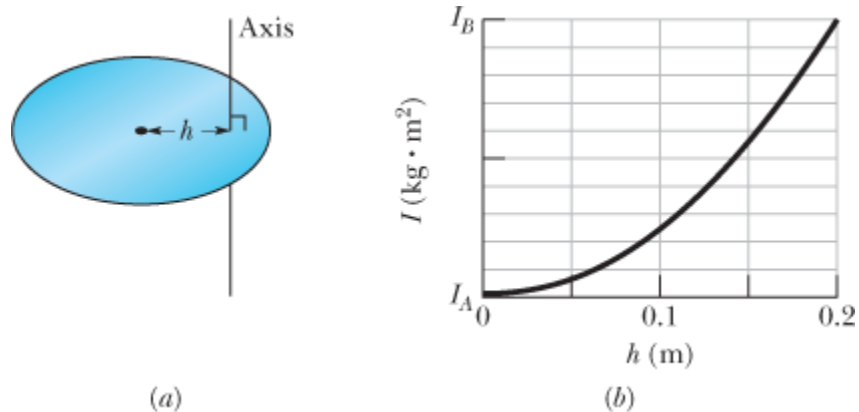


Figure 10-31 Problem 36.

- 37 **SSM** Calculate the rotational inertia of a meter stick, with mass  $0.56 \text{ kg}$ , about an axis perpendicular to the stick and located at the  $20 \text{ cm}$  mark. (Treat the stick as a thin rod.)

**Answer:**

$$0.097 \text{ kg} \cdot \text{m}^2$$

- 38 Figure 10-32 shows three  $0.0100 \text{ kg}$  particles that have been glued to a rod of length  $L = 6.00 \text{ cm}$  and negligible mass. The assembly can rotate around a perpendicular axis through point  $O$  at the left end. If we remove one particle (that is,  $33\%$  of the mass), by what percentage does the rotational inertia of the assembly around the rotation axis decrease when that removed particle is (a) the innermost one and (b) the outermost one?

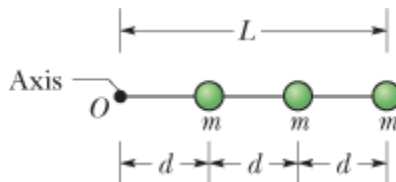


Figure 10-32 Problems 38 and 62.

- 39 Trucks can be run on energy stored in a rotating flywheel, with an electric motor getting the flywheel up to its top speed of  $200\pi \text{ rad/s}$ . One such flywheel is a solid, uniform cylinder with a mass of  $500 \text{ kg}$  and a radius of  $1.0 \text{ m}$ . (a) What is the kinetic energy of the flywheel after charging? (b) If the truck uses an average power of  $8.0 \text{ kW}$ , for how many minutes can it operate between chargings?

**Answer:**

(a)  $49 \text{ MJ}$ ; (b)  $1.0 \times 10^2 \text{ min}$

- 40 Figure 10-33 shows an arrangement of 15 identical disks that have been glued together in a rod-like shape of length  $L = 1.0000 \text{ m}$  and (total) mass  $M = 100.0 \text{ mg}$ . The disk arrangement can rotate about a perpendicular axis through its central disk at point  $O$ . (a) What is the rotational inertia of the arrangement about that axis? (b) If we approximated the arrangement as being a uniform rod of mass  $M$  and length  $L$ , what percentage error would we make in using the formula

in Table 10-2e to calculate the rotational inertia?

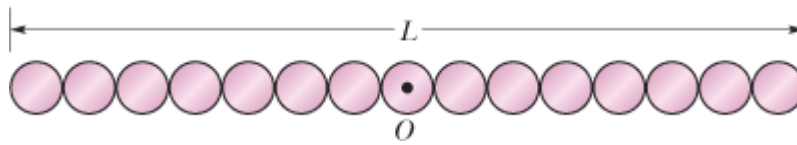


Figure 10-33 Problem 40.

- 41 **GO** In Fig. 10-34, two particles, each with mass  $m = 0.85$  kg, are fastened to each other, and to a rotation axis at  $O$ , by two thin rods, each with length  $d = 5.6$  cm and mass  $M = 1.2$  kg. The combination rotates around the rotation axis with the angular speed  $\omega = 0.30$  rad/s. Measured about  $O$ , what are the combination's (a) rotational inertia and (b) kinetic energy?

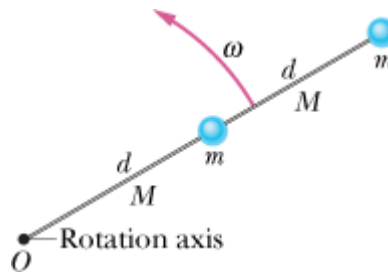


Figure 10-34 Problem 41.

**Answer:**

(a)  $0.023 \text{ kg} \cdot \text{m}^2$ ; (b)  $1.1 \text{ mJ}$

- 42 The masses and coordinates of four particles are as follows: 50 g,  $x = 2.0$  cm,  $y = 2.0$  cm; 25 g,  $x = 0$ ,  $y = 4.0$  cm; 25 g,  $x = -3.0$  cm,  $y = -3.0$  cm; 30 g,  $x = -2.0$  cm,  $y = 4.0$  cm. What are the rotational inertias of this collection about the (a)  $x$ , (b)  $y$ , and (c)  $z$  axes? (d) Suppose the answers to (a) and (b) are  $A$  and  $B$ , respectively. Then what is the answer to (c) in terms of  $A$  and  $B$ ?
- 43 **SSM WWW** The uniform solid block in Fig. 10-35 has mass  $0.172$  kg and edge lengths  $a = 3.5$  cm,  $b = 8.4$  cm, and  $c = 1.4$  cm. Calculate its rotational inertia about an axis through one corner and perpendicular to the large faces.

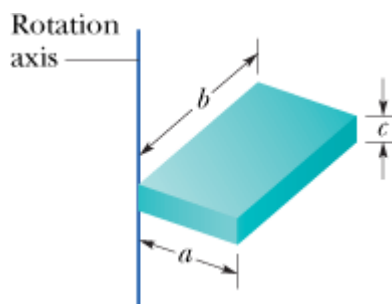


Figure 10-35 Problem 43.

**Answer:**

$4.7 \times 10^{-4} \text{ kg} \cdot \text{m}^2$



- 44 Four identical particles of mass 0.50 kg each are placed at the vertices of a 2.0 m × 2.0 m square and held there by four massless rods, which form the sides of the square. What is the rotational inertia of this rigid body about an axis that (a) passes through the midpoints of opposite sides and lies in the plane of the square, (b) passes through the midpoint of one of the sides and is perpendicular to the plane of the square, and (c) lies in the plane of the square and passes through two diagonally opposite particles?

### sec. 10-8 Torque

- 45 **SSM ILW** The body in Fig. 10-36 is pivoted at  $O$ , and two forces act on it as shown. If  $r_1 = 1.30$  m,  $r_2 = 2.15$  m,  $F_1 = 4.20$  N,  $F_2 = 4.90$  N,  $\theta_1 = 75.0^\circ$ , and  $\theta_2 = 60.0^\circ$ , what is the net torque about the pivot?

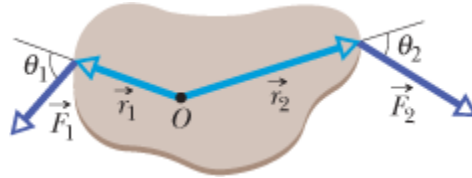


Figure 10-36 Problem 45.

**Answer:**

- 3.85 N · m

- 46 The body in Fig. 10-37 is pivoted at  $O$ . Three forces act on it:  $F_A = 10$  N at point  $A$ , 8.0 m from  $O$ ;  $F_B = 16$  N at  $B$ , 4.0 m from  $O$ ; and  $F_C = 19$  N at  $C$ , 3.0 m from  $O$ . What is the net torque about  $O$ ?

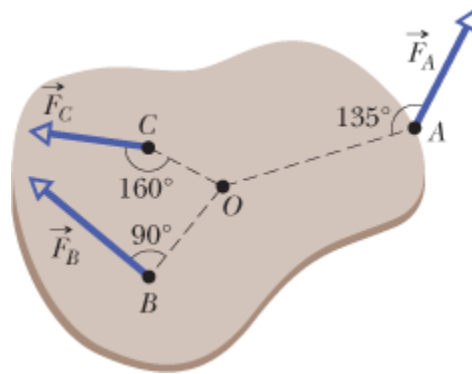


Figure 10-37 Problem 46.

- 47 **SSM** A small ball of mass 0.75 kg is attached to one end of a 1.25-m-long massless rod, and the other end of the rod is hung from a pivot. When the resulting pendulum is  $30^\circ$  from the vertical, what is the magnitude of the gravitational torque calculated about the pivot?

**Answer:**

4.6 N · m

- 48 The length of a bicycle pedal arm is 0.152 m, and a downward force of 111 N is applied to the pedal by the rider. What is the magnitude of the torque about the pedal arm's pivot when the arm is at angle (a)  $30^\circ$ , (b)  $90^\circ$ , and (c)  $180^\circ$  with the vertical?

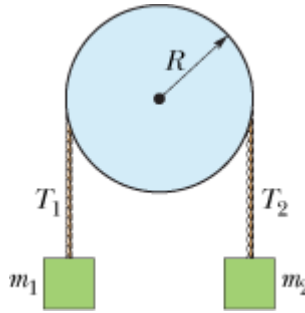
### sec. 10-9 Newton's Second Law for Rotation

- 49 **SSM ILW** During the launch from a board, a diver's angular speed about her center of mass changes from zero to  $6.20 \text{ rad/s}$  in  $220 \text{ ms}$ . Her rotational inertia about her center of mass is  $12.0 \text{ kg} \cdot \text{m}^2$ . During the launch, what are the magnitudes of (a) her average angular acceleration and (b) the average external torque on her from the board?

**Answer:**

(a)  $28.2 \text{ rad/s}^2$ ; (b)  $338 \text{ N} \cdot \text{m}$

- 50 If a  $32.0 \text{ N} \cdot \text{m}$  torque on a wheel causes angular acceleration  $25.0 \text{ rad/s}^2$ , what is the wheel's rotational inertia?
- 51 **GO** In Fig. 10-38, block 1 has mass,  $m_1 = 460 \text{ g}$ , block 2 has mass  $m_2 = 500 \text{ g}$ , and the pulley, which is mounted on a horizontal axle with negligible friction, has radius  $R = 5.00 \text{ cm}$ . When released from rest, block 2 falls  $75.0 \text{ cm}$  in  $5.00 \text{ s}$  without the cord slipping on the pulley. (a) What is the magnitude of the acceleration of the blocks? What are (b) tension  $T_2$  and (c) tension  $T_1$ ? (d) What is the magnitude of the pulley's angular acceleration? (e) What is its rotational inertia?



**Figure 10-38** Problems 51 and 83.

**Answer:**

(a)  $6.00 \text{ cm/s}^2$ ; (b)  $4.87 \text{ N}$ ; (c)  $4.54 \text{ N}$ ; (d)  $1.20 \text{ rad/s}^2$ ; (e)  $0.0138 \text{ kg} \cdot \text{m}^2$

- 52 **GO** In Fig. 10-39, a cylinder having a mass of  $2.0 \text{ kg}$  can rotate about its central axis through point  $O$ . Forces are applied as shown:  $F_1 = 6.0 \text{ N}$ ,  $F_2 = 4.0 \text{ N}$ ,  $F_3 = 2.0 \text{ N}$ , and  $F_4 = 5.0 \text{ N}$ . Also,  $r = 5.0 \text{ cm}$  and  $R = 12 \text{ cm}$ . Find the (a) magnitude and (b) direction of the angular acceleration of the cylinder. (During the rotation, the forces maintain their same angles relative to the cylinder.)

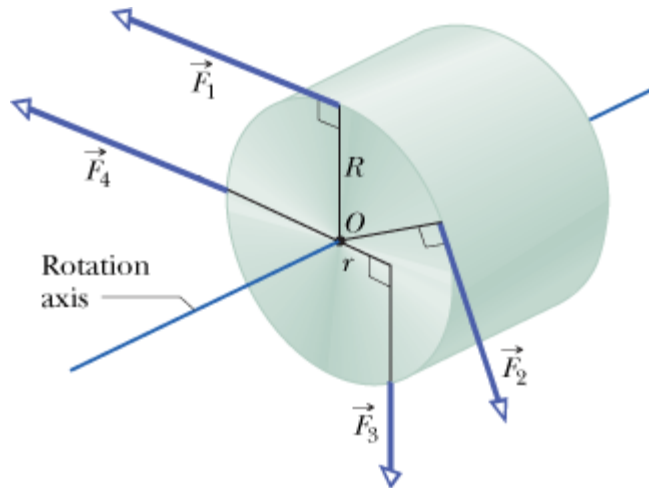



Figure 10-39 Problem 52.

••53  Figure 10-40 shows a uniform disk that can rotate around its center like a merry-go-round. The disk has a radius of 2.00 cm and a mass of 20.0 grams and is initially at rest. Starting at time  $t = 0$ , two forces are to be applied tangentially to the rim as indicated, so that at time  $t = 1.25$  s the disk has an angular velocity of 250 rad/s counterclockwise. Force  $\vec{F}_1$  has a magnitude of 0.100 N. What is magnitude  $F_2$ ?

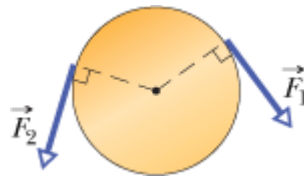



Figure 10-40 Problem 53.

**Answer:**

0.140 N

••54  In a judo foot-sweep move, you sweep your opponent's left foot out from under him while pulling on his gi (uniform) toward that side. As a result, your opponent rotates around his right foot and onto the mat. Figure 10-41 shows a simplified diagram of your opponent as you face him, with his left foot swept out. The rotational axis is through point  $O$ . The gravitational force  $\vec{F}_g$  on him effectively acts at his center of mass, which is a horizontal distance  $d = 28$  cm from point  $O$ . His mass is 70 kg, and his rotational inertia about point  $O$  is  $65 \text{ kg}\cdot\text{m}^2$ . What is the magnitude of his initial angular acceleration about point  $O$  if your pull  $\vec{F}_a$  on his gi is (a) negligible and (b) horizontal with a magnitude of 300 N and applied at height  $h = 1.4$  m?

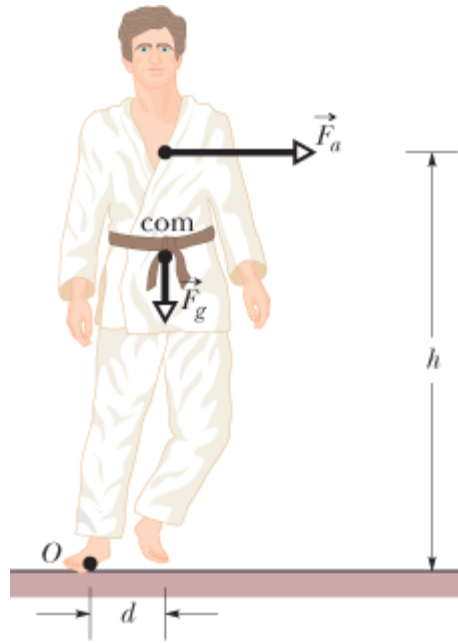


Figure 10-41 Problem 54.

••55GO In Fig. 10-42a an irregularly shaped plastic plate with uniform thickness and density (mass per unit volume) is to be rotated around an axle that is perpendicular to the plate face and through point  $O$ . The rotational inertia of the plate about that axle is measured with the following method. A circular disk of mass  $0.500\text{ kg}$  and radius  $2.00\text{ cm}$  is glued to the plate, with its center aligned with point  $O$  (Fig. 10-42b). A string is wrapped around the edge of the disk the way a string is wrapped around a top. Then the string is pulled for  $5.00\text{ s}$ . As a result, the disk and plate are rotated by a constant force of  $0.400\text{ N}$  that is applied by the string tangentially to the edge of the disk. The resulting angular speed is  $114\text{ rad/s}$ . What is the rotational inertia of the plate about the axle?

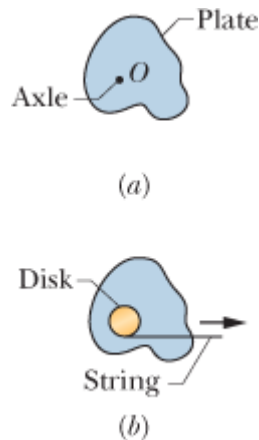


Figure 10-42 Problem 55.

**Answer:**

$$2.51 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

••56 Figure 10-43 shows particles 1 and 2, each of mass  $m$ , attached to the ends of a rigid massless rod

of length  $L_1 + L_2$ , with  $L_1 = 20$  cm and  $L_2 = 80$  cm. The rod is held horizontally on the fulcrum and then released. What are the magnitudes of the initial accelerations of (a) particle 1 and (b) particle 2?

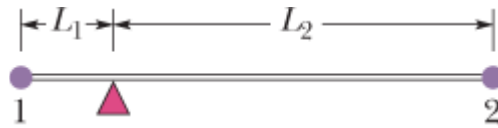


Figure 10-43 Problem 56.

- 57 A pulley, with a rotational inertia of  $1.0 \times 10^{-3} \text{ kg} \cdot \text{m}^2$  about its axle and a radius of 10 cm, is acted on by a force applied tangentially at its rim. The force magnitude varies in time as  $F = 0.50t + 0.30t^2$ , with  $F$  in newtons and  $t$  in seconds. The pulley is initially at rest. At  $t = 3.0$  s what are its (a) angular acceleration and (b) angular speed?

**Answer:**

(a)  $4.2 \times 10^2 \text{ rad/s}^2$ ; (b)  $5.0 \times 10^2 \text{ rad/s}$

### sec. 10-10 Work and Rotational Kinetic Energy

- 58 (a) If  $R = 12$  cm,  $M = 400$  g, and  $m = 50$  g in Fig. 10-18, find the speed of the block after it has descended 50 cm starting from rest. Solve the problem using energy conservation principles. (b) Repeat (a) with  $R = 5.0$  cm.
- 59 An automobile crankshaft transfers energy from the engine to the axle at the rate of 100 hp ( $= 74.6$  kW) when rotating at a speed of 1800 rev/min. What torque (in newton-meters) does the crankshaft deliver?
- 60 A thin rod of length 0.75 m and mass 0.42 kg is suspended freely from one end. It is pulled to one side and then allowed to swing like a pendulum, passing through its lowest position with angular speed 4.0 rad/s. Neglecting friction and air resistance, find (a) the rod's kinetic energy at its lowest position and (b) how far above that position the center of mass rises.
- 61 A 32.0 kg wheel, essentially a thin hoop with radius 1.20 m, is rotating at 280 rev/min. It must be brought to a stop in 15.0 s. (a) How much work must be done to stop it? (b) What is the required average power?


**Answer:**

(a) - 19.8 kJ; (b) 1.32 kW

- 62 In Fig. 10-32, three 0.0100 kg particles have been glued to a rod of length  $L = 6.00$  cm and negligible mass and can rotate around a perpendicular axis through point  $O$  at one end. How much work is required to change the rotational rate (a) from 0 to 20.0 rad/s, (b) from 20.0 rad/s to 40.0 rad/s, and (c) from 40.0 rad/s to 60.0 rad/s? (d) What is the slope of a plot of the assembly's kinetic energy (in joules) versus the square of its rotation rate (in radians-squared per second-squared)?
- 63 **SSM ILW** A meter stick is held vertically with one end on the floor and is then allowed to fall. Find the speed of the other end just before it hits the floor, assuming that the end on the floor does not slip. (*Hint:* Consider the stick to be a thin rod and use the conservation of energy principle.)

**Answer:**

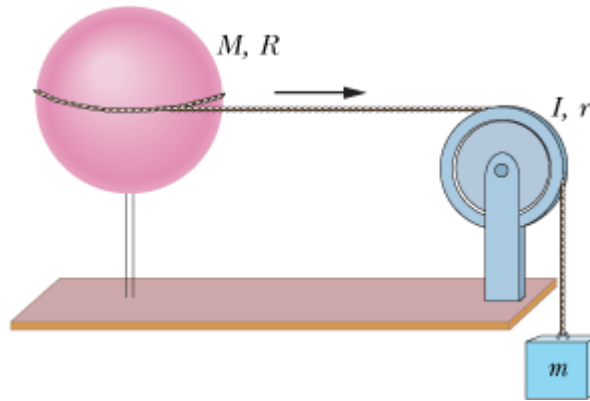
5.42 m/s

- 64 A uniform cylinder of radius 10 cm and mass 20 kg is mounted so as to rotate freely about a horizontal axis that is parallel to and 5.0 cm from the central longitudinal axis of the cylinder. (a) What is the rotational inertia of the cylinder about the axis of rotation? (b) If the cylinder is released from rest with its central longitudinal axis at the same height as the axis about which the cylinder rotates, what is the angular speed of the cylinder as it passes through its lowest position?
- 65  A tall, cylindrical chimney falls over when its base is ruptured. Treat the chimney as a thin rod of length 55.0 m. At the instant it makes an angle of  $35.0^\circ$  with the vertical as it falls, what are (a) the radial acceleration of the top, and (b) the tangential acceleration of the top. (*Hint:* Use energy considerations, not a torque.) (c) At what angle  $\theta$  is the tangential acceleration equal to  $g$ ?


**Answer:**

(a)  $5.32 \text{ m/s}^2$ ; (b)  $8.43 \text{ m/s}^2$ ; (c)  $41.8^\circ$

- 66 A uniform spherical shell of mass  $M = 4.5 \text{ kg}$  and radius  $R = 8.5 \text{ cm}$  can rotate about a vertical axis on frictionless bearings (Fig. 10-44). A massless cord passes around the equator of the shell, over a pulley of rotational inertia  $I = 3.0 \times 10^{-3} \text{ kg} \cdot \text{m}^2$  and radius  $r = 5.0 \text{ cm}$ , and is attached to a small object of mass  $m = 0.60 \text{ kg}$ . There is no friction on the pulley's axle; the cord does not slip on the pulley. What is the speed of the object when it has fallen 82 cm after being released from rest? Use energy considerations.



**Figure 10-44** Problem 66.

- 67  Figure 10-45 shows a rigid assembly of a thin hoop (of mass  $m$  and radius  $R = 0.150 \text{ m}$ ) and a thin radial rod (of mass  $m$  and length  $L = 2.00R$ ). The assembly is upright, but if we give it a slight nudge, it will rotate around a horizontal axis in the plane of the rod and hoop, through the lower end of the rod. Assuming that the energy given to the assembly in such a nudge is negligible, what would be the assembly's angular speed about the rotation axis when it passes through the upside-down (inverted) orientation?

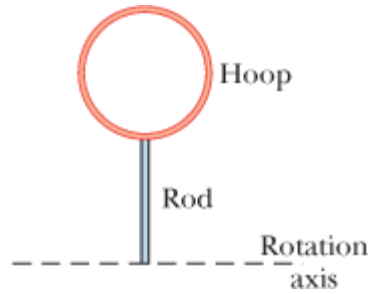


Figure 10-45 Problem 67.

**Answer:**

9.82 rad/s

### Additional Problems

- 68 Two uniform solid spheres have the same mass of 1.65 kg, but one has a radius of 0.226 m and the other has a radius of 0.854 m. Each can rotate about an axis through its center. (a) What is the magnitude  $\tau$  of the torque required to bring the smaller sphere from rest to an angular speed of 317 rad/s in 15.5 s? (b) What is the magnitude  $F$  of the force that must be applied tangentially at the sphere's equator to give that torque? What are the corresponding values of (c)  $\tau$  and (d)  $F$  for the larger sphere?
- 69 In Fig. 10-46, a small disk of radius  $r = 2.00$  cm has been glued to the edge of a larger disk of radius  $R = 4.00$  cm so that the disks lie in the same plane. The disks can be rotated around a perpendicular axis through point  $O$  at the center of the larger disk. The disks both have a uniform density (mass per unit volume) of  $1.40 \times 10^3$  kg/m<sup>3</sup> and a uniform thickness of 5.00 mm. What is the rotational inertia of the two-disk assembly about the rotation axis through  $O$ ?

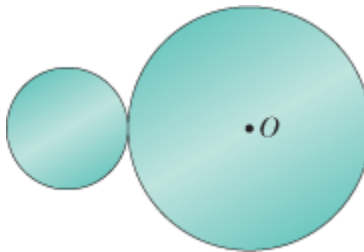


Figure 10-46 Problem 69.

**Answer:**

$6.16 \times 10^{-5} \text{ kg} \cdot \text{m}^2$

- 70 A wheel, starting from rest, rotates with a constant angular acceleration of  $2.00 \text{ rad/s}^2$ . During a certain 3.00 s interval, it turns through 90.0 rad. (a) What is the angular velocity of the wheel at the start of the 3.00 s interval? (b) How long has the wheel been turning before the start of the 3.00 s interval?
- 71 **SSM** In Fig. 10-47, two 6.20 kg blocks are connected by a massless string over a pulley of radius 2.40 cm and rotational inertia  $7.40 \times 10^{-4} \text{ kg} \cdot \text{m}^2$ . The string does not slip on the pulley; it is not known whether there is friction between the table and the sliding block; the pulley's axis is frictionless. When this system is released from rest, the pulley turns through 0.130 rad in 91.0 ms and the acceleration of the blocks is constant. What are (a) the magnitude of the pulley's angular

acceleration, (b) the magnitude of either block's acceleration, (c) string tension  $T_1$ , and (d) string tension  $T_2$ ?

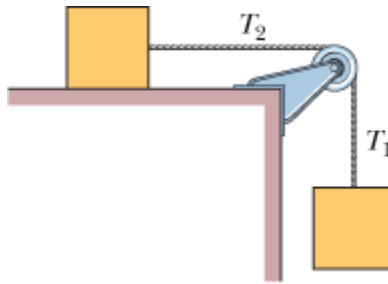


Figure 10-47 Problem 71.

**Answer:**

(a)  $31.4 \text{ rad/s}^2$ ; (b)  $0.754 \text{ m/s}^2$ ; (c)  $56.1 \text{ N}$ ; (d)  $55.1 \text{ N}$

- 72** Attached to each end of a thin steel rod of length  $1.20 \text{ m}$  and mass  $6.40 \text{ kg}$  is a small ball of mass  $1.06 \text{ kg}$ . The rod is constrained to rotate in a horizontal plane about a vertical axis through its midpoint. At a certain instant, it is rotating at  $39.0 \text{ rev/s}$ . Because of friction, it slows to a stop in  $32.0 \text{ s}$ . Assuming a constant retarding torque due to friction, compute (a) the angular acceleration, (b) the retarding torque, (c) the total energy transferred from mechanical energy to thermal energy by friction, and (d) the number of revolutions rotated during the  $32.0 \text{ s}$ . (e) Now suppose that the retarding torque is known not to be constant. If any of the quantities (a), (b), (c), and (d) can still be computed without additional information, give its value.
- 73** A uniform helicopter rotor blade is  $7.80 \text{ m}$  long, has a mass of  $110 \text{ kg}$ , and is attached to the rotor axle by a single bolt. (a) What is the magnitude of the force on the bolt from the axle when the rotor is turning at  $320 \text{ rev/min}$ ? (*Hint:* For this calculation the blade can be considered to be a point mass at its center of mass. Why?) (b) Calculate the torque that must be applied to the rotor to bring it to full speed from rest in  $6.70 \text{ s}$ . Ignore air resistance. (The blade cannot be considered to be a point mass for this calculation. Why not? Assume the mass distribution of a uniform thin rod.) (c) How much work does the torque do on the blade in order for the blade to reach a speed of  $320 \text{ rev/min}$ ?

**Answer:**

(a)  $4.81 \times 10^5 \text{ N}$ ; (b)  $1.12 \times 10^4 \text{ N} \cdot \text{m}$ ; (c)  $1.25 \times 10^6 \text{ J}$

- 74** *Racing disks.* Figure 10-48 shows two disks that can rotate about their centers like a merry-go-round. At time  $t = 0$ , the reference lines of the two disks have the same orientation. Disk A is already rotating, with a constant angular velocity of  $9.5 \text{ rad/s}$ . Disk B has been stationary but now begins to rotate at a constant angular acceleration of  $2.2 \text{ rad/s}^2$ . (a) At what time  $t$  will the reference lines of the two disks momentarily have the same angular displacement  $\theta$ ? (b) Will that time  $t$  be the first time since  $t = 0$  that the reference lines are momentarily aligned?

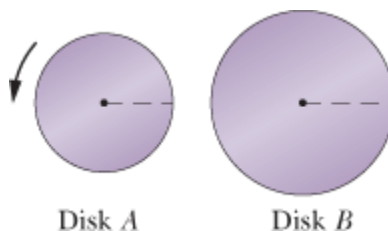






Figure 10-48 Problem 74.

- 75  A high-wire walker always attempts to keep his center of mass over the wire (or rope). He normally carries a long, heavy pole to help: If he leans, say, to his right (his com moves to the right) and is in danger of rotating around the wire, he moves the pole to his left (its com moves to the left) to slow the rotation and allow himself time to adjust his balance. Assume that the walker has a mass of 70.0 kg and a rotational inertia of about the wire. What is the magnitude of his angular acceleration about the wire if his com is 5.0 cm to the right of the wire and (a) he carries no pole and (b) the 14.0 kg pole he carries has its com 10 cm to the left of the wire?

**Answer:**


(a) 2.3 rad/s<sup>2</sup>; (b) 1.4 rad/s<sup>2</sup>

- 76 Starting from rest at  $t = 0$ , a wheel undergoes a constant angular acceleration. When  $t = 2.0$  s, the angular velocity of the wheel is 5.0 rad/s. The acceleration continues until  $t = 20$  s, when it abruptly ceases. Through what angle does the wheel rotate in the interval  $t = 0$  to  $t = 40$  s?

- 77  A record turntable rotating at  $33\frac{1}{3}$  rev/min slows down and stops in 30 s after the motor is turned off. (a) Find its (constant) angular acceleration in revolutions per minute-squared. (b) How many revolutions does it make in this time?

**Answer:**

(a) - 67 rev/min<sup>2</sup>; (b) 8.3 rev

- 78  A rigid body is made of three identical thin rods, each with length  $L = 0.600$  m, fastened together in the form of a letter **H** (Fig. 10-49). The body is free to rotate about a horizontal axis that runs along the length of one of the legs of the **H**. The body is allowed to fall from rest from a position in which the plane of the **H** is horizontal. What is the angular speed of the body when the plane of the **H** is vertical?

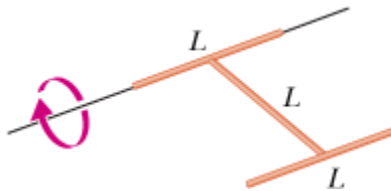



Figure 10-49 Problem 78.

- 79  (a) Show that the rotational inertia of a solid cylinder of mass  $M$  and radius  $R$  about its central axis is equal to the rotational inertia of a thin hoop of mass  $M$  and radius  $R/\sqrt{2}$  about its central axis. (b) Show that the rotational inertia  $I$  of any given body of mass  $M$  about any given axis is equal to the rotational inertia of an *equivalent hoop* about that axis, if the hoop has the same mass  $M$  and a radius  $k$  given by

$$k = \sqrt{\frac{I}{M}}$$

The radius  $k$  of the equivalent hoop is called the *radius of gyration* of the given body.

- 80 A disk rotates at constant angular acceleration, from angular position  $\theta_1 = 10.0$  rad to angular position  $\theta_2 = 70.0$  rad in 6.00 s. Its angular velocity at  $\theta_2$  is 15.0 rad/s. (a) What was its angular velocity at  $\theta_1$ ? (b) What is the angular acceleration? (c) At what angular position was the disk

initially at rest? (d) Graph  $\theta$  versus time  $t$  and angular speed  $\omega$  versus  $t$  for the disk, from the beginning of the motion (let  $t = 0$  then).

- 81 The thin uniform rod in Fig. 10-50 has length 2.0 m and can pivot about a horizontal, frictionless pin through one end. It is released from rest at angle  $\theta = 40^\circ$  above the horizontal. Use the principle of conservation of energy to determine the angular speed of the rod as it passes through the horizontal position.

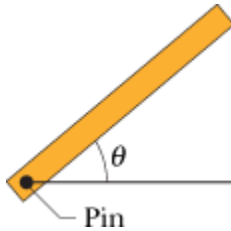



Figure 10-50 Problem 81.


**Answer:**

3.1 rad/s

- 82  George Washington Gale Ferris, Jr., a civil engineering graduate from Rensselaer Polytechnic Institute, built the original Ferris wheel for the 1893 World's Columbian Exposition in Chicago. The wheel, an astounding engineering construction at the time, carried 36 wooden cars, each holding up to 60 passengers, around a circle 76 m in diameter. The cars were loaded 6 at a time, and once all 36 cars were full, the wheel made a complete rotation at constant angular speed in about 2 min. Estimate the amount of work that was required of the machinery to rotate the passengers alone.
- 83 In Fig. 10-38, two blocks, of mass  $m_1 = 400$  g and  $m_2 = 600$  g, are connected by a massless cord that is wrapped around a uniform disk of mass  $M = 500$  g and radius  $R = 12.0$  cm. The disk can rotate without friction about a fixed horizontal axis through its center; the cord cannot slip on the disk. The system is released from rest. Find (a) the magnitude of the acceleration of the blocks, (b) the tension  $T_1$  in the cord at the left, and (c) the tension  $T_2$  in the cord at the right.


**Answer:**

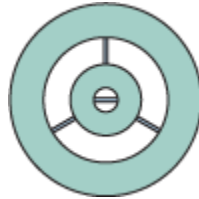
(a)  $1.57 \text{ m/s}^2$ ; (b) 4.55 N; (c) 4.94 N

- 84  At 7:14 A.M. on June 30, 1908, a huge explosion occurred above remote central Siberia, at latitude  $61^\circ$  N and longitude  $102^\circ$  E; the fireball thus created was the brightest flash seen by anyone before nuclear weapons. The *Tunguska Event*, which according to one chance witness "covered an enormous part of the sky," was probably the explosion of a *stony asteroid* about 140 m wide. (a) Considering only Earth's rotation, determine how much later the asteroid would have had to arrive to put the explosion above Helsinki at longitude  $25^\circ$  E. This would have obliterated the city. (b) If the asteroid had, instead, been a *metallic asteroid*, it could have reached Earth's surface. How much later would such an asteroid have had to arrive to put the impact in the Atlantic Ocean at longitude  $20^\circ$  W? (The resulting tsunamis would have wiped out coastal civilization on both sides of the Atlantic.)
- 85 A golf ball is launched at an angle of  $20^\circ$  to the horizontal, with a speed of 60 m/s and a rotation rate of 90 rad/s. Neglecting air drag, determine the number of revolutions the ball makes by the time it reaches maximum height.

**Answer:**

30 rev

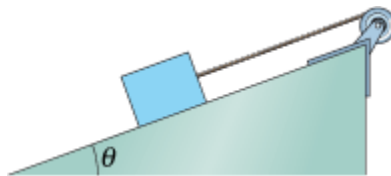
- 86  Figure 10-51 shows a flat construction of two circular rings that have a common center and are held together by three rods of negligible mass. The construction, which is initially at rest, can rotate around the common center (like a merry-go-round), where another rod of negligible mass lies. The mass, inner radius, and outer radius of the rings are given in the following table. A tangential force of magnitude 12.0 N is applied to the outer edge of the outer ring for 0.300 s. What is the change in the angular speed of the construction during that time interval?



**Figure 10-51** Problem 86.

| Ring | Mass (kg) | Inner Radius (m) | Outer Radius (m) |
|------|-----------|------------------|------------------|
| 1    | 0.120     | 0.0160           | 0.0450           |
| 2    | 0.240     | 0.0900           | 0.1400           |

- 87 In Fig. 10-52, a wheel of radius 0.20 m is mounted on a frictionless horizontal axle. A massless cord is wrapped around the wheel and attached to a 2.0 kg box that slides on a frictionless surface inclined at angle  $\theta = 20^\circ$  with the horizontal. The box accelerates down the surface at  $2.0 \text{ m/s}^2$ . What is the rotational inertia of the wheel about the axle?



**Figure 10-52** Problem 87.

**Answer:**

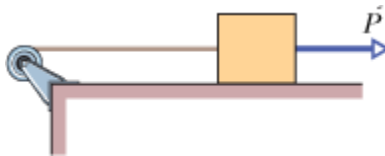
$0.054 \text{ kg} \cdot \text{m}^2$

- 88 A thin spherical shell has a radius of 1.90 m. An applied torque of  $960 \text{ N} \cdot \text{m}$  gives the shell an angular acceleration of  $6.20 \text{ rad/s}^2$  about an axis through the center of the shell. What are (a) the rotational inertia of the shell about that axis and (b) the mass of the shell?
- 89 A bicyclist of mass 70 kg puts all his mass on each downward-moving pedal as he pedals up a steep road. Take the diameter of the circle in which the pedals rotate to be 0.40 m, and determine the magnitude of the maximum torque he exerts about the rotation axis of the pedals.

**Answer:**

$$1.4 \times 10^2 \text{ N} \cdot \text{m}$$

- 90 The flywheel of an engine is rotating at 25.0 rad/s. When the engine is turned off, the flywheel slows at a constant rate and stops in 20.0 s. Calculate (a) the angular acceleration of the flywheel, (b) the angle through which the flywheel rotates in stopping, and (c) the number of revolutions made by the flywheel in stopping.
- 91 **SSM** In Fig. 10-18a, a wheel of radius 0.20 m is mounted on a frictionless horizontal axis. The rotational inertia of the wheel about the axis is  $0.40 \text{ kg} \cdot \text{m}^2$ . A massless cord wrapped around the wheel's circumference is attached to a 6.0 kg box. The system is released from rest. When the box has a kinetic energy of 6.0 J, what are (a) the wheel's rotational kinetic energy and (b) the distance the box has fallen?
- 92 Our Sun is  $2.3 \times 10^4$  ly (light-years) from the center of our Milky Way galaxy and is moving in a circle around that center at a speed of 250 km/s. (a) How long does it take the Sun to make one revolution about the galactic center? (b) How many revolutions has the Sun completed since it was formed about  $4.5 \times 10^9$  years ago?
- 93 **SSM** A wheel of radius 0.20 m is mounted on a frictionless horizontal axis. The rotational inertia of the wheel about the axis is  $0.050 \text{ kg} \cdot \text{m}^2$ . A massless cord wrapped around the wheel is attached to a 2.0 kg block that slides on a horizontal frictionless surface. If a horizontal force of magnitude  $P = 3.0 \text{ N}$  is applied to the block as shown in Fig. 10-53, what is the magnitude of the angular acceleration of the wheel? Assume the cord does not slip on the wheel.



**Figure 10-53** Problem 93.

**Answer:**

$$4.6 \text{ rad/s}^2$$

- 94 A car starts from rest and moves around a circular track of radius 30.0 m. Its speed increases at the constant rate of  $0.500 \text{ m/s}^2$ . (a) What is the magnitude of its *net* linear acceleration 15.0 s later? (b) What angle does this net acceleration vector make with the car's velocity at this time?
- 95 The rigid body shown in Fig. 10-54 consists of three particles connected by massless rods. It is to be rotated about an axis perpendicular to its plane through point  $P$ . If  $M = 0.40 \text{ kg}$ ,  $a = 30 \text{ cm}$ , and  $b = 50 \text{ cm}$ , how much work is required to take the body from rest to an angular speed of 5.0 rad/s?

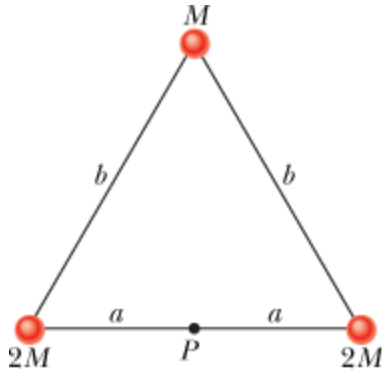


Figure 10-54 Problem 95.

**Answer:**

2.6 J

- 96 *Beverage engineering.* The pull tab was a major advance in the engineering design of beverage containers. The tab pivots on a central bolt in the can's top. When you pull upward on one end of the tab, the other end presses downward on a portion of the can's top that has been scored. If you pull upward with a 10 N force, approximately what is the magnitude of the force applied to the scored section? (You will need to examine a can with a pull tab.)
- 97 Figure 10-55 shows a propeller blade that rotates at 2000 rev/min about a perpendicular axis at point B. Point A is at the outer tip of the blade, at radial distance 1.50 m. (a) What is the difference in the magnitudes  $a$  of the centripetal acceleration of point A and of a point at radial distance 0.150 m? (b) Find the slope of a plot of  $a$  versus radial distance along the blade.

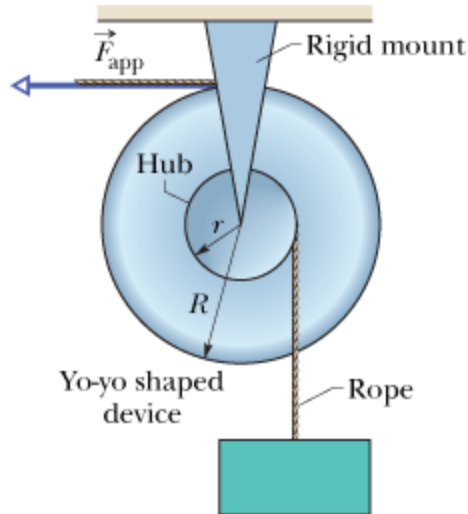


Figure 10-55 Problem 97.

**Answer:**

(a)  $5.92 \times 10^4 \text{ m/s}^2$ ; (b)  $4.39 \times 10^4 \text{ s}^{-2}$

- 98 A yo-yo-shaped device mounted on a horizontal frictionless axis is used to lift a 30 kg box as shown in Fig. 10-56. The outer radius  $R$  of the device is 0.50 m, and the radius  $r$  of the hub is 0.20 m. When a constant horizontal force  $\vec{F}_{\text{app}}$  of magnitude 140 N is applied to a rope wrapped around the outside of the device, the box, which is suspended from a rope wrapped around the hub, has an upward acceleration of magnitude  $0.80 \text{ m/s}^2$ . What is the rotational inertia of the device about its axis of rotation?



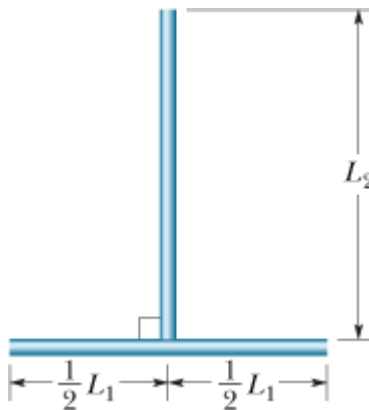
**Figure 10-56** Problem 98.

- 99** A small ball with mass 1.30 kg is mounted on one end of a rod 0.780 m long and of negligible mass. The system rotates in a horizontal circle about the other end of the rod at 5010 rev/min. (a) Calculate the rotational inertia of the system about the axis of rotation. (b) There is an air drag of  $2.30 \times 10^{-2}$  N on the ball, directed opposite its motion. What torque must be applied to the system to keep it rotating at constant speed?

**Answer:**

(a)  $0.791 \text{ kg} \cdot \text{m}^2$ ; (b)  $1.79 \times 10^{-2} \text{ N} \cdot \text{m}$

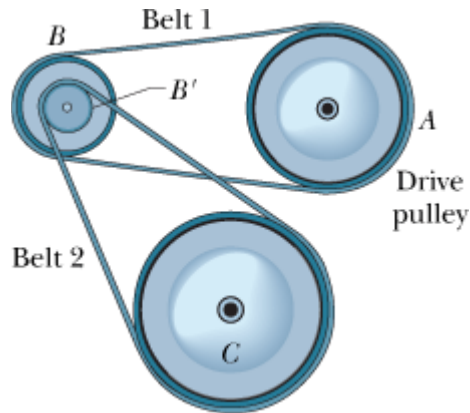
- 100** Two thin rods (each of mass 0.20 kg) are joined together to form a rigid body as shown in Fig. 10-57. One of the rods has length  $L_1 = 0.40$  m, and the other has length  $L_2 = 0.50$  m. What is the rotational inertia of this rigid body about (a) an axis that is perpendicular to the plane of the paper and passes through the center of the shorter rod and (b) an axis that is perpendicular to the plane of the paper and passes through the center of the longer rod?



**Figure 10-57** Problem 100.

- 101** In Fig. 10-58, four pulleys are connected by two belts. Pulley A (radius 15 cm) is the drive pulley, and it rotates at 10 rad/s. Pulley B (radius 10 cm) is connected by belt 1 to pulley A. Pulley B' (radius 5 cm) is concentric with pulley B and is rigidly attached to it. Pulley C (radius 25 cm) is connected by belt 2 to pulley B'. Calculate (a) the linear speed of a point on belt 1, (b) the angular

speed of pulley  $B$ , (c) the angular speed of pulley  $B'$ , (d) the linear speed of a point on belt 2, and (e) the angular speed of pulley  $C$ . (*Hint*: If the belt between two pulleys does not slip, the linear speeds at the rims of the two pulleys must be equal.)

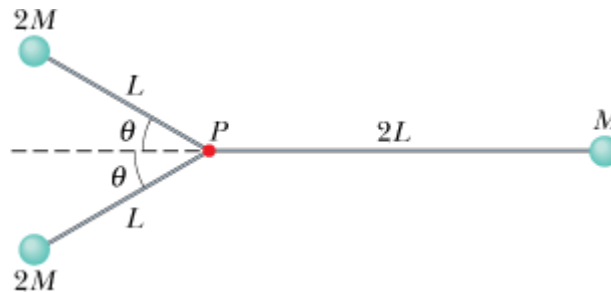


**Figure 10-58** Problem 101.

**Answer:**

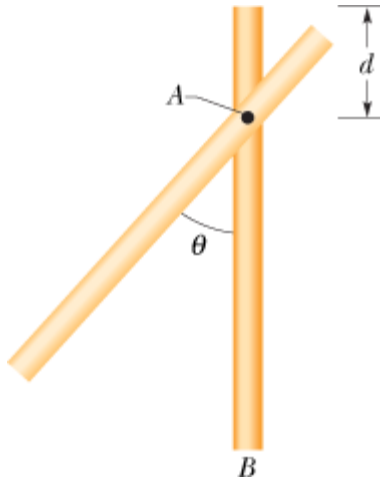
(a)  $1.5 \times 10^2$  cm/s; (b) 15 rad/s; (c) 15 rad/s; (d) 75 cm/s; (e) 3.0 rad/s

**102** The rigid object shown in Fig. 10-59 consists of three balls and three connecting rods, with  $M = 1.6$  kg,  $L = 0.60$  m, and  $\theta = 30^\circ$ . The balls may be treated as particles, and the connecting rods have negligible mass. Determine the rotational kinetic energy of the object if it has an angular speed of 1.2 rad/s about (a) an axis that passes through point  $P$  and is perpendicular to the plane of the figure and (b) an axis that passes through point  $P$ , is perpendicular to the rod of length  $2L$ , and lies in the plane of the figure.



**Figure 10-59** Problem 102.

**103** In Fig. 10-60, a thin uniform rod (mass 3.0 kg, length 4.0 m) rotates freely about a horizontal axis  $A$  that is perpendicular to the rod and passes through a point at distance  $d = 1.0$  m from the end of the rod. The kinetic energy of the rod as it passes through the vertical position is 20 J. (a) What is the rotational inertia of the rod about axis  $A$ ? (b) What is the (linear) speed of the end  $B$  of the rod as the rod passes through the vertical position? (c) At what angle  $\theta$  will the rod momentarily stop in its upward swing?

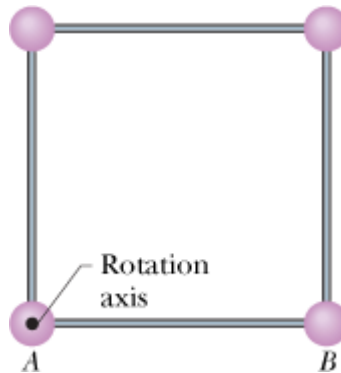


**Figure 10-60** Problem 103.

**Answer:**

(a)  $7.0 \text{ kg} \cdot \text{m}^2$ ; (b)  $7.2 \text{ m/s}$ ; (c)  $71^\circ$

- 104** Four particles, each of mass,  $0.20 \text{ kg}$ , are placed at the vertices of a square with sides of length  $0.50 \text{ m}$ . The particles are connected by rods of negligible mass. This rigid body can rotate in a vertical plane about a horizontal axis  $A$  that passes through one of the particles. The body is released from rest with rod  $AB$  horizontal (Fig. 10-61). (a) What is the rotational inertia of the body about axis  $A$ ? (b) What is the angular speed of the body about axis  $A$  when rod  $AB$  swings through the vertical position?



**Figure 10-61** Problem 104.



## sec. 11-2 Rolling as Translation and Rotation Combined

- **1** A car travels at  $80 \text{ km/h}$  on a level road in the positive direction of an  $x$  axis. Each tire has a



diameter of 66 cm. Relative to a woman riding in the car and in unit-vector notation, what are the velocity  $\vec{v}$  at the (a) center, (b) top, and (c) bottom of the tire and the magnitude  $a$  of the acceleration at the (d) center, (e) top, and (f) bottom of each tire? Relative to a hitchhiker sitting next to the road and in unit-vector notation, what are the velocity  $\vec{v}$  at the (g) center, (h) top, and (i) bottom of the tire and the magnitude  $a$  of the acceleration at the (j) center, (k) top, and (l) bottom of each tire?

**Answer:**

(a) 0; (b)  $(22 \text{ m/s})\hat{i}$ ; (c)  $(-22 \text{ m/s})\hat{i}$ ; (d) 0; (e)  $1.5 \times 10^3 \text{ m/s}^2$ ; (f)  $1.5 \times 10^3 \text{ m/s}^2$ ; (g)  $(22 \text{ m/s})\hat{i}$ ; (h)  $(44 \text{ m/s})\hat{i}$ ; (i) 0; (j) 0; (k)  $1.5 \times 10^3 \text{ m/s}^2$ ; (l)  $1.5 \times 10^3 \text{ m/s}^2$

- 2 An automobile traveling at 80.0 km/h has tires of 75.0 cm diameter. (a) What is the angular speed of the tires about their axles? (b) If the car is brought to a stop uniformly in 30.0 complete turns of the tires (without skidding), what is the magnitude of the angular acceleration of the wheels? (c) How far does the car move during the braking?

### sec. 11-4 The Forces of Rolling

- 3 **SSM** A 140 kg hoop rolls along a horizontal floor so that the hoop's center of mass has a speed of 0.150 m/s. How much work must be done on the hoop to stop it?

**Answer:**

-3.15 J

- 4 A uniform solid sphere rolls down an incline. (a) What must be the incline angle if the linear acceleration of the center of the sphere is to have a magnitude of  $0.10g$ ? (b) If a frictionless block were to slide down the incline at that angle, would its acceleration magnitude be more than, less than, or equal to  $0.10g$ ? Why?
- 5 **ILW** A 1000 kg car has four 10 kg wheels. When the car is moving, what fraction of its total kinetic energy is due to rotation of the wheels about their axles? Assume that the wheels have the same rotational inertia as uniform disks of the same mass and size. Why do you not need to know the radius of the wheels?

**Answer:**

0.020

- 6 Figure 11-30 gives the speed  $v$  versus time  $t$  for a 0.500 kg object of radius 6.00 cm that rolls smoothly down a  $30^\circ$  ramp. The scale on the velocity axis is set by  $v_s = 4.0 \text{ m/s}$ . What is the rotational inertia of the object?

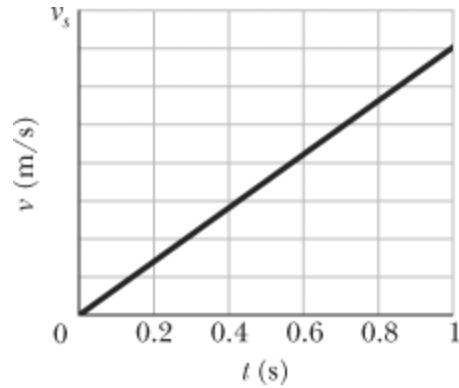


Figure 11-30 Problem 6.

- 7 **ILW** In Fig. 11-31, a solid cylinder of radius 10 cm and mass 12 kg starts from rest and rolls without slipping a distance  $L = 6.0$  m down a roof that is inclined at the angle  $\theta = 30^\circ$ . (a) What is the angular speed of the cylinder about its center as it leaves the roof? (b) The roof's edge is at height  $H = 5.0$  m. How far horizontally from the roof's edge does the cylinder hit the level ground?

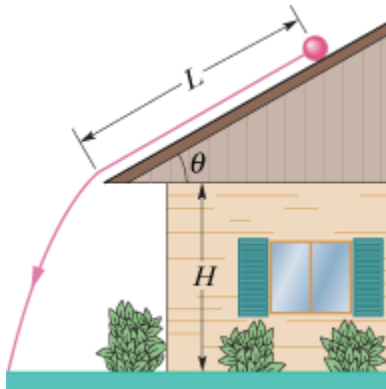


Figure 11-31 Problem 7.

**Answer:**

- (a) 63 rad/s; (b) 4.0 m
- 8 Figure 11-32 shows the potential energy  $U(x)$  of a solid ball that can roll along an  $x$  axis. The scale on the  $U$  axis is set by  $U_s = 100$  J. The ball is uniform, rolls smoothly, and has a mass of 0.400 kg. It is released at  $x = 7.0$  m headed in the negative direction of the  $x$  axis with a mechanical energy of 75 J. (a) If the ball can reach  $x = 0$  m, what is its speed there, and if it cannot, what is its turning point? Suppose, instead, it is headed in the positive direction of the  $x$  axis when it is released at  $x = 7.0$  m with 75 J. (b) If the ball can reach  $x = 13$  m, what is its speed there, and if it cannot, what is its turning point?

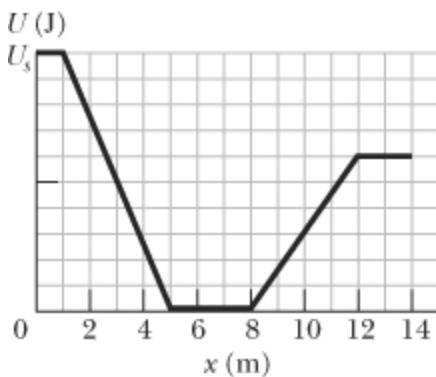



Figure 11-32 Problem 8.

- 9  In Fig. 11-33, a solid ball rolls smoothly from rest (starting at height  $H = 6.0$  m) until it leaves the horizontal section at the end of the track, at height  $h = 2.0$  m. How far horizontally from point A does the ball hit the floor?

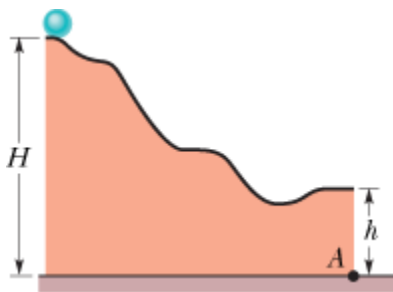


Figure 11-33 Problem 9.

**Answer:**

4.8m

- 10 A hollow sphere of radius  $0.15$  m, with rotational inertia  $I = 0.040$  kg·m<sup>2</sup> about a line through its center of mass, rolls without slipping up a surface inclined at  $30^\circ$  to the horizontal. At a certain initial position, the sphere's total kinetic energy is  $20$  J. (a) How much of this initial kinetic energy is rotational? (b) What is the speed of the center of mass of the sphere at the initial position? When the sphere has moved  $1.0$  m up the incline from its initial position, what are (c) its total kinetic energy and (d) the speed of its center of mass?
- 11 In Fig. 11-34, a constant horizontal force  $\vec{F}_{\text{app}}$  of magnitude  $10$  N is applied to a wheel of mass  $10$  kg and radius  $0.30$  m. The wheel rolls smoothly on the horizontal surface, and the acceleration of its center of mass has magnitude  $0.60$  m/s<sup>2</sup>. (a) In unit-vector notation, what is the frictional force on the wheel? (b) What is the rotational inertia of the wheel about the rotation axis through its center of mass?

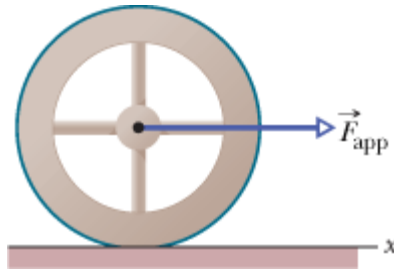


Figure 11-34 Problem 11.

Answer:

(a)  $(-4.0 \text{ N})\hat{i}$ ; (b)  $0.60 \text{ kg} \cdot \text{m}^2$

- 12 GO In Fig. 11-35, a solid brass ball of mass  $0.280 \text{ g}$  will roll smoothly along a loop-the-loop track when released from rest along the straight section. The circular loop has radius  $R = 14.0 \text{ cm}$ , and the ball has radius  $r \ll R$ . (a) What is  $h$  if the ball is on the verge of leaving the track when it reaches the top of the loop? If the ball is released at height  $h = 6.00R$ , what are the (b) magnitude and (c) direction of the horizontal force component acting on the ball at point  $Q$ ?

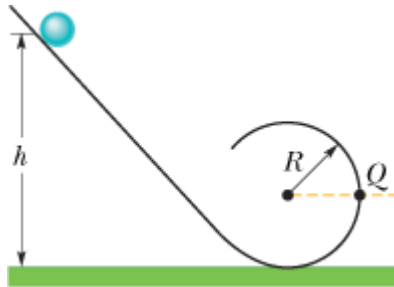


Figure 11-35 Problem 12.

- 13 Nonuniform ball. In Fig. 11-36, a ball of mass  $M$  and radius  $R$  rolls smoothly from rest down a ramp and onto a circular loop of radius  $0.48 \text{ m}$ . The initial height of the ball is  $h = 0.36 \text{ m}$ . At the loop bottom, the magnitude of the normal force on the ball is  $2.00Mg$ . The ball consists of an outer spherical shell (of a certain uniform density) that is glued to a central sphere (of a different uniform density). The rotational inertia of the ball can be expressed in the general form  $I = \beta MR^2$ , but  $\beta$  is not  $0.4$  as it is for a ball of uniform density. Determine  $\beta$ .



Figure 11-36 Problem 13.

Answer:

0.50

- 14 GO In Fig. 11-37, a small, solid, uniform ball is to be shot from point  $P$  so that it rolls smoothly

along a horizontal path, up along a ramp, and onto a plateau. Then it leaves the plateau horizontally to land on a game board, at a horizontal distance  $d$  from the right edge of the plateau. The vertical heights are  $h_1 = 5.00$  cm and  $h_2 = 1.60$  cm. With what speed must the ball be shot at point  $P$  for it to land at  $d = 6.00$  cm?

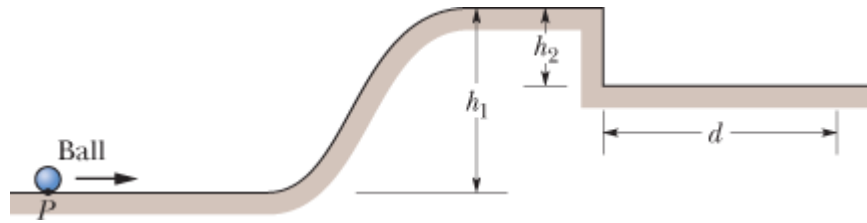



Figure 11-37 Problem 14.

- 15  A bowler throws a bowling ball of radius  $R = 11$  cm along a lane. The ball (Fig. 11-38) slides on the lane with initial speed  $v_{\text{com},0} = 8.5$  m/s and initial angular speed  $\omega_0 = 0$ . The coefficient of kinetic friction between the ball and the lane is 0.21. The kinetic frictional force  $\vec{f}_k$  acting on the ball causes a linear acceleration of the ball while producing a torque that causes an angular acceleration of the ball. When speed  $v_{\text{com}}$  has decreased enough and angular speed  $\omega$  has increased enough, the ball stops sliding and then rolls smoothly. (a) What then is  $v_{\text{com}}$  in terms of  $\omega$ ? During the sliding, what are the ball's (b) linear acceleration and (c) angular acceleration? (d) How long does the ball slide? (e) How far does the ball slide? (f) What is the linear speed of the ball when smooth rolling begins?

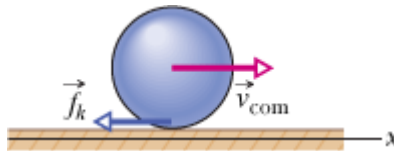


Figure 11-38 Problem 15.

**Answer:**

(a)  $-(0.11 \text{ m})\omega$  (b)  $-2.1 \text{ m/s}^2$ ; (c)  $-47 \text{ rad/s}^2$ ; (d)  $1.2 \text{ s}$ ; (e)  $8.6 \text{ m}$ ; (f)  $6.1 \text{ m/s}$

- 16 *Nonuniform cylindrical object.* In Fig. 11-39, a cylindrical object of mass  $M$  and radius  $R$  rolls smoothly from rest down a ramp and onto a horizontal section. From there it rolls off the ramp and onto the floor, landing a horizontal distance  $d = 0.506$  m from the end of the ramp. The initial height of the object is  $H = 0.90$  m; the end of the ramp is at height  $h = 0.10$  m. The object consists of an outer cylindrical shell (of a certain uniform density) that is glued to a central cylinder (of a different uniform density). The rotational inertia of the object can be expressed in the general form  $I = \beta MR^2$ , but  $\beta$  is not 0.5 as it is for a cylinder of uniform density. Determine  $\beta$ .

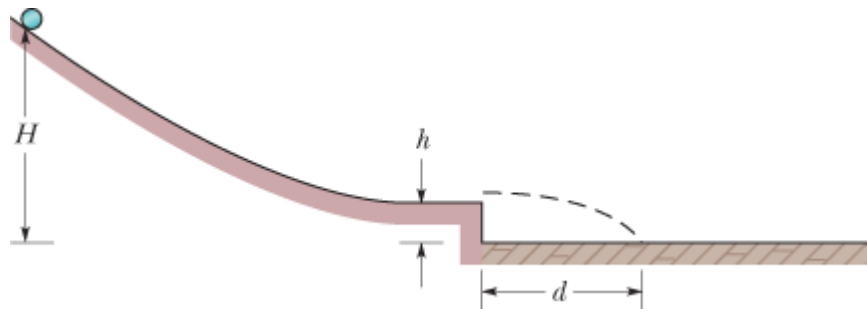




Figure 11-39 Problem 16.

### sec. 11-5 The Yo-Yo

- 17  **SSM** A yo-yo has a rotational inertia of  $950 \text{ g}\cdot\text{cm}^2$  and a mass of 120 g. Its axle radius is 3.2 mm, and its string is 120 cm long. The yo-yo rolls from rest down to the end of the string. (a) What is the magnitude of its linear acceleration? (b) How long does it take to reach the end of the string? As it reaches the end of the string, what are its (c) linear speed, (d) translational kinetic energy, (e) rotational kinetic energy, and (f) angular speed?

**Answer:**

(a)  $13 \text{ cm/s}^2$ ; (b) 4.4 s; (c) 55 cm/s; (d) 18 ml; (e) 1.4 J; (f) 27 rev/s

- 18  In 1980, over San Francisco Bay, a large yo-yo was released from a crane. The 116 kg yo-yo consisted of two uniform disks of radius 32 cm connected by an axle of radius 3.2 cm. What was the magnitude of the acceleration of the yo-yo during (a) its fall and (b) its rise? (c) What was the tension in the cord on which it rolled? (d) Was that tension near the cord's limit of 52 kN? Suppose you build a scaled-up version of the yo-yo (same shape and materials but larger). (e) Will the magnitude of your yo-yo's acceleration as it falls be greater than, less than, or the same as that of the San Francisco yo-yo? (f) How about the tension in the cord?

### sec. 11-6 Torque Revisited

- 19 In unit-vector notation, what is the net torque about the origin on a flea located at coordinates (0, -4.0 m, 5.0 m) when forces  $\vec{F}_1 = (3.0 \text{ N})\hat{k}$  and  $\vec{F}_2 = (-2.0 \text{ N})\hat{j}$  act on the flea?

**Answer:**

$(-2.0 \text{ N}\cdot\text{m})\hat{i}$

- 20 A plum is located at coordinates (-2.0 m, 0, 4.0 m). In unit-vector notation, what is the torque about the origin on the plum if that torque is due to a force  $\vec{F}$  whose only component is (a)  $F_x = 6.0 \text{ N}$ , (b)  $F_x = -6.0 \text{ N}$ , (c)  $F_z = 6.0 \text{ N}$ , and (d)  $F_z = -6.0 \text{ N}$ ?
- 21 In unit-vector notation, what is the torque about the origin on a particle located at coordinates (0, -4.0 m, 3.0 m) if that torque is due to (a) force  $\vec{F}_1$  with components  $F_{1x} = 2.0 \text{ N}$ ,  $F_{1y} = F_{1z} = 0$ , and (b) force  $\vec{F}_2$  components  $F_{2x} = 0$ ,  $F_{2y} = 2.0 \text{ N}$ ,  $F_{2z} = 4.0 \text{ N}$ ?

**Answer:**

(a)  $(6.0 \text{ N}\cdot\text{m})\hat{j} + (8.0 \text{ N}\cdot\text{m})\hat{k}$ ; (b)  $(-22 \text{ N}\cdot\text{m})\hat{i}$

- 22 A particle moves through an xyz coordinate system while a force acts on the particle. When the particle has the position vector  $\vec{r} = (2.00 \text{ m})\hat{i} - (3.00 \text{ m})\hat{j} + (2.00 \text{ m})\hat{k}$ , the force is given by  $\vec{F} = F_x\hat{i} + (7.00 \text{ N})\hat{j} - (6.00 \text{ N})\hat{k}$  and the corresponding torque about the origin is  $\vec{\tau} = (4.00 \text{ N}\cdot\text{m})\hat{i} + (2.00 \text{ N}\cdot\text{m})\hat{j} - (1.00 \text{ N}\cdot\text{m})\hat{k}$ . Determine  $F_x$ .

- 23 Force  $\vec{F} = (2.0 \text{ N})\hat{i} - (3.0 \text{ N})\hat{k}$  acts on a pebble with position vector  $\vec{r} = (0.50 \text{ m})\hat{j} - (2.0 \text{ m})\hat{k}$  relative to the origin. In unit-vector notation, what is the resulting torque on the pebble about (a) the origin and (b) the point (2.0 m, 0, -3.0 m)?

**Answer:**

(a)  $(-1.5 \text{ N} \cdot \text{m})\hat{i} - (4.0 \text{ N} \cdot \text{m})\hat{j} - (1.0 \text{ N} \cdot \text{m})\hat{k}$ ; (b)  $(-1.5 \text{ N} \cdot \text{m})\hat{i} - (4.0 \text{ N} \cdot \text{m})\hat{j} - (1.0 \text{ N} \cdot \text{m})\hat{k}$

- 24 In unit-vector notation, what is the torque about the origin on a jar of jalapeño peppers located at coordinates (3.0 m, -2.0 m, 4.0 m) due to (a) force  $\vec{F}_1 = (3.0 \text{ N})\hat{i} - (4.0 \text{ N})\hat{j} + (5.0 \text{ N})\hat{k}$ , (b) force  $\vec{F}_2 = (-3.0 \text{ N})\hat{i} - (4.0 \text{ N})\hat{j} - (5.0 \text{ N})\hat{k}$ , and (c) the vector sum of  $\vec{F}_1$  and  $\vec{F}_2$ ? (d) Repeat part (c) for the torque about the point with coordinates (3.0 m, 2.0 m, 4.0 m).

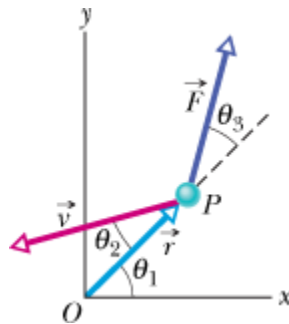
- 25 **SSM** Force  $\vec{F} = (-8.0 \text{ N})\hat{i} + (6.0 \text{ N})\hat{j}$  acts on a particle with position vector  $\vec{r} = (3.0 \text{ m})\hat{i} + (4.0 \text{ m})\hat{j}$ . What are (a) the torque on the particle about the origin, in unit-vector notation, and (b) the angle between the directions of  $\vec{r}$  and  $\vec{F}$ ?

**Answer:**

(a)  $(50 \text{ N} \cdot \text{m})\hat{k}$ ; (b)  $90^\circ$

## sec. 11-7 Angular Momentum

- 26 At the instant of Fig. 11-40, a 2.0 kg particle  $P$  has a position vector  $\vec{r}$  of magnitude 3.0 m and angle  $\theta_1 = 45^\circ$  and a velocity vector  $\vec{v}$  of magnitude 4.0 m/s and angle  $\theta_2 = 30^\circ$ . Force  $\vec{F}$ , of magnitude 2.0 N and angle  $\theta_3 = 30^\circ$ , acts on  $P$ . All three vectors lie in the  $xy$  plane. About the origin, what are the (a) magnitude and (b) direction of the angular momentum of  $P$  and the (c) magnitude and (d) direction of the torque acting on  $P$ ?



**Figure 11-40** Problem 26.

- 27 **SSM** At one instant, force  $\vec{F} = 4.0\hat{j} \text{ N}$  acts on a 0.25 kg object that has position vector  $\vec{r} = (2.0\hat{i} - 2.0\hat{k}) \text{ m}$  and velocity vector  $\vec{v} = (-5.0\hat{i} + 5.0\hat{k}) \text{ m/s}$ . About the origin and in unit-vector notation, what are (a) the object's angular momentum and (b) the torque acting on the

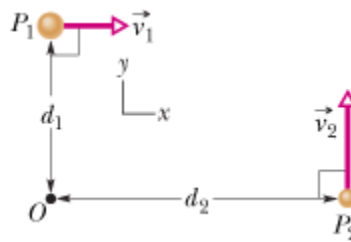
object?

**Answer:**

(a) 0; (b)  $(8.0 \text{ N} \cdot \text{m})\hat{i} + (8.0 \text{ N} \cdot \text{m})\hat{k}$

- 28 A 2.0 kg particle-like object moves in a plane with velocity components  $v_x = 30 \text{ m/s}$  and  $v_y = 60 \text{ m/s}$  as it passes through the point with  $(x, y)$  coordinates of  $(3.0, -4.0) \text{ m}$ . Just then, in unit-vector notation, what is its angular momentum relative to (a) the origin and (b) the point located at  $(-2.0, -2.0) \text{ m}$ ?

- 29 **ILW** In the instant of Fig. 11-41, two particles move in an  $xy$  plane. Particle  $P_1$  has mass  $6.5 \text{ kg}$  and speed  $v_1 = 2.2 \text{ m/s}$ , and it is at distance  $d_1 = 1.5 \text{ m}$  from point  $O$ . Particle  $P_2$  has mass  $3.1 \text{ kg}$  and speed  $v_2 = 3.6 \text{ m/s}$ , and it is at distance  $d_2 = 2.8 \text{ m}$  from point  $O$ . What are the (a) magnitude and (b) direction of the net angular momentum of the two particles about  $O$ ?



**Figure 11-41** Problem 29.

**Answer:**

(a)  $9.8 \text{ kg} \cdot \text{m}^2/\text{s}$ ; (b)  $+z$  direction

- 30 At the instant the displacement of a 2.00 kg object relative to the origin is

$$\vec{d} = (2.00 \text{ m})\hat{i} + (4.00 \text{ m})\hat{j} - (3.00 \text{ m})\hat{k},$$

its velocity is

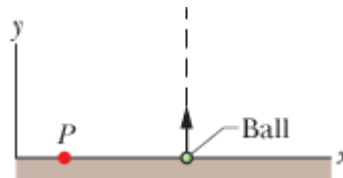
$$\vec{v} = -(6.00 \text{ m/s})\hat{i} + (3.00 \text{ m/s})\hat{j} + (3.00 \text{ m/s})\hat{k}$$

and it is subject to a force

$$\vec{F} = (6.00 \text{ N})\hat{i} - (8.00 \text{ N})\hat{j} + (4.00 \text{ N})\hat{k}.$$

Find (a) the acceleration of the object, (b) the angular momentum of the object about the origin, (c) the torque about the origin acting on the object, and (d) the angle between the velocity of the object and the force acting on the object.

- 31 In Fig. 11-42, a 0.400 kg ball is shot directly upward at initial speed 40.0 m/s. What is its angular momentum about  $P$ , 2.00 m horizontally from the launch point, when the ball is (a) at maximum height and (b) halfway back to the ground? What is the torque on the ball about  $P$  due to the gravitational force when the ball is (c) at maximum height and (d) halfway back to the ground?



**Figure 11-42** Problem 31.



**Answer:**

(a) 0; (b)  $-22.6 \text{ kg} \cdot \text{m}^2/\text{s}$ ; (c)  $-7.84 \text{ N} \cdot \text{m}$ ; (d)  $-7.84 \text{ N} \cdot \text{m}$

### sec. 11-8 Newton's Second Law in Angular Form

•32 A particle is acted on by two torques about the origin:  $\vec{\tau}_1$  has a magnitude of  $2.0 \text{ N} \cdot \text{m}$  and is directed in the positive direction of the  $x$  axis, and  $\vec{\tau}_2$  has a magnitude of  $4.0 \text{ N} \cdot \text{m}$  and is directed in the negative direction of the  $y$  axis. In unit-vector notation, find  $d\vec{\ell}/dt$ , where  $\vec{\ell}$  is the angular momentum of the particle about the origin.

•33 **SSM WWW ILW** At time  $t = 0$ , a  $3.0 \text{ kg}$  particle with velocity  $\vec{v} = (5.0 \text{ m/s})\hat{i} - (6.0 \text{ m/s})\hat{j}$  is at  $x = 3.0 \text{ m}$ ,  $y = 8.0 \text{ m}$ . It is pulled by a  $7.0 \text{ N}$  force in the negative  $x$  direction. About the origin, what are (a) the particle's angular momentum, (b) the torque acting on the particle, and (c) the rate at which the angular momentum is changing?

**Answer:**

(a)  $(-1.7 \times 10^2 \text{ kg} \cdot \text{m}^2/\text{s})\hat{k}$ ; (b)  $(+56 \text{ N} \cdot \text{m})\hat{k}$ ; (c)  $(+56 \text{ kg} \cdot \text{m}^2/\text{s}^2)\hat{k}$

•34 A particle is to move in an  $xy$  plane, clockwise around the origin as seen from the positive side of the  $z$  axis. In unit-vector notation, what torque acts on the particle if the magnitude of its angular momentum about the origin is (a)  $4.0 \text{ kg} \cdot \text{m}^2/\text{s}$ , (b)  $4.0t^2 \text{ kg} \cdot \text{m}^2/\text{s}$ , (c)  $4.0\sqrt{t} \text{ kg} \cdot \text{m}^2/\text{s}$ , and (d)  $4.0/t^2 \text{ kg} \cdot \text{m}^2/\text{s}$ ?

•35 At time  $t$ , the vector  $\vec{r} = 4.0t^2\hat{i} - (2.0t + 6.0t^2)\hat{j}$  gives the position of a  $3.0 \text{ kg}$  particle relative to the origin of an  $xy$  coordinate system ( $\vec{r}$  is in meters and  $t$  is in seconds). (a) Find an expression for the torque acting on the particle relative to the origin. (b) Is the magnitude of the particle's angular momentum relative to the origin increasing, decreasing, or unchanging?

**Answer:**

(a)  $48t\hat{k} \text{ N} \cdot \text{m}$ ; (b) increasing

### sec. 11-10 The Angular Momentum of a Rigid Body Rotating About a Fixed Axis

•36 Figure 11-43 shows three rotating, uniform disks that are coupled by belts. One belt runs around the rims of disks  $A$  and  $C$ . Another belt runs around a central hub on disk  $A$  and the rim of disk  $B$ . The belts move smoothly without slippage on the rims and hub. Disk  $A$  has radius  $R$ ; its hub has radius  $0.5000R$ ; disk  $B$  has radius  $0.2500R$ ; and disk  $C$  has radius  $2.000R$ . Disks  $B$  and  $C$  same density (mass per unit volume) and thickness. What is the ratio of the magnitude of the angular momentum of disk  $C$  to that of disk  $B$ ?

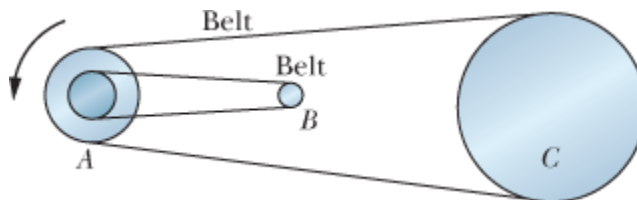


Figure 11-43 Problem 36.

- 37 **GO** In Fig. 11-44, three particles of mass  $m = 23$  g are fastened to three rods of length  $d = 12$  cm and negligible mass. The rigid assembly rotates around point  $O$  at the angular speed  $\omega = 0.85$  rad/s. About  $O$ , what are (a) the rotational inertia of the assembly, (b) the magnitude of the angular momentum of the middle particle, and (c) the magnitude of the angular momentum of the assembly?

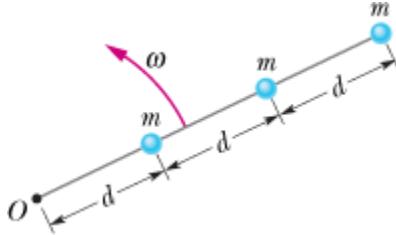


Figure 11-44 Problem 37.

**Answer:**

- (a)  $4.6 \times 10^{-3} \text{ kg} \cdot \text{m}^2$ ; (b)  $1.1 \times 10^{-3} \text{ kg} \cdot \text{m}^2/\text{s}$ ; (c)  $3.9 \times 10^{-3} \text{ kg} \cdot \text{m}^2/\text{s}$
- 38 A sanding disk with rotational inertia  $1.2 \times 10^{-3} \text{ kg} \cdot \text{m}^2$  is attached to an electric drill whose motor delivers a torque of magnitude  $16 \text{ N} \cdot \text{m}$  about the central axis of the disk. About that axis and with the torque applied for  $33 \text{ ms}$ , what is the magnitude of the (a) angular momentum and (b) angular velocity of the disk?
- 39 **SSM** The angular momentum of a flywheel having a rotational inertia of  $0.140 \text{ kg} \cdot \text{m}^2$  about its central axis decreases from  $3.00$  to  $0.800 \text{ kg} \cdot \text{m}^2/\text{s}$  in  $1.50 \text{ s}$ . (a) What is the magnitude of the average torque acting on the flywheel about its central axis during this period? (b) Assuming a constant angular acceleration, through what angle does the flywheel turn? (c) How much work is done on the wheel? (d) What is the average power of the flywheel?

**Answer:**

- (a)  $1.47 \text{ N} \cdot \text{m}$ ; (b)  $20.4 \text{ rad}$ ; (c)  $-29.9 \text{ J}$ ; (d)  $19.9 \text{ W}$
- 40 A disk with a rotational inertia of  $7.00 \text{ kg} \cdot \text{m}^2$  rotates like a merry-go-round while undergoing a variable torque given by  $\tau = (5.00 + 2.00t) \text{ N} \cdot \text{m}$ . At time  $t = 1.00 \text{ s}$ , its angular momentum is  $5.00 \text{ kg} \cdot \text{m}^2/\text{s}$ . What is its angular momentum at  $t = 3.00 \text{ s}$ ?
- 41 **GO** Figure 11-45 shows a rigid structure consisting of a circular hoop of radius  $R$  and mass  $m$ , and a square made of four thin bars, each of length  $R$  and mass  $m$ . The rigid structure rotates at a constant speed about a vertical axis, with a period of rotation of  $2.5 \text{ s}$ . Assuming  $R = 0.50 \text{ m}$  and  $m = 2.0 \text{ kg}$ , calculate (a) the structure's rotational inertia about the axis of rotation and (b) its angular momentum about that axis.

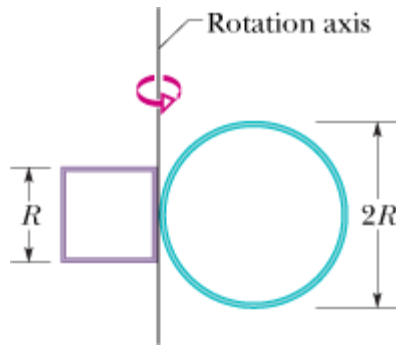


Figure 11-45 Problem 41.

Answer:

(a)  $1.6 \text{ kg} \cdot \text{m}^2$ ; (b)  $4.0 \text{ kg} \cdot \text{m}^2/\text{s}$

••42 Figure 11-46 gives the torque  $\tau$  that acts on an initially stationary disk that can rotate about its center like a merry-go-round.

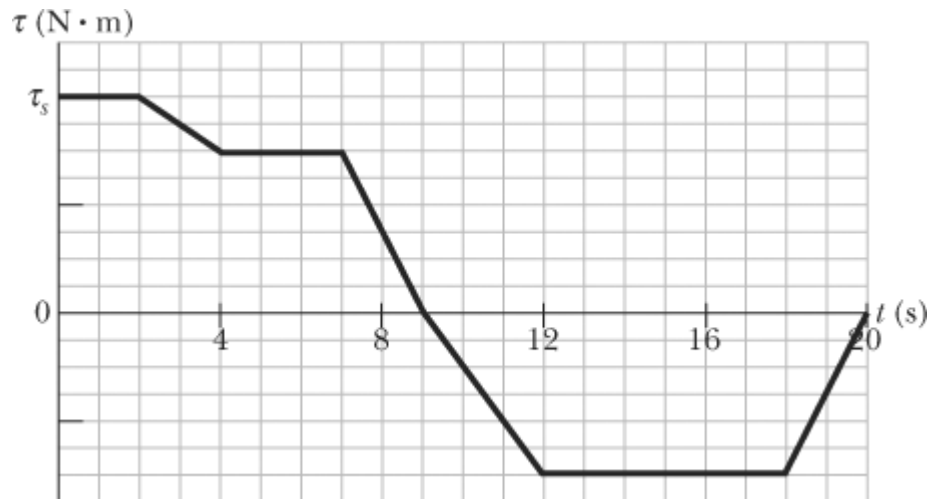


Figure 11-46 Problem 42.

The scale on the  $\tau_s = 4.0 \text{ N}\cdot\text{m}$ . What is the angular momentum of the disk about the rotation axis at times (a)  $t = 7.0 \text{ s}$  and (b)  $t = 20 \text{ s}$ ?

### sec. 11-11 Conservation of Angular Momentum

•43 In Fig. 11-47, two skaters, each of mass  $50 \text{ kg}$ , approach each other along parallel paths separated by  $3.0 \text{ m}$ . They have opposite velocities of  $1.4 \text{ m/s}$  each. One skater carries one end of a long pole of negligible mass, and the other skater grabs the other end as she passes. The skaters then rotate around the center of the pole. Assume that the friction between skates and ice is negligible. What are (a) the radius of the circle, (b) the angular speed of the skaters, and (c) the kinetic energy of the two-skater system? Next, the skaters pull along the pole until they are separated by  $1.0 \text{ m}$ . What then are (d) their angular speed and (e) the kinetic energy of the system? (f) What provided the energy for the increased kinetic energy?

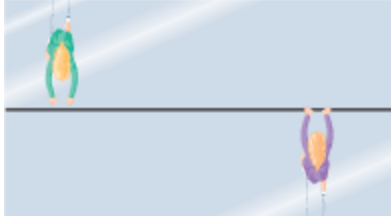


Figure 11-47 Problem 43.

**Answer:**

- (a) 1.5 m; (b) 0.93 rad/s; (c) 98 J; (d) 8.4 rad/s; (e)  $8.8 \times 10^2$  J; (f) internal energy of the skaters
- 44 A Texas cockroach of mass 0.17 kg runs counterclockwise around the rim of a lazy Susan (a circular disk mounted on a vertical axle) that has radius 15 cm, rotational inertia  $5.0 \times 10^{-3} \text{ kg}\cdot\text{m}^2$ , and frictionless bearings. The cockroach's speed (relative to the ground) is 2.0 m/s, and the lazy Susan turns clockwise with angular speed  $\omega_0 = 2.8 \text{ rad/s}$ . The cockroach finds a bread crumb on the rim and, of course, stops. (a) What is the angular speed of the lazy Susan after the cockroach stops? (b) Is mechanical energy conserved as it stops?
  - 45 **SSM WWW** A man stands on a platform that is rotating (without friction) with an angular speed of 1.2 rev/s; his arms are outstretched and he holds a brick in each hand. The rotational inertia of the system consisting of the man, bricks, and platform about the central vertical axis of the platform is  $6.0 \text{ kg}\cdot\text{m}^2$ . If by moving the bricks the man decreases the rotational inertia of the system to  $2.0 \text{ kg}\cdot\text{m}^2$ , what are (a) the resulting angular speed of the platform and (b) the ratio of the new kinetic energy of the system to the original kinetic energy? (c) What source provided the added kinetic energy?

**Answer:**

- (a) 3.6 rev/s; (b) 3.0; (c) forces on the bricks from the man transferred energy from the man's internal energy to kinetic energy
- 46 The rotational inertia of a collapsing spinning star drops to its initial value. What is the ratio of the new rotational kinetic energy to the initial rotational kinetic energy?
  - 47 **SSM** A track is mounted on a large wheel that is free to turn with negligible friction about a vertical axis (Fig. 11-48). A toy train of mass  $m$  is placed on the track and, with the system initially at rest, the train's electrical power is turned on. The train reaches speed 0.15 m/s with respect to the track. What is the angular speed of the wheel if its mass is  $1.1m$  and its radius is 0.43 m? (Treat the wheel as a hoop, and neglect the mass of the spokes and hub.)

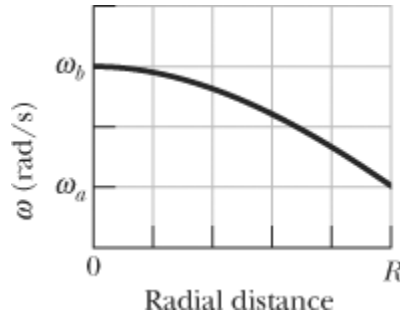


Figure 11-48 Problem 47.

**Answer:**

- 0.17 rad/s
- 48 A Texas cockroach first rides at the center of a circular disk that rotates freely like a merry-go-

round without external torques. The cockroach then walks out to the edge of the disk, at radius  $R$ . Figure 11-49 gives the angular speed  $\omega$  of the cockroach–disk system during the walk. The scale on the  $\omega$  axis is set by  $\omega_a = 5.0$  rad/s and  $\omega_b = 6.0$  rad/s. When the cockroach is on the edge at radius  $R$ , what is the ratio of the bug's rotational inertia to that of the disk, both calculated about the rotation axis?



**Figure 11-49** Problem 48.

- 49 Two disks are mounted (like a merry-go-round) on low-friction bearings on the same axle and can be brought together so that they couple and rotate as one unit. The first disk, with rotational inertia  $3.30 \text{ kg}\cdot\text{m}^2$  about its central axis, is set spinning counterclockwise at 450 rev/min. The second disk, with rotational inertia  $6.60 \text{ kg}\cdot\text{m}^2$  about its central axis, is set spinning counterclockwise at 900 rev/min. They then couple together. (a) What is their angular speed after coupling? If instead the second disk is set spinning clockwise at 900 rev/min, what are their (b) angular speed and (c) direction of rotation after they couple together?

**Answer:**

(a) 750 rev/min; (b) 450 rev/min; (c) clockwise

- 50 The rotor of an electric motor has rotational inertia  $I_m = 2.0 \times 10^{-3} \text{ kg}\cdot\text{m}^2$  about its central axis. The motor is used to change the orientation of the space probe in which it is mounted. The motor axis is mounted along the central axis of the probe; the probe has rotational inertia  $I_p = 12 \text{ kg}\cdot\text{m}^2$  about this axis. Calculate the number of revolutions of the rotor required to turn the probe through  $30^\circ$  about its central axis.
- 51 **SSM ILW** A wheel is rotating freely at angular speed 800 rev/min on a shaft whose rotational inertia is negligible. A second wheel, initially at rest and with twice the rotational inertia of the first, is suddenly coupled to the same shaft. (a) What is the angular speed of the resultant combination of the shaft and two wheels? (b) What fraction of the original rotational kinetic energy is lost?

**Answer:**

(a) 267 rev/min; (b) 0.667

- 52 **GO** A cockroach of mass  $m$  lies on the rim of a uniform disk of mass  $4.00m$  that can rotate freely about its center like a merry-go-round. Initially the cockroach and disk rotate together with an angular velocity of 0.260 rad/s. Then the cockroach walks halfway to the center of the disk. (a) What then is the angular velocity of the cockroach–disk system? (b) What is the ratio  $K/K_0$  of the new kinetic energy of the system to its initial kinetic energy? (c) What accounts for the change in the kinetic energy?
- 53 **GO** A uniform thin rod of length 0.500 m and mass 4.00 kg can rotate in a horizontal plane about a

vertical axis through its center. The rod is at rest when a 3.00 g bullet traveling in the rotation plane is fired into one end of the rod. As viewed from above, the bullet's path makes angle  $\theta = 60.0^\circ$  with the rod (Fig. 11-50). If the bullet lodges in the rod and the angular velocity of the rod is 10 rad/s immediately after the collision, what is the bullet's speed just before impact?

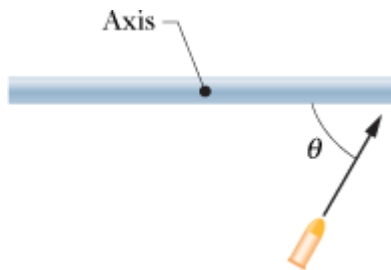



Figure 11-50 Problem 53.

**Answer:**

$$1.3 \times 10^3 \text{ m/s}$$

- 54  Figure 11-51 shows an overhead view of a ring that can rotate about its center like a merry-go-round. Its outer radius  $R_2$  is 0.800 m, its inner radius  $R_1$  is  $R_2/2.00$ , its mass  $M$  is 8.00 kg, and the mass of the crossbars at its center is negligible. It initially rotates at an angular speed of 8.00 rad/s with a cat of mass  $m = M/4.00$  on its outer edge, at radius  $R_2$ . By how much does the cat increase the kinetic energy of the cat–ring system if the cat crawls to the inner edge, at radius  $R_1$ ?

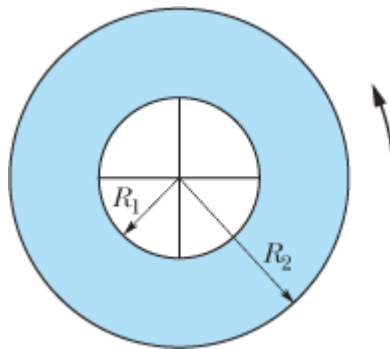



Figure 11-51 Problem 54.

- 55 A horizontal vinyl record of mass 0.10 kg and radius 0.10 m rotates freely about a vertical axis through its center with an angular speed of 4.7 rad/s. The rotational inertia of the record about its axis of rotation is  $5.0 \times 10^{-4} \text{ kg}\cdot\text{m}^2$ . A wad of wet putty of mass 0.020 kg drops vertically onto the record from above and sticks to the edge of the record. What is the angular speed of the record immediately after the putty sticks to it?

**Answer:**

$$3.4 \text{ rad/s}$$

- 56  In a long jump, an athlete leaves the ground with an initial angular momentum that tends to rotate her body forward, threatening to ruin her landing. To counter this tendency, she rotates her outstretched arms to “take up” the angular momentum (Fig. 11-18). In 0.700 s, one arm sweeps through 0.500 rev and the other arm sweeps through 1.000 rev. Treat each arm as a thin

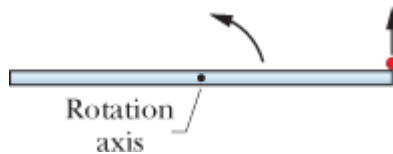
rod of mass 4.0 kg and length 0.60 m, rotating around one end. In the athlete's reference frame, what is the magnitude of the total angular momentum of the arms around the common rotation axis through the shoulders?

- 57 A uniform disk of mass  $10m$  and radius  $3.0r$  can rotate freely about its fixed center like a merry-go-round. A smaller uniform disk of mass  $m$  and radius  $r$  lies on top of the larger disk, concentric with it. Initially the two disks rotate together with an angular velocity of  $20 \text{ rad/s}$ . Then a slight disturbance causes the smaller disk to slide outward across the larger disk, until the outer edge of the smaller disk catches on the outer edge of the larger disk. Afterward, the two disks again rotate together (without further sliding). (a) What then is their angular velocity about the center of the larger disk? (b) What is the ratio  $K/K_0$  of the new kinetic energy of the two-disk system to the system's initial kinetic energy?

**Answer:**

(a)  $18 \text{ rad/s}$ ; (b)  $0.92$

- 58 A horizontal platform in the shape of a circular disk rotates on a frictionless bearing about a vertical axle through the center of the disk. The platform has a mass of  $150 \text{ kg}$ , a radius of  $2.0 \text{ m}$ , and a rotational inertia of  $300 \text{ kg}\cdot\text{m}^2$  about the axis of rotation. A  $60 \text{ kg}$  student walks slowly from the rim of the platform toward the center. If the angular speed of the system is  $1.5 \text{ rad/s}$  when the student starts at the rim, what is the angular speed when she is  $0.50 \text{ m}$  from the center?
- 59 Figure 11-52 is an overhead view of a thin uniform rod of length  $0.800 \text{ m}$  and mass  $M$  rotating horizontally at angular speed  $20.0 \text{ rad/s}$  about an axis through its center. A particle of mass  $M/3.00$  initially attached to one end is ejected from the rod and travels along a path that is perpendicular to the rod at the instant of ejection. If the particle's speed  $v_p$  is  $6.00 \text{ m/s}$  greater than the speed of the rod end just after ejection, what is the value of  $v_p$ ?



**Figure 11-52** Problem 59.

**Answer:**

$11.0 \text{ m/s}$

- 60 In Fig. 11-53, a  $1.0 \text{ g}$  bullet is fired into a  $0.50 \text{ kg}$  block attached to the end of a  $0.60 \text{ m}$  nonuniform rod of mass  $0.50 \text{ kg}$ . The block-rod-bullet system then rotates in the plane of the figure, about a fixed axis at  $A$ . The rotational inertia of the rod alone about that axis at  $A$  is  $0.060 \text{ kg}\cdot\text{m}^2$ . Treat the block as a particle. (a) What then is the rotational inertia of the block-rod-bullet system about point  $A$ ? (b) If the angular speed of the system about  $A$  just after impact is  $4.5 \text{ rad/s}$ , what is the bullet's speed just before impact?

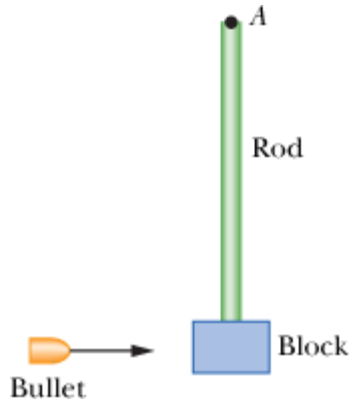


Figure 11-53 Problem 60.

- 61 The uniform rod (length 0.60 m, mass 1.0 kg) in Fig. 11-54 rotates in the plane of the figure about an axis through one end, with a rotational inertia of  $0.12 \text{ kg}\cdot\text{m}^2$ . As the rod swings through its lowest position, it collides with a 0.20 kg putty wad that sticks to the end of the rod. If the rod's angular speed just before collision is  $2.4 \text{ rad/s}$ , what is the angular speed of the rod–putty system immediately after collision?

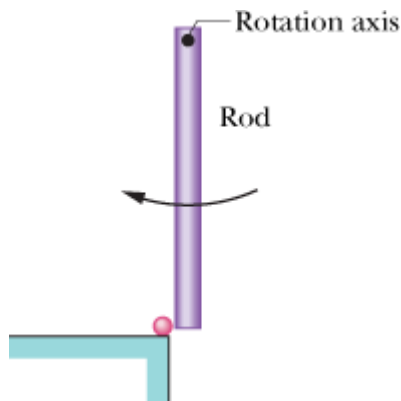


Figure 11-54 Problem 61.

**Answer:**

1.5 rad/s

- 62 ~~62~~ During a jump to his partner, an aerialist is to make a quadruple somersault lasting a time  $t = 1.87 \text{ s}$ . For the first and last quarter-revolution, he is in the extended orientation shown in Fig. 11-55, with rotational inertia  $I_1 = 19.9 \text{ kg}\cdot\text{m}^2$  around his center of mass (the dot). During the rest of the flight he is in a tight tuck, with rotational inertia  $I_2 = 3.93 \text{ kg}\cdot\text{m}^2$ . What must be his angular speed  $\omega_2$  around his center of mass during the tuck?



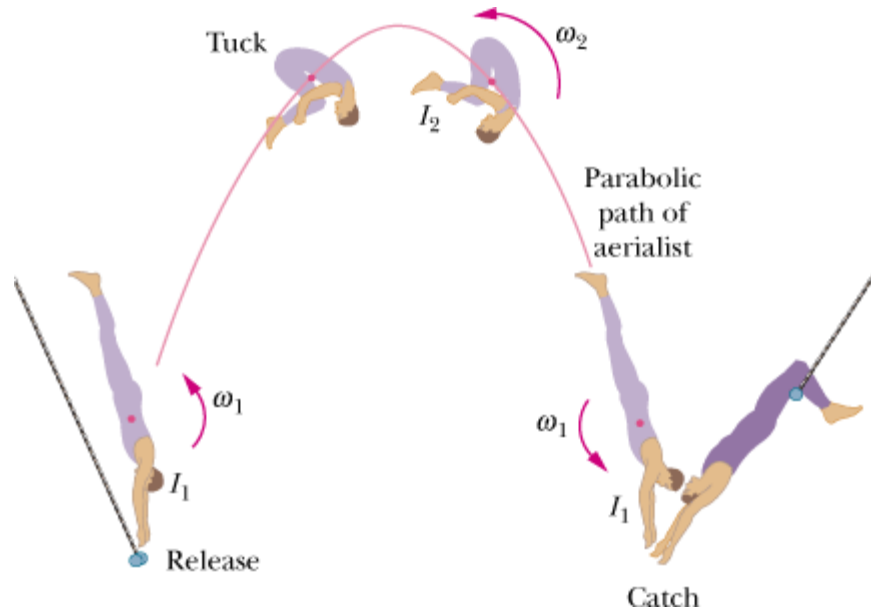


Figure 11-55 Problem 62.

- 63 GO In Fig. 11-56, a 30 kg child stands on the edge of a stationary merry-go-round of radius 2.0 m. The rotational inertia of the merry-go-round about its rotation axis is  $150 \text{ kg}\cdot\text{m}^2$ . The child catches a ball of mass 1.0 kg thrown by a friend. Just before the ball is caught, it has a horizontal velocity  $\vec{v}$  of magnitude 12 m/s, at angle  $= 37^\circ$  with a line tangent to the outer edge of the merry-go-round, as shown. What is the angular speed of the merry-go-round just after the ball is caught?

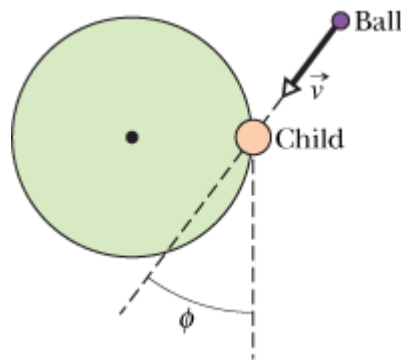


Figure 11-56 Problem 63.

**Answer:**

0.070 rad/s

- 64 ~~SSM~~ A ballerina begins a tour jeté (Fig. 11-19a) with angular speed  $\omega_i$  and a rotational inertia consisting of two parts:  $I_{\text{leg}} = 1.44 \text{ kg}\cdot\text{m}^2$  for her leg extended outward at angle  $\theta = 90.0^\circ$  to her body and  $I_{\text{trunk}} = 0.660 \text{ kg}\cdot\text{m}^2$  for the rest of her body (primarily her trunk). Near her maximum height she holds both legs at angle  $\theta = 30.0^\circ$  to her body and has angular speed  $\omega_f$  (Fig. 11-19b). Assuming that  $I_{\text{trunk}}$  has not changed, what is the ratio  $\omega_f/\omega_i$ ?

- 65 SSM WWW Two 2.00 kg balls are attached to the ends of a thin rod of length 50.0 cm and

negligible mass. The rod is free to rotate in a vertical plane without friction about a horizontal axis through its center. With the rod initially horizontal (Fig. 11-57), a 50.0 g wad of wet putty drops onto one of the balls, hitting it with a speed of 3.00 m/s and then sticking to it. (a) What is the angular speed of the system just after the putty wad hits? (b) What is the ratio of the kinetic energy of the system after the collision to that of the putty wad just before? (c) Through what angle will the system rotate before it momentarily stops?

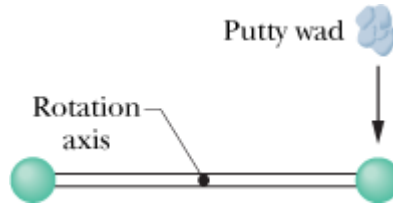


Figure 11-57 Problem 65.

**Answer:**

(a) 0.148 rad/s; (b) 0.0123; (c)  $181^\circ$

- 66 In Fig. 11-58, a small 50 g block slides down a frictionless surface through height  $h = 20$  cm and then sticks to a uniform rod of mass 100 g and length 40 cm. The rod pivots about point  $O$  through angle  $\theta$  before momentarily stopping. Find  $\theta$ .

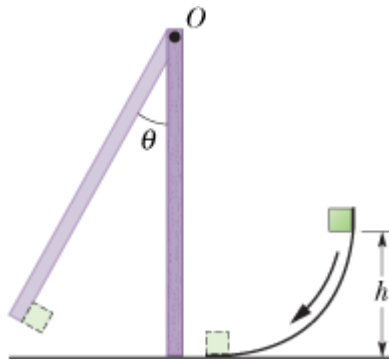


Figure 11-58 Problem 66.

- 67 Figure 11-59 is an overhead view of a thin uniform rod of length 0.600 m and mass  $M$  rotating horizontally at 80.0 rad/s counterclockwise about an axis through its center. A particle of mass  $M/3.00$  and traveling horizontally at speed 40.0 m/s hits the rod and sticks. The particle's path is perpendicular to the rod at the instant of the hit, at a distance  $d$  from the rod's center. (a) At what value of  $d$  are rod and particle stationary after the hit? (b) In which direction do rod and particle rotate if  $d$  is greater than this value?

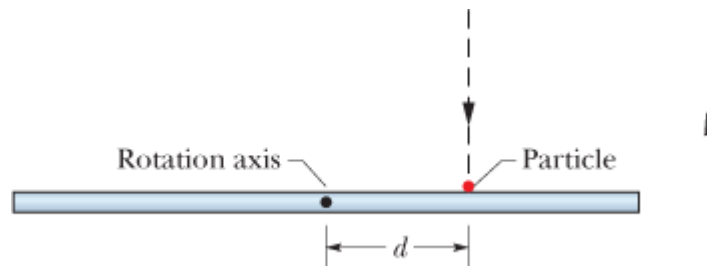


Figure 11-59 Problem 67.

**Answer:**

(a) 0.180 m; (b) clockwise

### sec. 11-12 Precession of a Gyroscope

- 68 A top spins at 30 rev/s about an axis that makes an angle of  $30^\circ$  with the vertical. The mass of the top is 0.50 kg, its rotational inertia about its central axis is  $5.0 \times 10^{-4} \text{ kg}\cdot\text{m}^2$ , and its center of mass is 4.0 cm from the pivot point. If the spin is clockwise from an overhead view, what are the (a) precession rate and (b) direction of the precession as viewed from overhead?
- 69 A certain gyroscope consists of a uniform disk with a 50 cm radius mounted at the center of an axle that is 11 cm long and of negligible mass. The axle is horizontal and supported at one end. If the disk is spinning around the axle at 1000 rev/min, what is the precession rate?

**Answer:**

0.041 rad/s

### Additional Problems

- 70 A uniform solid ball rolls smoothly along a floor, then up a ramp inclined at  $15.0^\circ$ . It momentarily stops when it has rolled 1.50 m along the ramp. What was its initial speed?
- 71 **SSM** In Fig. 11-60, a constant horizontal force  $\vec{F}_{\text{app}}$  of magnitude 12 N is applied to a uniform solid cylinder by fishing line wrapped around the cylinder. The mass of the cylinder is 10 kg, its radius is 0.10 m, and the cylinder rolls smoothly on the horizontal surface. (a) What is the magnitude of the acceleration of the center of mass of the cylinder? (b) What is the magnitude of the angular acceleration of the cylinder about the center of mass? (c) In unit-vector notation, what is the frictional force acting on the cylinder?

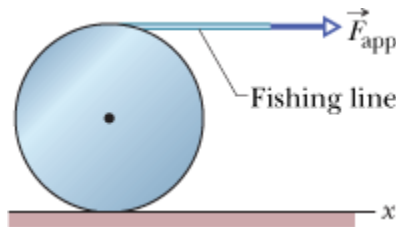


Figure 11-60 Problem 71.

**Answer:**

(a)  $1.6 \text{ m/s}^2$ ; (b)  $16 \text{ rad/s}^2$ ; (c)  $(4.0 \text{ N})\hat{i}$

- 72 A thin-walled pipe rolls along the floor. What is the ratio of its translational kinetic energy to its rotational kinetic energy about the central axis parallel to its length?
- 73 **SSM** A 3.0 kg toy car moves along an  $x$  axis with a velocity given by  $\vec{v} = -2.0t^3\hat{i} \text{ m/s}$ , with  $t$  in seconds. For  $t > 0$ , what are (a) the angular momentum  $\vec{L}$  of the car and (b) the torque  $\vec{\tau}$  on the car, both calculated about the origin? What are (c)  $\vec{L}$  and (d)  $\vec{\tau}$  about the point (2.0 m, 5.0 m, 0)? What are (e)  $\vec{L}$  and (f)  $\vec{\tau}$  about the point (2.0 m, -5.0 m, 0)?

**Answer:**

(a) 0; (b) 0; (c)  $-30r^3\hat{k} \text{ kg} \cdot \text{m}^2/\text{s}$ ; (d)  $-90r^2\hat{k} \text{ N} \cdot \text{m}$ ; (e)  $30r^3\hat{k} \text{ kg} \cdot \text{m}^2/\text{s}$ ; (f)  $90r^2 \text{ k N} \cdot \text{m}$

**74** A wheel rotates clockwise about its central axis with an angular momentum of  $600 \text{ kg}\cdot\text{m}^2/\text{s}$ . At time  $t = 0$ , a torque of magnitude  $50 \text{ N}\cdot\text{m}$  is applied to the wheel to reverse the rotation. At what time  $t$  is the angular speed zero?

**75 SSM** In a playground, there is a small merry-go-round of radius  $1.20 \text{ m}$  and mass  $180 \text{ kg}$ . Its radius of gyration (see Problem 79 of Chapter 10) is  $91.0 \text{ cm}$ . A child of mass  $44.0 \text{ kg}$  runs at a speed of  $3.00 \text{ m/s}$  along a path that is tangent to the rim of the initially stationary merry-go-round and then jumps on. Neglect friction between the bearings and the shaft of the merry-go-round. Calculate (a) the rotational inertia of the merry-go-round about its axis of rotation, (b) the magnitude of the angular momentum of the running child about the axis of rotation of the merry-go-round, and (c) the angular speed of the merry-go-round and child after the child has jumped onto the merry-go-round.

**Answer:**

(a)  $149 \text{ kg} \cdot \text{m}^2$ ; (b)  $158 \text{ kg} \cdot \text{m}^2/\text{s}$ ; (c)  $0.744 \text{ rad/s}$

**76** A uniform block of granite in the shape of a book has face dimensions of  $20 \text{ cm}$  and  $15 \text{ cm}$  and a thickness of  $1.2 \text{ cm}$ . The density (mass per unit volume) of granite is  $2.64 \text{ g/cm}^3$ . The block rotates around an axis that is perpendicular to its face and halfway between its center and a corner. Its angular momentum about that axis is  $0.104 \text{ kg}\cdot\text{m}^2/\text{s}$ . What is its rotational kinetic energy about that axis?

**77 SSM** Two particles, each of mass  $2.90 \times 10^{-4} \text{ kg}$  and speed  $5.46 \text{ m/s}$ , travel in opposite directions along parallel lines separated by  $4.20 \text{ cm}$ . (a) What is the magnitude  $L$  of the angular momentum of the two-particle system around a point midway between the two lines? (b) Does the value of  $L$  change if the point about which it is calculated is not midway between the lines? If the direction of travel for one of the particles is reversed, what would be (c) the answer to part (a) and (d) the answer to part (b)?

**Answer:**

(a)  $6.65 \times 10^{-5} \text{ kg} \cdot \text{m}^2/\text{s}$ ; (b) no; (c) 0; (d) yes

**78** A wheel of radius  $0.250 \text{ m}$ , which is moving initially at  $43.0 \text{ m/s}$ , rolls to a stop in  $225 \text{ m}$ . Calculate the magnitudes of (a) its linear acceleration and (b) its angular acceleration. (c) The wheel's rotational inertia is  $0.155 \text{ kg}\cdot\text{m}^2$  about its central axis. Calculate the magnitude of the torque about the central axis due to friction on the wheel.

**79** Wheels  $A$  and  $B$  in Fig. 11-61 are connected by a belt that does not slip. The radius of  $B$  is  $3.00$  times the radius of  $A$ . What would be the ratio of the rotational inertias  $I_A/I_B$  if the two wheels had (a) the same angular momentum about their central axes and (b) the same rotational kinetic energy?

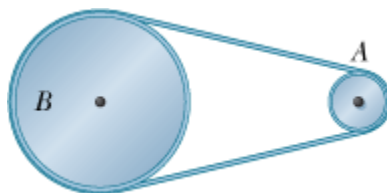


Figure 11-61 Problem 79.

Answer:

(a) 0.333; (b) 0.111

80

A 2.50 kg particle that is moving horizontally over a floor with velocity  $(-3.00 \text{ m/s})\hat{j}$  undergoes a completely inelastic collision with a 4.00 kg particle that is moving horizontally over the floor with velocity  $(4.50 \text{ m/s})\hat{i}$ . The collision occurs at  $xy$  coordinates  $(-0.500 \text{ m}, -0.100 \text{ m})$ . After the collision and in unit-vector notation, what is the angular momentum of the stuck-together particles with respect to the origin?

81

**SSM** A uniform wheel of mass 10.0 kg and radius 0.400 m is mounted rigidly on a massless axle through its center (Fig. 11-62). The radius of the axle is 0.200 m, and the rotational inertia of the wheel–axle combination about its central axis is  $0.600 \text{ kg}\cdot\text{m}^2$ . The wheel is initially at rest at the top of a surface that is inclined at angle  $\theta = 30.0^\circ$  with the horizontal; the axle rests on the surface while the wheel extends into a groove in the surface without touching the surface. Once released, the axle rolls down along the surface smoothly and without slipping. When the wheel–axle combination has moved down the surface by 2.00 m, what are (a) its rotational kinetic energy and (b) its translational kinetic energy?

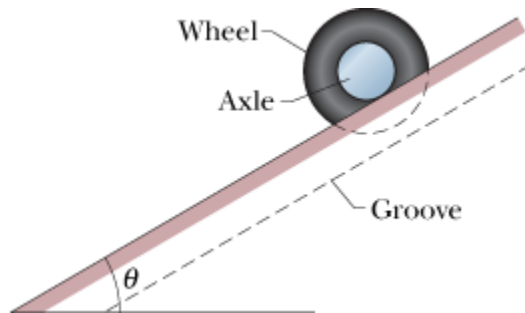


Figure 11-62 Problem 81.

Answer:

(a) 58.8 J; (b) 39.2 J

82

A uniform rod rotates in a horizontal plane about a vertical axis through one end. The rod is 6.00 m long, weighs 10.0 N, and rotates at 240 rev/min. Calculate (a) its rotational inertia about the axis of rotation and (b) the magnitude of its angular momentum about that axis.

83

A solid sphere of weight 36.0 N rolls up an incline at an angle of  $30.0^\circ$ . At the bottom of the incline the center of mass of the sphere has a translational speed of 4.90 m/s. (a) What is the kinetic energy of the sphere at the bottom of the incline? (b) How far does the sphere travel up along the incline? (c) Does the answer to (b) depend on the sphere's mass?

Answer:

(a) 61.7 J; (b) 3.43 m; (c) no

84

Suppose that the yo-yo in Problem 17, instead of rolling from rest, is thrown so that its initial speed down the string is 1.3 m/s. (a) How long does the yo-yo take to reach the end of the

string? As it reaches the end of the string, what are its (b) total kinetic energy, (c) linear speed, (d) translational kinetic energy, (e) angular speed, and (f) rotational kinetic energy?

- 85 A girl of mass  $M$  stands on the rim of a frictionless merry-go-round of radius  $R$  and rotational inertia  $I$  that is not moving. She throws a rock of mass  $m$  horizontally in a direction that is tangent to the outer edge of the merry-go-round. The speed of the rock, relative to the ground, is  $v$ . Afterward, what are (a) the angular speed of the merry-go-round and (b) the linear speed of the girl?

**Answer:**

(a)  $mvR/(I + MR^2)$ ; (b)  $mvR^2/(I + MR^2)$

- 86 At time  $t = 0$ , a 2.0 kg particle has the position vector  $\vec{r} = (4.0 \text{ m})\hat{i} - (2.0 \text{ m})\hat{j}$  relative to the origin. Its velocity is given by  $\vec{v} = (-6.0t^2 \text{ m/s})\hat{i}$  for  $t \geq 0$  in seconds. About the origin, what are (a) the particle's angular momentum  $\vec{L}$  and (b) the torque  $\vec{\tau}$  acting on the particle, both in unit-vector notation and for  $t > 0$ ? About the point  $(-2.0 \text{ m}, -3.0 \text{ m}, 0)$ , what are (c)  $\vec{L}$  and (d)  $\vec{\tau}$  for  $t > 0$ ?

- 87 If Earth's polar ice caps fully melted and the water returned to the oceans, the oceans would be deeper by about 30 m. What effect would this have on Earth's rotation? Make an estimate of the resulting change in the length of the day.

**Answer:**

rotational speed would decrease; day would be about 0.8 s longer

- 88 A 1200 kg airplane is flying in a straight line at 80 m/s, 1.3 km above the ground. What is the magnitude of its angular momentum with respect to a point on the ground directly under the path of the plane?
- 89 With axle and spokes of negligible mass and a thin rim, a certain bicycle wheel has a radius of 0.350 m and weighs 37.0 N; it can turn on its axle with negligible friction. A man holds the wheel above his head with the axle vertical while he stands on a turntable that is free to rotate without friction; the wheel rotates clockwise, as seen from above, with an angular speed of 57.7 rad/s, and the turntable is initially at rest. The rotational inertia of *wheel + man + turntable* about the common axis of rotation is  $2.10 \text{ kg}\cdot\text{m}^2$ . The man's free hand suddenly stops the rotation of the wheel (relative to the turntable). Determine the resulting (a) angular speed and (b) direction of rotation of the system.

**Answer:**

(a) 12.7 rad/s; (b) clockwise

- 90 For an 84 kg person standing at the equator, what is the magnitude of the angular momentum about Earth's center due to Earth's rotation?
- 91 A small solid sphere with radius 0.25 cm and mass 0.56 g rolls without slipping on the inside of a large fixed hemisphere with radius 15 cm and a vertical axis of symmetry. The sphere starts at the top from rest. (a) What is its kinetic energy at the bottom? (b) What fraction of its kinetic energy at the bottom is associated with rotation about an axis through its com? (c) What is the magnitude of the normal force on the hemisphere from the sphere when the sphere reaches the bottom?

**Answer:**

(a) 0.81 mJ; (b) 0.29; (c)  $1.3 \times 10^{-2}$  N

**92**An automobile has a total mass of 1700 kg. It accelerates from rest to 40 km/h in 10 s. Assume each wheel is a uniform 32 kg disk. Find, for the end of the 10 s interval, (a) the rotational kinetic energy of each wheel about its axle, (b) the total kinetic energy of each wheel, and (c) the total kinetic energy of the automobile.

**93**A body of radius  $R$  and mass  $m$  is rolling smoothly with speed  $v$  on a horizontal surface. It then rolls up a hill to a maximum height  $h$ . (a) If  $h = 3v^2/4g$ , what is the body's rotational inertia about the rotational axis through its center of mass? (b) What might the body be?

**Answer:**

(a)  $mR^2/2$ ; (b) a solid circular cylinder

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## sec. 12-4 The Center of Gravity

- 1 Because  $g$  varies so little over the extent of most structures, any structure's center of gravity effectively coincides with its center of mass. Here is a fictitious example where  $g$  varies more significantly. Figure 12-23 shows an array of six particles, each with mass  $m$ , fixed to the edge of a rigid structure of negligible mass. The distance between adjacent particles along the edge is 2.00 m. The following table gives the value of  $g$  ( $\text{m/s}^2$ ) at each particle's location. Using the coordinate system shown, find (a) the  $x$  coordinate  $x_{\text{com}}$  and (b) the  $y$  coordinate  $y_{\text{com}}$  of the center of mass of the six-particle system. Then find (c) the  $x$  coordinate  $x_{\text{cog}}$  and (d) the  $y$  coordinate  $y_{\text{cog}}$  of the center of gravity of the six-particle system.

| Particle | $g$  |
|----------|------|
| 1        | 8.00 |
| 2        | 7.80 |
| 3        | 7.60 |
| 4        | 7.40 |
| 5        | 7.60 |
| 6        | 7.80 |

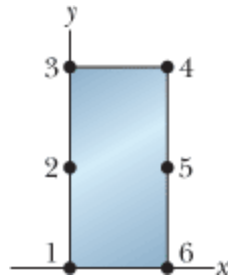


Figure 12-23 Problem 1.

**Answer:**

(a) 1.00 m; (b) 2.00 m; (c) 0.987 m; (d) 1.97 m

### sec. 12-5 Some Examples of Static Equilibrium

- 2 An automobile with a mass of 1360 kg has 3.05 m between the front and rear axles. Its center of gravity is located 1.78 m behind the front axle. With the automobile on level ground, determine the magnitude of the force from the ground on (a) each front wheel (assuming equal forces on the front wheels) and (b) each rear wheel (assuming equal forces on the rear wheels).
- 3 **SSM WWW** In Fig. 12-24, a uniform sphere of mass  $m = 0.85$  kg and radius  $r = 4.2$  cm is held in place by a massless rope attached to a frictionless wall a distance  $L = 8.0$  cm above the center of the sphere. Find (a) the tension in the rope and (b) the force on the sphere from the wall.

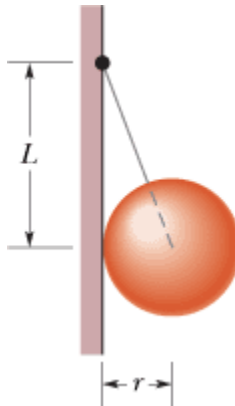


Figure 12-24 Problem 3.

**Answer:**

(a) 9.4 N; (b) 4.4 N

- 4 An archer's bow is drawn at its midpoint until the tension in the string is equal to the force exerted by the archer. What is the angle between the two halves of the string?
- 5 **ILW** A rope of negligible mass is stretched horizontally between two supports that are 3.44 m apart. When an object of weight 3160 N is hung at the center of the rope, the rope is observed to sag by 35.0 cm. What is the tension in the rope?

**Answer:**



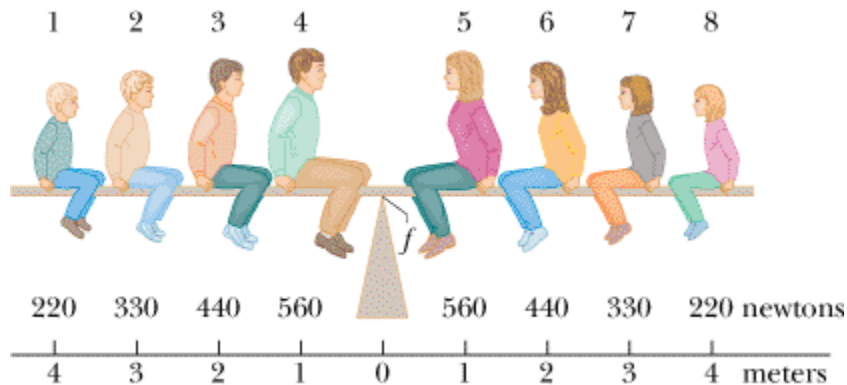
7.92 kN

- 6 A scaffold of mass 60 kg and length 5.0 m is supported in a horizontal position by a vertical cable at each end. A window washer of mass 80 kg stands at a point 1.5 m from one end. What is the tension in (a) the nearer cable and (b) the farther cable?
- 7 A 75 kg window cleaner uses a 10 kg ladder that is 5.0 m long. He places one end on the ground 2.5 m from a wall, rests the upper end against a cracked window, and climbs the ladder. He is 3.0 m up along the ladder when the window breaks. Neglect friction between the ladder and window and assume that the base of the ladder does not slip. When the window is on the verge of breaking, what are (a) the magnitude of the force on the window from the ladder, (b) the magnitude of the force on the ladder from the ground, and (c) the angle (relative to the horizontal) of that force on the ladder?

**Answer:**

(a)  $2.8 \times 10^2$  N; (b)  $8.8 \times 10^2$  N; (c)  $71^\circ$

- 8 A physics Brady Bunch, whose weights in newtons are indicated in Fig. 12-25, is balanced on a seesaw. What is the number of the person who causes the largest torque about the rotation axis at *fulcrum*  $f$  directed (a) out of the page and (b) into the page?



**Figure 12-25** Problem 8.

- 9 **SSM** A meter stick balances horizontally on a knife-edge at the 50.0 cm mark. With two 5.00 g coins stacked over the 12.0 cm mark, the stick is found to balance at the 45.5 cm mark. What is the mass of the meter stick?

**Answer:**

74.4 g

- 10 **GO** The system in Fig. 12-26 is in equilibrium, with the string in the center exactly horizontal. Block  $A$  weighs 40 N, block  $B$  weighs 50 N, and angle is  $35^\circ$ . Find (a) tension  $T_1$ , (b) tension  $T_2$ , (c) tension  $T_3$ , and (d) angle  $\theta$ .

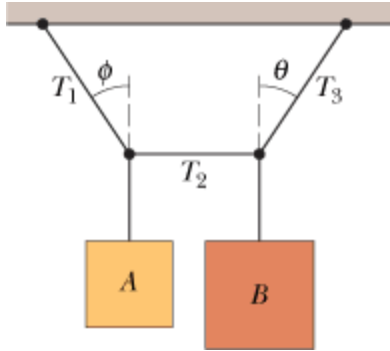


Figure 12-26 Problem 10.

- 11 **SSM** Figure 12-27 shows a diver of weight 580 N standing at the end of a diving board with a length of  $L = 4.5$  m and negligible mass. The board is fixed to two pedestals (supports) that are separated by distance  $d = 1.5$  m. Of the forces acting on the board, what are the (a) magnitude and (b) direction (up or down) of the force from the left pedestal and the (c) magnitude and (d) direction (up or down) of the force from the right pedestal? (e) Which pedestal (left or right) is being stretched, and (f) which pedestal is being compressed?

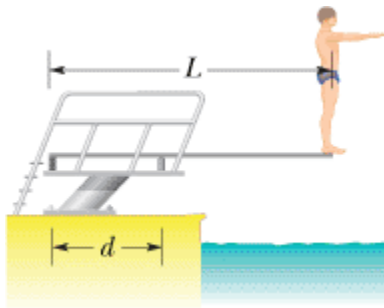


Figure 12-27 Problem 11.

**Answer:**

- (a) 1.2 kN; (b) down; (c) 1.7 kN; (d) up; (e) left; (f) right
- 12 In Fig. 12-28, trying to get his car out of mud, a man ties one end of a rope around the front bumper and the other end tightly around a utility pole 18 m away. He then pushes sideways on the rope at its midpoint with a force of 550 N, displacing the center of the rope 0.30 m, but the car barely moves. What is the magnitude of the force on the car from the rope? (The rope stretches somewhat.)

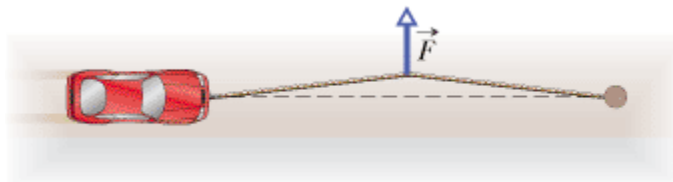


Figure 12-28 Problem 12.

- 13 Figure 12-29 shows the anatomical structures in the lower leg and foot that are involved in standing on tiptoe, with the heel raised slightly off the floor so that the foot effectively contacts the floor only at point  $P$ . Assume distance  $a = 5.0$  cm, distance  $b = 15$  cm, and the person's weight  $W$

= 900 N. Of the forces acting on the foot, what are the (a) magnitude and (b) direction (up or down) of the force at point A from the calf muscle and the (c) magnitude and (d) direction (up or down) of the force at point B from the lower leg bones?

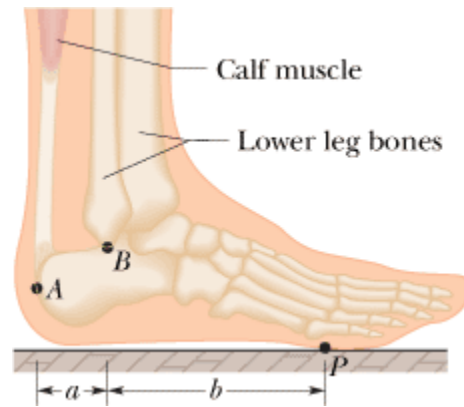


Figure 12-29 Problem 13.

**Answer:**

(a) 2.7 kN; (b) up; (c) 3.6 kN; (d) down

- 14 In Fig. 12-30, a horizontal scaffold, of length 2.00 m and uniform mass 50.0 kg, is suspended from a building by two cables. The scaffold has dozens of paint cans stacked on it at various points. The total mass of the paint cans is 75.0 kg. The tension in the cable at the right is 722 N. How far horizontally from *that* cable is the center of mass of the system of paint cans?



Figure 12-30 Problem 14.

- 15 **ILW** Forces  $\vec{F}_1$ ,  $\vec{F}_2$ , and  $\vec{F}_3$  act on the structure of Fig. 12-31, shown in an overhead view. We wish to put the structure in equilibrium by applying a fourth force, at a point such as P. The fourth force has vector components  $\vec{F}_h$  and  $\vec{F}_v$ . We are given that  $a = 2.0$  m,  $b = 3.0$  m,  $c = 1.0$  m,  $F_1 = 20$  N,  $F_2 = 10$  N, and  $F_3 = 5.0$  N. Find (a)  $F_h$ , (b)  $F_v$ , and (c) d.

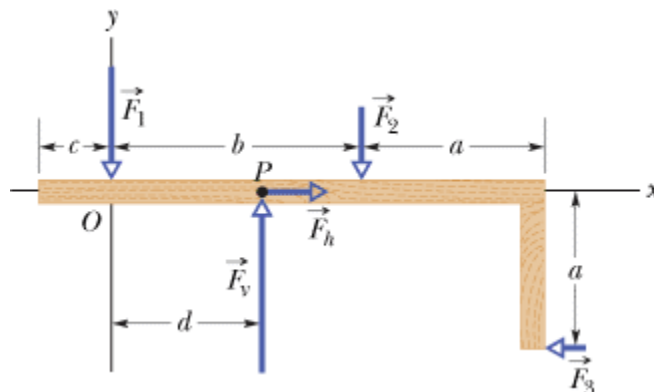
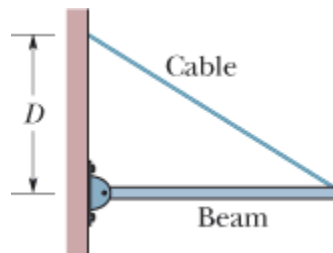


Figure 12-31 Problem 15.

**Answer:**

(a) 5.0 N; (b) 30 N; (c) 1.3 m


- 16 A uniform cubical crate is 0.750 m on each side and weighs 500 N. It rests on a floor with one edge against a very small, fixed obstruction. At what least height above the floor must a horizontal force of magnitude 350 N be applied to the crate to tip it?
- 17 In Fig. 12-32, a uniform beam of weight 500 N and length 3.0 m is suspended horizontally. On the left it is hinged to a wall; on the right it is supported by a cable bolted to the wall at distance  $D$  above the beam. The least tension that will snap the cable is 1200 N. (a) What value of  $D$  corresponds to that tension? (b) To prevent the cable from snapping, should  $D$  be increased or decreased from that value?

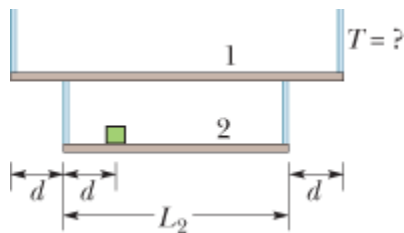


**Figure 12-32** Problem 17.

**Answer:**

(a) 0.64 m; (b) increased

- 18  In Fig. 12-33, horizontal scaffold 2, with uniform mass  $m_2 = 30.0$  kg and length  $L_2 = 2.00$  m, hangs from horizontal scaffold 1, with uniform mass  $m_1 = 50.0$  kg. A 20.0 kg box of nails lies on scaffold 2, centered at distance  $d = 0.500$  m from the left end. What is the tension  $T$  in the cable indicated?



**Figure 12-33** Problem 18.

- 19 To crack a certain nut in a nutcracker, forces with magnitudes of at least 40 N must act on its shell from both sides. For the nutcracker of Fig. 12-34, with distances  $L = 12$  cm and  $d = 2.6$  cm, what are the force components  $F_{\perp}$  (perpendicular to the handles) corresponding to that 40 N?

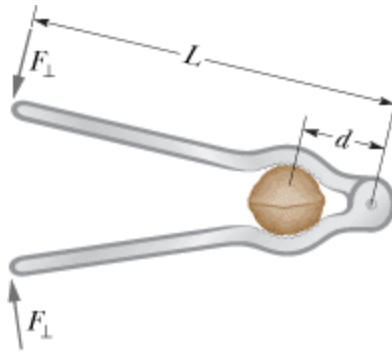


Figure 12-34 Problem 19.

Answer:

8.7 N

- 20 A bowler holds a bowling ball ( $M = 7.2 \text{ kg}$ ) in the palm of his hand (Fig. 12-35). His upper arm is vertical; his lower arm ( $1.8 \text{ kg}$ ) is horizontal. What is the magnitude of (a) the force of the biceps muscle on the lower arm and (b) the force between the bony structures at the elbow contact point?

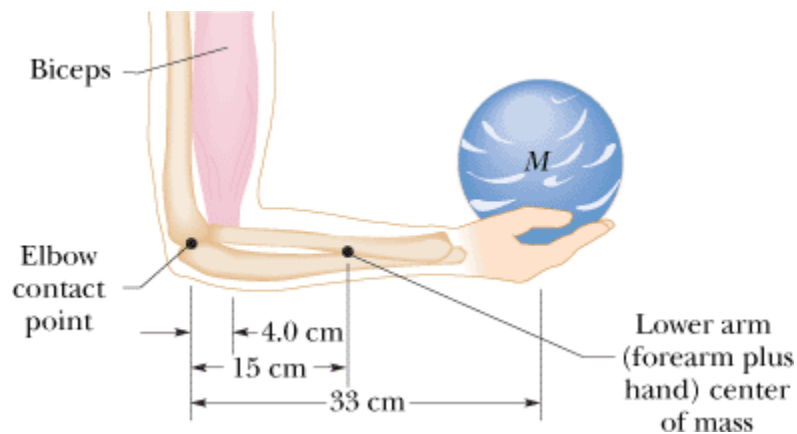


Figure 12-35 Problem 20.

- 21 **ILW** The system in Fig. 12-36 is in equilibrium. A concrete block of mass  $225 \text{ kg}$  hangs from the end of the uniform strut of mass  $45.0 \text{ kg}$ . For angles  $\phi = 30.0^\circ$  and  $\theta = 45.0^\circ$ , find (a) the tension  $T$  in the cable and the (b) horizontal and (c) vertical components of the force on the strut from the hinge.

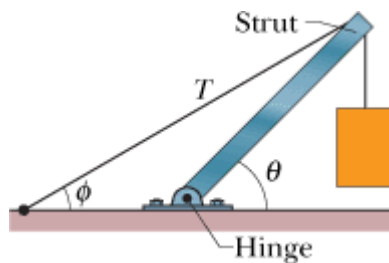

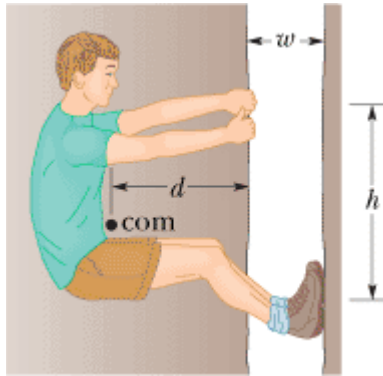


Figure 12-36 Problem 21.

**Answer:**

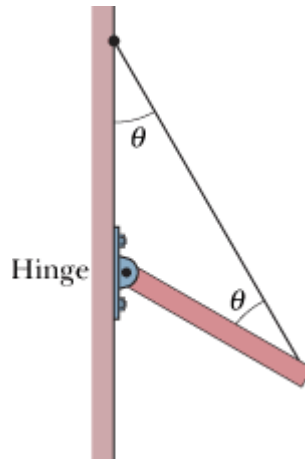
(a) 6.63 kN; (b) 5.74 kN; (c) 5.96 kN

- 22  In Fig. 12-37, a 55 kg rock climber is in a lie-back climb along a fissure, with hands pulling on one side of the fissure and feet pressed against the opposite side. The fissure has width  $w = 0.20$  m, and the center of mass of the climber is a horizontal distance  $d = 0.40$  m from the fissure. The coefficient of static friction between hands and rock is  $\mu_1 = 0.40$ , and between boots and rock it is  $\mu_2 = 1.2$ . (a) What is the least horizontal pull by the hands and push by the feet that will keep the climber stable? (b) For the horizontal pull of (a), what must be the vertical distance  $h$  between hands and feet? If the climber encounters wet rock, so that  $\mu_1$  and  $\mu_2$  are reduced, what happens to (c) the answer to (a) and (d) the answer to (b)?



**Figure 12-37** Problem 22.


- 23 In Fig. 12-38, one end of a uniform beam of weight 222 N is hinged to a wall; the other end is supported by a wire that makes angles  $\theta = 30.0^\circ$  with both wall and beam. Find (a) the tension in the wire and the (b) horizontal and (c) vertical components of the force of the hinge on the beam.



**Figure 12-38** Problem 23.

**Answer:**

(a) 192 N; (b) 96.1 N; (c) 55.5 N

- 24  In Fig. 12-39, a climber with a weight of 533.8 N is held by a belay rope connected to her climbing harness and belay device; the force of the rope on her has a line of action through her center of mass. The indicated angles are  $\theta = 40.0^\circ$  and  $\phi = 30.0^\circ$ . If her feet are on the verge of sliding on the vertical wall, what is the coefficient of static friction between her climbing shoes and the wall?

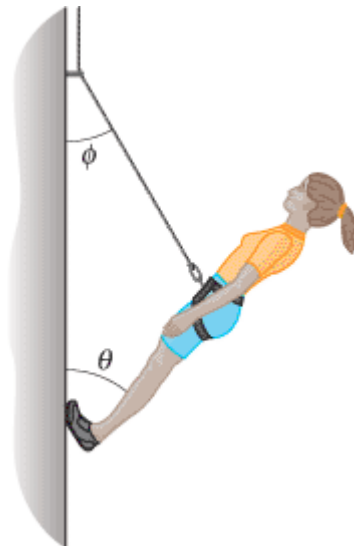
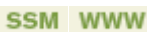


Figure 12-39 Problem 24.

- 25  In Fig. 12-40, what magnitude of (constant) force  $\vec{F}$  applied horizontally at the axle of the wheel is necessary to raise the wheel over an obstacle of height  $h = 3.00$  cm? The wheel's radius is  $r = 6.00$  cm, and its mass is  $m = 0.800$  kg.

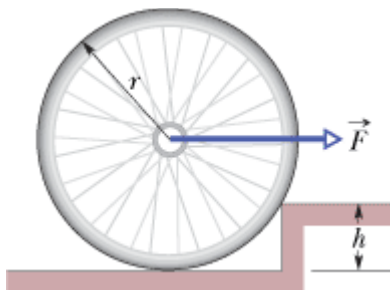



Figure 12-40 Problem 25.

**Answer:**

13.6 N

- 26  In Fig. 12-41, a climber leans out against a vertical ice wall that has negligible friction. Distance  $a$  is 0.914 m and distance  $L$  is 2.10 m. His center of mass is distance  $d = 0.940$  m from the feet–ground contact point. If he is on the verge of sliding, what is the coefficient of static friction between feet and ground?

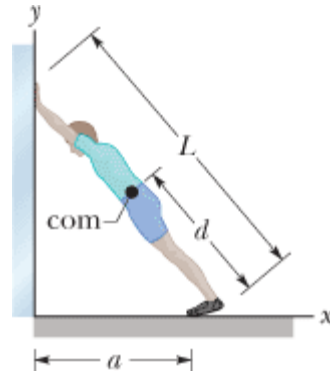


Figure 12-41 Problem 26.

- 27 In Fig. 12-42, a 15 kg block is held in place via a pulley system. The person's upper arm is vertical; the forearm is at angle  $\theta = 30^\circ$  with the horizontal. Forearm and hand together have a mass of 2.0 kg, with a center of mass at distance  $d_1 = 15$  cm from the contact point of the forearm bone and the upper-arm bone (humerus). The triceps muscle pulls vertically upward on the forearm at distance  $d_2 = 2.5$  cm behind that contact point. Distance  $d_3$  is 35 cm. What are the (a) magnitude and (b) direction (up or down) of the force on the forearm from the triceps muscle and the (c) magnitude and (d) direction (up or down) of the force on the forearm from the humerus?

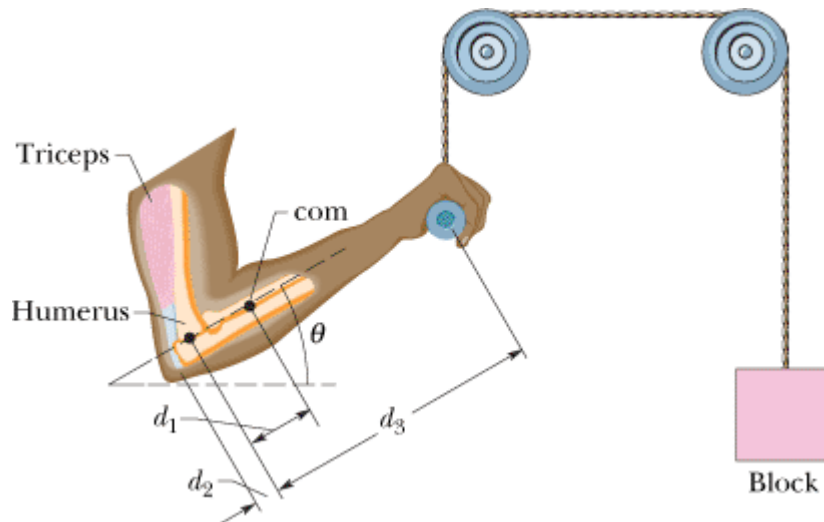


Figure 12-42 Problem 27.

Answer:

(a) 1.9 kN; (b) up; (c) 2.1 kN; (d) down

- 28 **GO** In Fig. 12-43, suppose the length  $L$  of the uniform bar is 3.00 m and its weight is 200 N. Also, let the block's weight  $W = 300$  N and the angle  $\theta = 30.0^\circ$ . The wire can withstand a maximum tension of 500 N. (a) What is the maximum possible distance  $x$  before the wire breaks? With the block placed at this maximum  $x$ , what are the (b) horizontal and (c) vertical components of the force on the bar from the hinge at  $A$ ?



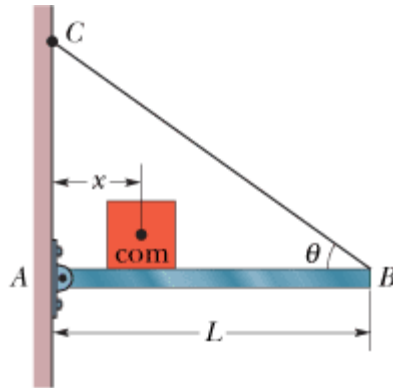


Figure 12-43 Problems 28 and 34.

- 29 A door has a height of 2.1 m along a  $y$  axis that extends vertically upward and a width of 0.91 m along an  $x$  axis that extends outward from the hinged edge of the door. A hinge 0.30 m from the top and a hinge 0.30 m from the bottom each support half the door's mass, which is 27 kg. In unit-vector notation, what are the forces on the door at (a) the top hinge and (b) the bottom hinge?

Answer:

(a)  $(-80 \text{ N})\hat{i} + (1.3 \times 10^2 \text{ N})\hat{j}$ ; (b)  $(80 \text{ N})\hat{i} + (1.3 \times 10^2 \text{ N})\hat{j}$

- 30 In Fig. 12-44, a 50.0 kg uniform square sign, of edge length  $L = 2.00$  m, is hung from a horizontal rod of length  $d_h = 3.00$  m and negligible mass. A cable is attached to the end of the rod and to a point on the wall at distance  $d_v = 4.00$  m above the point where the rod is hinged to the wall. (a) What is the tension in the cable? What are the (b) magnitude and (c) direction (left or right) of the horizontal component of the force on the rod from the wall, and the (d) magnitude and (e) direction (up or down) of the vertical component of this force?

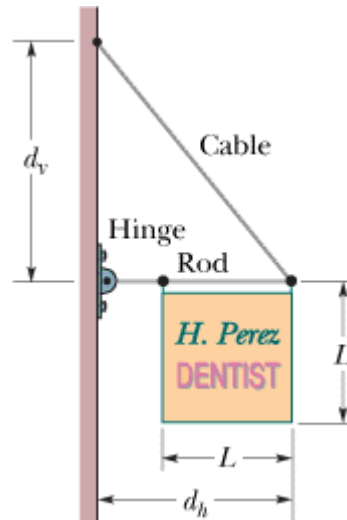


Figure 12-44 Problem 30.

- 31 GO In Fig. 12-45, a nonuniform bar is suspended at rest in a horizontal position by two massless cords. One cord makes the angle  $\theta = 36.9^\circ$  with the vertical; the other makes the angle  $= 53.1^\circ$  with the vertical. If the length  $L$  of the bar is 6.10 m, compute the distance  $x$  from the left end of the bar to its center of mass.

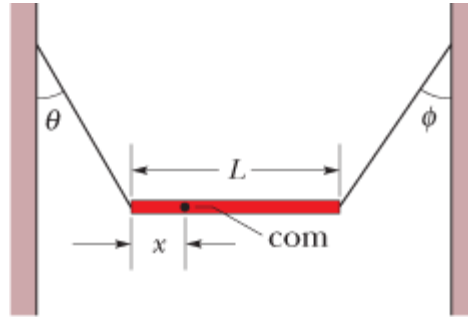


Figure 12-45 Problem 31.

Answer:

2.20 m

- 32 In Fig. 12-46, the driver of a car on a horizontal road makes an emergency stop by applying the brakes so that all four wheels lock and skid along the road. The coefficient of kinetic friction between tires and road is 0.40. The separation between the front and rear axles is  $L = 4.2$  m, and the center of mass of the car is located at distance  $d = 1.8$  m behind the front axle and distance  $h = 0.75$  m above the road. The car weighs 11 kN. Find the magnitude of (a) the braking acceleration of the car, (b) the normal force on each rear wheel, (c) the normal force on each front wheel, (d) the braking force on each rear wheel, and (e) the braking force on each front wheel. (*Hint:* Although the car is not in translational equilibrium, it *is* in rotational equilibrium.)

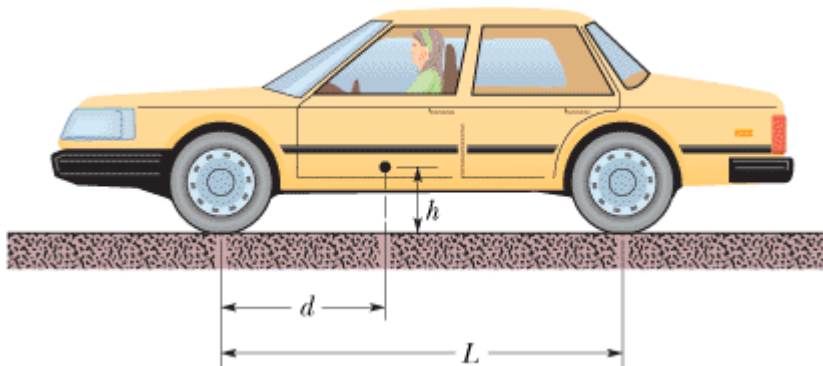
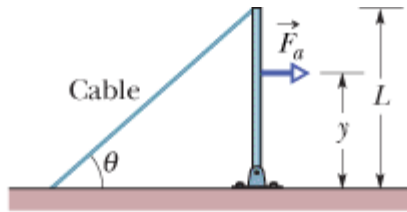
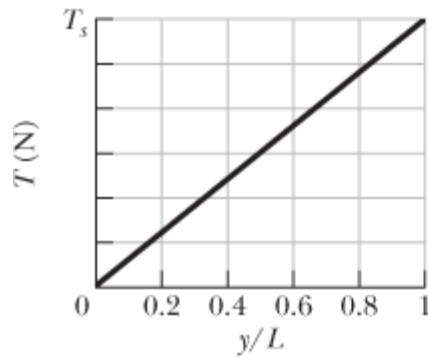


Figure 12-46 Problem 32.

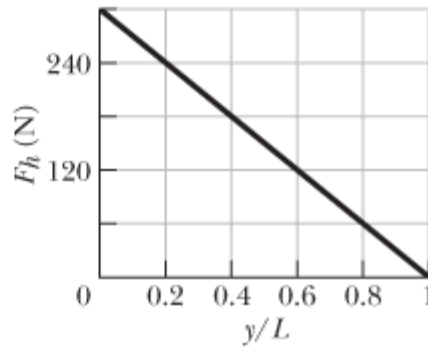
- 33 Figure 12-47a shows a vertical uniform beam of length  $L$  that is hinged at its lower end. A horizontal force  $\vec{F}_a$  is applied to the beam at distance  $y$  from the lower end. The beam remains vertical because of a cable attached at the upper end, at angle  $\theta$  with the horizontal. Figure 12-47b gives the tension  $T$  in the cable as a function of the position of the applied force given as a fraction  $y/L$  of the beam length. The scale of the  $T$  axis is set by  $T_s = 600$  N. Figure 12-47c gives the magnitude  $F_h$  of the horizontal force on the beam from the hinge, also as a function of  $y/L$ . Evaluate (a) angle  $\theta$  and (b) the magnitude of  $\vec{F}_a$ .



(a)



(b)



(c)

Figure 12-47 Problem 33.


**Answer:**

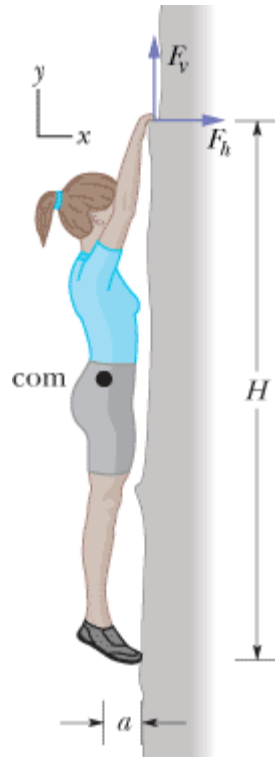
(a)  $60.0^\circ$ ; (b) 300 N

- 34 In Fig. 12-43, a thin horizontal bar  $AB$  of negligible weight and length  $L$  is hinged to a vertical wall at  $A$  and supported at  $B$  by a thin wire  $BC$  that makes an angle  $\theta$  with the horizontal. A block of weight  $W$  can be moved anywhere along the bar; its position is defined by the distance  $x$  from the wall to its center of mass. As a function of  $x$ , find (a) the tension in the wire, and the (b) horizontal and (c) vertical components of the force on the bar from the hinge at  $A$ .
- 35 **SSM WWW** A cubical box is filled with sand and weighs 890 N. We wish to “roll” the box by pushing horizontally on one of the upper edges. (a) What minimum force is required? (b) What minimum coefficient of static friction between box and floor is required? (c) If there is a more efficient way to roll the box, find the smallest possible force that would have to be applied directly to the box to roll it. (*Hint:* At the onset of tipping, where is the normal force located?)

**Answer:**

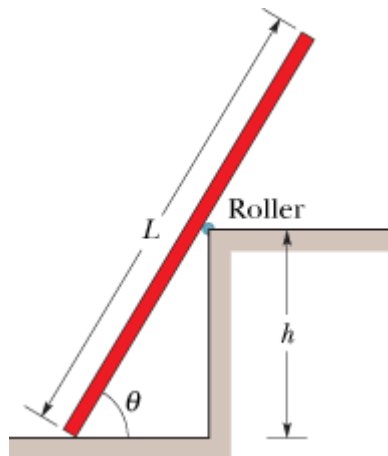
(a) 445 N; (b) 0.50; (c) 315 N

- 36  Figure 12-48 shows a 70 kg climber hanging by only the *crimp hold* of one hand on the edge of a shallow horizontal ledge in a rock wall. (The fingers are pressed down to gain purchase.) Her feet touch the rock wall at distance  $H = 2.0$  m directly below her crimped fingers but do not provide any support. Her center of mass is distance  $a = 0.20$  m from the wall. Assume that the force from the ledge supporting her fingers is equally shared by the four fingers. What are the values of the (a) horizontal component  $F_h$  and (b) vertical component  $F_v$  of the force on *each* fingertip?



**Figure 12-48** Problem 36.

- 37 In Fig. 12-49, a uniform plank, with a length  $L$  of 6.10 m and a weight of 445 N, rests on the ground and against a frictionless roller at the top of a wall of height  $h = 3.05$  m. The plank remains in equilibrium for any value of  $\theta \geq 70^\circ$  but slips if  $\theta < 70^\circ$ . Find the coefficient of static friction between the plank and the ground.



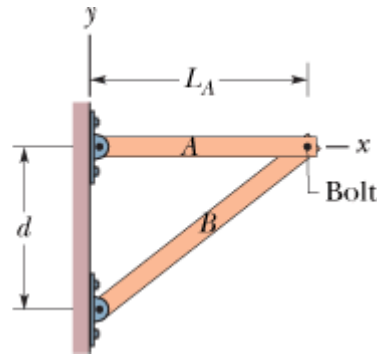
**Figure 12-49** Problem 37.

**Answer:**

0.34

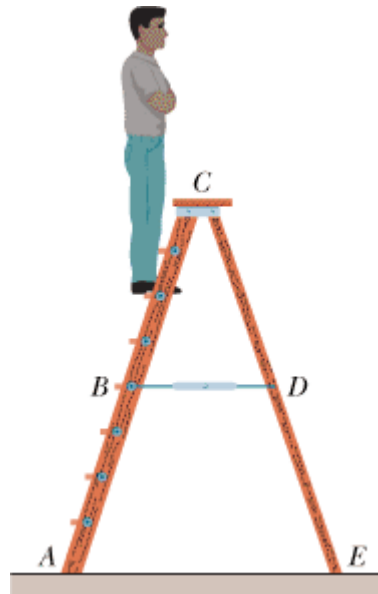
- 38 In Fig. 12-50, uniform beams  $A$  and  $B$  are attached to a wall with hinges and loosely bolted together (there is no torque of one on the other). Beam  $A$  has length  $L_A = 2.40$  m and mass 54.0

kg; beam  $B$  has mass  $68.0$  kg. The two hinge points are separated by distance  $d = 1.80$  m. In unit-vector notation, what is the force on (a) beam  $A$  due to its hinge, (b) beam  $A$  due to the bolt, (c) beam  $B$  due to its hinge, and (d) beam  $B$  due to the bolt?



**Figure 12-50** Problem 38.

- 39 For the stepladder shown in Fig. 12-51, sides  $AC$  and  $CE$  are each  $2.44$  m long and hinged at  $C$ . Bar  $BD$  is a tie-rod  $0.762$  m long, halfway up. A man weighing  $854$  N climbs  $1.80$  m along the ladder. Assuming that the floor is frictionless and neglecting the mass of the ladder, find (a) the tension in the tie-rod and the magnitudes of the forces on the ladder from the floor at (b)  $A$  and (c)  $E$ . (*Hint*: Isolate parts of the ladder in applying the equilibrium conditions.)



**Figure 12-51** Problem 39.

**Answer:**

(a)  $211$  N; (b)  $534$  N; (c)  $320$  N

- 40 Figure 12-52a shows a horizontal uniform beam of mass  $m_b$  and length  $L$  that is supported on the left by a hinge attached to a wall and on the right by a cable at angle  $\theta$  with the horizontal. A package of mass  $m_p$  is positioned on the beam at a distance  $x$  from the left end. The total mass is  $m_b + m_p = 61.22$  kg. Figure 12-52b gives the tension  $T$  in the cable as a function of the package's position given as a fraction  $x/L$  of the beam length. The scale of the  $T$  axis is set by  $T_a = 500$  N

and  $T_b = 700$  N. Evaluate (a) angle  $\theta$ , (b) mass  $m_b$ , and (c) mass  $m_p$ .

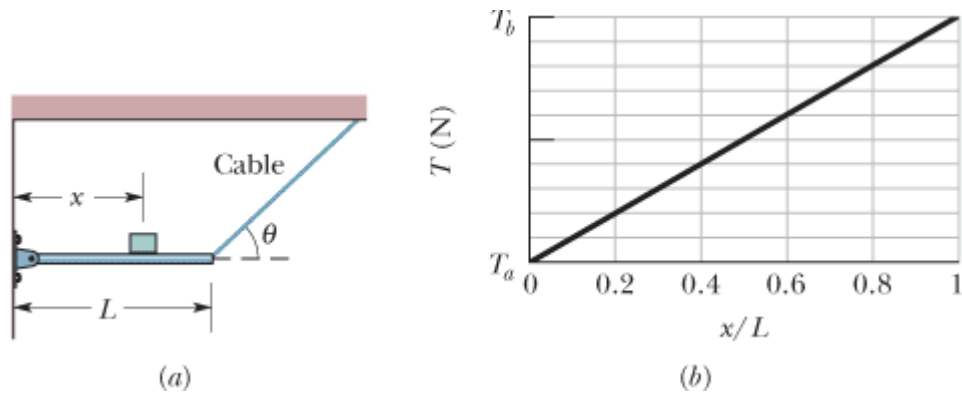


Figure 12-52 Problem 40.

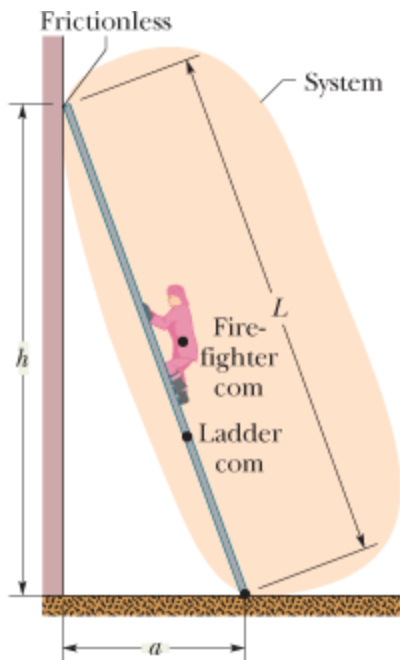
- 41 A crate, in the form of a cube with edge lengths of 1.2 m, contains a piece of machinery; the center of mass of the crate and its contents is located 0.30 m above the crate's geometrical center. The crate rests on a ramp that makes an angle  $\theta$  with the horizontal. As  $\theta$  is increased from zero, an angle will be reached at which the crate will either tip over or start to slide down the ramp. If the coefficient of static friction  $\mu_s$  between ramp and crate is 0.60, (a) does the crate tip or slide and (b) at what angle  $\theta$  does this occur? If  $\mu_s = 0.70$ , (c) does the crate tip or slide and (d) at what angle  $\theta$  does this occur? (*Hint: At the onset of tipping, where is the normal force located?*)

**Answer:**

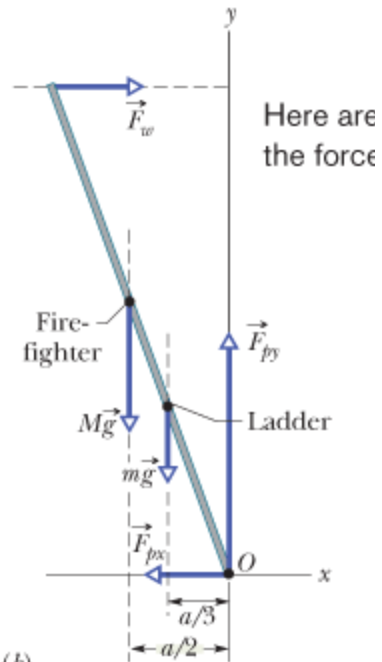
(a) slides; (b)  $31^\circ$ ; (c) tips; (d)  $34^\circ$

- In Fig. 12-5 and the associated sample problem, let the coefficient of static friction  $\mu_s$  between the  
42 ladder and the pavement be 0.53. How far (in percent) up the ladder must the firefighter go to put the ladder on the verge of sliding?

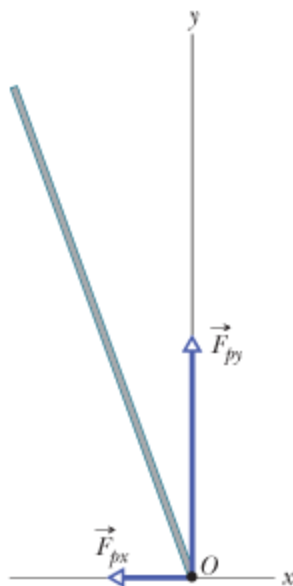




(a)

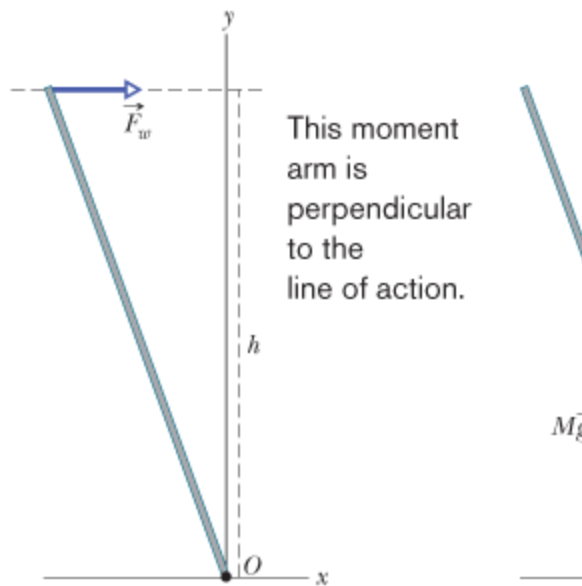


(b)



(c)

Choosing the rotation axis here eliminates the torques due to these forces.



Here too.

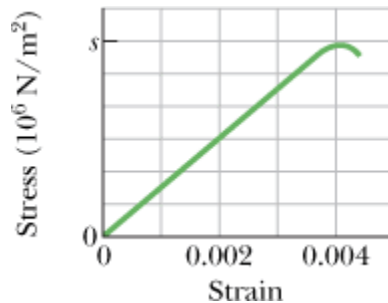
These horizontal forces balance.

## sec. 12-7 Elasticity

- 43 **SSM ILW** A horizontal aluminum rod 4.8 cm in diameter projects 5.3 cm from a wall. A 1200 kg object is suspended from the end of the rod. The shear modulus of aluminum is  $3.0 \times 10^{10} \text{ N/m}^2$ . Neglecting the rod's mass, find (a) the shear stress on the rod and (b) the vertical deflection of the end of the rod.

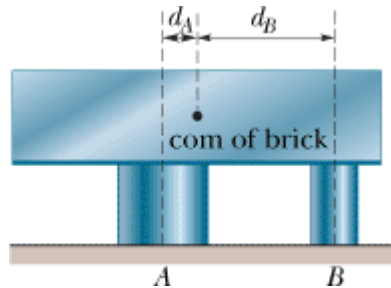
**Answer:**

- (a)  $6.5 \times 10^6 \text{ N/m}^2$ ; (b)  $1.1 \times 10^{-5} \text{ m}$
- 44 Figure 12-53 shows the stress–strain curve for a material. The scale of the stress axis is set by  $s = 300$ , in units of  $10^6 \text{ N/m}^2$ . What are (a) the Young's modulus and (b) the approximate yield strength for this material?




**Figure 12-53** Problem 44.

- 45 In Fig. 12-54, a lead brick rests horizontally on cylinders *A* and *B*. The areas of the top faces of the cylinders are related by  $A_A = 2A_B$ ; the Young's moduli of the cylinders are related by  $E_A = 2E_B$ . The cylinders had identical lengths before the brick was placed on them. What fraction of the brick's mass is supported (a) by cylinder *A* and (b) by cylinder *B*? The horizontal distances between the center of mass of the brick and the centerlines of the cylinders are  $d_A$  for cylinder *A* and  $d_B$  for cylinder *B*. (c) What is the ratio  $d_A/d_B$ ?



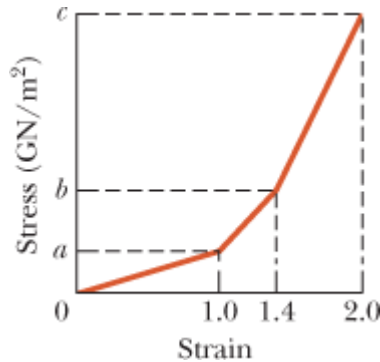
**Figure 12-54** Problem 45.

**Answer:**

- (a) 0.80; (b) 0.20; (c) 0.25
- 46  Figure 12-55 shows an approximate plot of stress versus strain for a spider-web thread, out to the point of breaking at a strain of 2.00. The vertical axis scale is set by values  $a = 0.12 \text{ GN/m}^2$ ,  $b = 0.30 \text{ GN/m}^2$ , and  $c = 0.80 \text{ GN/m}^2$ . Assume that the thread has an initial length of 0.80 cm, an initial cross-sectional area of  $8.0 \times 10^{-12} \text{ m}^2$ , and (during stretching) a constant volume. Assume also that when the single thread snares a flying insect, the insect's kinetic energy is

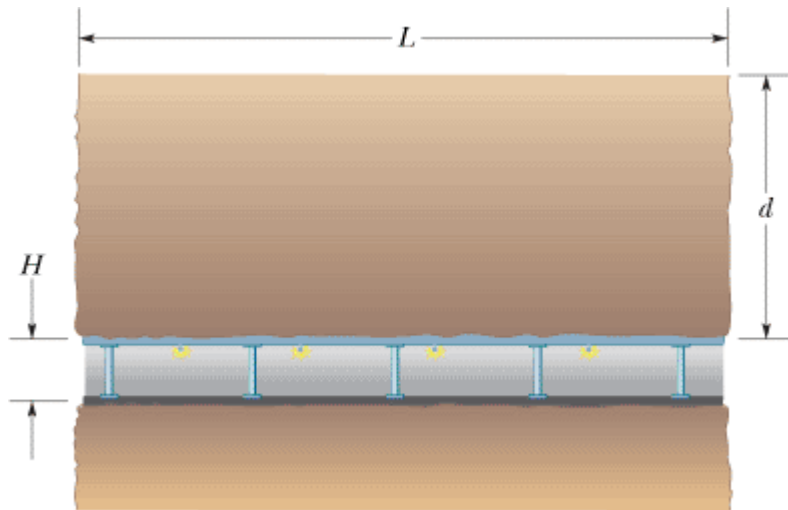


transferred to the stretching of the thread. (a) How much kinetic energy would put the thread on the verge of breaking? What is the kinetic energy of (b) a fruit fly of mass 6.00 mg and speed 1.70 m/s and (c) a bumble bee of mass 0.388 g and speed 0.420 m/s? Would (d) the fruit fly and (e) the bumble bee break the thread?



**Figure 12-55** Problem 46.

- 47 A tunnel of length  $L = 150$  m, height  $H = 7.2$  m, and width 5.8 m (with a flat roof) is to be constructed at distance  $d = 60$  m beneath the ground. (See Fig. 12-56.) The tunnel roof is to be supported entirely by square steel columns, each with a cross-sectional area of  $960 \text{ cm}^2$ . The mass of  $1.0 \text{ cm}^3$  of the ground material is 2.8 g. (a) What is the total weight of the ground material the columns must support? (b) How many columns are needed to keep the compressive stress on each column at one-half its ultimate strength?



**Figure 12-56** Problem 47.

**Answer:**

(a)  $1.4 \times 10^9$  N; (b) 75

- 48 Figure 12-57 shows the stress versus strain plot for an aluminum wire that is stretched by a machine pulling in opposite directions at the two ends of the wire. The scale of the stress axis is set by  $s = 7.0$ , in units of  $10^7 \text{ N/m}^2$ . The wire has an initial length of 0.800 m and an initial cross-sectional area of  $2.00 \times 10^{-6} \text{ m}^2$ . How much work does the force from the machine do on the wire to produce a strain of  $1.00 \times 10^{-3}$ ?

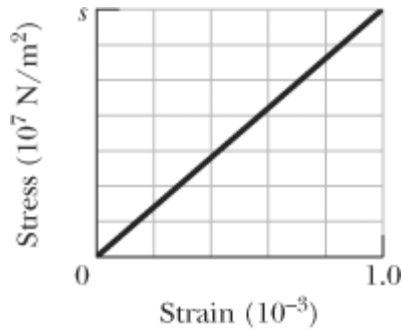



Figure 12-57 Problem 48.

- 49  In Fig. 12-58, a 103 kg uniform log hangs by two steel wires, *A* and *B*, both of radius 1.20 mm. Initially, wire *A* was 2.50 m long and 2.00 mm shorter than wire *B*. The log is now horizontal. What are the magnitudes of the forces on it from (a) wire *A* and (b) wire *B*? (c) What is the ratio  $d_A/d_B$ ?

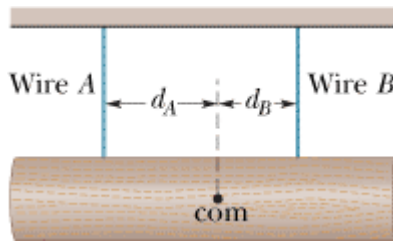


Figure 12-58 Problem 49.

Answer:

(a) 866 N; (b) 143 N; (c) 0.165


- 50  Figure 12-59 represents an insect caught at the midpoint of a spider-web thread. The thread breaks under a stress of  $8.20 \times 10^8 \text{ N/m}^2$  and a strain of 2.00. Initially, it was horizontal and had a length of 2.00 cm and a cross-sectional area of  $8.00 \times 10^{-12} \text{ m}^2$ . As the thread was stretched under the weight of the insect, its volume remained constant. If the weight of the insect puts the thread on the verge of breaking, what is the insect's mass? (A spider's web is built to break if a potentially harmful insect, such as a bumble bee, becomes snared in the web.)



Figure 12-59 Problem 50.

- 51 Figure 12-60 is an overhead view of a rigid rod that turns about a vertical axle until the identical rubber stoppers *A* and *B* are forced against rigid walls at distances  $r_A = 7.0 \text{ cm}$  and  $r_B = 4.0 \text{ cm}$  from the axle. Initially the stoppers touch the walls without being compressed. Then force  $\vec{F}$  of magnitude 220 N is applied perpendicular to the rod at a distance  $R = 5.0 \text{ cm}$  from the axle. Find the magnitude of the force compressing (a) stopper *A* and (b) stopper *B*.

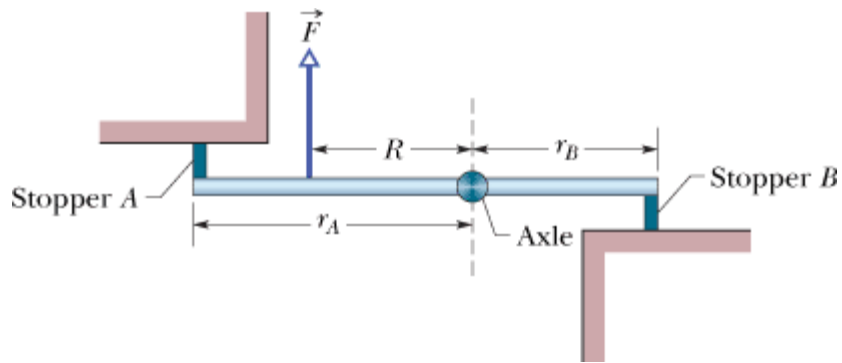


Figure 12-60 Problem 51.

**Answer:**

(a)  $1.2 \times 10^2$  N; (b) 68 N

### Additional Problems

**52** After a fall, a 95 kg rock climber finds himself dangling from the end of a rope that had been 15 m long and 9.6 mm in diameter but has stretched by 2.8 cm. For the rope, calculate (a) the strain, (b) the stress, and (c) the Young's modulus.

**53 SSM** In Fig. 12-61, a rectangular slab of slate rests on a bedrock surface inclined at angle  $\theta = 26^\circ$ . The slab has length  $L = 43$  m, thickness  $T = 2.5$  m, and width  $W = 12$  m, and  $1.0 \text{ cm}^3$  of it has a mass of 3.2 g. The coefficient of static friction between slab and bedrock is 0.39. (a) Calculate the component of the gravitational force on the slab parallel to the bedrock surface. (b) Calculate the magnitude of the static frictional force on the slab. By comparing (a) and (b), you can see that the slab is in danger of sliding. This is prevented only by chance protrusions of bedrock. (c) To stabilize the slab, bolts are to be driven perpendicular to the bedrock surface (two bolts are shown). If each bolt has a cross-sectional area of  $6.4 \text{ cm}^2$  and will snap under a shearing stress of  $3.6 \times 10^8 \text{ N/m}^2$ , what is the minimum number of bolts needed? Assume that the bolts do not affect the normal force.

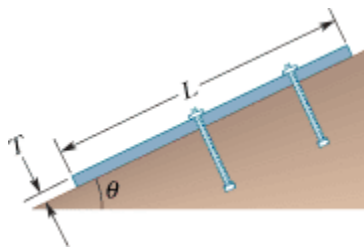


Figure 12-61 Problem 53.

**Answer:**

(a)  $1.8 \times 10^7$  N; (b)  $1.4 \times 10^7$  N; (c) 16

**54** A uniform ladder whose length is 5.0 m and whose weight is 400 N leans against a frictionless vertical wall. The coefficient of static friction between the level ground and the foot of the ladder is 0.46. What is the greatest distance the foot of the ladder can be placed from the base of the wall without the ladder immediately slipping?

- 55 **SSM** In Fig. 12-62, block *A* (mass 10 kg) is in equilibrium, but it would slip if block *B* (mass 5.0 kg) were any heavier. For angle  $\theta = 30^\circ$ , what is the coefficient of static friction between block *A* and the surface below it?

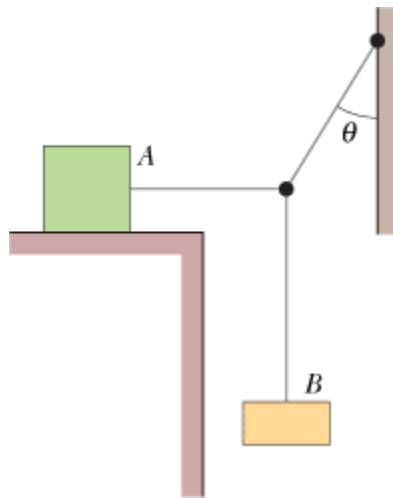


Figure 12-62 Problem 55.

**Answer:**

0.29

- 56 Figure 12-63*a* shows a uniform ramp between two buildings that allows for motion between the buildings due to strong winds. At its left end, it is hinged to the building wall; at its right end, it has a roller that can roll along the building wall. There is no vertical force on the roller from the building, only a horizontal force with magnitude  $F_h$ . The horizontal distance between the buildings is  $D = 4.00$  m. The rise of the ramp is  $h = 0.490$  m. A man walks across the ramp from the left. Figure 12-63*b* gives  $F_h$  as a function of the horizontal distance  $x$  of the man from the building at the left. The scale of the  $F_h$  axis is set by  $a = 20$  kN and  $b = 25$  kN. What are the masses of (a) the ramp and (b) the man?

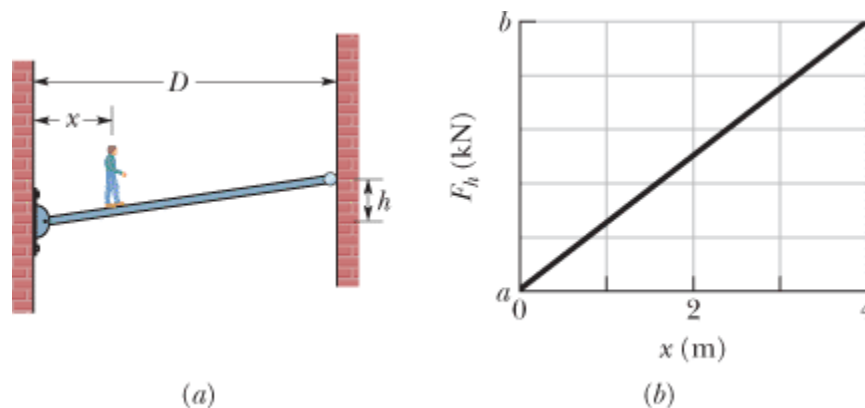
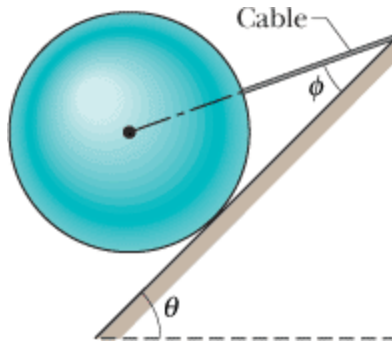


Figure 12-63 Problem 56.

- 57 In Fig. 12-64, a 10 kg sphere is supported on a frictionless plane inclined at angle  $\theta = 45^\circ$  from the horizontal. Angle  $\theta$  is  $25^\circ$ . Calculate the tension in the cable.

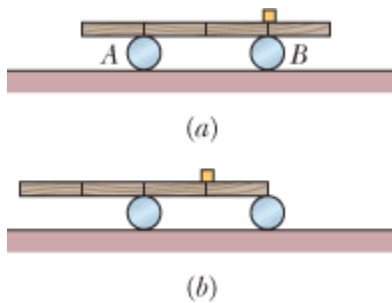


**Figure 12-64** Problem 57.

**Answer:**

76 N

- 58** In Fig. 12-65*a*, a uniform 40.0 kg beam is centered over two rollers. Vertical lines across the beam mark off equal lengths. Two of the lines are centered over the rollers; a 10.0 kg package of tamales is centered over roller *B*. What are the magnitudes of the forces on the beam from (a) roller *A* and (b) roller *B*? The beam is then rolled to the left until the right-hand end is centered over roller *B* (Fig. 12-65*b*). What now are the magnitudes of the forces on the beam from (c) roller *A* and (d) roller *B*? Next, the beam is rolled to the right. Assume that it has a length of 0.800 m. (e) What horizontal distance between the package and roller *B* puts the beam on the verge of losing contact with roller *A*?



**Figure 12-65** Problem 58.

- 59 SSM** In Fig. 12-66, an 817 kg construction bucket is suspended by a cable *A* that is attached at *O* to two other cables *B* and *C*, making angles  $\theta_1 = 51.0^\circ$  and  $\theta_2 = 66.0^\circ$  with the horizontal. Find the tensions in (a) cable *A*, (b) cable *B*, and (c) cable *C*. (*Hint:* To avoid solving two equations in two unknowns, position the axes as shown in the figure.)

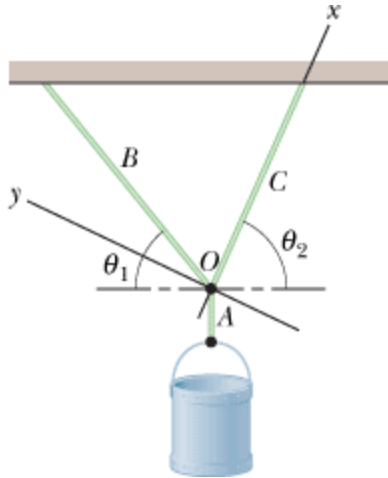


Figure 12-66 Problem 59.

**Answer:**

(a) 8.01 kN; (b) 3.65 kN; (c) 5.66 kN

- 60 In Fig. 12-67, a package of mass  $m$  hangs from a short cord that is tied to the wall via cord 1 and to the ceiling via cord 2. Cord 1 is at angle  $= 40^\circ$  with the horizontal; cord 2 is at angle  $\theta$ . (a) For what value of  $\theta$  is the tension in cord 2 minimized? (b) In terms of  $mg$ , what is the minimum tension in cord 2?

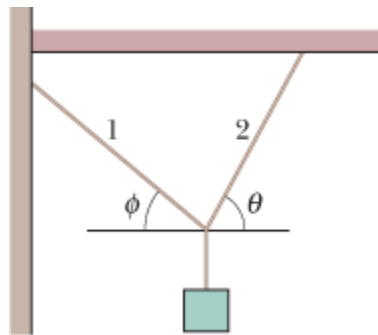


Figure 12-67 Problem 60.

- 61 **ILW** The force  $\vec{F}$  in Fig. 12-68 keeps the 6.40 kg block and the pulleys in equilibrium. The pulleys have negligible mass and friction. Calculate the tension  $T$  in the upper cable. (*Hint:* When a cable wraps halfway around a pulley as here, the magnitude of its net force on the pulley is twice the tension in the cable.)

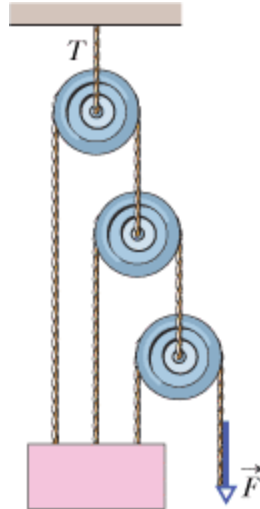


Figure 12-68 Problem 61.

**Answer:**

71.7 N

- 62 A mine elevator is supported by a single steel cable 2.5 cm in diameter. The total mass of the elevator cage and occupants is 670 kg. By how much does the cable stretch when the elevator hangs by (a) 12 m of cable and (b) 362 m of cable? (Neglect the mass of the cable.)
- 63 ~~63~~ Four bricks of length  $L$ , identical and uniform, are stacked on top of one another (Fig. 12-69) in such a way that part of each extends beyond the one beneath. Find, in terms of  $L$ , the maximum values of (a)  $a_1$ , (b)  $a_2$ , (c)  $a_3$ , (d)  $a_4$ , and (e)  $h$ , such that the stack is in equilibrium.

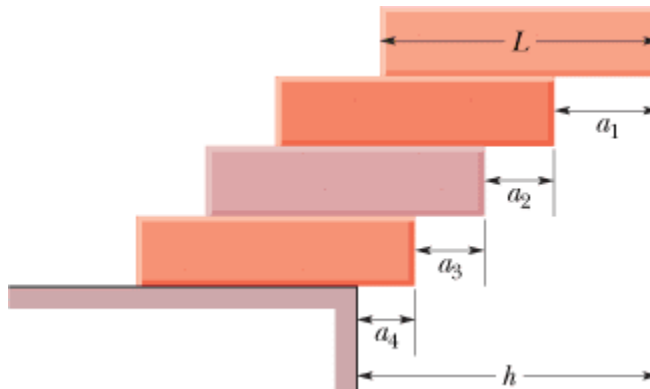


Figure 12-69 Problem 63.

**Answer:**

(a)  $L/2$ ; (b)  $L/4$ ; (c)  $L/6$ ; (d)  $L/8$ ; (e)  $25L/24$

- 64 In Fig. 12-70, two identical, uniform, and frictionless spheres, each of mass  $m$ , rest in a rigid rectangular container. A line connecting their centers is at  $45^\circ$  to the horizontal. Find the magnitudes of the forces on the spheres from (a) the bottom of the container, (b) the left side of the container, (c) the right side of the container, and (d) each other. (*Hint:* The force of one sphere on

the other is directed along the center-center line.)

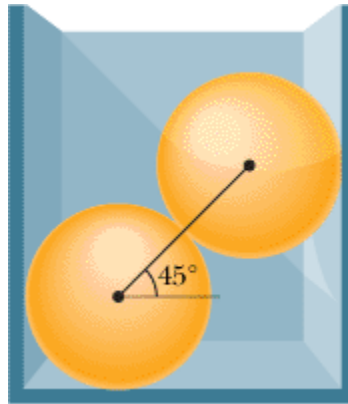


Figure 12-70 Problem 64.

- 65 In Fig. 12-71, a uniform beam with a weight of 60 N and a length of 3.2 m is hinged at its lower end, and a horizontal force  $\vec{F}$  of magnitude 50 N acts at its upper end. The beam is held vertical by a cable that makes angle  $\theta = 25^\circ$  with the ground and is attached to the beam at height  $h = 2.0$  m. What are (a) the tension in the cable and (b) the force on the beam from the hinge in unit-vector notation?

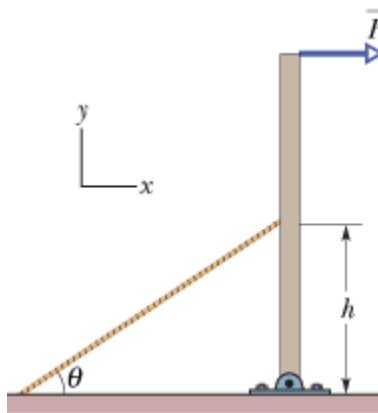
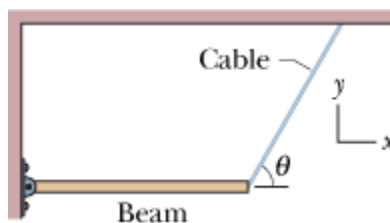


Figure 12-71 Problem 65.

**Answer:**

(a) 88 N; (b)  $(30\hat{i} + 97\hat{j})$  N

- 66 A uniform beam is 5.0 m long and has a mass of 53 kg. In Fig. 12-72, the beam is supported in a horizontal position by a hinge and a cable, with angle  $\theta = 60^\circ$ . In unit-vector notation, what is the force on the beam from the hinge?





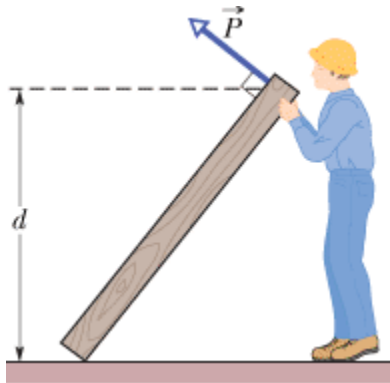
**Figure 12-72** Problem 66.

- 67 A solid copper cube has an edge length of 85.5 cm. How much stress must be applied to the cube to reduce the edge length to 85.0 cm? The bulk modulus of copper is  $1.4 \times 10^{11} \text{ N/m}^2$ .

**Answer:**

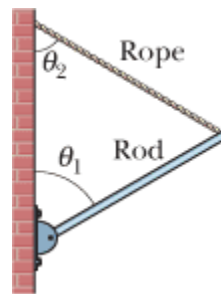
$$2.4 \times 10^9 \text{ N/m}^2$$

- 68 A construction worker attempts to lift a uniform beam off the floor and raise it to a vertical position. The beam is 2.50 m long and weighs 500 N. At a certain instant the worker holds the beam momentarily at rest with one end at distance  $d = 1.50 \text{ m}$  above the floor, as shown in Fig. 12-73, by exerting a force  $\vec{P}$  on the beam, perpendicular to the beam. (a) What is the magnitude  $P$ ? (b) What is the magnitude of the (net) force of the floor on the beam? (c) What is the minimum value the coefficient of static friction between beam and floor can have in order for the beam not to slip at this instant?



**Figure 12-73** Problem 68.

- 69 **SSM** In Fig. 12-74, a uniform rod of mass  $m$  is hinged to a building at its lower end, while its upper end is held in place by a rope attached to the wall. If angle  $\theta_1 = 60^\circ$ , what value must angle  $\theta_2$  have so that the tension in the rope is equal to  $mg/2$ ?



**Figure 12-74** Problem 69.

**Answer:**

$$60^\circ$$

- 70 A 73 kg man stands on a level bridge of length  $L$ . He is at distance  $L/4$  from one end. The bridge is uniform and weighs 2.7 kN. What are the magnitudes of the vertical forces on the bridge from its

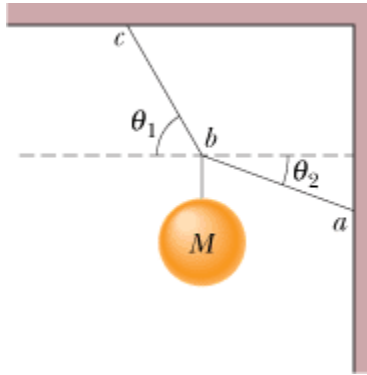
supports at (a) the end farther from him and (b) the nearer end?

- 71 SSM** A uniform cube of side length 8.0 cm rests on a horizontal floor. The coefficient of static friction between cube and floor is  $\mu$ . A horizontal pull  $\vec{P}$  is applied perpendicular to one of the vertical faces of the cube, at a distance 7.0 cm above the floor on the vertical midline of the cube face. The magnitude of  $\vec{P}$  is gradually increased. During that increase, for what values of  $\mu$  will the cube eventually (a) begin to slide and (b) begin to tip? (*Hint: At the onset of tipping, where is the normal force located?*)

**Answer:**

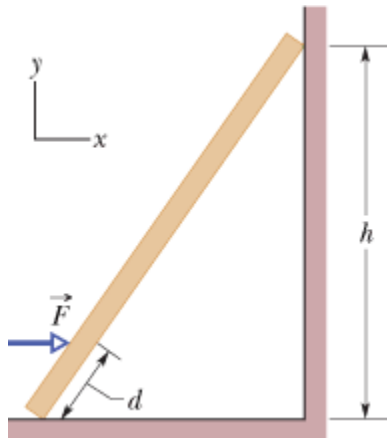
(a)  $\mu < 0.57$ ; (b)  $\mu > 0.57$

- 72** The system in Fig. 12-75 is in equilibrium. The angles are  $\theta_1 = 60^\circ$  and  $\theta_2 = 20^\circ$ , and the ball has mass  $M = 2.0$  kg. What is the tension in (a) string  $ab$  and (b) string  $bc$ ?



**Figure 12-75** Problem 72.

- 73 SSM** A uniform ladder is 10 m long and weighs 200 N. In Fig. 12-76, the ladder leans against a vertical, frictionless wall at height  $h = 8.0$  m above the ground. A horizontal force  $\vec{F}$  is applied to the ladder at distance  $d = 2.0$  m from its base (measured along the ladder). (a) If force magnitude  $F = 50$  N, what is the force of the ground on the ladder, in unit-vector notation? (b) If  $F = 150$  N, what is the force of the ground on the ladder, also in unit-vector notation? (c) Suppose the coefficient of static friction between the ladder and the ground is 0.38; for what minimum value of the force magnitude  $F$  will the base of the ladder just barely start to move toward the wall?



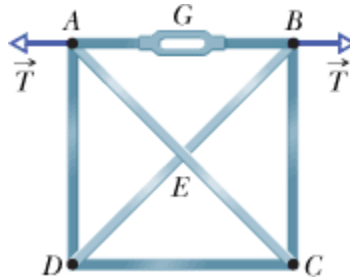
**Figure 12-76** Problem 73.

**Answer:**

(a)  $(35\hat{i} + 200\hat{j})$  N; (b)  $(-45\hat{i} + 200\hat{j})$  N; (c)  $1.9 \times 10^2$  N

**74** A pan balance is made up of a rigid, massless rod with a hanging pan attached at each end. The rod is supported at and free to rotate about a point not at its center. It is balanced by unequal masses placed in the two pans. When an unknown mass  $m$  is placed in the left pan, it is balanced by a mass  $m_1$  placed in the right pan; when the mass  $m$  is placed in the right pan, it is balanced by a mass  $m_2$  in the left pan. Show that  $m = \sqrt{m_1 m_2}$ .

**75** The rigid square frame in Fig. 12-77 consists of the four side bars  $AB$ ,  $BC$ ,  $CD$ , and  $DA$  plus two diagonal bars  $AC$  and  $BD$ , which pass each other freely at  $E$ . By means of the turnbuckle  $G$ , bar  $AB$  is put under tension, as if its ends were subject to horizontal, outward forces  $\vec{T}$  of magnitude 535 N. (a) Which of the other bars are in tension? What are the magnitudes of (b) the forces causing the tension in those bars and (c) the forces causing compression in the other bars? (*Hint:* Symmetry considerations can lead to considerable simplification in this problem.)

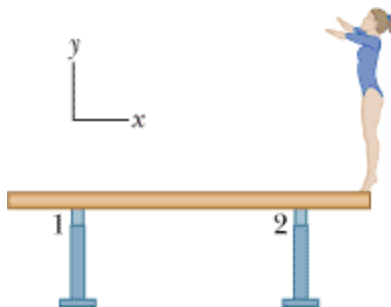


**Figure 12-77** Problem 75.

**Answer:**

(a)  $BC$ ,  $CD$ ,  $DA$ ; (b) 535 N; (c) 757 N

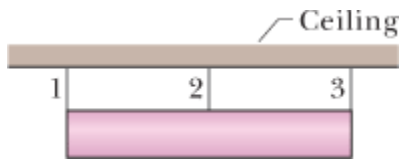
**76** A gymnast with mass 46.0 kg stands on the end of a uniform balance beam as shown in Fig. 12-78. The beam is 5.00 m long and has a mass of 250 kg (excluding the mass of the two supports). Each support is 0.540 m from its end of the beam. In unit-vector notation, what are the forces on the beam due to (a) support 1 and (b) support 2?



**Figure 12-78** Problem 76.

**77** Figure 12-79 shows a 300 kg cylinder that is horizontal. Three steel wires support the cylinder from a ceiling. Wires 1 and 3 are attached at the ends of the cylinder, and wire 2 is attached at the

center. The wires each have a cross-sectional area of  $2.00 \times 10^{-6} \text{ m}^2$ . Initially (before the cylinder was put in place) wires 1 and 3 were 2.0000 m long and wire 2 was 6.00 mm longer than that. Now (with the cylinder in place) all three wires have been stretched. What is the tension in (a) wire 1 and (b) wire 2?

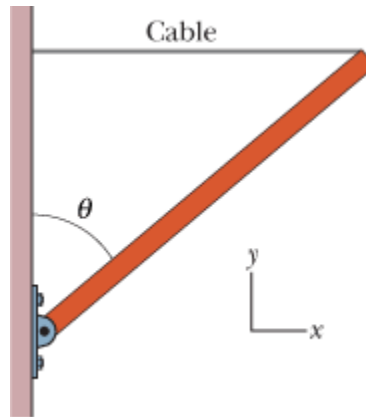


**Figure 12-79** Problem 77.


**Answer:**

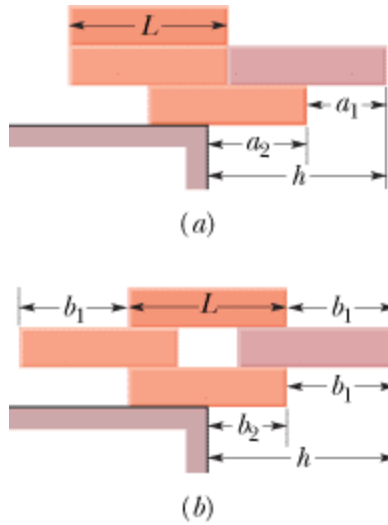
(a) 1.38 kN; (b) 180 N

**78** In Fig. 12-80, a uniform beam of length 12.0 m is supported by a horizontal cable and a hinge at angle  $\theta = 50.0^\circ$ . The tension in the cable is 400 N. In unit-vector notation, what are (a) the gravitational force on the beam and (b) the force on the beam from the hinge?



**Figure 12-80** Problem 78.

**79**  Four bricks of length  $L$ , identical and uniform, are stacked on a table in two ways, as shown in Fig. 12-81 (compare with Problem 63). We seek to maximize the overhang distance  $h$  in both arrangements. Find the optimum distances  $a_1$ ,  $a_2$ ,  $b_1$ , and  $b_2$ , and calculate  $h$  for the two arrangements.



**Figure 12-81** Problem 79.

**Answer:**

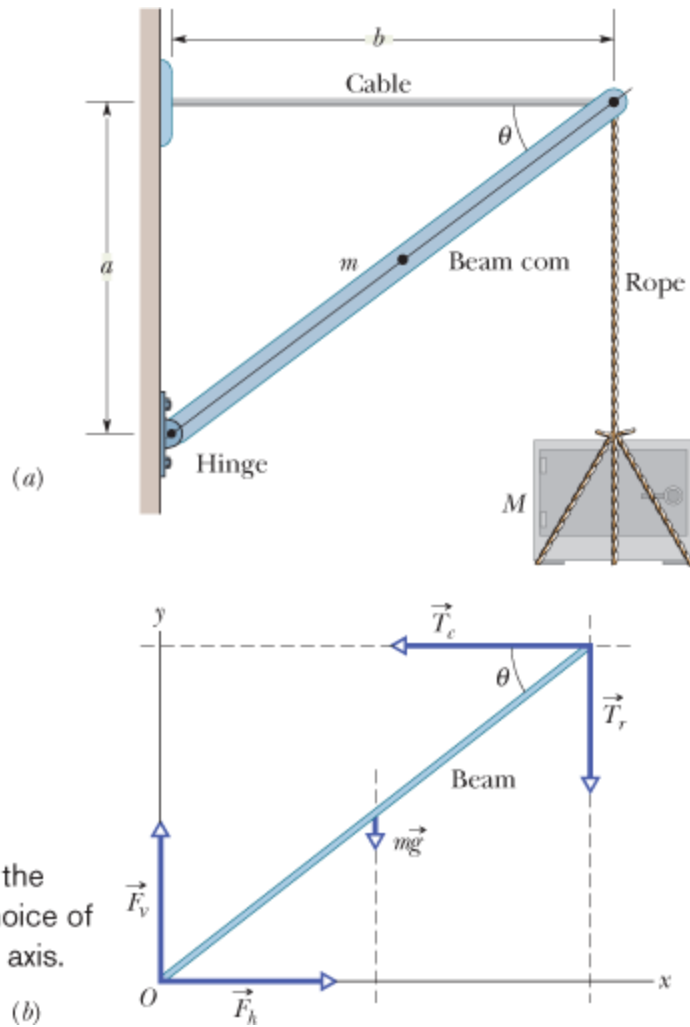
(a)  $a_1 = L/2$ ,  $a_2 = 5L/8$ ,  $h = 9L/8$ ; (b)  $b_1 = 2L/3$ ,  $b_2 = L/2$ ,  $h = 7L/6$

- 80** A cylindrical aluminum rod, with an initial length of 0.8000 m and radius 1000.0  $\mu\text{m}$ , is clamped in place at one end and then stretched by a machine pulling parallel to its length at its other end. Assuming that the rod's density (mass per unit volume) does not change, find the force magnitude that is required of the machine to decrease the radius to 999.9  $\mu\text{m}$ . (The yield strength is not exceeded.)
- 81** A beam of length  $L$  is carried by three men, one man at one end and the other two supporting the beam between them on a crosspiece placed so that the load of the beam is equally divided among the three men. How far from the beam's free end is the crosspiece placed? (Neglect the mass of the crosspiece.)

**Answer:**

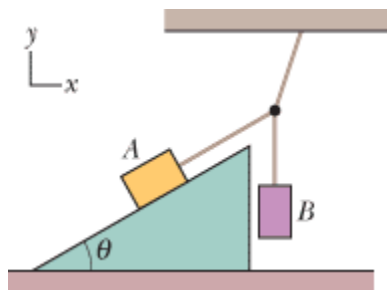
$L/4$

- 82** If the (square) beam in Fig. 12-6a and the associated sample problem is of Douglas fir, what must be its thickness to keep the compressive stress on it to  $\frac{1}{6}$  of its ultimate strength?



**Figure 12-6**(a) A heavy safe is hung from a boom consisting of a horizontal steel cable and a uniform beam. (b) A free-body diagram for the beam.

83 Figure 12-82 shows a stationary arrangement of two crayon boxes and three cords. Box A has a mass of 11.0 kg and is on a ramp at angle  $\theta = 30.0^\circ$ ; box B has a mass of 7.00 kg and hangs on a cord. The cord connected to box A is parallel to the ramp, which is frictionless. (a) What is the tension in the upper cord, and (b) what angle does that cord make with the horizontal?



**Figure 12-82** Problem 83.

**Answer:**

(a) 106 N; (b) 64.0°