sec. 28-3 The Definition of $\vec{B}$

•1 SSM ILW A proton traveling at 23.0° with respect to the direction of a magnetic field of strength 2.60 mT experiences a magnetic force of 6.50×10^{-17} N. Calculate (a) the proton's speed and (b) its kinetic energy in electron-volts.

Answer:

(a) 400 km/s; (b) 835 eV

•2 A particle of mass 10 g and charge 80 μC moves through a uniform magnetic field, in a region where the free-fall acceleration is $-9.8\hat{j}$ m/s^2. The velocity of the particle is a constant $20\hat{i}$ km/s, which is perpendicular to the magnetic field. What, then, is the magnetic field?

•3 An electron that has velocity

$$\vec{v} = \left(2.0 \times 10^6 \text{ m/s}\right)\hat{i} + \left(3.0 \times 10^6 \text{ m/s}\right)\hat{j}$$

moves through the uniform magnetic field $\vec{B} = (0.030\text{ T})\hat{i} - (0.15\text{ T})\hat{j}$ (a) Find the force on the electron due to the magnetic field. (b) Repeat your calculation for a proton having the same velocity.

Answer:

(a) $(6.2 \times 10^{-14}\text{ N})\hat{k}$; (b) $(-6.2 \times 10^{-14}\text{ N})\hat{k}$

•4 An alpha particle travels at a velocity $\vec{v}$ of magnitude 550 m/s through a uniform magnetic field $\vec{B}$ of magnitude 0.045 T. (An alpha particle has a charge of $+3.2 \times 10^{-19}$ C and a mass of 6.6×10^{-27} kg.) The angle between $\vec{v}$ and $\vec{B}$ is 52°. What is the magnitude of (a) the force $\vec{F}_B$ acting on the particle due to the field and (b) the acceleration of the particle due to $\vec{F}_B$? (c) Does the speed of the particle increase, decrease, or remain the same?

••5

An electron moves through a uniform magnetic field given by $\vec{B} = B_x\hat{i} + (3.0B_x)\hat{j}$. At a particular instant, the electron has velocity $\vec{v} = \left(2.0\hat{i} + 4.0\hat{j}\right)\text{m/s}$ and the magnetic force acting on it is $\left(6.4 \times 10^{-19}\text{ N}\right)\hat{k}$. Find B_x.

Answer:

- 2.0 T

••6

GO A proton moves through a uniform magnetic field given by $\vec{B} = \left(10\hat{i} - 20\hat{j} + 30\hat{k}\right)$mT. At time t_1, the proton has a velocity given by $\vec{v} = v_x\hat{i} + v_y\hat{j} + (2.0\text{ km/s})\hat{k}$ and the magnetic force on the proton is $\vec{F}_B = (4.0 \times 10^{-17}\,\mathrm{N})\hat{\mathrm{i}} + (2.0 \times 10^{-17}\,\mathrm{N})\hat{\mathrm{j}}$. At that instant, what are (a) v_x and (b) v_y?

sec. 28-4 Crossed Fields: Discovery of the Electron

•7
An electron has an initial velocity of $(12.0\hat{\mathrm{j}} + 15.0\hat{\mathrm{k}})$ km/s and a constant acceleration of $(2.00 \times 10^{12}\,\mathrm{m/s^2})\hat{\mathrm{i}}$ in a region in which uniform electric and magnetic fields are present. If $\vec{B} = (400\ \mu\mathrm{T})\hat{\mathrm{i}}$, find the electric field $\vec{E}$.

Answer:

$(-11.4\ \mathrm{V/m})\hat{\mathrm{i}} - (6.00\ \mathrm{V/m})\hat{\mathrm{j}} + (4.80\ \mathrm{V/m})\hat{\mathrm{k}}$

•8 An electric field of 1.50 kV/m and a perpendicular magnetic field of 0.400 T act on a moving electron to produce no net force. What is the electron's speed?

•9 ILW In Fig. 28-31, an electron accelerated from rest through potential difference V_1 = 1.00 kV enters the gap between two allel plates having separation d = 20.0 mm and potential difference V_2 = 100 V. The lower plate is at the lower potential. Neglect fringing and assume that the electron's velocity vector is perpendicular to the electric field vector between the plates. In unit-vector notation, what uniform magnetic field allows the electron to travel in a straight line in the gap?

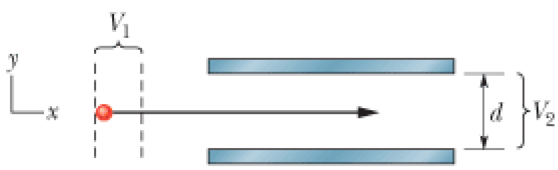


Figure 28-31 Problem 9.

Answer:

$-(0.267\ \mathrm{mT})\hat{\mathrm{k}}$

••10 A proton travels through uniform magnetic and electric fields. The magnetic field is $\vec{B} = -2.50\hat{\mathrm{i}}\ \mathrm{mT}$. At one instant the velocity of the proton is $\vec{v} = 2000\hat{\mathrm{j}}\ \mathrm{m/s}$. At that instant and in unit-vector notation, what is the net force acting on the proton if the electric field is (a) $4.00\hat{\mathrm{k}}\ \mathrm{V/m}$, (b) $-4.00\hat{\mathrm{k}}\ \mathrm{V/m}$, and (c) $4.00\hat{\mathrm{i}}\ \mathrm{V/m}$?

••11 An ion source is producing ^{6}Li ions, which have charge $+e$ and mass 9.99 10^{-27} kg. The ions are accelerated by a potential difference of 10 kV and pass horizontally into a region in which there is a uniform vertical magnetic field of magnitude B = 1.2 T. Calculate the strength of the smallest electric field, to be set up over the same region, that will allow the ^{6}Li ions to pass through undeflected.

Answer:

0.68 MV/m

•••12 GO At time t_1, an electron is sent along the positive direction of an x axis, through both an electric field $\vec{E}$ and a magnetic field $\vec{B}$, with $\vec{E}$ directed parallel to the y axis. Figure 28-32 gives the y component $F_{net,y}$ of the net force on the electron due to the two fields, as a function of the electron's speed v at time t_1. The scale of the velocity axis is set by $v_s = 100.0$ m/s. The x and z components of the net force are zero at t_1. Assuming $B_x = 0$, find (a) the magnitude E and (b) $\vec{B}$ in unit-vector notation.

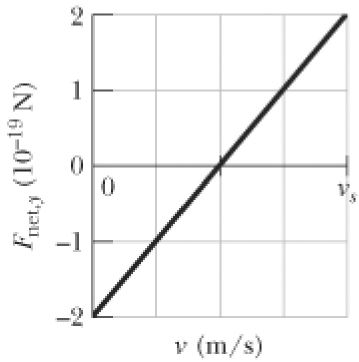


Figure 28-32 Problem 12.

sec. 28-5 Crossed Fields: The Hall Effect

•13 A strip of copper 150 μm thick and 4.5 mm wide is placed in a uniform magnetic field $\vec{B}$ of magnitude 0.65 T, with $\vec{B}$ perpendicular to the strip. A current $i = 23$ A is then sent through the strip such that a Hall potential difference V appears across the width of the strip. Calculate V. (The number of charge carriers per unit volume for copper is 8.47×10^{28} electrons/m^3.)

Answer:

7.4 μV

•14 A metal strip 6.50 cm long, 0.850 cm wide, and 0.760 mm thick moves with constant velocity $\vec{v}$ through a uniform magnetic field $B = 1.20$ mT directed perpendicular to the strip, as shown in Fig. 28-33. A potential difference of 3.90 μV is measured between points x and y across the strip. Calculate the speed v.

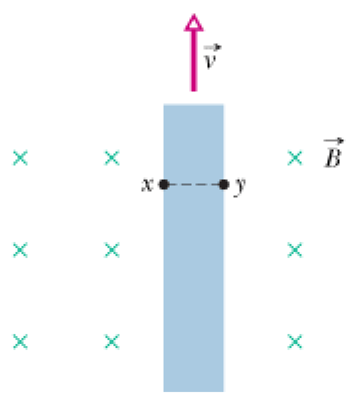


Figure 28-33 Problem 14.

••15 In Fig. 28-34, a conducting rectangular solid of dimensions $d_x = 5.00$ m, $d_y = 3.00$ m, and $d_z =$

2.00 m moves at constant velocity $\vec{v} = (20.0 \text{ m/s})\hat{i}$ through a uniform magnetic field $\vec{B} = (30.0 \text{ mT})\hat{j}$. What are the resulting (a) electric field within the solid, in unit-vector notation, and (b) potential difference across the solid?

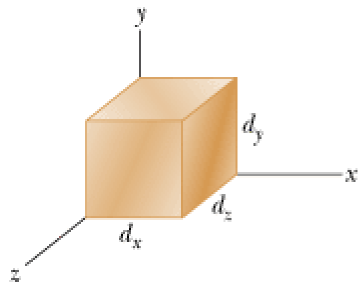


Figure 28-34 Problems 15 and 16.

Answer:

(a) (- 600 mV/m)$\hat{k}$; (b) 1.20 V

•••16 GO Figure 28-34 shows a metallic block, with its faces parallel to coordinate axes. The block is in a uniform magnetic field of magnitude 0.020 T. One edge length of the block is 25 cm; the block is *not* drawn to scale. The block is moved at 3.0 m/s parallel to each axis, in turn, and the resulting potential difference V that appears across the block is measured. With the motion parallel to the y axis, V = 12 mV; with the motion parallel to the z axis, V = 18 mV; with the motion parallel to the x axis, V = 0. What are the block lengths (a) d_x, (b) d_y, and (c) d_z?

sec. 28-6 A Circulating Charged Particle

•17 An alpha particle can be produced in certain radioactive decays of nuclei and consists of two protons and two neutrons. The particle has a charge of $q = +2e$ and a mass of 4.00 u, where u is the atomic mass unit, with 1 u = 1.661 × 10^{-27} kg. Suppose an alpha particle travels in a circular path of radius 4.50 cm in a uniform magnetic field with B = 1.20 T. Calculate (a) its speed, (b) its period of revolution, (c) its kinetic energy, and (d) the potential difference through which it would have to be accelerated to achieve this energy.

Answer:

(a) 2.60 × 10^6 m/s; (b) 0.109 μs; (c) 0.140 MeV; (d) 70.0 kV

•18 GO In Fig. 28-35, a particle moves along a circle in a region of uniform magnetic field of magnitude B = 4.00 mT. The particle is either a proton or an electron (you must decide which). It experiences a magnetic force of magnitude 3.20 × 10^{-15} N. What are (a) the particle's speed, (b) the radius of the circle, and (c) the period of the motion?

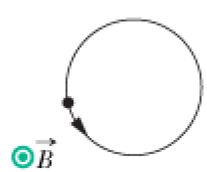


Figure 28-35 Problem 18.

•19 A certain particle is sent into a uniform magnetic field, with the particle's velocity vector perpendicular to the direction of the field. Figure 28-36 gives the period T of the particle's motion versus the *inverse* of the field magnitude B. The vertical axis scale is set by T_s = 40.0 ns, and the horizontal axis scale is set by $B_s^{-1} = 5.0\,\mathrm{T}^{-1}$. What is the ratio m/q of the particle's mass to the magnitude of its charge?

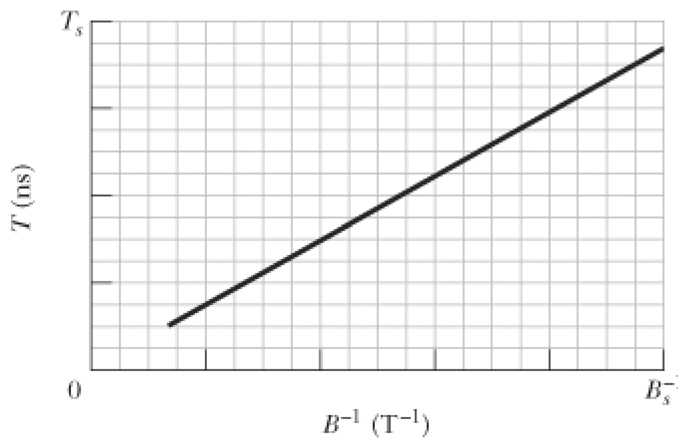


Figure 28-36 Problem 19.

Answer:

1.2×10^{-9} kg/C

•20 An electron is accelerated from rest through potential difference V and then enters a region of uniform magnetic field, where it undergoes uniform circular motion. Figure 28-37 gives the radius r of that motion versus $V^{1/2}$. The vertical axis scale is set by r_s = 3.0 mm, and the horizontal axis scale is set by $V_s^{1/2} = 40.0\,\mathrm{V}^{1/2}$. What is the magnitude of the magnetic field?

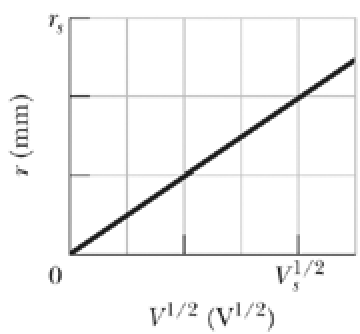


Figure 28-37 Problem 20.

•21 SSM An electron of kinetic energy 1.20 keV circles in a plane perpendicular to a uniform magnetic field. The orbit radius is 25.0 cm. Find (a) the electron's speed, (b) the magnetic field magnitude, (c) the circling frequency, and (d) the period of the motion.

Answer:

(a) 2.05×10^7 m/s; (b) 467 μT; (c) 13.1 MHz; (d) 76.3 ns

•22 In a nuclear experiment a proton with kinetic energy 1.0 MeV moves in a circular path in a uniform magnetic field. What energy must (a) an alpha particle ($q = +2e$, $m = 4.0$ u) and (b) a deuteron ($q = +e$, $m = 2.0$ u) have if they are to circulate in the same circular path?

•23 What uniform magnetic field, applied perpendicular to a beam of electrons moving at 1.30×10^6 m/s, is required to make the electrons travel in a circular arc of radius 0.350 m?

Answer:

21.1 μT

•24 An electron is accelerated from rest by a potential difference of 350 V. It then enters a uniform magnetic field of magnitude 200 mT with its velocity perpendicular to the field. Calculate (a) the speed of the electron and (b) the radius of its path in the magnetic field.

•25 (a) Find the frequency of revolution of an electron with an energy of 100 eV in a uniform magnetic field of magnitude 35.0 μT. (b) Calculate the radius of the path of this electron if its velocity is perpendicular to the magnetic field.

Answer:

(a) 0.978 MHz; (b) 96.4 cm

••26 In Fig. 28-38, a charged particle moves into a region of uniform magnetic field $\vec{B}$, goes through half a circle, and then exits that region. The particle is either a proton or an electron (you must decide which). It spends 130 ns in the region. (a) What is the magnitude of $\vec{B}$? (b) If the particle is sent back through the magnetic field (along the same initial path) but with 2.00 times its previous kinetic energy, how much time does it spend in the field during this trip?

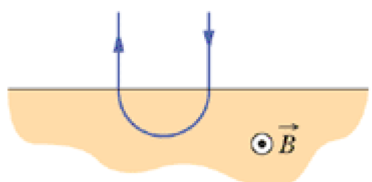


Figure 28-38 Problem 26.

••27 A mass spectrometer (Fig. 28-12) is used to separate uranium ions of mass 3.92×10^{-25} kg and charge 3.20×10^{-19} C from related species. The ions are accelerated through a potential difference of 100 kV and then pass into a uniform magnetic field, where they are bent in a path of radius 1.00 m. After traveling through 180° and passing through a slit of width 1.00 mm and height 1.00 cm, they are collected in a cup. (a) What is the magnitude of the (perpendicular) magnetic field in the separator? If the machine is used to separate out 100 mg of material per hour, calculate (b) the current of the desired ions in the machine and (c) the thermal energy produced in the cup in 1.00 h.

Answer:

(a) 495 mT; (b) 22.7 mA; (c) 8.17 MJ

••28 A particle undergoes uniform circular motion of radius 26.1 μm in a uniform magnetic field. The magnetic force on the particle has a magnitude of 1.60×10^{-17} N. What is the kinetic energy of the particle?

•29 An electron follows a helical path in a uniform magnetic field of magnitude 0.300 T. The pitch of the path is 6.00 μm, and the magnitude of the magnetic force on the electron is 2.00×10^{-15} N.

What is the electron's speed?

Answer:

65.3 km/s

••30 GO In Fig. 28-39, an electron with an initial kinetic energy of 4.0 keV enters region 1 at time $t = 0$. That region contains a uniform magnetic field directed into the page, with magnitude 0.010 T. The electron goes through a half-circle and then exits region 1, headed toward region 2 across a gap of 25.0 cm. There is an electric potential difference $\Delta V = 2000$ V across the gap, with a polarity such that the electron's speed increases uniformly as it traverses the gap. Region 2 contains a uniform magnetic field directed out of the page, with magnitude 0.020 T. The electron goes through a half-circle and then leaves region 2. At what time t does it leave?

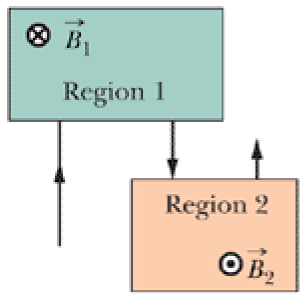


Figure 28-39 Problem 30.

••31 A particular type of fundamental particle decays by transforming into an electron e^- and a positron e^+. Suppose the decaying particle is at rest in a uniform magnetic field $\vec{B}$ of magnitude 3.53 mT and the e^- and e^+ move away from the decay point in paths lying in a plane perpendicular to $\vec{B}$. How long after the decay do the e^- and e^+ collide?

Answer:

5.07 ns

••32 A source injects an electron of speed $v = 1.5 \times 10^7$ m/s into a uniform magnetic field of magnitude $B = 1.0\ 10^{-3}$ T. The velocity of the electron makes an angle $\theta = 10°$ with the direction of the magnetic field. Find the distance d from the point of injection at which the electron next crosses the field line that passes through the injection point.

••33 SSM WWW A positron with kinetic energy 2.00 keV is projected into a uniform magnetic field $\vec{B}$ of magnitude 0.100 T, with its velocity vector making an angle of 89.0° with $\vec{B}$. Find (a) the period, (b) the pitch p, and (c) the radius r of its helical path.

Answer:

(a) 0.358 ns; (b) 0.166 mm; (c) 1.51 mm

••34 An electron follows a helical path in a uniform magnetic field given by $\vec{B} = (20\hat{i} - 50\hat{j} - 30\hat{k})$ mT. At time $t = 0$, the electron's velocity is given by $\vec{v} = (20\hat{i} - 30\hat{j} + 50\hat{k})$ m/s (a) What is the angle between $\vec{v}$ and $\vec{B}$? The electron's velocity changes with time. Do (b) its speed and (c) the angle change with time? (d) What is the radius of the helical path?

sec. 28-7 Cyclotrons and Synchrotrons

••35 A proton circulates in a cyclotron, beginning approximately at rest at the center. Whenever it passes through the gap between dees, the electric potential difference between the dees is 200 V. (a) By how much does its kinetic energy increase with each passage through the gap? (b) What is its kinetic energy as it completes 100 passes through the gap? Let r_{100} be the radius of the proton's circular path as it completes those 100 passes and enters a dee, and let r_{101} be its next radius, as it enters a dee the next time. (c) By what percentage does the radius increase when it changes from r_{100} to r_{101}? That is, what is

$$\text{percentage increase} = \frac{r_{101} - r_{100}}{r_{100}} 100\% \text{ ?}$$

Answer:

(a) 200 eV; (b) 20.0 keV; (c) 0.499%

••36 A cyclotron with dee radius 53.0 cm is operated at an oscillator frequency of 12.0 MHz to accelerate protons. (a) What magnitude B of magnetic field is required to achieve resonance? (b) At that field magnitude, what is the kinetic energy of a proton emerging from the cyclotron? Suppose, instead, that B = 1.57 T. (c) What oscillator frequency is required to achieve resonance now? (d) At that frequency, what is the kinetic energy of an emerging proton?

••37 Estimate the total path length traveled by a deuteron in a cyclotron of radius 53 cm and operating frequency 12 MHz during the (entire) acceleration process. Assume that the accelerating potential between the dees is 80 kV.

Answer:

2.4×10^2 m

••38 In a certain cyclotron a proton moves in a circle of radius 0.500 m. The magnitude of the magnetic field is 1.20 T. (a) What is the oscillator frequency? (b) What is the kinetic energy of the proton, in electron-volts?

sec. 28-8 Magnetic Force on a Current-Carrying Wire

•39 SSM A horizontal power line carries a current of 5000 A from south to north. Earth's magnetic field (60.0 μT) is directed toward the north and inclined downward at 70.0° to the horizontal. Find the (a) magnitude and (b) direction of the magnetic force on 100 m of the line due to Earth's field.

Answer:

(a) 28.2 N; (b) horizontally west

•40 A wire 1.80 m long carries a current of 13.0 A and makes an angle of 35.0° with a uniform magnetic field of magnitude B = 1.50 T. Calculate the magnetic force on the wire.

•41 ILW A 13.0 g wire of length L = 62.0 cm is suspended by a pair of flexible leads in a uniform magnetic field of magnitude 0.440 T (Fig. 28-40). What are the (a) magnitude and (b) direction (left or right) of the current required to remove the tension in the supporting leads?

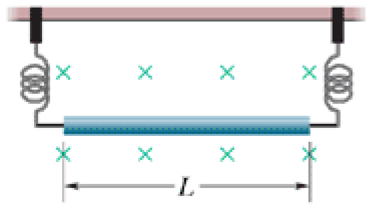


Figure 28-40 Problem 41.

Answer:

(a) 467 mA;(b) right

•**42**The bent wire shown in Fig. 28-41 lies in a uniform magnetic field. Each straight section is 2.0 m long and makes an angle of $\theta = 60°$ with the x axis, and the wire carries a current of 2.0 A. What is the net magnetic force on the wire in unit-vector notation if the magnetic field is given by (a) $4.0\hat{k}$ T and (b) $4.0\hat{i}$ T?

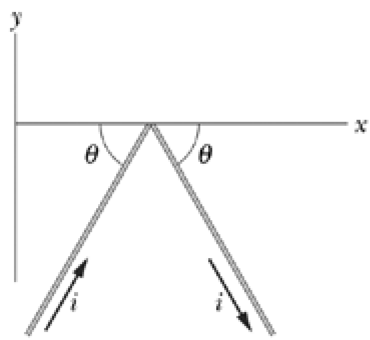


Figure 28-41 Problem 42.

•**43**A single-turn current loop, carrying a current of 4.00 A, is in the shape of a right triangle with sides 50.0, 120, and 130 cm. The loop is in a uniform magnetic field of magnitude 75.0 mT whose direction is parallel to the current in the 130 cm side of the loop. What is the magnitude of the magnetic force on (a) the 130 cm side, (b) the 50.0 cm side, and (c) the 120 cm side? (d) What is the magnitude of the net force on the loop?

Answer:

(a) 0;(b) 0.138 N; (c) 0.138 N; (d) 0

••**44**Figure 28-42 shows a wire ring of radius $a = 1.8$ cm that is perpendicular to the general direction of a radially symmetric, diverging magnetic field. The magnetic field at the ring is everywhere of the same magnitude $B = 3.4$ mT, and its direction at the ring everywhere makes an angle $\theta = 20°$ with a normal to the plane of the ring. The twisted lead wires have no effect on the problem. Find the magnitude of the force the field exerts on the ring if the ring carries a current $i = 4.6$ mA.

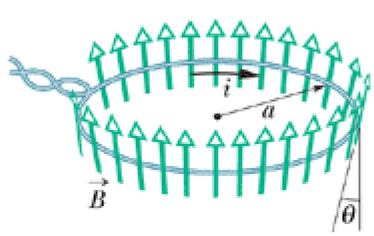


Figure 28-42 Problem 44.

••45 A wire 50.0 cm long carries a 0.500 A current in the positive direction of an x axis through a magnetic field $\vec{B} = (3.00\,\mathrm{mT})\hat{\mathrm{j}} + (10.0\,\mathrm{mT})\hat{\mathrm{k}}$. In unit-vector notation, what is the magnetic force on the wire?

Answer:

$(-2.50\,\mathrm{mN})\hat{\mathrm{j}}\,(0.750\,\mathrm{mN})\hat{\mathrm{K}}$

••46 In Fig. 28-43, a metal wire of mass m = 24.1 mg can slide with negligible friction on two horizontal parallel rails separated by distance d = 2.56 cm. The track lies in a vertical uniform magnetic field of magnitude 56.3 mT. At time t = 0, device G is connected to the rails, producing a constant current i = 9.13 mA in the wire and rails (even as the wire moves). At t = 61.1 ms, what are the wire's (a) speed and (b) direction of motion (left or right)?

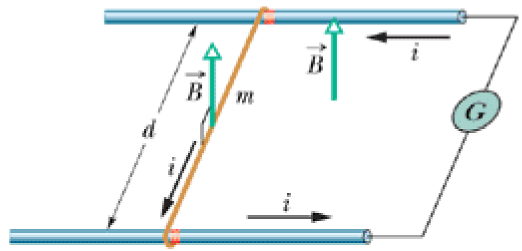


Figure 28-43 Problem 46.

•••47 GO A 1.0 kg copper rod rests on two horizontal rails 1.0 m apart and carries a current of 50 A from one rail to the other. The coefficient of static friction between rod and rails is 0.60. What are the (a) magnitude and (b) angle (relative to the vertical) of the smallest magnetic field that puts the rod on the verge of sliding?

Answer:

(a) 0.12 T; (b) 31°

•••48 A long, rigid conductor, lying along an x axis, carries a current of 5.0 A in the negative x direction. A magnetic field $\vec{B}$ is present, given by $\vec{B} = 3.0\hat{\mathrm{i}} + 8.0x^2\hat{\mathrm{j}}$ with x in meters and $\vec{B}$ in milliteslas. Find, in unit-vector notation, the force on the 2.0 m segment of the conductor that lies between x = 1.0 m and x = 3.0 m.

sec. 28-9 Torque on a Current Loop

•49 SSM Figure 28-44 shows a rectangular 20-turn coil of wire, of dimensions 10 cm by 5.0 cm. It carries a current of 0.10 A and is hinged along one long side. It is mounted in the xy plane, at angle θ = 30° to the direction of a uniform magnetic field of magnitude 0.50 T. In unit-vector notation, what is the torque acting on the coil about the hinge line?

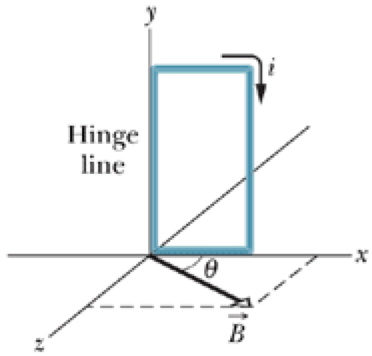


Figure 28-44 Problem 49.

Answer:

$(-4.3 \times 10^{-3}\ \mathrm{N \cdot m})\hat{j}$

••50An electron moves in a circle of radius $r = 5.29 \times 10^{-11}$ m with speed 2.19×10^{6} m/s. Treat the circular path as a current loop with a constant current equal to the ratio of the electron's charge magnitude to the period of the motion. If the circle lies in a uniform magnetic field of magnitude $B = 7.10$ mT, what is the maximum possible magnitude of the torque produced on the loop by the field?

••51Figure 28-45 shows a wood cylinder of mass $m = 0.250$ kg and length $L = 0.100$ m, with $N = 10.0$ turns of wire wrapped around it longitudinally, so that the plane of the wire coil contains the long central axis of the cylinder. The cylinder is released on a plane inclined at an angle θ to the horizontal, with the plane of the coil parallel to the incline plane. If there is a vertical uniform magnetic field of magnitude 0.500 T, what is the least current i through the coil that keeps the cylinder from rolling down the plane?

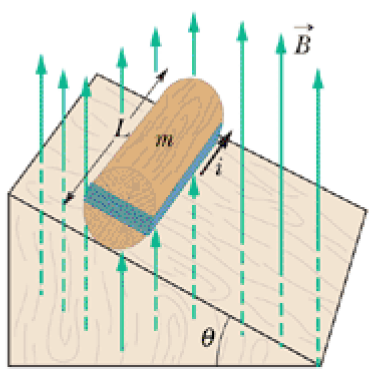


Figure 28-45 Problem 51.

Answer:

2.45 A

••52In Fig. 28-46, a rectangular loop carrying current lies in the plane of a uniform magnetic field of magnitude 0.040 T. The loop consists of a single turn of flexible conducting wire that is wrapped around a flexible mount such that the dimensions of the rectangle can be changed. (The total length of the wire is not changed.) As edge length x is varied from approximately zero to its maximum value of approximately 4.0 cm, the magnitude τ of the torque on the loop changes. The maximum value of τ is 4.80×10^{-8} N·m. What is the current in the loop?

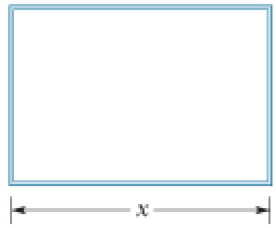


Figure 28-46 Problem 52.

••53Prove that the relation $\tau = NiAB \sin\theta$ holds not only for the rectangular loop of Fig. 28-19 but also for a closed loop of any shape. (*Hint:* Replace the loop of arbitrary shape with an assembly of adjacent long, thin, approximately rectangular loops that are nearly equivalent to the loop of arbitrary shape as far as the distribution of current is concerned.)

sec. 28-10 The Magnetic Dipole Moment

•54A magnetic dipole with a dipole moment of magnitude 0.020 J/T is released from rest in a uniform magnetic field of magnitude 52 mT. The rotation of the dipole due to the magnetic force on it is unimpeded. When the dipole rotates through the orientation where its dipole moment is aligned with the magnetic field, its kinetic energy is 0.80 mJ. (a) What is the initial angle between the dipole moment and the magnetic field? (b) What is the angle when the dipole is next (momentarily) at rest?

•55 SSM Two concentric, circular wire loops, of radii r_1 = 20.0 cm and r_2 = 30.0 cm, are located in an *xy* plane; each carries a clockwise current of 7.00 A (Fig. 28-47). (a) Find the magnitude of the net magnetic dipole moment of the system. (b) Repeat for reversed current in the inner loop.

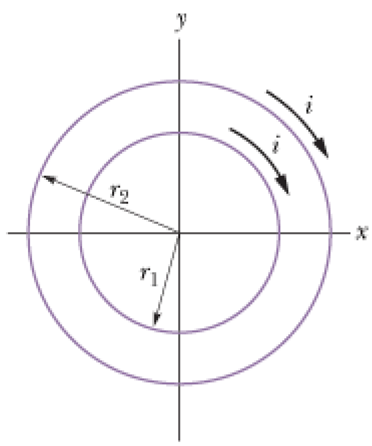


Figure 28-47 Problem 55.

Answer:

(a) 2.86 A· m^2; (b) 1.10 A · m^2

•56A circular wire loop of radius 15.0 cm carries a current of 2.60 A. It is placed so that the normal to its plane makes an angle of 41.0° with a uniform magnetic field of magnitude 12.0 T. (a) Calculate the magnitude of the magnetic dipole moment of the loop. (b) What is the magnitude of the torque acting on the loop?

•57 SSM A circular coil of 160 turns has a radius of 1.90 cm. (a) Calculate the current that results in a magnetic dipole moment of magnitude 2.30 A · m^2. (b) Find the maximum magnitude of the torque that the coil, carrying this current, can experience in a uniform 35.0 mT magnetic field.

Answer:

(a) 12.7 A; (b) 0.0805 N · m

•58 The magnetic dipole moment of Earth has magnitude 8.00×10^{22} J/T. Assume that this is produced by charges flowing in Earth's molten outer core. If the radius of their circular path is 3500 km, calculate the current they produce.

•59 A current loop, carrying a current of 5.0 A, is in the shape of a right triangle with sides 30, 40, and 50 cm. The loop is in a uniform magnetic field of magnitude 80 mT whose direction is parallel to the current in the 50 cm side of the loop. Find the magnitude of (a) the magnetic dipole moment of the loop and (b) the torque on the loop.

Answer:

(a) 0.30 A · m^2; (b) 0.024 N · m

••60 Figure 28-48 shows a current loop *ABCDEFA* carrying a current *i* = 5.00 A. The sides of the loop are parallel to the coordinate axes shown, with *AB* = 20.0 cm, *BC* = 30.0 cm, and *FA* = 10.0 cm. In unit-vector notation, what is the magnetic dipole moment of this loop? (*Hint:* Imagine equal and opposite currents *i* in the line segment *AD*; then treat the two rectangular loops *ABCDA* and *ADEFA*.)

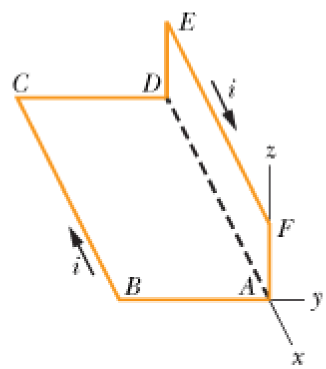


Figure 28-48 Problem 60.

••61 SSM The coil in Fig. 28-49 carries current *i* = 2.00 A in the direction indicated, is parallel to an *xz* plane, has 3.00 turns and an area of 4.00×10^{-3} m^2, and lies in a uniform magnetic field $\vec{B} = (2.00\hat{i} - 3.00\hat{j} - 4.00\hat{k})$ mT What are (a) the orientation energy of the coil in the magnetic field and (b) the torque (in unit-vector notation) on the coil due to the magnetic field?

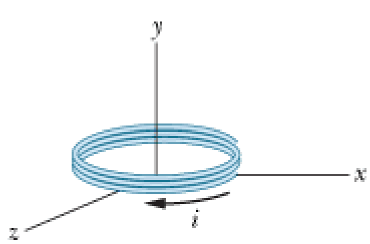


Figure 28-49 Problem 61.

Answer:

(a) - 72.0 μJ; (b) (96.0$\hat{\mathrm{i}}$ 48.0 $\hat{\mathrm{k}}$) μN · m

••62 In Fig. 28-50*a*, two concentric coils, lying in the same plane, carry currents in opposite directions. The current in the larger coil 1 is fixed. Current i_2 in coil 2 can be varied. Figure 28-50*b* gives the net magnetic moment of the two-coil system as a function of i_2. The vertical axis scale is set by $\mu_{net,s} = 2.0 \times 10^{-5}$ A · m^2 and the horizontal axis scale is set by i_{2s} = 10.0 mA. If the current in coil 2 is then reversed, what is the magnitude of the net magnetic moment of the two-coil system when i_2 = 7.0 mA?

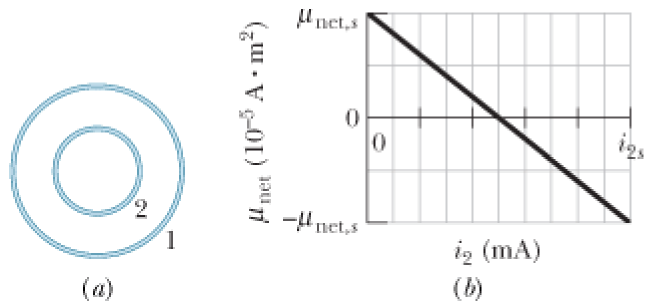


Figure 28-50 Problem 62.

••63A circular loop of wire having a radius of 8.0 cm carries a current of 0.20 A. A vector of unit length and parallel to the dipole moment $\vec{\mu}$ of the loop is given by $0.60\hat{\mathrm{i}} - 0.80\hat{\mathrm{j}}$. (This unit vector gives the orientation of the magnetic dipole moment vector.) If the loop is located in a uniform magnetic field given by $\vec{B} = (0.25\,\mathrm{T})\hat{\mathrm{i}} + (0.30\,\mathrm{T})\hat{\mathrm{k}}$find (a) the torque on the loop (in unit-vector notation) and (b) the orientation energy of the loop.

Answer:

(a) -(9.7 × 10^{-4} N · m)$\hat{\mathrm{i}}$ - (7.2 × 10^{-4} N · m)$\hat{\mathrm{j}}$ + (8.0 × 10^{-4} N· m)$\hat{\mathrm{k}}$; (b)- 6.0 × 10^{-4} J

••64 Figure 28-51 gives the orientation energy U of a magnetic dipole in an external magnetic field $\vec{B}$, as a function of angle between the directions of $\vec{B}$ and the dipole moment. The vertical axis scale is set by $U_s = 2.0 \times 10^{-4}$ J. The dipole can be rotated about an axle with negligible friction in order that to change . Counterclockwise rotation from = 0 yields positive values of , and clockwise rotations yield negative values. The dipole is to be released at angle = 0 with a rotational kinetic energy of 6.7 × 10^{-4} J, so that it rotates counterclockwise. To what maximum value of Φ will it rotate? (In the language of Section 28-6, what value is the turning point in the potential well of Fig. 28-51?)

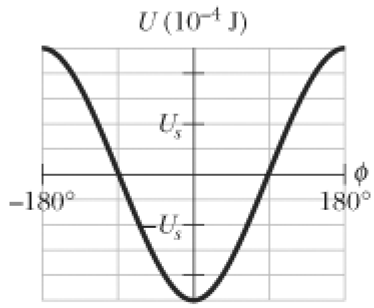


Figure 28-51 Problem 64.

••65 SSM ILW A wire of length 25.0 cm carrying a current of 4.51 mA is to be formed into a circular coil and placed in a uniform magnetic field $\vec{B}$ of magnitude 5.71 mT. If the torque on the coil from the field is maximized, what are (a) the angle between $\vec{B}$ and the coil's magnetic dipole moment and (b) the number of turns in the coil? (c) What is the magnitude of that maximum torque?

Answer:

(a) 90°; (b) 1; (c) 1.28×10^{7} N · m

Additional Problems

66 A proton of charge + e and mass m enters a uniform magnetic field $\vec{B} = B\hat{i}$ with an initial velocity $\vec{v} = v_{0x}\hat{i} + v_{0y}\hat{j}$. Find an expression in unit-vector notation for its velocity $\vec{v}$ at any later time t.

67 A stationary circular wall clock has a face with a radius of 15 cm. Six turns of wire are wound around its perimeter; the wire carries a current of 2.0 A in the clockwise direction. The clock is located where there is a constant, uniform external magnetic field of magnitude 70 mT (but the clock still keeps perfect time). At exactly 1:00 P.M., the hour hand of the clock points in the direction of the external magnetic field. (a) After how many minutes will the minute hand point in the direction of the torque on the winding due to the magnetic field? (b) Find the torque magnitude.

Answer:

(a) 20 min; (b) 5.9 10^{-2} N·m

68 A wire lying along a y axis from y = 0 to y = 0.250 m carries a current of 2.00 mA in the negative direction of the axis. The wire fully lies in a nonuniform magnetic field that is given by $\vec{B} = (0.300\ \text{T/m})y\hat{i} + (0.400\ \text{T/m})y\hat{j}$. In unit-vector notation, what is the magnetic force on the wire?

69 Atom 1 of mass 35 u and atom 2 of mass 37 u are both singly ionized with a charge of +e. After being introduced into a mass spectrometer (Fig. 28-12) and accelerated from rest through a potential difference V = 7.3 kV, each ion follows a circular path in a uniform magnetic field of magnitude B = 0.50 T. What is the distance Δx between the points where the ions strike the detector?

Answer:

8.2 mm

70 An electron with kinetic energy 2.5 keV moving along the positive direction of an x axis enters a region in which a uniform electric field of magnitude 10 kV/m is in the negative direction of the y axis. A uniform magnetic field $\vec{B}$ is to be set up to keep the electron moving along the x axis, and the direction of $\vec{B}$ is to be chosen to minimize the required magnitude of $\vec{B}$. In unit-vector notation, what $\vec{B}$ should be set up?

71 Physicist S. A. Goudsmit devised a method for measuring the mass of heavy ions by timing their period of revolution in a known magnetic field. A singly charged ion of iodine makes 7.00 rev in a 45.0 mT field in 1.29 ms. Calculate its mass in atomic mass units.

Answer:

127 u

72 A beam of electrons whose kinetic energy is K emerges from a thin-foil "window" at the end of an accelerator tube. A metal plate at distance d from this window is perpendicular to the direction of the emerging beam (Fig. 28-52). (a) Show that we can prevent the beam from hitting the plate if we apply a uniform magnetic field such that

$$B \geq \sqrt{\frac{2mK}{e^2 d^2}},$$

in which m and e are the electron mass and charge. (b) How should $\vec{B}$ be oriented?

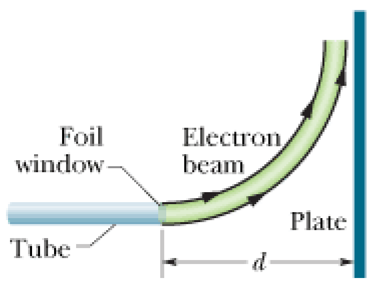


Figure 28-52 Problem 72.

73 SSM At time $t = 0$, an electron with kinetic energy 12 keV moves through $x = 0$ in the positive direction of an x axis that is parallel to the horizontal component of Earth's magnetic field $\vec{B}$. The field's vertical component is downward and has magnitude 55.0 μT. (a) What is the magnitude of the electron's acceleration due to $\vec{B}$? (b) What is the electron's distance from the x axis when the electron reaches coordinate $x = 20$ cm?

Answer:

(a) 6.3×10^{14} m/s^2; (b) 3.0 mm

74 GO A particle with charge 2.0 C moves through a uniform magnetic field. At one instant the

velocity of the particle is $\left(2.0\hat{i} + 4.0\hat{j} + 6.0\hat{k}\right)$ m/s and the magnetic force on the particle is $\left(4.0\hat{i} - 20\hat{j} + 12\hat{k}\right)$ N. The x and y components of the magnetic field are equal. What is $\vec{B}$?

75 A proton, a deuteron ($q = +e$, m = 2.0 u), and an alpha particle ($q = +2e$, m = 4.0 u) all having the same kinetic energy enter a region of uniform magnetic field $\vec{B}$, moving perpendicular to $\vec{B}$. What is the ratio of (a) the radius r_d of the deuteron path to the radius r_p of the proton path and (b) the radius r_a of the alpha particle path to r_p?

Answer:

(a) 1.4; (b) 1.0

76 Bainbridge's mass spectrometer, shown in Fig. 28-53, separates ions having the same velocity. The ions, after entering through slits, S_1 and S_2, pass through a velocity selector composed of an electric field produced by the charged plates P and P′, and a magnetic field $\vec{B}$ perpendicular to the electric field and the ion path. The ions that then pass undeviated through the crossed $\vec{E}$ and $\vec{B}$ fields enter into a region where a second magnetic field $\vec{B}'$ exists, where they are made to follow circular paths. A photographic plate (or a modern detector) registers their arrival. Show that, for the ions, $q/m = E/rBB'$, where r is the radius of the circular orbit.

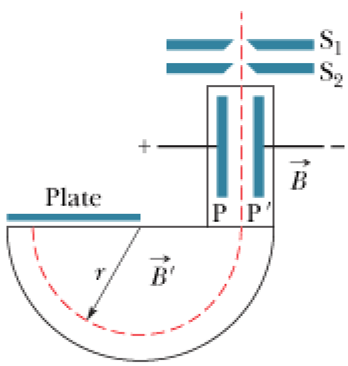


Figure 28-53 Problem 76.

77 SSM In Fig. 28-54, an electron moves at speed v = 100 m/s along an x axis through uniform electric and magnetic fields. The magnetic field $\vec{B}$ is directed into the page and has magnitude 5.00 T. In unit-vector notation, what is the electric field?

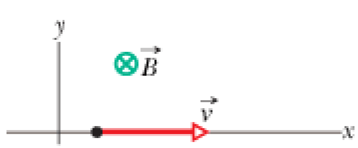


Figure 28-54 Problem 77.

Answer:

(- 500 V/m)$\hat{j}$

78 (a) In Fig. 28-8, show that the ratio of the Hall electric field magnitude E to the magnitude E_C of

the electric field responsible for moving charge (the current) along the length of the strip is

$$\frac{E}{E_C} = \frac{B}{ne\rho},$$

where ρ is the resistivity of the material and n is the number density of the charge carriers. (b) Compute this ratio numerically for Problem 13. (See Table 26-1.)

79 SSM A proton, a deuteron ($q = +e$, $m = 2.0$ u), and an alpha particle ($q = 2e$, $m = 4.0$ u) are accelerated through the same potential difference and then enter the same region of uniform magnetic field $\vec{B}$, moving perpendicular to $\vec{B}$. What is the ratio of (a) the proton's kinetic energy K_p to the alpha particle's kinetic energy K_a and (b) the deuteron's kinetic energy K_d to K_a? If the radius of the proton's circular path is 10 cm, what is the radius of (c) the deuteron's path and (d) the alpha particle's path?

Answer:

(a) 0.50; (b) 0.50; (c) 14 cm; (d) 14 cm

80 An electron in an old-fashioned TV camera tube is moving at 7.20×10^6 m/s in a magnetic field of strength 83.0 mT. What is the (a) maximum and (b) minimum magnitude of the force acting on the electron due to the field? (c) At one point the electron has an acceleration of magnitude 4.90×10^{14} m/s^2. What is the angle between the electron's velocity and the magnetic field?

81 A 5.0 μC particle moves through a region containing the uniform magnetic field $-20\hat{i}$ mT and the uniform electric field $300\hat{j}$ V/m. At a certain instant the velocity of the particle is $\left(17\hat{i} - 11\hat{j} + 7.0\hat{k}\right)$ km/s. At that instant and in unit-vector notation, what is the net electromagnetic force (the sum of the electric and magnetic forces) on the particle?

Answer:

$(0.80\hat{j} - 1.1\hat{k})$ mN

82 In a Hall-effect experiment, a current of 3.0 A sent lengthwise through a conductor 1.0 cm wide, 4.0 cm long, and 10 μm thick produces a transverse (across the width) Hall potential difference of 10 μV when a magnetic field of 1.5 T is passed perpendicularly through the thickness of the conductor. From these data, find (a) the drift velocity of the charge carriers and (b) the number density of charge carriers. (c) Show on a diagram the polarity of the Hall potential difference with assumed current and magnetic field directions, assuming also that the charge carriers are electrons.

83 SSM A particle of mass 6.0 g moves at 4.0 km/s in an xy plane, in a region with a uniform magnetic field given by $5.0\hat{i}$ mT. At one instant, when the particle's velocity is directed 37° counterclockwise from the positive direction of the x axis, the magnetic force on the particle is $0.48\hat{k}$ N. What is the particle's charge?

Answer:

-40 mC

84 A wire lying along an x axis from $x = 0$ to $x = 1.00$ m carries a current of 3.00 A in the positive x direction. The wire is immersed in a nonuniform magnetic field that is given by $\hat{B} = \left(4.00\ \mathrm{T/m^2}\right)x^2\hat{i} - \left(0.600\ \mathrm{T/m^2}\right)x^2\hat{j}$. In unit-vector notation, what is the magnetic force on

the wire?

85 At one instant, $\vec{v} = (-2.00\hat{i} + 4.00\hat{j} - 6.00\hat{k})$ m/s is the velocity of a proton in a uniform magnetic field $\vec{B} = (2.00\hat{i} - 4.00\hat{j} + 8.00\hat{k})$ mT. At that instant, what are (a) the magnetic force $\vec{F}$ acting on the proton, in unit-vector notation, (b) the angle between $\vec{v}$ and $\vec{F}$, and (c) the angle between $\vec{v}$ and $\vec{B}$?

Answer:

(a) $(12.8\hat{i} + 6.41\hat{j}) \times 10^{-22}$ N; (b) 90°; (c) 173°

86 An electron has velocity $\vec{v} = (32\hat{i} + 40\hat{j})$ km/s as it enters a uniform magnetic field $\vec{B} = 60\hat{i}\ \mu$T. What are (a) the radius of the helical path taken by the electron and (b) the pitch of that path? (c) To an observer looking into the magnetic field region from the entrance point of the electron, does the electron spiral clockwise or counterclockwise as it moves?

sec. 29-2 Calculating the Magnetic Field Due to a Current

•**1** A surveyor is using a magnetic compass 6.1 m below a power line in which there is a steady current of 100 A. (a) What is the magnetic field at the site of the compass due to the power line? (b) Will this field interfere seriously with the compass reading? The horizontal component of Earth's magnetic field at the site is 20 μT.

Answer:

(a) 3.3 μT; (b) yes

•**2** Figure 29-34*a* shows an element of length ds = 1.00 μm in a very long straight wire carrying current. The current in that element sets up a differential magnetic field $d\vec{B}$ at points in the surrounding space. Figure 29-34*b* gives the magnitude dB of the field for points 2.5 cm from the element, as a function of angle θ between the wire and a straight line to the point. The vertical scale is set by dB_s = 60.0 pT. What is the magnitude of the magnetic field set up by the entire wire at perpendicular distance 2.5 cm from the wire?

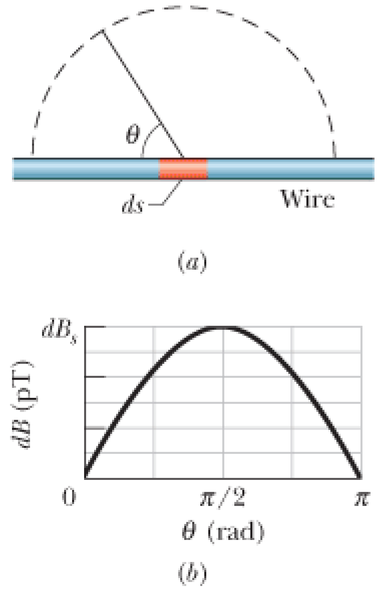


Figure 29-34 Problem 2.

•3 SSM At a certain location in the Philippines, Earth's magnetic field of 39 μT is horizontal and directed due north. Suppose the net field is zero exactly 8.0 cm above a long, straight, horizontal wire that carries a constant current. What are the (a) magnitude and (b) direction of the current?

Answer:

(a) 16 A; (b) east

•4 A straight conductor carrying current i = 5.0 A splits into identical semicircular arcs as shown in Fig. 29-35. What is the magnetic field at the center C of the resulting circular loop?

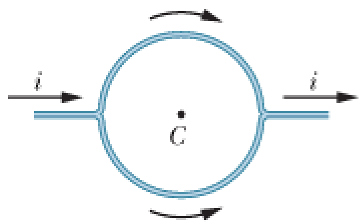


Figure 29-35 Problem 4.

•5 In Fig. 29-36, a current i = 10 A is set up in a long hairpin conductor formed by bending a wire into a semicircle of radius R = 5.0 mm. Point b is midway between the straight sections and so distant from the semicircle that each straight section can be approximated as being an infinite wire. What are the (a) magnitude and (b) direction (into or out of the page) of $\vec{B}$ at a and the (c) magnitude and (d) direction of $\vec{B}$ at b?

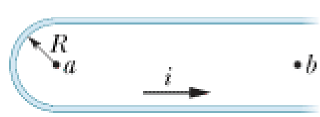


Figure 29-36 Problem 5.

Answer:

(a) 1.0 mT; (b) out; (c) 0.80 mT; (d) out

•6In Fig. 29-37, point *P* is at perpendicular distance *R* = 2.00 cm from a very long straight wire carrying a current. The magnetic field $\vec{B}$ set up at point *P* is due to contributions from all the identical currentlength elements $i\,d\vec{s}$ along the wire. What is the distance *s* to the element making (a) the greatest contribution to field $\vec{B}$ and (b) 10.0% of the greatest contribution?

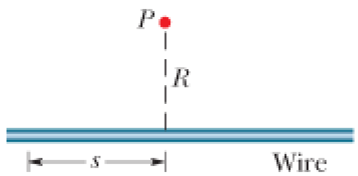


Figure 29-37 Problem 6.

•7 GO In Fig. 29-38, two circular arcs have radii *a* = 13.5 cm and *b* = 10.7 cm, subtend angle θ = 74.0°, carry current *i* = 0.411 A, and share the same center of curvature *P*. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at *P*?

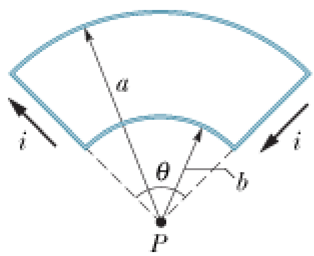


Figure 29-38 Problem 7.

Answer:

(a) 0.102 μT; (b) out

•8In Fig. 29-39, two semicircular arcs have radii R_2 = 7.80 cm and R_1 = 3.15 cm, carry current *i* = 0.281 A, and share the same center of curvature *C*. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at *C*?

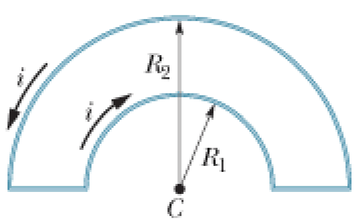


Figure 29-39 Problem 8.

•9 SSM Two long straight wires are parallel and 8.0 cm apart. They are to carry equal currents such that the magnetic field at a point halfway between them has magnitude 300 μT. (a) Should the currents be in the same or opposite directions? (b) How much current is needed?

Answer:

(a) opposite; (b) 30 A

•10 In Fig. 29-40, a wire forms a semicircle of radius R = 9.26 cm and two (radial) straight segments each of length L = 13.1 cm. The wire carries current i = 34.8 mA. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at the semicircle's center of curvature C?

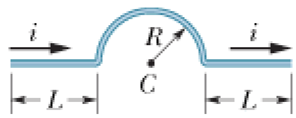


Figure 29-40 Problem 10.

•11 In Fig. 29-41, two long straight wires are perpendicular to the page and separated by distance d_1 = 0.75 cm. Wire 1 carries 6.5 A into the page. What are the (a) magnitude and (b) direction (into or out of the page) of the current in wire 2 if the net magnetic field due to the two currents is zero at point P located at distance d_2 = 1.50 cm from wire 2?

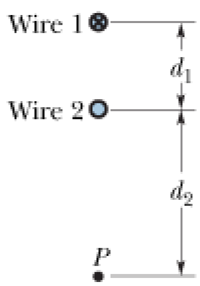


Figure 29-41 Problem 11.

Answer:

(a) 4.3 A; (b) out

•12 In Fig. 29-42, two long straight wires at separation d = 16.0 cm carry currents i_1 = 3.61 mA and i_2 = 3.00i_1 out of the page. (a) Where on the x axis is the net magnetic field equal to zero? (b) If the two currents are doubled, is the zero-field point shifted toward wire 1, shifted toward wire 2, or unchanged?

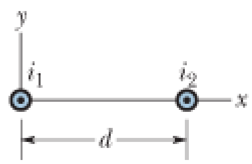


Figure 29-42 Problem 12.

••13 In Fig. 29-43, point P_1 is at distance R = 13.1 cm on the perpendicular bisector of a straight wire of length L = 18.0 cm carrying current i = 58.2 mA. (Note that the wire is *not* long.) What is the magnitude of the magnetic field at P_1 due to i?

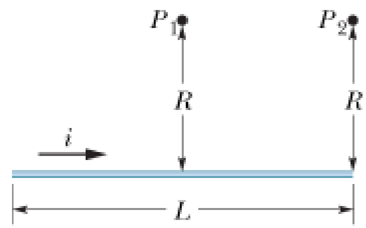


Figure 29-43 Problems 13 and 17.

Answer:

50.3 nT

••14 Equation 29-4 gives the magnitude B of the magnetic field set up by a current in an *infinitely long* straight wire, at a point P at perpendicular distance R from the wire. Suppose that point P is actually at perpendicular distance R from the midpoint of a wire with a *finite* length L. Using Eq. 29-4 to calculate B then results in a certain percentage error. What value must the ratio L/R exceed if the percentage error is to be less than 1.00%? That is, what L/R gives

$$\frac{(B \text{ from Eq. } 29-4)-(B \text{ actual})}{(B \text{ actual})}(100\%)=1.00\% \text{ ?}$$

••15 Figure 29-44 shows two current segments. The lower segment carries a current of $i_1 = 0.40$ A and includes a semicircular arc with radius 5.0 cm, angle 180°, and center point P. The upper segment carries current $i_2 = 2i_1$ and includes a circular arc with radius 4.0 cm, angle 120°, and the same center point P. What are the (a) magnitude and (b) direction of the net magnetic field $\vec{B}$ at P for the indicated current directions? What are the (c) magnitude and (d) direction of $\vec{B}$ if i_1 is reversed?

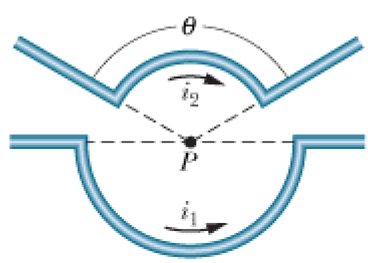


Figure 29-44 Problem 15.

Answer:

(a) 1.7 μT; (b) into; (c) 6.7 μT; (d) into

••16 GO In Fig. 29-45, two concentric circular loops of wire carrying current in the same direction lie in the same plane. Loop 1 has radius 1.50 cm and carries 4.00 mA. Loop 2 has radius 2.50 cm and carries 6.00 mA. Loop 2 is to be rotated about a diameter while the net magnetic field $\vec{B}$ set up by the two loops at their common center is measured. Through what angle must loop 2 be rotated so that the magnitude of that net field is 100 nT?

Figure 29-45 Problem 16.

••17 SSM In Fig. 29-43, point P_2 is at perpendicular distance R = 25.1 cm from one end of a straight wire of length L = 13.6 cm carrying current i = 0.693 A. (Note that the wire is *not* long.) What is the magnitude of the magnetic field at P_2?

Answer:

132 nT

••18A current is set up in a wire loop consisting of a semicircle of radius 4.00 cm, a smaller concentric semicircle, and two radial straight lengths, all in the same plane. Figure 29-46*a* shows the arrangement but is not drawn to scale. The magnitude of the magnetic field produced at the center of curvature is 47.25 μT. The smaller semicircle is then flipped over (rotated) until the loop is again entirely in the same plane (Fig. 29-46*b*). The magnetic field produced at the (same) center of curvature now has magnitude 15.75 μT, and its direction is reversed. What is the radius of the smaller semicircle?

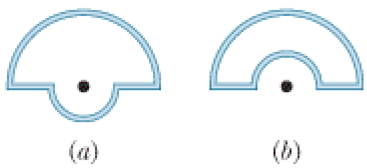


Figure 29-46 Problem 18.

••19One long wire lies along an x axis and carries a current of 30 A in the positive x direction. A second long wire is perpendicular to the xy plane, passes through the point (0, 4.0 m, 0), and carries a current of 40 A in the positive z direction. What is the magnitude of the resulting magnetic field at the point (0, 2.0 m, 0)?

Answer:

5.0 μT

••20In Fig. 29-47, part of a long insulated wire carrying current i = 5.78 mA is bent into a circular section of radius R = 1.89 cm. In unit-vector notation, what is the magnetic field at the center of curvature C if the circular section (a) lies in the plane of the page as shown and (b) is perpendicular to the plane of the page after being rotated 90° counterclockwise as indicated?

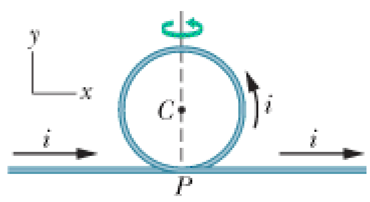


Figure 29-47 Problem 20.

••21 GO Figure 29-48 shows two very long straight wires (in cross section) that each carry a current of 4.00 A directly out of the page. Distance d_1 = 6.00 m and distance d_2 = 4.00 m. What is the magnitude of the net magnetic field at point P, which lies on a perpendicular bisector to the wires?

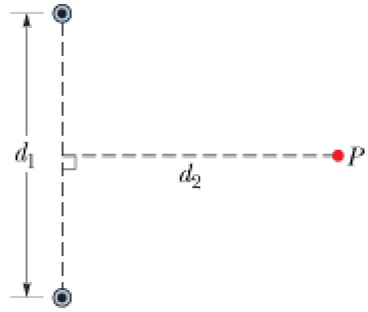


Figure 29-48 Problem 21.

Answer:

256 nT

••22 Figure 29-49*a* shows, in cross section, two long, parallel wires carrying current and separated by distance L. The ratio i_1/i_2 of their currents is 4.00; the directions of the currents are not indicated. Figure 29-49*b* shows the y component B_y of their net magnetic field along the x axis to the right of wire 2. The vertical scale is set by B_{ys} = 4.0 nT, and the horizontal scale is set by x_s = 20.0 cm. (a) At what value of $x > 0$ is B_y maximum? (b) If i_2 = 3 mA, what is the value of that maximum? What is the direction (into or out of the page) of (c) i_1 and (d) i_2?

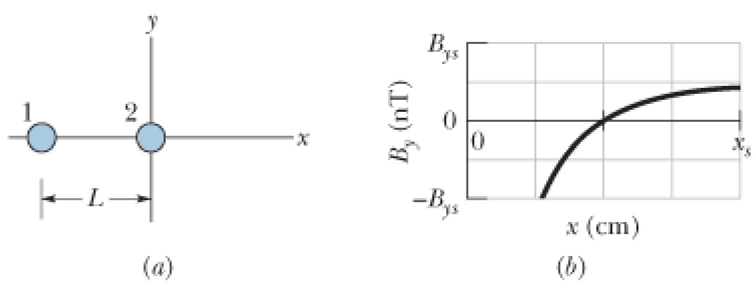


Figure 29-49 Problem 22.

••23 ILW Figure 29-50 shows a snapshot of a proton moving at velocity $\vec{v} = (-200\ \mathrm{m/s})\hat{\mathrm{j}}$ toward a long straight wire with current i = 350 mA. At the instant shown, the proton's distance from the wire is d = 2.89 cm. In unit-vector notation, what is the magnetic force on the proton due to the current?

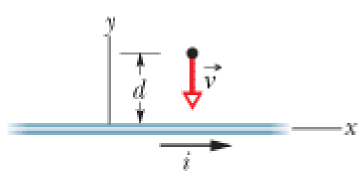


Figure 29-50 Problem 23.

Answer:

$$\left(-7.75 \times 10^{-23}\ \mathrm{N}\right)\hat{\mathrm{i}}$$

••24 GO Figure 29-51 shows, in cross section, four thin wires that are parallel, straight, and very long. They carry identical currents in the directions indicated. Initially all four wires are at distance d = 15.0 cm from the origin of the coordinate system, where they create a net magnetic field $\vec{B}$. (a) To what value of x must you move wire 1 along the x axis in order to rotate $\vec{B}$ counterclockwise $\vec{B}$ by 30°? (b) With wire 1 in that new position, to what value of x must you move wire 3 along the x axis to rotate $\vec{B}$ by 30° back to its initial orientation?

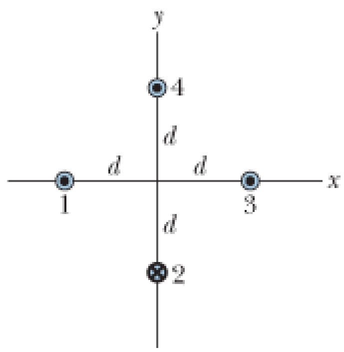


Figure 29-51 Problem 24.

••25 SSM A wire with current i = 3.00 A is shown in Fig. 29-52. Two semi-infinite straight sections, both tangent to the same circle, are connected by a circular arc that has a central angle θ and runs along the circumference of the circle. The arc and the two straight sections all lie in the same plane. If B = 0 at the circle's center, what is θ?

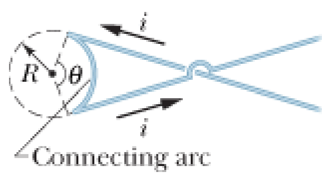


Figure 29-52 Problem 25.

Answer:

2.00 rad

••26 In Fig. 29-53*a*, wire 1 consists of a circular arc and two radial lengths; it carries current i_1 = 0.50 A in the direction indicated. Wire 2, shown in cross section, is long, straight, and perpendicular to the plane of the figure. Its distance from the center of the arc is equal to the radius R of the arc, and it carries a current i_2 that can be varied. The two currents set up a net magnetic field $\vec{B}$ at the center of the arc. Figure 29-53*b* gives the square of the field's magnitude B^2 plotted versus the square of the current i_2^2. The vertical scale is set by $B_s^2 = 10.0 \times 10^{-10}\ \mathrm{T}^2$. What angle is subtended by the arc?

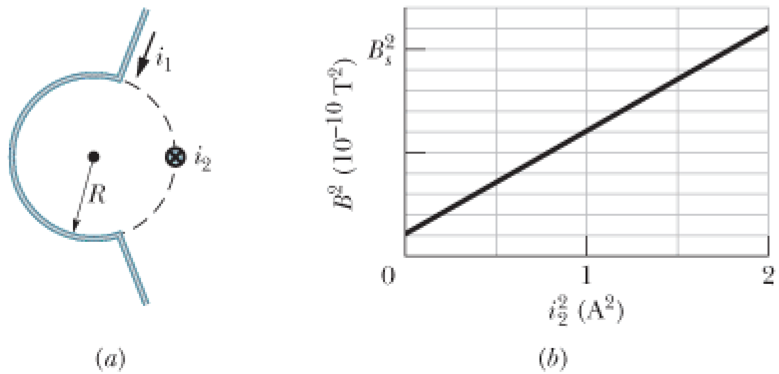


Figure 29-53 Problem 26.

••27 In Fig. 29-54, two long straight wires (shown in cross section) carry currents i_1 = 30.0 mA and i_2 = 40.0 mA directly out of the page. They are equal distances from the origin, where they set up a magnetic field $\vec{B}$. To what value must current i_1 be changed in order to rotate $\vec{B}$ 20.0° clockwise?

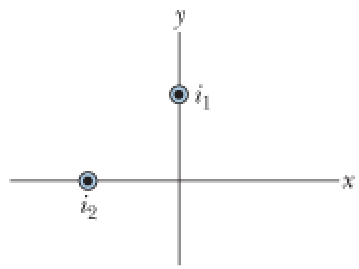


Figure 29-54 Problem 27.

Answer:

61.3 mA

••28 Figure 29-55*a* shows two wires, each carrying a current. Wire 1 consists of a circular arc of radius R and two radial lengths; it carries current i_1 = 2.0 A in the direction indicated. Wire 2 is long and straight; it carries a current i_2 that can be varied; and it is at distance $R/2$ from the center of the arc. The net magnetic field $\vec{B}$ due to the two currents is measured at the center of curvature of the arc. Figure 29-55*b* is a plot of the component of $\vec{B}$ in the direction perpendicular to the figure as a function of current i_2. The horizontal scale is set by i_{2s} = 1.00 A. What is the angle subtended by the arc?

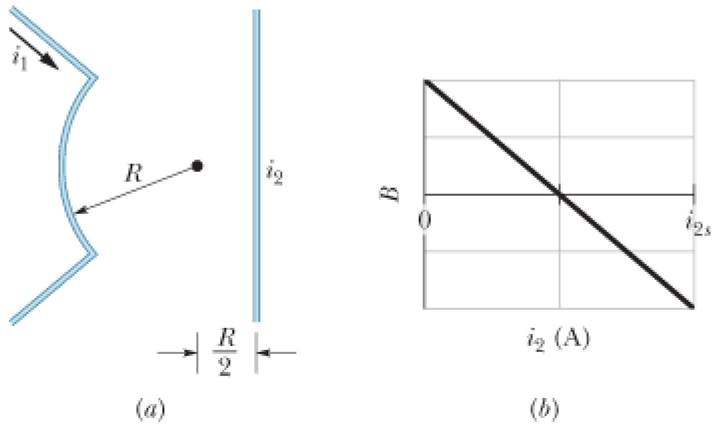


Figure 29-55 Problem 28.

••29 SSM In Fig. 29-56, four long straight wires are perpendicular to the page, and their cross sections form a square of edge length a = 20 cm. The currents are out of the page in wires 1 and 4 and into the page in wires 2 and 3, and each wire carries 20 A. In unit-vector notation, what is the net magnetic field at the square's center?

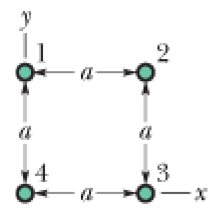


Figure 29-56 Problems 29, 37, and 40.

Answer:

$$(80\ \mu\mathrm{T})\hat{\mathrm{j}}$$

•••30 Two long straight thin wires with current lie against an equally long plastic cylinder, at radius R = 20.0 cm from the cylinder's central axis. Figure 29-57*a* shows, in cross section, the cylinder and wire 1 but not wire 2. With wire 2 fixed in place, wire 1 is moved around the cylinder, from angle $\theta_1 = 0°$ to angle $\theta_1 = 180°$, through the first and second quadrants of the *xy* coordinate system. The net magnetic field $\vec{B}$ at the center of the cylinder is measured as a function of θ_1. Figure 29-57*b* gives the *x* component B_x of that field as a function of θ_1 (the vertical scale is set by B_{xs} = 6.0 μT), and Fig. 29-57*c* gives the *y* component B_y (the vertical scale is set by B_{ys} = 4.0 μT). (a) At what angle θ_2 is wire 2 located? What are the (b) size and (c) direction (into or out of the page) of the current in wire 1 and the (d) size and (e) direction of the current in wire 2?

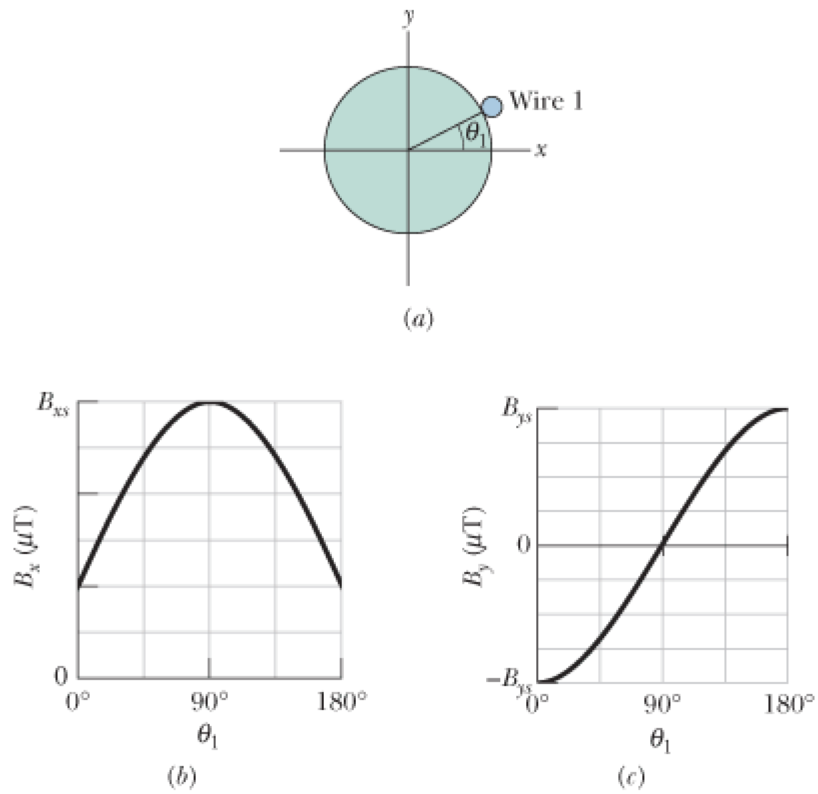


Figure 29-57 Problem 30.

•••31 In Fig. 29-58, length a is 4.7 cm (short) and current i is 13 A. What are the (a) magnitude and (b) direction (into or out of the page) of the magnetic field at point P?

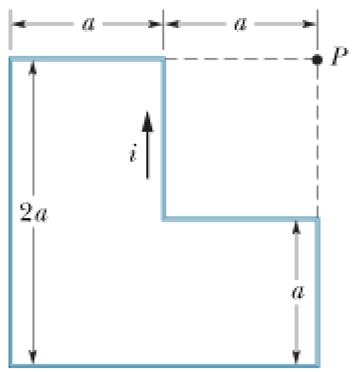


Figure 29-58 Problem 31.

Answer:

(a) 20 μT; (b) into

•••32 The current-carrying wire loop in Fig. 29-59a lies all in one plane and consists of a semicircle of radius 10.0 cm, a smaller semicircle with the same center, and two radial lengths. The smaller semicircle is rotated out of that plane by angle θ, until it is perpendicular to the plane (Fig. 29-59b). Figure 29-59c gives the magnitude of the net magnetic field at the center of curvature versus angle θ. The vertical scale is set by $B_a = 10.0\ \mu$T and $B_b = 12.0\ \mu$T. What is the radius of the smaller semicircle?

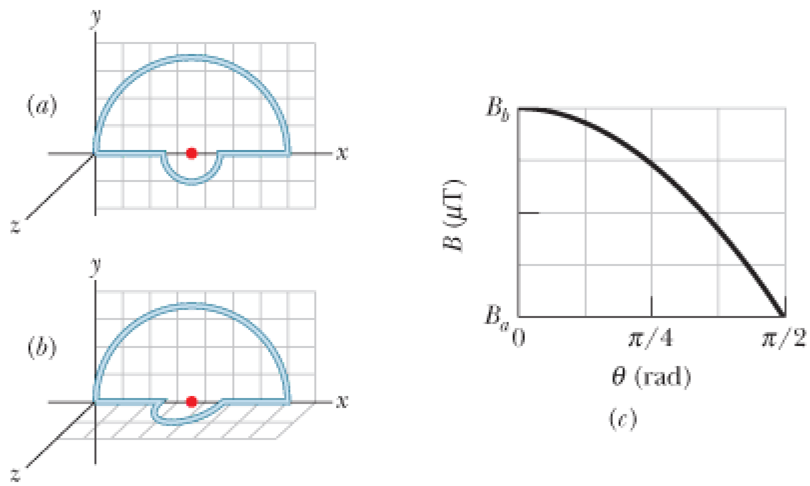


Figure 29-59 Problem 32.

•••33 SSM ILW Figure 29-60 shows a cross section of a long thin ribbon of width w = 4.91 cm that is carrying a uniformly distributed total current i = 4.61 μA into the page. In unit-vector notation, what is the magnetic field $\vec{B}$ at a point P in the plane of the ribbon at a distance d = 2.16 cm from its edge? (*Hint:* Imagine the ribbon as being constructed from many long, thin, parallel wires.)

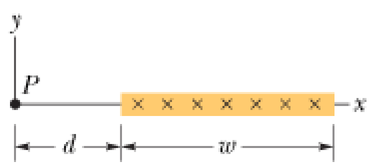


Figure 29-60 Problem 33.

Answer:

$(22.3\,\mathrm{pT})\hat{\mathrm{j}}$

•••34 Figure 29-61 shows, in cross section, two long straight wires held against a plastic cylinder of radius 20.0 cm. Wire 1 carries current i_1 = 60.0 mA out of the page and is fixed in place at the left side of the cylinder. Wire 2 carries current i_2 = 40.0 mA out of the page and can be moved around the cylinder. At what (positive) angle θ_2 should wire 2 be positioned such that, at the origin, the net magnetic field due to the two currents has magnitude 80.0 nT?

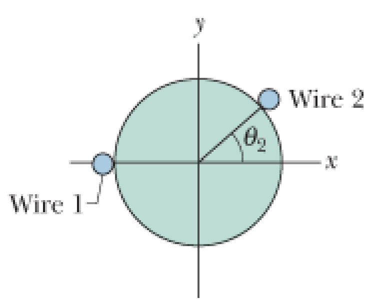


Figure 29-61 Problem 34.

sec. 29-3 Force Between Two Parallel Currents

•35 SSM Figure 29-62 shows wire 1 in cross section; the wire is long and straight, carries a current of 4.00 mA out of the page, and is at distance d_1 = 2.40 cm from a surface. Wire 2, which is parallel to wire 1 and also long, is at horizontal distance d_2 = 5.00 cm from wire 1 and carries a current of 6.80 mA into the page. What is the x component of the magnetic force *per unit length* on wire 2 due to wire 1?

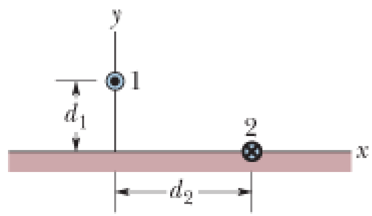


Figure 29-62 Problem 35.

Answer:

88.4 pN/m

••36 In Fig. 29-63, five long parallel wires in an xy plane are separated by distance d = 8.00 cm, have lengths of 10.0 m, and carry identical currents of 3.00 A out of the page. Each wire experiences a magnetic force due to the other wires. In unit-vector notation, what is the net magnetic force on (a) wire 1, (b) wire 2, (c) wire 3, (d) wire 4, and (e) wire 5?

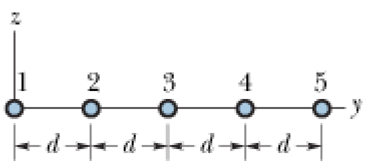


Figure 29-63 Problems 36 and 39.

••37 GO In Fig. 29-56, four long straight wires are perpendicular to the page, and their cross sections form a square of edge length a = 13.5 cm. Each wire carries 7.50 A, and the currents are out of the page in wires 1 and 4 and into the page in wires 2 and 3. In unit-vector notation, what is the net magnetic force *per meter of wire length* on wire 4?

Answer:

$$(-125\ \mu\mathrm{N/m})\hat{\mathrm{i}} + (41.7\ \mu\mathrm{N/m})\hat{\mathrm{j}}$$

••38 Figure 29-64*a* shows, in cross section, three current-carrying wires that are long, straight, and parallel to one another. Wires 1 and 2 are fixed in place on an x axis, with separation d. Wire 1 has a current of 0.750 A, but the direction of the current is not given. Wire 3, with a current of 0.250 A out of the page, can be moved along the x axis to the right of wire 2. As wire 3 is moved, the magnitude of the net magnetic force $\vec{F}_2$ on wire 2 due to the currents in wires 1 and 3 changes. The x component of that force is F_{2x} and the value per unit length of wire 2 is F_{2x}/L_2. Figure 29-64*b* gives F_{2x}/L_2 versus the position x of wire 3. The plot has an asymptote F_{2x}/L_2 = -0.627 μN/m as $x \to \infty$. The horizontal scale is set by x_s = 12.0 cm. What are the (a) size and (b) direction (into or out of the page) of the current in wire 2?

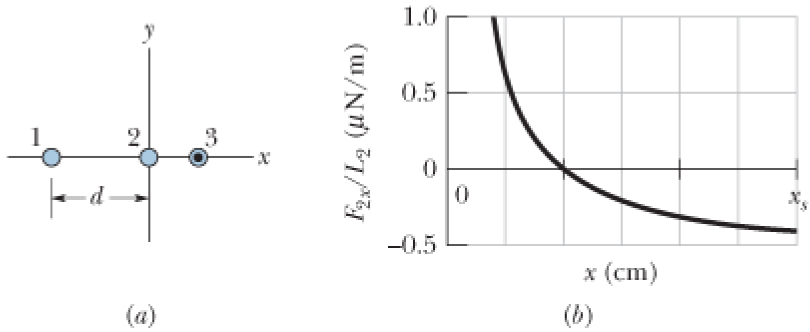


Figure 29-64 Problem 38.

••39 In Fig. 29-63, five long parallel wires in an *xy* plane are separated by distance d = 50.0 cm. The currents into the page are i_1 = 2.00 A, i_3 = 0.250 A, i_4 = 4.00 A, and i_5 = 2.00 A; the current out of the page is i_2 = 4.00 A. What is the magnitude of the net force *per unit length* acting on wire 3 due to the currents in the other wires?

Answer:

800 nN/m

••40 In Fig. 29-56, four long straight wires are perpendicular to the page, and their cross sections form a square of edge length a = 8.50 cm. Each wire carries 15.0 A, and all the currents are out of the page. In unit-vector notation, what is the net magnetic force *per meter of wire length* on wire 1?

•••41 ILW In Fig. 29-65, a long straight wire carries a current i_1 = 30.0 A and a rectangular loop carries current i_2 = 20.0 A. Take a = 1.00 cm, b = 8.00 cm, and L = 30.0 cm. In unit-vector notation, what is the net force on the loop due to i_1?

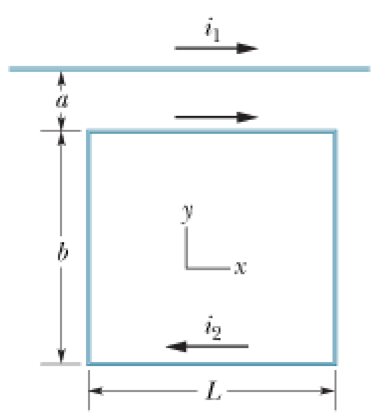


Figure 29-65 Problem 41.

Answer:

$(3.20\ \text{mN})\hat{j}$

sec. 29-4 Ampere's Law

•42 In a particular region there is a uniform current density of 15 A/m^2 in the positive z direction. What is the value of $\oint \vec{B} \cdot d\vec{s}$ when that line integral is calculated along the three straight-line segments from (x, y, z) coordinates ($4d$, 0, 0) to ($4d$, $3d$, 0) to (0, 0, 0) to ($4d$, 0, 0), where d = 20 cm?

•43 Figure 29-66 shows a cross section across a diameter of a long cylindrical conductor of radius a = 2.00 cm carrying uniform current 170 A. What is the magnitude of the current's magnetic field at radial distance (a) 0, (b) 1.00 cm, (c) 2.00 cm (wire's surface), and (d) 4.00 cm?

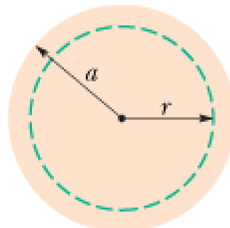


Figure 29-66 Problem 43.

Answer:

(a) 0; (b) 0.850 mT; (c) 1.70 mT; (d) 0.850 mT

•44 Figure 29-67 shows two closed paths wrapped around two conducting loops carrying currents i_1 = 5.0 A and i_2 = 3.0 A. What is the value of the integral $\oint \vec{B} \cdot d\vec{s}$ for (a) path 1 and (b) path 2?

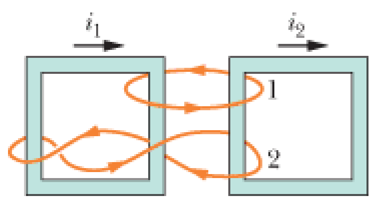


Figure 29-67 Problem 44.

•45 SSM Each of the eight conductors in Fig. 29-68 carries 2.0 A of current into or out of the page. Two paths are indicated for the line integral $\oint \vec{B} \cdot d\vec{s}$. What is the value of the integral for (a) path 1 and (b) path 2?

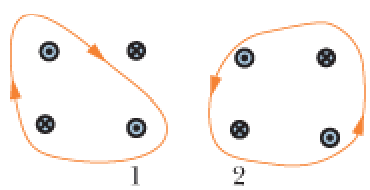


Figure 29-68 Problem 45.

Answer:

(a) -2.5 μT·m; (b) 0

•46 Eight wires cut the page perpendicularly at the points shown in Fig. 29-69. A wire labeled with the integer k (k = 1, 2, …, 8) carries the current ki, where i = 4.50 mA. For those wires with odd k, the current is out of the page; for those with even k, it is into the page. Evaluate $\oint \vec{B} \cdot d\vec{s}$ along the closed path in the direction shown.

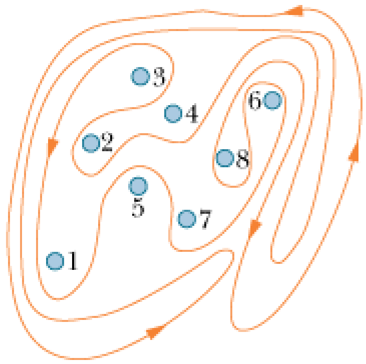


Figure 29-69 Problem 46.

••47 ILW The current density $\vec{J}$ inside a long, solid, cylindrical wire of radius $a = 3.1$ mm is in the direction of the central axis, and its magnitude varies linearly with radial distance r from the axis according to $J = J_0 r/a$, where $J_0 = 310\ \text{A/m}^2$. Find the magnitude of the magnetic field at (a) $r = 0$, (b) $r = a/2$, and (c) $r = a$.

Answer:

(a) 0; (b) 0.10 μT; (c) 0.40 μT

••48 In Fig. 29-70, a long circular pipe with outside radius $R = 2.6$ cm carries a (uniformly distributed) current $i = 8.00$ mA into the page. A wire runs parallel to the pipe at a distance of $3.00R$ from center to center. Find the (a) magnitude and (b) direction (into or out of the page) of the current in the wire such that the net magnetic field at point P has the same magnitude as the net magnetic field at the center of the pipe but is in the opposite direction.

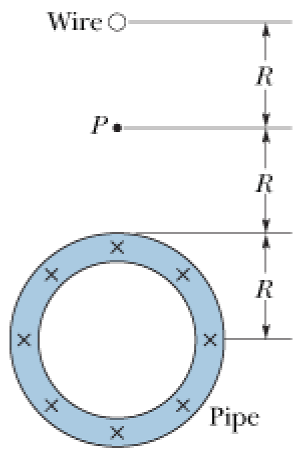


Figure 29-70 Problem 48.

sec. 29-5 Solenoids and Toroids

•49 A toroid having a square cross section, 5.00 cm on a side, and an inner radius of 15.0 cm has 500 turns and carries a current of 0.800 A. (It is made up of a square solenoid—instead of a round one as in Fig. 29-16—bent into a doughnut shape.) What is the magnetic field inside the toroid at (a) the inner radius and (b) the outer radius?

Answer:

(a) 533 μT; (b) 400 μT

•**50** A solenoid that is 95.0 cm long has a radius of 2.00 cm and a winding of 1200 turns; it carries a current of 3.60 A. Calculate the magnitude of the magnetic field inside the solenoid.

•**51** A 200-turn solenoid having a length of 25 cm and a diameter of 10 cm carries a current of 0.29 A. Calculate the magnitude of the magnetic field $\vec{B}$ inside the solenoid.

Answer:

0.30 mT

•**52** A solenoid 1.30 m long and 2.60 cm in diameter carries a current of 18.0 A. The magnetic field inside the solenoid is 23.0 mT. Find the length of the wire forming the solenoid.

••**53** A long solenoid has 100 turns/cm and carries current *i*. An electron moves within the solenoid in a circle of radius 2.30 cm perpendicular to the solenoid axis. The speed of the electron is 0.0460*c* (*c* = speed of light). Find the current *i* in the solenoid.

Answer:

0.272 A

••**54** An electron is shot into one end of a solenoid. As it enters the uniform magnetic field within the solenoid, its speed is 800 m/s and its velocity vector makes an angle of 30° with the central axis of the solenoid. The solenoid carries 4.0 A and has 8000 turns along its length. How many revolutions does the electron make along its helical path within the solenoid by the time it emerges from the solenoid's opposite end? (In a real solenoid, where the field is not uniform at the two ends, the number of revolutions would be slightly less than the answer here.)

••**55** SSM WWW ILW A long solenoid with 10.0 turns/cm and a radius of 7.00 cm carries a current of 20.0 mA. A current of 6.00 A exists in a straight conductor located along the central axis of the solenoid. (a) At what radial distance from the axis will the direction of the resulting magnetic field be at 45.0° to the axial direction? (b) What is the magnitude of the magnetic field there?

Answer:

(a) 4.77 cm; (b) 35.5 μT

sec. 29-6 A Current-Carrying Coil as a Magnetic Dipole

•**56** Figure 29-71 shows an arrangement known as a Helmholtz coil. It consists of two circular coaxial coils, each of 200 turns and radius R = 25.0 cm, separated by a distance $s = R$. The two coils carry equal currents i = 12.2 mA in the same direction. Find the magnitude of the net magnetic field at P, midway between the coils.

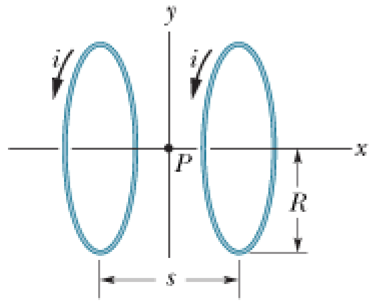


Figure 29-71 Problems 56 and 90.

•57 SSM A student makes a short electromagnet by winding 300 turns of wire around a wooden cylinder of diameter d = 5.0 cm. The coil is connected to a battery producing a current of 4.0 A in the wire. (a) What is the magnitude of the magnetic dipole moment of this device? (b) At what axial distance $z \gg d$ will the magnetic field have the magnitude 5.0 μT (approximately one-tenth that of Earth's magnetic field)?

Answer:

(a) 2.4 $A \cdot m^2$; (b) 46 cm

•58 Figure 29-72*a* shows a length of wire carrying a current i and bent into a circular coil of one turn. In Fig. 29-72*b* the same length of wire has been bent to give a coil of two turns, each of half the original radius. (a) If B_a and B_b are the magnitudes of the magnetic fields at the centers of the two coils, what is the ratio B_b/B_a? (b) What is the ratio μ_b/μ_a of the dipole moment magnitudes of the coils?

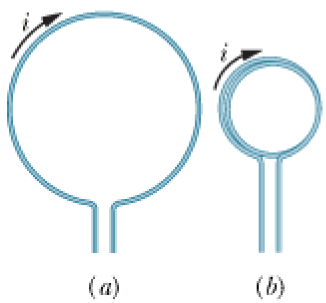


Figure 29-72 Problem 58.

•59 SSM What is the magnitude of the magnetic dipole moment of the solenoid described in Problem 51?

Answer:

0.47 $A \cdot m^2$

••60 GO In Fig. 29-73*a*, two circular loops, with different currents but the same radius of 4.0 cm, are centered on a y axis. They are initially separated by distance L = 3.0 cm, with loop 2 positioned at the origin of the axis. The currents in the two loops produce a net magnetic field at the origin, with y component B_y. That component is to be measured as loop 2 is gradually moved in the positive direction of the y axis. Figure 29-73*b* gives B_y as a function of the position y of loop 2. The curve approaches an asymptote of B_y = 7.20 μT as $y \to \infty$. The horizontal scale is set by y_s = 10.0 cm. What are (a) current i_1 in loop 1 and (b) current i_2 in loop 2?

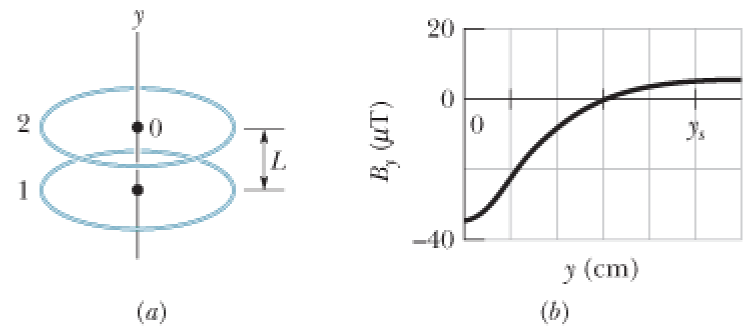


Figure 29-73 Problem 60.

••61A circular loop of radius 12 cm carries a current of 15 A. A flat coil of radius 0.82 cm, having 50 turns and a current of 1.3 A, is concentric with the loop. The plane of the loop is perpendicular to the plane of the coil. Assume the loop's magnetic field is uniform across the coil. What is the magnitude of (a) the magnetic field produced by the loop at its center and (b) the torque on the coil due to the loop?

Answer:

(a) 79 μT;(b) 1.1×10^{-6} N·m

••62In Fig. 29-74, current i = 56.2 mA is set up in a loop having two radial lengths and two semicircles of radii a = 5.72 cm and b = 9.36 cm with a common center P. What are the (a) magnitude and (b) direction (into or out of the page) of the magnetic field at P and the (c) magnitude and (d) direction of the loop's magnetic dipole moment?

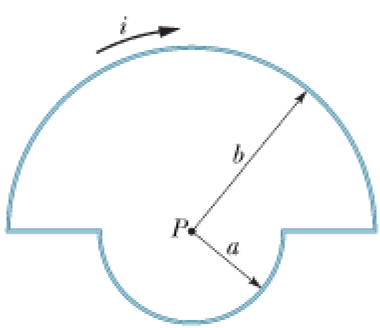


Figure 29-74 Problem 62.

••63In Fig. 29-75, a conductor carries 6.0 A along the closed path *abcdefgha* running along 8 of the 12 edges of a cube of edge length 10 cm. (a) Taking the path to be a combination of three square current loops (*bcfgb*, *abgha*, and *cdefc*), find the net magnetic moment of the path in unit-vector notation. (b) What is the magnitude of the net magnetic field at the *xyz* coordinates of (0, 5.0 m, 0)?

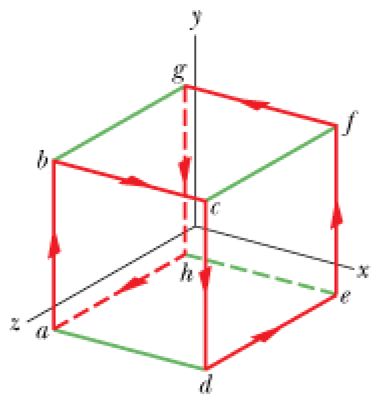


Figure 29-75 Problem 63.

Answer:

(a) $\left(0.060\ \mathrm{A}\cdot\mathrm{m}^2\right)\hat{\mathrm{j}}$; (b) $(96\,\mathrm{pT})\hat{\mathrm{j}}$

Additional Problems

64In Fig. 29-76, a closed loop carries current i = 200 mA. The loop consists of two radial straight wires and two concentric circular arcs of radii 2.00 m and 4.00 m. The angle θ is $\pi/4$ rad. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at the center of curvature P?

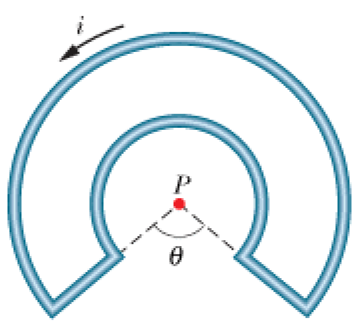


Figure 29-76 Problem 64.

65A cylindrical cable of radius 8.00 mm carries a current of 25.0 A, uniformly spread over its cross-sectional area. At what distance from the center of the wire is there a point within the wire where the magnetic field magnitude is 0.100 mT?

Answer:

1.28 mm

66Two long wires lie in an xy plane, and each carries a current in the positive direction of the x axis. Wire 1 is at y = 10.0 cm and carries 6.00 A; wire 2 is at y = 5.00 cm and carries 10.0 A. (a) In unit-vector notation, what is the net magnetic field $\vec{B}$ at the origin? (b) At what value of y does $\vec{B} = 0$? (c) If the current in wire 1 is reversed, at what value of y does $\vec{B} = 0$?

67Two wires, both of length L, are formed into a circle and a square, and each carries current i. Show that the square produces a greater magnetic field at its center than the circle produces at its center.

68 A long straight wire carries a current of 50 A. An electron, traveling at 1.0×10^7 m/s, is 5.0 cm from the wire. What is the magnitude of the magnetic force on the electron if the electron velocity is directed (a) toward the wire, (b) parallel to the wire in the direction of the current, and (c) perpendicular to the two directions defined by (a) and (b)?

69 Three long wires are parallel to a *z* axis, and each carries a current of 10 A in the positive *z* direction. Their points of intersection with the *xy* plane form an equilateral triangle with sides of 50 cm, as shown in Fig. 29-77. A fourth wire (wire *b*) passes through the midpoint of the base of the triangle and is parallel to the other three wires. If the net magnetic force on wire *a* is zero, what are the (a) size and (b) direction (+*z* or -*z*) of the current in wire *b*?

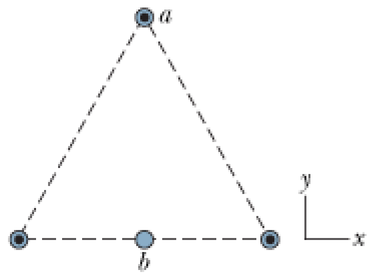


Figure 29-77 Problem 69.

Answer:

(a) 15 A; (b) -*z*

70 Figure 29-78 shows a closed loop with current $i = 2.00$ A. The loop consists of a half-circle of radius 4.00 m, two quarter-circles each of radius 2.00 m, and three radial straight wires. What is the magnitude of the net magnetic field at the common center of the circular sections?

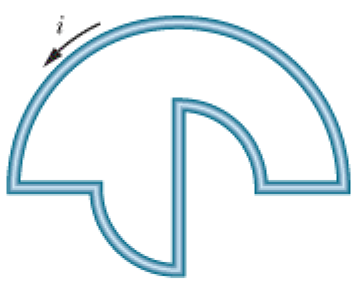


Figure 29-78 Problem 70.

71 A 10-gauge bare copper wire (2.6 mm in diameter) can carry a current of 50 A without overheating. For this current, what is the magnitude of the magnetic field at the surface of the wire?

Answer:

7.7 mT

72 A long vertical wire carries an unknown current. Coaxial with the wire is a long, thin, cylindrical conducting surface that carries a current of 30 mA upward. The cylindrical surface has a radius of 3.0 mm. If the magnitude of the magnetic field at a point 5.0 mm from the wire is 1.0 μT, what are the (a) size and (b) direction of the current in the wire?

73 Figure 29-79 shows a cross section of a long cylindrical conductor of radius $a = 4.00$ cm containing a long cylindrical hole of radius $b = 1.50$ cm. The central axes of the cylinder and hole are parallel and are distance $d = 2.00$ cm apart; current $i = 5.25$ A is uniformly distributed over the tinted area. (a) What is the magnitude of the magnetic field at the center of the hole? (b) Discuss the two special cases $b = 0$ and $d = 0$.

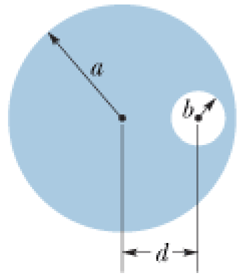


Figure 29-79 Problem 73.

Answer:

(a) 15.3 μT

74 The magnitude of the magnetic field 88.0 cm from the axis of a long straight wire is 7.30 μT. What is the current in the wire?

75 SSM Figure 29-80 shows a wire segment of length $\Delta s = 3.0$ cm, centered at the origin, carrying current $i = 2.0$ A in the positive y direction (as part of some complete circuit). To calculate the magnitude of the magnetic field $\vec{B}$ produced by the segment at a point several meters from the origin, we can use $B = (\mu_0/4\pi)i\,\Delta s\,(\sin\theta)/r^2$ as the Biot–Savart law. This is because r and θ are essentially constant over the segment. Calculate $\vec{B}$ (in unit-vector notation) at the (x, y, z) coordinates (a) (0, 0, 5.0 m), (b) (0, 6.0 m, 0), (c) (7.0 m, 7.0 m, 0), and (d) (-3.0 m, -4.0 m, 0).

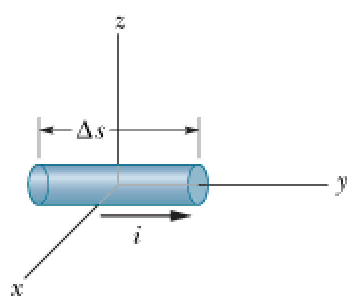


Figure 29-80 Problem 75.

Answer:

(a) $\left(0.24\hat{\mathrm{i}}\right)\mathrm{nT}$; (b) 0; (c) $\left(-43\hat{\mathrm{k}}\right)\mathrm{pT}$; (d) $\left(-0.14\hat{\mathrm{k}}\right)\mathrm{nT}$

76 GO Figure 29-81 shows, in cross section, two long parallel wires spaced by distance $d = 10.0$ cm; each carries 100 A, out of the page in wire 1. Point P is on a perpendicular bisector of the line connecting the wires. In unit-vector notation, what is the net magnetic field at P if the current in wire 2 is (a) out of the page and (b) into the page?

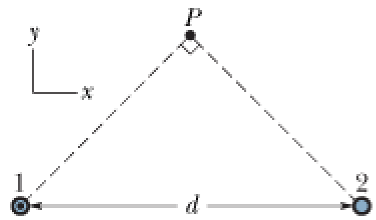


Figure 29-81 Problem 76.

77In Fig. 29-82, two infinitely long wires carry equal currents *i*. Each follows a 90° arc on the circumference of the same circle of radius *R*. Show that the magnetic field $\vec{B}$ at the center of the circle is the same as the field $\vec{B}$ a distance *R* below an infinite straight wire carrying a current *i* to the left.

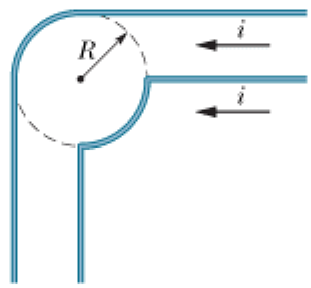


Figure 29-82 Problem 77.

78A long wire carrying 100 A is perpendicular to the magnetic field lines of a uniform magnetic field of magnitude 5.0 mT. At what distance from the wire is the net magnetic field equal to zero?

79A long, hollow, cylindrical conductor (with inner radius 2.0 mm and outer radius 4.0 mm) carries a current of 24 A distributed uniformly across its cross section. A long thin wire that is coaxial with the cylinder carries a current of 24 A in the opposite direction. What is the magnitude of the magnetic field (a) 1.0 mm, (b) 3.0 mm, and (c) 5.0 mm from the central axis of the wire and cylinder?

Answer:

(a) 4.8 mT; (b) 0.93 mT; (c) 0

80A long wire is known to have a radius greater than 4.0 mm and to carry a current that is uniformly distributed over its cross section. The magnitude of the magnetic field due to that current is 0.28 mT at a point 4.0 mm from the axis of the wire, and 0.20 mT at a point 10 mm from the axis of the wire. What is the radius of the wire?

81 SSM Figure 29-83 shows a cross section of an infinite conducting sheet carrying a current per unit *x*-length of λ; the current emerges perpendicularly out of the page. (a) Use the Biot–Savart law and symmetry to show that for all points *P* above the sheet and all points *P'* below it, the magnetic field $\vec{B}$ is parallel to the sheet and directed as shown. (b) Use Ampere's law to prove that $B = \frac{1}{2}\mu_0\lambda$ at all points *P* and *P'*.

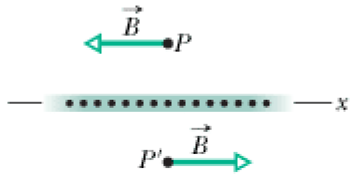


Figure 29-83 Problem 81.

82Figure 29-84 shows, in cross section, two long parallel wires that are separated by distance d = 18.6 cm. Each carries 4.23 A, out of the page in wire 1 and into the page in wire 2. In unit-vector notation, what is the net magnetic field at point P at distance R = 34.2 cm, due to the two currents?

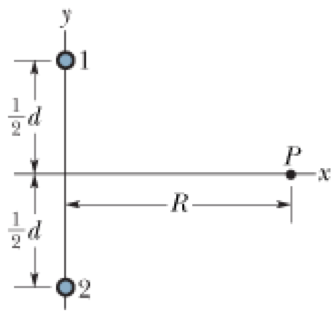


Figure 29-84 Problem 82.

83 **SSM** In unit-vector notation, what is the magnetic field at point P in Fig. 29-85 if i = 10 A and a = 8.0 cm? (Note that the wires are *not* long.)

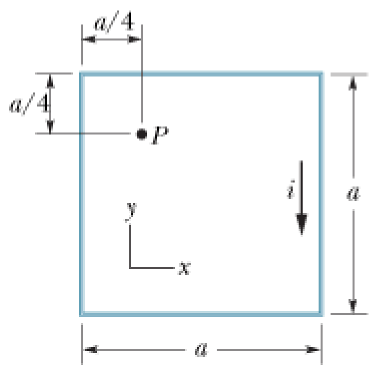


Figure 29-85 Problem 83.

Answer:

$$(-0.20\,\mathrm{mT})\hat{\mathrm{k}}$$

84Three long wires all lie in an xy plane parallel to the x axis. They are spaced equally, 10 cm apart. The two outer wires each carry a current of 5.0 A in the positive x direction. What is the magnitude of the force on a 3.0 m section of either of the outer wires if the current in the center wire is 3.2 A (a) in the positive x direction and (b) in the negative x direction?

85 **SSM** Figure 29-86 shows a cross section of a hollow cylindrical conductor of radii a and b, carrying a uniformly distributed current i. (a) Show that the magnetic field magnitude $B(r)$ for the radial distance r in the range $b < r < a$ is given by

$$B = \frac{\mu_0 i}{2\pi\left(a^2 - b^2\right)} \frac{r^2 - b^2}{r}.$$

(b) Show that when $r = a$, this equation gives the magnetic field magnitude B at the surface of a long straight wire carrying current i; when $r = b$, it gives zero magnetic field; and when $b = 0$, it gives the magnetic field inside a solid conductor of radius a carrying current i. (c) Assume that a = 2.0 cm, b = 1.8 cm, and i = 100 A, and then plot $B(r)$ for the range $0 < r < 6$ cm.

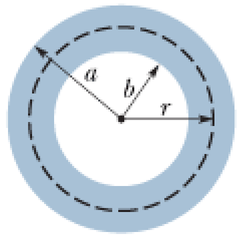


Figure 29-86 Problem 85.

86 Show that the magnitude of the magnetic field produced at the center of a rectangular loop of wire of length L and width W, carrying a current i, is

$$B = \frac{2\mu_0 i}{\pi} \frac{\left(L^2 + W^2\right)^{1/2}}{LW}.$$

87 Figure 29-87 shows a cross section of a long conducting coaxial cable and gives its radii (a, b, c). Equal but opposite currents i are uniformly distributed in the two conductors. Derive expressions for $B(r)$ with radial distance r in the ranges (a) $r < c$, (b) $c < r < b$, (c) $b < r < a$, and (d) $r > a$. (e) Test these expressions for all the special cases that occur to you. (f) Assume that a = 2.0 cm, b = 1.8 cm, c = 0.40 cm, and i = 120 A and plot the function $B(r)$ over the range $0 < r < 3$ cm.

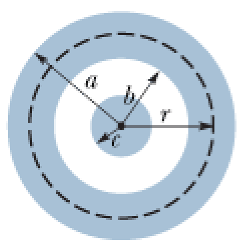


Figure 29-87 Problem 87.

Answer:

(a) $\mu_0 ir/2\pi c^2$; (b) $\mu_0 i/2\pi r$; (c) $\mu_0 i(a^2 - r^2)/2\pi(a^2 - b^2)r$; (d) 0

88 Figure 29-88 is an idealized schematic drawing of a rail gun. Projectile P sits between two wide rails of circular cross section; a source of current sends current through the rails and through the (conducting) projectile (a fuse is not used). (a) Let w be the distance between the rails, R the radius of each rail, and i the current. Show that the force on the projectile is directed to the right along the rails and is given approximately by

$$F = \frac{i^2 \mu_0}{2\pi} \ln \frac{w + R}{R}.$$

(b) If the projectile starts from the left end of the rails at rest, find the speed v at which it is expelled at the right. Assume that $i = 450$ kA, $w = 12$ mm, $R = 6.7$ cm, $L = 4.0$ m, and the projectile mass is 10 g.

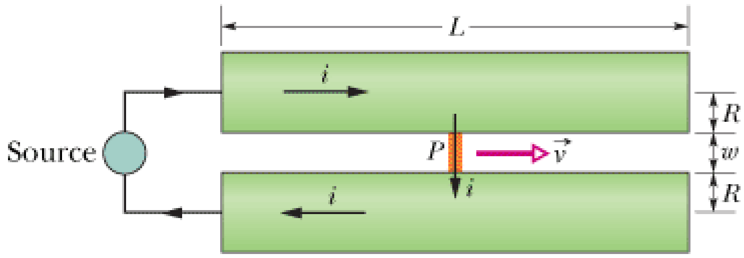


Figure 29-88 Problem 88.

89A square loop of wire of edge length a carries current i. Show that, at the center of the loop, the magnitude of the magnetic field produced by the current is

$$B = \frac{2\sqrt{2}\,\mu_0 i}{\pi a}.$$

90In Fig. 29-71, an arrangement known as Helmholtz coils consists of two circular coaxial coils, each of N turns and radius R, separated by distance s. The two coils carry equal currents i in the same direction. (a) Show that the first derivative of the magnitude of the net magnetic field of the coils (dB/dx) vanishes at the midpoint P regardless of the value of s. Why would you expect this to be true from symmetry? (b) Show that the second derivative (d^2B/dx^2) also vanishes at P, provided $s = R$. This accounts for the uniformity of B near P for this particular coil separation.

91 SSM A square loop of wire of edge length a carries current i. Show that the magnitude of the magnetic field produced at a point on the central perpendicular axis of the loop and a distance x from its center is

$$B(x) = \frac{4\mu_0 i a^2}{\pi\left(4x^2 + a^2\right)\left(4x^2 + 2a^2\right)^{1/2}}.$$

Prove that this result is consistent with the result shown in Problem 89.

92Show that if the thickness of a toroid is much smaller than its radius of curvature (a very skinny toroid), then Eq. 29-24 for the field inside a toroid reduces to Eq. 29-23 for the field inside a solenoid. Explain why this result is to be expected.

93 SSM Show that a uniform magnetic field $\vec{B}$ cannot drop abruptly to zero (as is suggested by the lack of field lines to the right of point a in Fig. 29-89) as one moves perpendicular to $\vec{B}$, say along the horizontal arrow in the figure. (*Hint:* Apply Ampere's law to the rectangular path shown by the dashed lines.) In actual magnets, "fringing" of the magnetic field lines always occurs, which means that $\vec{B}$ approaches zero in a gradual manner. Modify the field lines in the figure to indicate a more realistic situation.

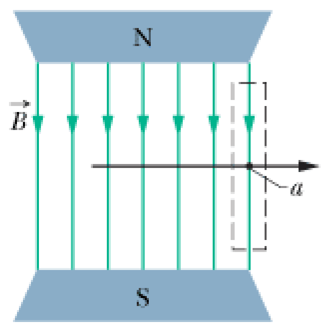


Figure 29-89 Problem 93.

sec. 30-4 Lenz's Law

•1 In Fig. 30-31, a circular loop of wire 10 cm in diameter (seen edge-on) is placed with its normal $\vec{N}$ at an angle $\theta = 30°$ with the direction of a uniform magnetic field $\vec{B}$ of magnitude 0.50 T. The loop is then rotated such that $\vec{N}$ rotates in a cone about the field direction at the rate 100 rev/min; angle θ remains unchanged during the process. What is the emf induced in the loop?

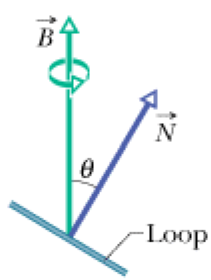


Figure 30-31 Problem 1.

Answer:

0

•2 A certain elastic conducting material is stretched into a circular loop of 12.0 cm radius. It is placed with its plane perpendicular to a uniform 0.800 T magnetic field. When released, the radius of the loop starts to shrink at an instantaneous rate of 75.0 cm/s. What emf is induced in the loop at that instant?

•3 SSM WWW In Fig. 30-32, a 120-turn coil of radius 1.8 cm and resistance 5.3 Ω is coaxial with a solenoid of 220 turns/cm and diameter 3.2 cm. The solenoid current drops from 1.5 A to zero in time interval Δt = 25 ms. What current is induced in the coil during Δt?

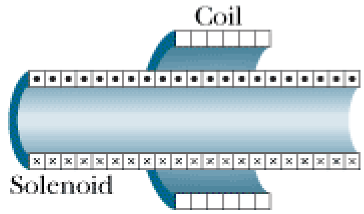


Figure 30-32 Problem 3.

Answer:

30 mA

•4A wire loop of radius 12 cm and resistance 8.5 Ω is located in a uniform magnetic field $\vec{B}$ that changes in magnitude as given in Fig. 30-33. The vertical axis scale is set by B_s = 0.50 T, and the horizontal axis scale is set by t_s = 6.00 s. The loop's plane is perpendicular to $\vec{B}$. What emf is induced in the loop during time intervals (a) 0 to 2.0 s, (b) 2.0 s to 4.0 s, and (c) 4.0 s to 6.0 s?

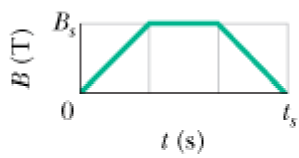


Figure 30-33 Problem 4.

•5In Fig. 30-34, a wire forms a closed circular loop, of radius R = 2.0 m and resistance 4.0 Ω. The circle is centered on a long straight wire; at time t = 0, the current in the long straight wire is 5.0 A rightward. Thereafter, the current changes according to i = 5.0 A - (2.0 A/s^2)t^2. (The straight wire is insulated; so there is no electrical contact between it and the wire of the loop.) What is the magnitude of the current induced in the loop at times t > 0?

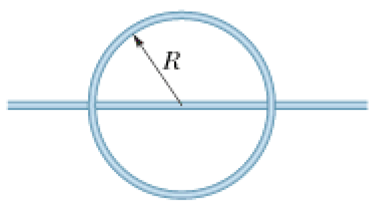


Figure 30-34 Problem 5.

Answer:

0

•6Figure 30-35a shows a circuit consisting of an ideal battery with emf $\mathscr{E}$ = 6.00 μ V, a resistance R, and a small wire loop of area 5.0 cm^2. For the time interval t = 10 s to t = 20 s, an external magnetic field is set up throughout the loop. The field is uniform, its direction is into the page in Fig. 30-35a, and the field magnitude is given by $B = at$, where B is in teslas, a is a constant, and t is in seconds. Figure 30-35b gives the current i in the circuit before, during, and after the external field is set up. The vertical axis scale is set by i_s = 2.0 mA. Find the constant a in the equation for the field magnitude.

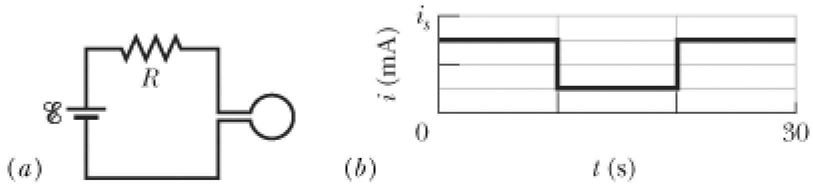


Figure 30-35 Problem 6.

•7In Fig. 30-36, the magnetic flux through the loop increases according to the relation $\Phi_B = 6.0t^2 + 7.0t$, where Φ_B is in milliwebers and t is in seconds. (a) What is the magnitude of the emf induced in the loop when $t = 2.0$ s? (b) Is the direction of the current through R to the right or left?

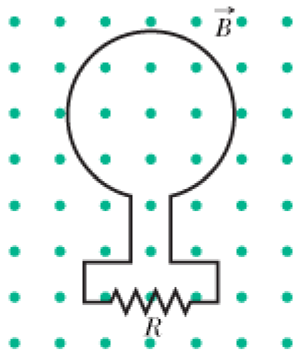


Figure 30-36 Problem 7.

Answer:

(a) 31 mV; (b) left

•8A uniform magnetic field $\vec{B}$ is perpendicular to the plane of a circular loop of diameter 10 cm formed from wire of diameter 2.5 mm and resistivity 1.69×10^{-8} Ω m. At what rate must the magnitude of $\vec{B}$ change to induce a 10 A current in the loop?

•9A small loop of area 6.8 mm^2 is placed inside a long solenoid that has 854 turns/cm and carries a sinusoidally varying current i of amplitude 1.28 A and angular frequency 212 rad/s. The central axes of the loop and solenoid coincide. What is the amplitude of the emf induced in the loop?

Answer:

0.198 mV

••10Figure 30-37 shows a closed loop of wire that consists of a pair of equal semicircles, of radius 3.7 cm, lying in mutually perpendicular planes. The loop was formed by folding a flat circular loop along a diameter until the two halves became perpendicular to each other. A uniform magnetic field $\vec{B}$ of magnitude 76 mT is directed perpendicular to the fold diameter and makes equal angles (of 45°) with the planes of the semicircles. The magnetic field is reduced to zero at a uniform rate during a time interval of 4.5 ms. During this interval, what are the (a) magnitude and (b) direction (clockwise or counterclockwise when viewed along the direction of $\vec{B}$) of the emf induced in the loop?

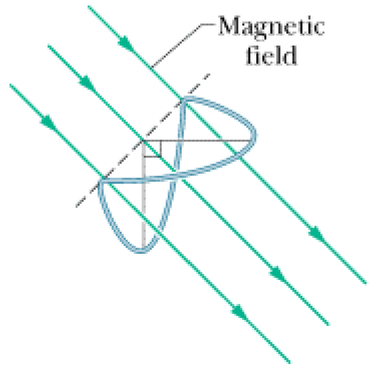


Figure 30-37 Problem 10.

••11A rectangular coil of N turns and of length a and width b is rotated at frequency f in a uniform magnetic field $\vec{B}$, as indicated in Fig. 30-38. The coil is connected to co-rotating cylinders, against which metal brushes slide to make contact. (a) Show that the emf induced in the coil is given (as a function of time t) by

$$\mathcal{E} = 2\pi fNaB\sin(2\pi ft) = \mathcal{E}_0\sin(2\pi ft). \quad (30\text{-}0)$$

This is the principle of the commercial alternating-current generator. (b) What value of Nab gives an emf with $\mathscr{E}_0$ = 150 V when the loop is rotated at 60.0 rev/s in a uniform magnetic field of 0.500 T?

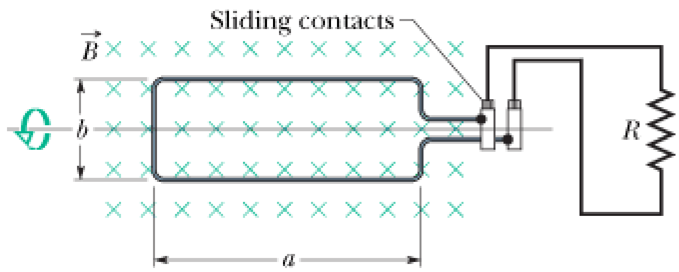


Figure 30-38 Problem 11.

Answer:

(b) 0.796 m^2

••12In Fig. 30-39, a wire loop of lengths L = 40.0 cm and W = 25.0 cm lies in a magnetic field $\vec{B}$. What are the (a) magnitude $\mathscr{E}$ and (b) direction (clockwise or counterclockwise—or “none”if $\mathscr{E}$ = 0) of the emf induced in the loop if $\vec{B} = (4.00 \times 10^{-2} T/m)y\hat{k}$? What are (c) $\mathscr{E}$ and (d) the direction if $\vec{B} = (6.00 \times 10^{-2}\ \mathrm{T/s})t\hat{k}$What are (e) $\mathscr{E}$ and (f) the direction if $\vec{B} = (8.00 \times 10^{-2}\ \mathrm{T/m.s})yt\hat{k}$What are (g) $\mathscr{E}$ and (h) the direction if $\vec{B} = (3.00 \times 10^{-2}\ \mathrm{T/m \cdot s})xt\hat{j}$What are (i) $\mathscr{E}$ and (j) the direction if $\vec{B} = (5.00 \times 10^{-2}\ \mathrm{T/m \cdot s})yt\hat{i}$?

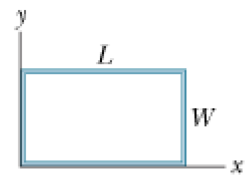


Figure 30-39 Problem 12.

••13 ILW One hundred turns of (insulated) copper wire are wrapped around a wooden cylindrical core of cross-sectional area 1.20×10^{-3} m^2. The two ends of the wire are connected to a resistor. The total resistance in the circuit is 13.0 Ω. If an externally applied uniform longitudinal magnetic field in the core changes from 1.60 T in one direction to 1.60 T in the opposite direction, how much charge flows through a point in the circuit during the change?

Answer:

29.5 mC

••14 In Fig. 30-40*a*, a uniform magnetic field $\vec{B}$ increases in magnitude with time *t* as given by Fig. 30-40*b*, where the vertical axis scale is set by B_s = 9.0 mT and the horizontal scale is set by t_s = 3.0 s. A circular conducting loop of area 8.0×10^{-4} m^2 lies in the field, in the plane of the page. The amount of charge *q* passing point *A* on the loop is given in Fig. 30-40*c* as a function of *t*, with the vertical axis scale set by q_s = 6.0 mC and the horizontal axis scale again set by t_s = 3.0 s. What is the loop's resistance?

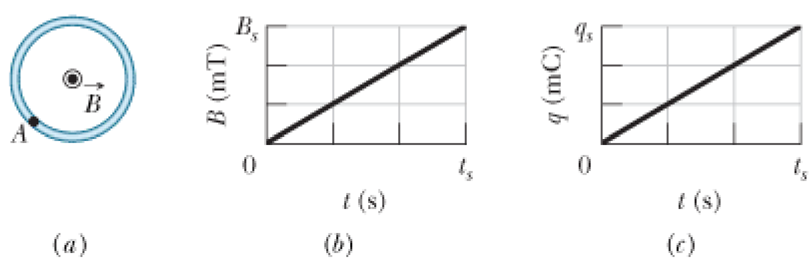


Figure 30-40 Problem 14.

••15 GO A square wire loop with 2.00 m sides is perpendicular to a uniform magnetic field, with half the area of the loop in the field as shown in Fig. 30-41. The loop contains an ideal battery with emf $\mathscr{E}$ = 20.0 V. If the magnitude of the field varies with time according to $B = 0.0420 - 0.870t$, with *B* in teslas and *t* in seconds, what are (a) the net emf in the circuit and (b) the direction of the (net) current around the loop?

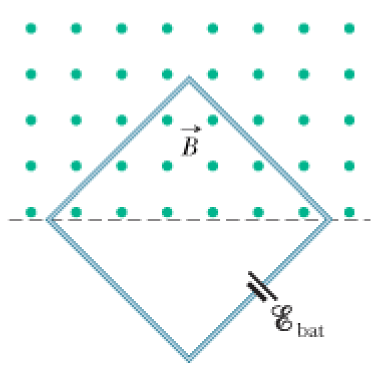


Figure 30-41 Problem 15.

Answer:

(a) 21.7 V; (b) counterclockwise

••16 Figure 30-42*a* shows a wire that forms a rectangle (W = 20 cm, H = 30 cm) and has a resistance of 5.0 mΩ. Its interior is split into three equal areas, with magnetic fields $\vec{B}_1$, $\vec{B}_2$, $\vec{B}_3$. The fields are uniform within each region and directly out of or into the page as indicated. Figure 30-42*b* gives the change in the *z* components B_z of the three fields with time *t*; the vertical axis scale is set by B_s = 4.0 μT and B_b = -2.5B_s, and the horizontal axis scale is set by t_s = 2.0 s. What are the (a) magnitude and (b) direction of the current induced in the wire?

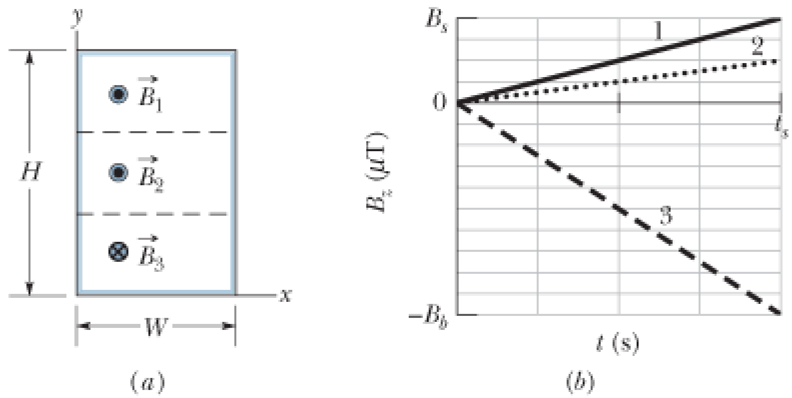


Figure 30-42 Problem 16.

••17 A small circular loop of area 2.00 cm^2 is placed in the plane of, and concentric with, a large circular loop of radius 1.00 m. The current in the large loop is changed at a constant rate from 200 A to -200 A (a change in direction) in a time of 1.00 s, starting at t = 0. What is the magnitude of the magnetic field $\vec{B}$ at the center of the small loop due to the current in the large loop at (a) t = 0, (b) t = 0.500 s, and (c) t = 1.00 s? (d) From t = 0 to t = 1.00 s, is $\vec{B}$ reversed? Because the inner loop is small, assume is uniform over its area. (e) What emf is induced in the small loop at t = 0.500 s?

Answer:

(a) 1.26 × 10^{-4} T; (b) 0; (c) 1.26 × $^{-4}$ T; (d) yes; (e) 5.04 × 10^{-8} V

••18 In Fig. 30-43, two straight conducting rails form a right angle. A conducting bar in contact with the rails starts at the vertex at time t = 0 and moves with a constant velocity of 5.20 m/s along them. A magnetic field with B = 0.350 T is directed out of the page. Calculate (a) the flux through the triangle formed by the rails and bar at t = 3.00 s and (b) the emf around the triangle at that time. (c) If the emf is $\mathscr{E} = at^n$, where a and n are constants, what is the value of n?

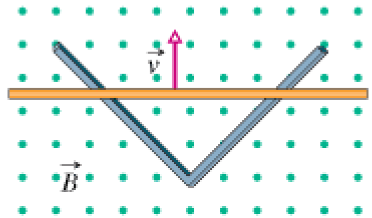


Figure 30-43 Problem 18.

••19 ILW An electric generator contains a coil of 100 turns of wire, each forming a rectangular loop 50.0 cm by 30.0 cm. The coil is placed entirely in a uniform magnetic field with magnitude B = 3.50 T and with $\vec{B}$ initially perpendicular to the coil's plane. What is the maximum value of the emf produced when the coil is spun at 1000 rev/min about an axis perpendicular to $\vec{B}$?

Answer:

5.50 kV

••20 At a certain place, Earth's magnetic field has magnitude B = 0.590 gauss and is inclined downward at an angle of 70.0° to the horizontal. A flat horizontal circular coil of wire with a radius of 10.0 cm has 1000 turns and a total resistance of 85.0 Ω. It is connected in series to a meter with 140 Ω resistance. The coil is flipped through a half-revolution about a diameter, so that it is again horizontal. How much charge flows through the meter during the flip?

••21 In Fig. 30-44, a stiff wire bent into a semicircle of radius a = 2.0 cm is rotated at constant angular speed 40 rev/s in a uniform 20 mT magnetic field. What are the (a) frequency and (b) amplitude of the emf induced in the loop?

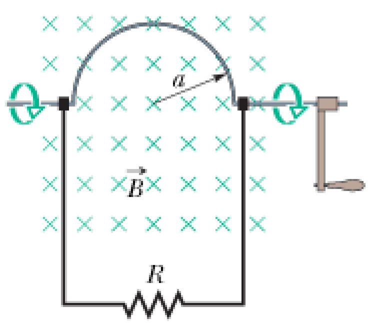


Figure 30-44 Problem 21.

Answer:

(a) 40 Hz; (b) 3.2 mV

••22 A rectangular loop (area = 0.15 m^2) turns in a uniform magnetic field, B = 0.20 T. When the angle between the field and the normal to the plane of the loop is $\pi/2$ rad and increasing at 0.60 rad/s, what emf is induced in the loop?

••23 SSM Figure 30-45 shows two parallel loops of wire having a common axis. The smaller loop (radius r) is above the larger loop (radius R) by a distance $x \gg R$. Consequently, the magnetic field due to the counterclockwise current i in the larger loop is nearly uniform throughout the smaller loop. Suppose that x is increasing at the constant rate $dx/dt = v$. (a) Find an expression for the magnetic flux through the area of the smaller loop as a function of x.(*Hint:* See Eq. 29-27.) In the smaller loop, find (b) an expression for the induced emf and (c) the direction of the induced current.

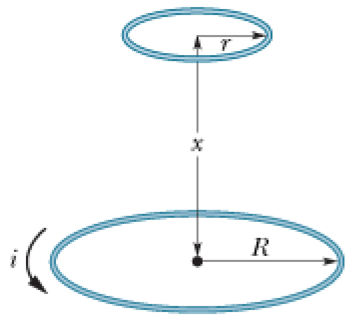


Figure 30-45 Problem 23.

Answer:

(a) $\mu_0 i R^2 \pi r^2/2x^3$; (b) $3\mu_0 i \pi R^2 r^2 v/2x^4$; (c) counterclockwise

••24 A wire is bent into three circular segments, each of radius r = 10 cm, as shown in Fig. 30-46. Each segment is a quadrant of a circle, *ab* lying in the *xy* plane, *bc* lying in the *yz* plane, and *ca* lying in the *zx* plane. (a) If a uniform magnetic field $\vec{B}$ points in the positive x direction, what is the magnitude of the emf developed in the wire when B increases at the rate of 3.0 mT/s? (b) What is the direction of the current in segment *bc*?

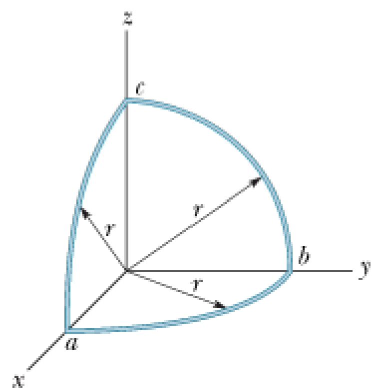


Figure 30-46 Problem 24.

••25 GO Two long, parallel copper wires of diameter 2.5 mm carry currents of 10 A in opposite directions. (a) Assuming that their central axes are 20 mm apart, calculate the magnetic flux per meter of wire that exists in the space between those axes. (b) What percentage of this flux lies inside the wires? (c) Repeat part (a) for parallel currents.

Answer:

(a) 13µWb/m; (b) 17%; (c) 0

•••26 For the wire arrangement in Fig. 30-47, a = 12.0 cm and b = 16.0 cm. The current in the long straight wire is $i = 4.50t^2 - 10.0t$, where i is in amperes and t is in seconds. (a) Find the emf in the square loop at t = 3.00 s. (b) What is the direction of the induced current in the loop?

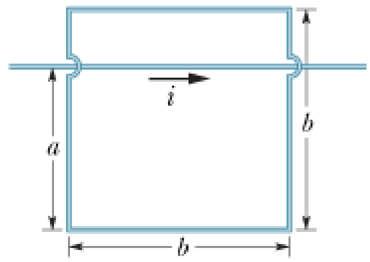


Figure 30-47 Problem 26.

••27 ILW As seen in Fig. 30-48, a square loop of wire has sides of length 2.0 cm. A magnetic field is directed out of the page; its magnitude is given by $B = 4.0t^2y$, where B is in teslas, t is in seconds, and y is in meters. At $t = 2.5$ s, what are the (a) magnitude and (b) direction of the emf induced in the loop?

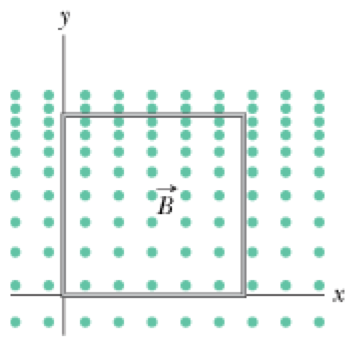


Figure 30-48 Problem 27.

Answer:

(a) 80μV; (b) clockwise

••28 In Fig. 30-49, a rectangular loop of wire with length $a = 2.2$ cm, width $b = 0.80$ cm, and resistance $R = 0.40\ \text{m}\Omega$ is placed near an infinitely long wire carrying current $i = 4.7$ A. The loop is then moved away from the wire at constant speed $v = 3.2$ mm/s. When the center of the loop is at distance $r = 1.5b$, what are (a) the magnitude of the magnetic flux through the loop and (b) the current induced in the loop?

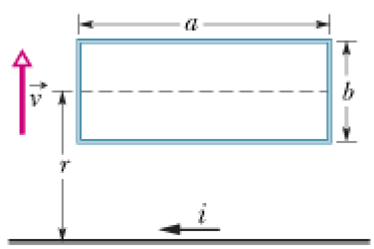


Figure 30-49 Problem 28.

sec. 30-5 Induction and Energy Transfers

•29 In Fig. 30-50, a metal rod is forced to move with constant velocity $\vec{v}$ along two parallel metal rails, connected with a strip of metal at one end. A magnetic field of magnitude $B = 0.350$ T points out of the page. (a) If the rails are separated by $L = 25.0$ cm and the speed of the rod is 55.0 cm/s, what emf is generated? (b) If the rod has a resistance of 18.0Ω and the rails and connector have negligible resistance, what is the current in the rod? (c) At what rate is energy being transferred to thermal energy?

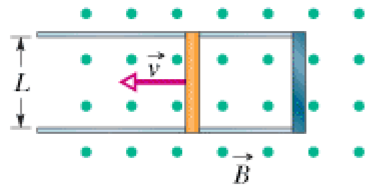


Figure 30-50 Problems 29 and 35.

Answer:

(a) 48.1 mV; (b) 2.67 mA; (c) 0.129 mW

••30 In Fig. 30-51*a*, a circular loop of wire is concentric with a solenoid and lies in a plane perpendicular to the solenoid's central axis. The loop has radius 6.00 cm. The solenoid has radius 2.00 cm, consists of 8000 turns/m, and has a current i_{sol} varying with time t as given in Fig. 30-51*b*, where the vertical axis scale is set by $i_s = 1.00$ A and the horizontal axis scale is set by $t_s = 2.0$ s. Figure 30-51*c* shows, as a function of time, the energy E_{th} that is transferred to thermal energy of the loop; the vertical axis scale is set by $E_s = 100.0$ nJ. What is the loop's resistance?

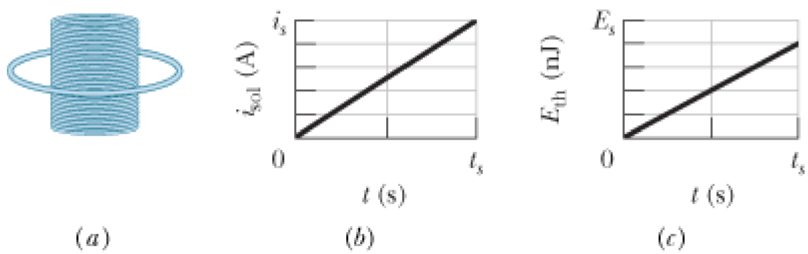


Figure 30-51 Problem 30.

•31 SSM ILW If 50.0 cm of copper wire (diameter = 1.00 mm) is formed into a circular loop and placed perpendicular to a uniform magnetic field that is increasing at the constant rate of 10.0 mT/s, at what rate is thermal energy generated in the loop?

Answer:

3.68μW

•32A loop antenna of area 2.00 cm^2 and resistance 5.21 $\mu\Omega$ is perpendicular to a uniform magnetic field of magnitude 17.0 μT. The field magnitude drops to zero in 2.96 ms. How much thermal energy is produced in the loop by the change in field?

••33 Figure 30-52 shows a rod of length $L = 10.0$ cm that is forced to move at constant speed $v = 5.00$ m/s along horizontal rails. The rod, rails, and connecting strip at the right form a conducting loop. The rod has resistance 0.400 Ω; the rest of the loop has negligible resistance. A current $i = 100$ A through the long straight wire at distance $a = 10.0$ mm from the loop sets up a (nonuniform) magnetic field through the loop. Find the (a) emf and (b) current induced in the loop. (c) At what rate is thermal energy generated in the rod? (d) What is the magnitude of the force that must be applied to the rod to make it move at constant speed? (e) At what rate does this force do work on the rod?

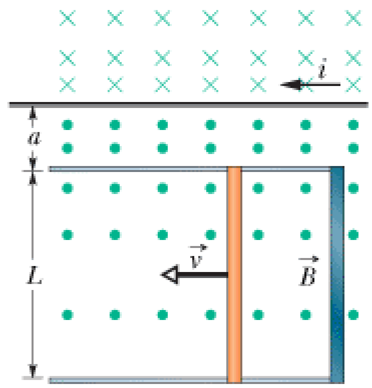


Figure 30-52 Problem 33.

Answer:

(a) 240μV; (b) 0.600 mA; (c) 0.144μW; (d) 2.87 × 10^{-8} N; (e) 0.144µW

••34In Fig. 30-53, a long rectangular conducting loop, of width L, resistance R, and mass m, is hung in a horizontal, uniform magnetic field $\vec{B}$ that is directed into the page and that exists only above line *aa*. The loop is then dropped; during its fall, it accelerates until it reaches a certain terminal speed v_t. Ignoring air drag, find an expression for v_t.

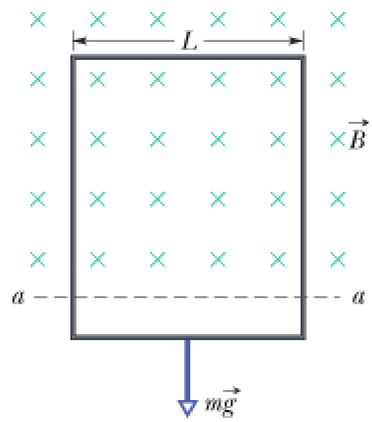


Figure 30-53 Problem 34.

••35The conducting rod shown in Fig. 30-50 has length L and is being pulled along horizontal, frictionless conducting rails at a constant velocity $\vec{v}$. The rails are connected at one end with a metal strip. A uniform magnetic field $\vec{B}$, directed out of the page, fills the region in which the rod moves. Assume that L = 10 cm, v = 5.0 m/s, and B = 1.2 T. What are the (a) magnitude and (b) direction (up or down the page) of the emf induced in the rod? What are the (c) size and (d) direction of the current in the conducting loop? Assume that the resistance of the rod is 0.40 Ω and that the resistance of the rails and metal strip is negligibly small. (e) At what rate is thermal energy being generated in the rod? (f) What external force on the rod is needed to maintain $\vec{v}$? (g) At what rate does this force do work on the rod?

Answer:

(a) 0.60 V; (b) up; (c) 1.5 A; (d) clockwise; (e) 0.90 W; (f) 0.18 N; (g) 0.90 W

sec. 30-6 Induced Electric Fields

•36 Figure 30-54 shows two circular regions R_1 and R_2 with radii r_1 = 20.0 cm and r_2 = 30.0 cm. In R_1 there is a uniform magnetic field of magnitude B_1 = 50.0 mT directed into the page, and in R_2 there is a uniform magnetic field of magnitude B_2 = 75.0 mT directed out of the page (ignore fringing). Both fields are decreasing at the rate of 8.50 mT/s. Calculate $\oint \vec{E} \cdot d\vec{s}$ for (a) path 1, (b) path 2, and (c) path 3.

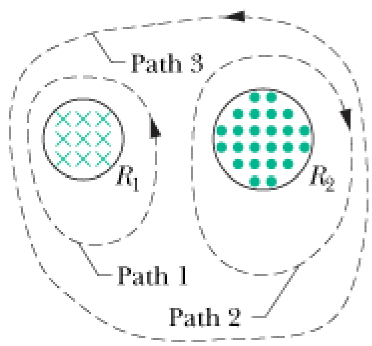


Figure 30-54 Problem 36.

•37 SSM ILW A long solenoid has a diameter of 12.0 cm. When a current i exists in its windings, a uniform magnetic field of magnitude B = 30.0 mT is produced in its interior. By decreasing i, the field is caused to decrease at the rate of 6.50 mT/s. Calculate the magnitude of the induced electric field (a) 2.20 cm and (b) 8.20 cm from the axis of the solenoid.

Answer:

(a) 71.5μV/m; (b) 143μV/m

••38 GO A circular region in an xy plane is penetrated by a uniform magnetic field in the positive direction of the z axis. The field's magnitude B (in teslas) increases with time t (in seconds) according to $B = at$, where a is a constant. The magnitude E of the electric field set up by that increase in the magnetic field is given by Fig. 30-55 versus radial distance r; the vertical axis scale is set by E_s = 300 μN/C, and the horizontal axis scale is set by r_s = 4.00 cm. Find a.

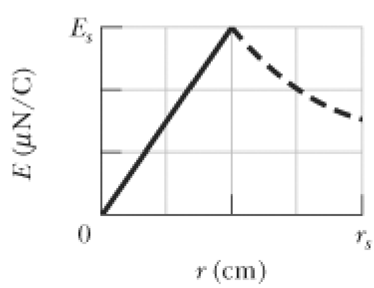


Figure 30-55 Problem 38.

••39 The magnetic field of a cylindrical magnet that has a pole-face diameter of 3.3 cm can be varied sinusoidally between 29.6 T and 30.0 T at a frequency of 15 Hz. (The current in a wire wrapped around a permanent magnet is varied to give this variation in the net field.) At a radial distance of 1.6 cm, what is the amplitude of the electric field induced by the variation?

Answer:

0.15 V/m

sec. 30-7 Inductors and Inductance

•40 The inductance of a closely packed coil of 400 turns is 8.0 mH. Calculate the magnetic flux through the coil when the current is 5.0 mA.

••41 A circular coil has a 10.0 cm radius and consists of 30.0 closely wound turns of wire. An externally produced magnetic field of magnitude 2.60 mT is perpendicular to the coil. (a) If no current is in the coil, what magnetic flux links its turns? (b) When the current in the coil is 3.80 A in a certain direction, the net flux through the coil is found to vanish. What is the inductance of the coil?

Answer:

(a) 2.45 mWb; (b) 0.645 mH

••42 Figure 30-56 shows a copper strip of width $W = 16.0$ cm that has been bent to form a shape that consists of a tube of radius $R = 1.8$ cm plus two parallel flat extensions. Current $i = 35$ mA is distributed uniformly across the width so that the tube is effectively a one-turn solenoid. Assume that the magnetic field outside the tube is negligible and the field inside the tube is uniform. What are (a) the magnetic field magnitude inside the tube and (b) the inductance of the tube (excluding the flat extensions)?

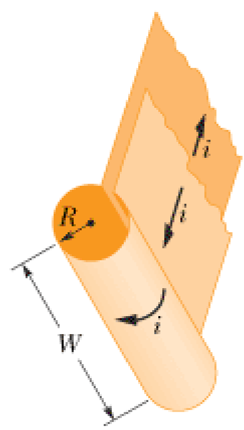


Figure 30-56 Problem 42.

••43 GO Two identical long wires of radius $a = 1.53$ mm are parallel and carry identical currents in opposite directions. Their center-to-center separation is $d = 14.2$ cm. Neglect the flux within the wires but consider the flux in the region between the wires. What is the inductance per unit length of the wires?

Answer:

1.81 μH/m

sec. 30-8 Self-Induction

••44 A 12 H inductor carries a current of 2.0 A. At what rate must the current be changed to produce a 60 V emf in the inductor?

•45 At a given instant the current and self-induced emf in an inductor are directed as indicated in Fig. 30-57. (a) Is the current increasing or decreasing? (b) The induced emf is 17 V, and the rate of change of the current is 25 kA/s; find the inductance.

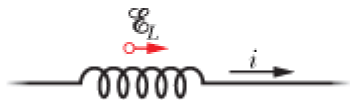


Figure 30-57 Problem 45.

Answer:

(a) decreasing; (b) 0.68 mH

••46 The current i through a 4.6 H inductor varies with time t as shown by the graph of Fig. 30-58, where the vertical axis scale is set by i_s = 8.0 A and the horizontal axis scale is set by t_s = 6.0 ms. The inductor has a resistance of 12 Ω. Find the magnitude of the induced emf $\mathscr{E}$ during time intervals (a) 0 to 2 ms, (b) 2 ms to 5 ms, and (c) 5 ms to 6 ms. (Ignore the behavior at the ends of the intervals.)

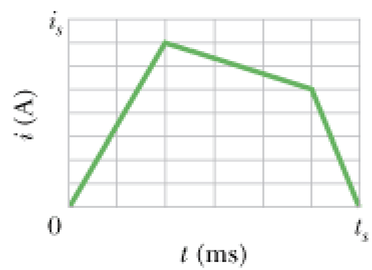


Figure 30-58 Problem 46.

••47*Inductors in series*. Two inductors L_1 and L_2 are connected in series and are separated by a large distance so that the magnetic field of one cannot affect the other. (a) Show that the equivalent inductance is given by

$$L_{eq} = L_1 + L_2.$$

(*Hint:* Review the derivations for resistors in series and capacitors in series. Which is similar here?) (b) What is the generalization of (a) for N inductors in series?

Answer:

(b) $L_{eq} = \Sigma L_j$, sum from j = 1 to $j = N$

••48*Inductors in parallel*. Two inductors L_1 and L_2 are connected in parallel and separated by a large distance so that the magnetic field of one cannot affect the other. (a) Show that the equivalent inductance is given by

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}.$$

(*Hint:* Review the derivations for resistors in parallel and capacitors in parallel. Which is similar here?) (b) What is the generalization of (a) for N inductors in parallel?

••49 The inductor arrangement of Fig. 30-59, with L_1 = 30.0 mH, L_2 = 50.0 mH, L_3 = 20.0 mH, and L_4 = 15.0 mH, is to be connected to a varying current source. What is the equivalent inductance of the arrangement? (First see Problems 47 and 48.)

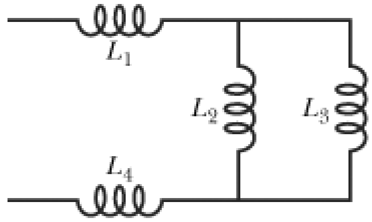


Figure 30-59 Problem 49.

Answer:

59.3 mH

sec. 30-9 *RL* Circuits

••50 The current in an *RL* circuit builds up to one-third of its steady-state value in 5.00 s. Find the inductive time constant.

•51 ILW The current in an *RL* circuit drops from 1.0 A to 10 mA in the first second following removal of the battery from the circuit. If *L* is 10 H, find the resistance *R* in the circuit.

Answer:

46 Ω

•52 The switch in Fig. 30-15 is closed on *a* at time $t = 0$. What is the ratio $\mathscr{E}_L/\mathscr{E}$ of the inductor's self-induced emf to the battery's emf (a) just after $t = 0$ and (b) at $t = 2.00\tau_L$? (c) At what multiple of τ_L will $\mathscr{E}_L/\mathscr{E} = 0.500$?

•53 SSM A solenoid having an inductance of 6.30 μH is connected in series with a 1.20 kΩ resistor. (a) If a 14.0 V battery is connected across the pair, how long will it take for the current through the resistor to reach 80.0% of its final value? (b) What is the current through the resistor at time $t = 1.0\tau_L$?

Answer:

(a) 8.45 ns; (b) 7.37 mA

•54 In Fig. 30-60, $\mathscr{E} = 100$ V, $R_1 = 10.0\ \Omega$, $R_2 = 20.0\ \Omega$, $R_3 = 30.0\Omega$, and $L = 2.00$ H. Immediately after switch S is closed, what are (a) i_1 and (b) i_2? (Let currents in the indicated directions have positive values and currents in the opposite directions have negative values.) A long time later, what are (c) i_1 and (d) i_2? The switch is then reopened. Just then, what are (e) i_1 and (f) i_2? A long time later, what are (g) i_1 and (h) i_2?

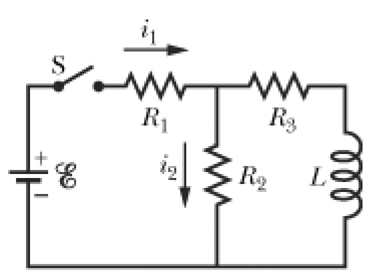


Figure 30-60 Problem 54.

•55 SSM A battery is connected to a series RL circuit at time $t = 0$. At what multiple of τ_L will the current be 0.100% less than its equilibrium value?

Answer:

6.91

•56In Fig. 30-61, the inductor has 25 turns and the ideal battery has an emf of 16 V. Figure 30-62 gives the magnetic flux Φ through each turn versus the current i through the inductor. The vertical axis scale is set by $\Phi_s = 4.0 \times 10^{-4}$ T · m^2, and the horizontal axis scale is set by $i_s = 2.00$ A. If switch S is closed at time $t = 0$, at what rate di/dt will the current be changing at $t = 1.5\ \tau_L$?

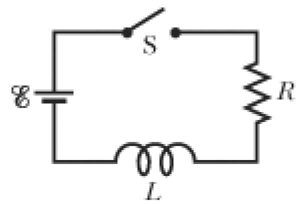


Figure 30-61 Problems 56, 80, 83, and 93.

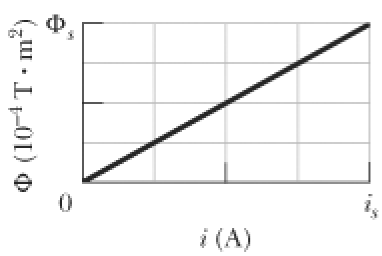


Figure 30-62 Problem 56.

••57 GO In Fig. 30-63, $R = 15\ \Omega$, $L = 5.0$ H, the ideal battery has $\mathscr{E} = 10$ V, and the fuse in the upper branch is an ideal 3.0 A fuse. It has zero resistance as long as the current through it remains less than 3.0 A. If the current reaches 3.0 A, the fuse "blows" and thereafter has infinite resistance. Switch S is closed at time $t = 0$. (a) When does the fuse blow? (*Hint:* Equation 30-41 does not apply. Rethink Eq. 30-39.) (b) Sketch a graph of the current i through the inductor as a function of time. Mark the time at which the fuse blows.

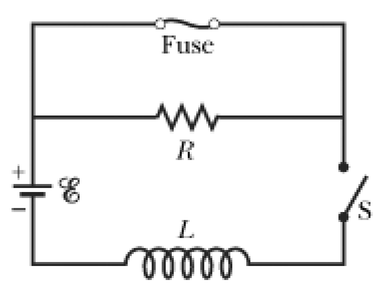


Figure 30-63 Problem 57.

Answer:

(a) 1.5 s

••58 GO Suppose the emf of the battery in the circuit shown in Fig. 30-16 varies with time t so that the current is given by $i(t)$ 3.0 + 5.0t, where i is in amperes and t is in seconds. Take $R = 4.0\ \Omega$ and $L = 6.0$ H, and find an expression for the battery emf as a function of t.(*Hint:* Apply the loop rule.)

•••59 SSM WWW In Fig. 30-64, after switch S is closed at time $t = 0$, the emf of the source is automatically adjusted to maintain a constant current i through S. (a) Find the current through the inductor as a function of time. (b) At what time is the current through the resistor equal to the current through the inductor?

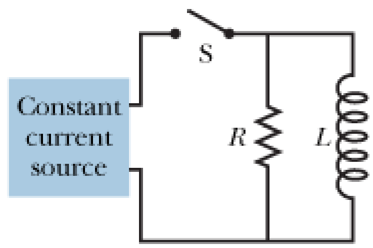


Figure 30-64 Problem 59.

Answer:

(a) $i[1 - \exp(-Rt/L)]$; (b) $(L/R)\ln 2$

•••60A wooden toroidal core with a square cross section has an inner radius of 10 cm and an outer radius of 12 cm. It is wound with one layer of wire (of diameter 1.0 mm and resistance per meter 0.020Ω/m). What are (a) the inductance and (b) the inductive time constant of the resulting toroid? Ignore the thickness of the insulation on the wire.

sec. 30-10 Energy Stored in a Magnetic Field

•61 SSM A coil is connected in series with a 10.0 k resistor. An ideal 50.0 V battery is applied across the two devices, and the current reaches a value of 2.00 mA after 5.00 ms. (a) Find the inductance of the coil. (b) How much energy is stored in the coil at this same moment?

Answer:

(a) 97.9 H; (b) 0.196 mJ

•62A coil with an inductance of 2.0 H and a resistance of 10 Ω is suddenly connected to an ideal battery with $\mathscr{E}$ = 100 V. At 0.10 s after the connection is made, what is the rate at which (a) energy is being stored in the magnetic field, (b) thermal energy is appearing in the resistance, and (c) energy is being delivered by the battery?

•63 ILW At $t = 0$, a battery is connected to a series arrangement of a resistor and an inductor. If the inductive time constant is 37.0 ms, at what time is the rate at which energy is dissipated in the resistor equal to the rate at which energy is stored in the inductor's magnetic field?

Answer:

25.6 ms

•64 At $t = 0$, a battery is connected to a series arrangement of a resistor and an inductor. At what multiple of the inductive time constant will the energy stored in the inductor's magnetic field be 0.500 its steady-state value?

••65 GO For the circuit of Fig. 30-16, assume that $\mathscr{E}$ = 10.0 V, R = 6.70 Ω, and L = 5.50 H. The ideal battery is connected at time t = 0. (a) How much energy is delivered by the battery during the first 2.00 s? (b) How much of this energy is stored in the magnetic field of the inductor? (c) How much of this energy is dissipated in the resistor?

Answer:

(a) 18.7 J; (b) 5.10 J; (c) 13.6 J

sec. 30-11 Energy Density of a Magnetic Field

•66 A circular loop of wire 50 mm in radius carries a current of 100 A. Find the (a) magnetic field strength and (b) energy density at the center of the loop.

•67 SSM A solenoid that is 85.0 cm long has a cross-sectional area of 17.0 cm^2. There are 950 turns of wire carrying a current of 6.60 A. (a) Calculate the energy density of the magnetic field inside the solenoid. (b) Find the total energy stored in the magnetic field there (neglect end effects).

Answer:

(a) 34.2 J/m^3; (b) 49.4 mJ

•68 A toroidal inductor with an inductance of 90.0 mH encloses a volume of 0.0200 m^3. If the average energy density in the toroid is 70.0 J/m^3, what is the current through the inductor?

•69 ILW What must be the magnitude of a uniform electric field if it is to have the same energy density as that possessed by a 0.50 T magnetic field?

Answer:

1.5×10^8 V/m

••70 GO Figure 30-65a shows, in cross section, two wires that are straight, parallel, and very long. The ratio i_1/i_2 of the current carried by wire 1 to that carried by wire 2 is 1/3. Wire 1 is fixed in place. Wire 2 can be moved along the positive side of the x axis so as to change the magnetic energy density u_B set up by the two currents at the origin. Figure 30-65b gives u_B as a function of the position x of wire 2. The curve has an asymptote of u_B = 1.96 nJ/m^3 as $x \to \infty$ and the horizontal axis scale is set by x_s = 60.0 cm. What is the value of (a) i_1 and (b) i_2?

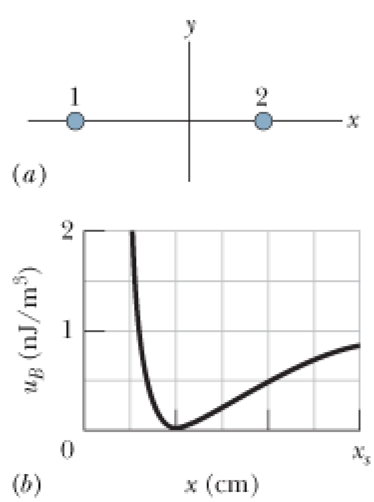


Figure 30-65 Problem 70.

•71 A length of copper wire carries a current of 10 A uniformly distributed through its cross section. Calculate the energy density of (a) the magnetic field and (b) the electric field at the surface of the wire. The wire diameter is 2.5 mm, and its resistance per unit length is 3.3 Ω/km.

Answer:

(a) 1.0 J/m^3; (b) 4.8 × 10^{-15} J/m^3

sec. 30-12 Mutual Induction

••72 Coil 1 has L_1 = 25 mH and N_1 = 100 turns. Coil 2 has L_2 = 40 mH and N_2 = 200 turns. The coils are fixed in place; their mutual inductance M is 3.0 mH. A 6.0 mA current in coil 1 is changing at the rate of 4.0 A/s. (a) What magnetic flux Φ_{12} links coil 1, and (b) what self-induced emf appears in that coil? (c) What magnetic flux Φ_{21} links coil 2, and (d) what mutually induced emf appears in that coil?

•73 SSM Two coils are at fixed locations. When coil 1 has no current and the current in coil 2 increases at the rate 15.0 A/s, the emf in coil 1 is 25.0 mV. (a) What is their mutual inductance? (b) When coil 2 has no current and coil 1 has a current of 3.60 A, what is the flux linkage in coil 2?

Answer:

(a) 1.67 mH; (b) 6.00 mWb

•74 Two solenoids are part of the spark coil of an automobile. When the current in one solenoid falls from 6.0 A to zero in 2.5 ms, an emf of 30 kV is induced in the other solenoid. What is the mutual inductance M of the solenoids?

••75 ILW A rectangular loop of N closely packed turns is positioned near a long straight wire as shown in Fig. 30-66. What is the mutual inductance M for the loop—wire combination if N = 100, a = 1.0 cm, b = 8.0 cm, and l = 30 cm?

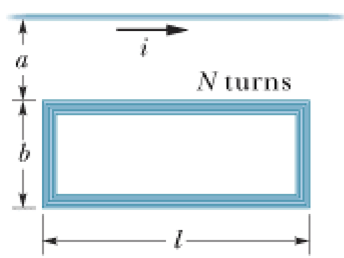


Figure 30-66 Problem 75.

Answer:

13μH

••76 A coil C of N turns is placed around a long solenoid S of radius R and n turns per unit length, as in Fig. 30-67. (a) Show that the mutual inductance for the coil–solenoid combination is given by $M = \mu_0 \pi R^2 nN$. (b) Explain why M does not depend on the shape, size, or possible lack of close packing of the coil.

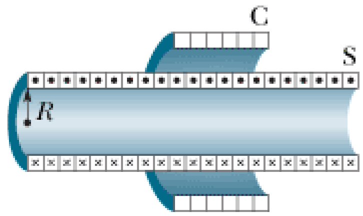


Figure 30-67 Problem 76.

••77 SSM Two coils connected as shown in Fig. 30-68 separately have inductances L_1 and L_2. Their mutual inductance is M. (a) Show that this combination can be replaced by a single coil of equivalent inductance given by

$$L_{eq} = L_1 + L_2 + 2M.$$

(b) How could the coils in Fig. 30-68 be reconnected to yield an equivalent inductance of

$$L_{eq} = L_1 + L_2 - 2M.$$

(This problem is an extension of Problem 47, but the requirement that the coils be far apart has been removed.)

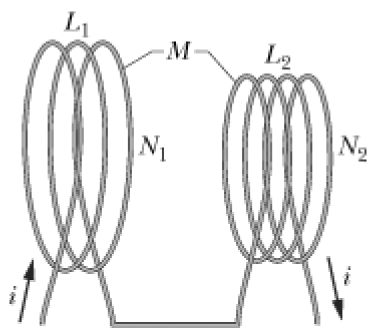


Figure 30-68 Problem 77.

Answer:

(b) have the turns of the two solenoids wrapped in opposite directions

Additional Problems

78 At time $t = 0$, a 12.0 V potential difference is suddenly applied to the leads of a coil of inductance 23.0 mH and a certain resistance R. At time $t = 0.150$ ms, the current through the inductor is changing at the rate of 280 A/s. Evaluate R.

79 SSM In Fig. 30-69, the battery is ideal and $\mathscr{E} = 10$ V, $R_1 = 5.0$, $R_2 = 10\ \Omega$, and $L = 5.0$ H. Switch S is closed at time $t = 0$. Just afterwards, what are (a) i_1, (b) i_2, (c) the current i_S through the switch, (d) the potential difference V_2 across resistor 2, (e) the potential difference V_L across the inductor, and (f) the rate of change di_2/dt? A long time later, what are (g) i_1, (h) i_2, (i) i_S, (j) V_2, (k) V_L, and (1) di_2/dt?

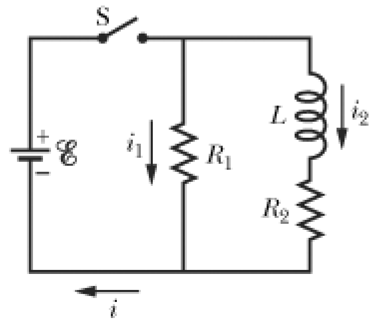


Figure 30-69 Problem 79.

Answer:

(a) 2.0 A; (b) 0; (c) 2.0 A; (d) 0; (e) 10 V; (f) 2.0 A/s; (g) 2.0 A; (h) 1.0 A; (i) 3.0 A; (j) 10 V; (k) 0; (l) 0

80 In Fig. 30-61, $R = 4.0\ k\Omega$, $L = 8.0$ mH, and the ideal battery has $\mathscr{E} = 20$ V. How long after switch S is closed is the current 2.0 mA?

81 SSM Figure 30-70*a* shows a rectangular conducting loop of resistance $R = 0.020\ \Omega$, height $H = 1.5$ cm, and length $D = 2.5$ cm being pulled at constant speed $v = 40$ cm/s through two regions of uniform magnetic field. Figure 30-70*b* gives the current *i* induced in the loop as a function of the position *x* of the right side of the loop. The vertical axis scale is set by $i_s = 3.0\ \mu A$. For example, a current equal to i_s is induced clockwise as the loop enters region 1. What are the (a) magnitude and (b) direction (into or out of the page) of the magnetic field in region 1? What are the (c) magnitude and (d) direction of the magnetic field in region 2?

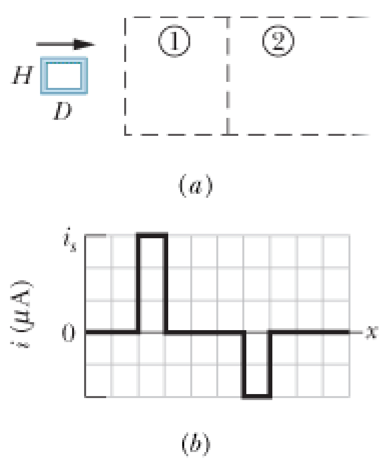


Figure 30-70 Problem 81.

Answer:

(a) 10μT; (b) out; (c) 3.3μT; (d) out

82 A uniform magnetic field $\vec{B}$ is perpendicular to the plane of a circular wire loop of radius *r*. The magnitude of the field varies with time according to $B = B_0 e^{-t/\tau}$, where B_0 and τ are constants. Find an expression for the emf in the loop as a function of time.

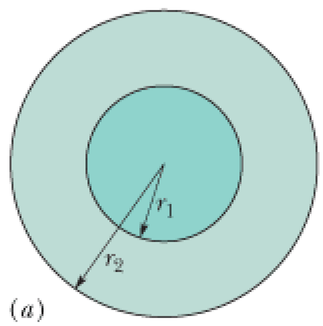
83 Switch S in Fig. 30-61 is closed at time $t = 0$, initiating the buildup of current in the 15.0 mH

inductor and the 20.0 Ω resistor. At what time is the emf across the inductor equal to the potential difference across the resistor?

Answer:

0.520 ms

84 Figure 30-71*a* shows two concentric circular regions in which uniform magnetic fields can change. Region 1, with radius r_1 = 1.0 cm, has an outward magnetic field $\vec{B}_1$ that is increasing in magnitude. Region 2, with radius r_2 = 2.0 cm, has an outward magnetic field $\vec{B}_2$ that may also be changing. Imagine that a conducting ring of radius R is centered on the two regions and then the emf $\mathscr{E}$ around the ring is determined. Figure 30-71*b* gives emf $\mathscr{E}$ as a function of the square R^2 of the ring's radius, to the outer edge of region 2. The vertical axis scale is set by $\mathscr{E}_s$ = 20.0 nV. What are the rates (a) dB_1/dt and (b) dB_2/dt? (c) Is the magnitude of $\vec{B}_2$ increasing, decreasing, or remaining constant?



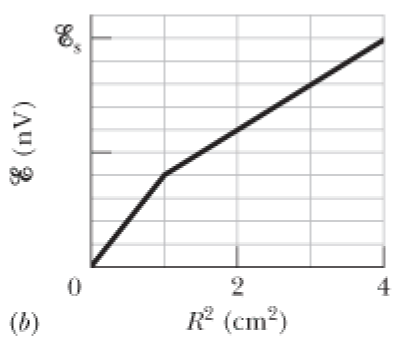


Figure 30-71 Problem 84.

85 SSM Figure 30-72 shows a uniform magnetic field $\vec{B}$ confined to a cylindrical volume of radius R. The magnitude of $\vec{B}$ is decreasing at a constant rate of 10 mT/s. In unit-vector notation, what is the initial acceleration of an electron released at (a) point *a* (radial distance r = 5.0 cm), (b) point *b* (r = 0), and (c) point *c* (r = 5.0 cm)?

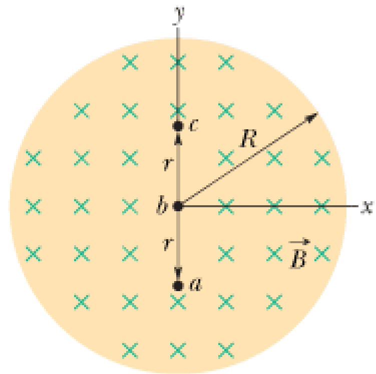


Figure 30-72 Problem 85.

Answer:

(a) $(4.4 \times 10^7\ \text{m/s}^2)\hat{\mathrm{i}}$; (b) 0; (c) $(-4.4 \times 10^7\ \text{m/s}^2)\hat{\mathrm{i}}$

86 GO In Fig. 30-73*a*, switch S has been closed on *A* long enough to establish a steady current in the inductor of inductance L_1 = 5.00 mH and the resistor of resistance R_1 = 25.0 Ω. Similarly, in Fig. 30-73*b*, switch S has been closed on *A* long enough to establish a steady current in the inductor of inductance L_2 = 3.00 mH and the resistor of resistance R_2 = 30.0 Ω. The ratio Φ_{02}/Φ_{01} of the magnetic flux through a turn in inductor 2 to that in inductor 1 is 1.50. At time t = 0, the two switches are closed on *B*. At what time t is the flux through a turn in the two inductors equal?

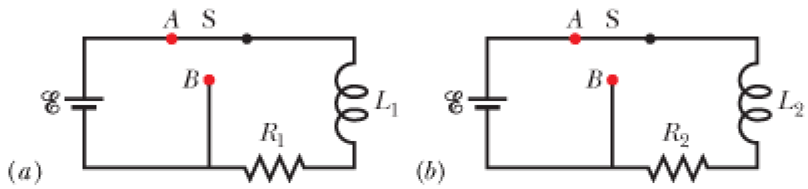


Figure 30-73 Problem 86.

87 SSM A square wire loop 20 cm on a side, with resistance 20 m Ω, has its plane normal to a uniform magnetic field of magnitude B = 2.0 T. If you pull two opposite sides of the loop away from each other, the other two sides automatically draw toward each other, reducing the area enclosed by the loop. If the area is reduced to zero in time Δt = 0.20 s, what are (a) the average emf and (b) the average current induced in the loop during Δt?

Answer:

(a) 0.40 V; (b) 20 A

88 A coil with 150 turns has a magnetic flux of 50.0 nT · m^2 through each turn when the current is 2.00 mA. (a) What is the inductance of the coil? What are the (b) inductance and (c) flux through each turn when the current is increased to 4.00 mA? (d) What is the maximum emf $\mathscr{E}$ across the coil when the current through it is given by i = (3.00 mA) cos (377t), with t in seconds?

89 A coil with an inductance of 2.0 H and a resistance of 10 Ω is suddenly connected to an ideal battery with $\mathscr{E}$ = 100 V. (a) What is the equilibrium current? (b) How much energy is stored in the magnetic field when this current exists in the coil?

Answer:

(a) 10 A; (b) 1.0×10^2 J

90 How long would it take, following the removal of the battery, for the potential difference across the resistor in an *RL* circuit (with $L = 2.00$ H, $R = 3.00\ \Omega$) to decay to 10.0% of its initial value?

91 SSM In the circuit of Fig. 30-74, $R_1 = 20$ kΩ, $R_2 = 20\ \Omega$, $L = 50$ mH, and the ideal battery has $\mathscr{E} = 40$ V. Switch S has been open for a long time when it is closed at time $t = 0$. Just after the switch is closed, what are (a) the current i_{bat} through the battery and (b) the rate di_{bat}/dt? At $t = 3.0$ ms, what are (c) i_{bat} and (d) di_{bat}/dt? A long time later, what are (e) i_{bat} and (f) di_{bat}/dt?

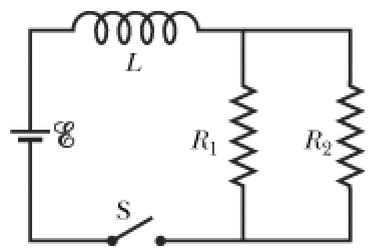


Figure 30-74 Problem 91.

Answer:

(a) 0; (b) 8.0×10^2 A/s; (c) 1.8 mA; (d) 4.4×10^2 A/s; (e) 4.0 mA; (f) 0

92 The flux linkage through a certain coil of 0.75Ω resistance would be 26 mWb if there were a current of 5.5 A in it. (a) Calculate the inductance of the coil. (b) If a 6.0 V ideal battery were suddenly connected across the coil, how long would it take for the current to rise from 0 to 2.5 A?

93 In Fig. 30-61, a 12.0 V ideal battery, a 20.0 Ω resistor, and an inductor are connected by a switch at time $t = 0$. At what rate is the battery transferring energy to the inductor's field at $t = 1.61\tau_L$?

Answer:

1.15 W

94 A long cylindrical solenoid with 100 turns/cm has a radius of 1.6 cm. Assume that the magnetic field it produces is parallel to its axis and is uniform in its interior. (a) What is its inductance per meter of length? (b) If the current changes at the rate of 13 A/s, what emf is induced per meter?

95 In Fig. 30-75, $R_1 = 8.0\ \Omega$, $R_2 = 10\ \Omega$, $L_1 = 0.30$ H, $L_2 = 0.20$ H, and the ideal battery has $\mathscr{E} = 6.0$ V. (a) Just after switch S is closed, at what rate is the current in inductor 1 changing? (b) When the circuit is in the steady state, what is the current in inductor 1?

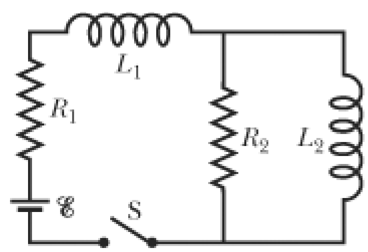


Figure 30-75 Problem 95.

Answer:

(a) 20 A/s; (b) 0.75 A

96 A square loop of wire is held in a uniform 0.24 T magnetic field directed perpendicular to the plane of the loop. The length of each side of the square is decreasing at a constant rate of 5.0 cm/s. What emf is induced in the loop when the length is 12 cm?

97 At time $t = 0$, a 45 V potential difference is suddenly applied to the leads of a coil with inductance $L = 50$ mH and resistance $R = 180\ \Omega$. At what rate is the current through the coil increasing at $t = 1.2$ ms?

Answer:

12 A/s

98 The inductance of a closely wound coil is such that an emf of 3.00 mV is induced when the current changes at the rate of 5.00 A/s. A steady current of 8.00 A produces a magnetic flux of 40.0 μWb through each turn. (a) Calculate the inductance of the coil. (b) How many turns does the coil have?

sec. 31-2 *LC* Oscillations, Qualitatively

•1 An oscillating *LC* circuit consists of a 75.0 mH inductor and a 3.60 μF capacitor. If the maximum charge on the capacitor is 2.90 μC, what are (a) the total energy in the circuit and (b) the maximum current?

Answer:

(a) 1.17 μJ; (b) 5.58 mA

•2 The frequency of oscillation of a certain *LC* circuit is 200 kHz. At time $t = 0$, plate *A* of the capacitor has maximum positive charge. At what earliest time $t > 0$ will (a) plate *A* again have maximum positive charge, (b) the other plate of the capacitor have maximum positive charge, and (c) the inductor have maximum magnetic field?

•3 In a certain oscillating *LC* circuit, the total energy is converted from electrical energy in the capacitor to magnetic energy in the inductor in 1.50 μs. What are (a) the period of oscillation and (b) the frequency of oscillation? (c) How long after the magnetic energy is a maximum will it be a maximum again?

Answer:

(a) 6.00 μs; (b) 167 kHz; (c) 3.00 μs

•4 What is the capacitance of an oscillating *LC* circuit if the maximum charge on the capacitor is 1.60 μC and the total energy is 140 μJ?

•5 In an oscillating *LC* circuit, $L = 1.10$ mH and $C = 4.00\ \mu$F. The maximum charge on the capacitor is 3.00 μC. Find the maximum current.

Answer:

45.2 mA

sec. 31-3 The Electrical–Mechanical Analogy

•6 A 0.50 kg body oscillates in SHM on a spring that, when extended 2.0 mm from its equilibrium position, has an 8.0 N restoring force. What are (a) the angular frequency of oscillation, (b) the period of oscillation, and (c) the capacitance of an *LC* circuit with the same period if *L* is 5.0 H?

••7 SSM The energy in an oscillating *LC* circuit containing a 1.25 H inductor is 5.70 μJ. The maximum charge on the capacitor is 175 μC. For a mechanical system with the same period, find the (a) mass, (b) spring constant, (c) maximum displacement, and (d) maximum speed.

Answer:

(a) 1.25 kg; (b) 372 N/m; (c) 1.75×10^{-4} m; (d) 3.02 mm/s

sec. 31-4 *LC* Oscillations, Quantitatively

•8 A single loop consists of inductors (L_1, L_2, …), capacitors (C_1, C_2, …), and resistors (R_1, R_2, …) connected in series as shown, for example, in Fig. 31-26*a*. Show that regardless of the sequence of these circuit elements in the loop, the behavior of this circuit is identical to that of the simple *LC* circuit shown in Fig. 31-26*b*. (*Hint:* Consider the loop rule and see Problem 47 in Chapter 30.)

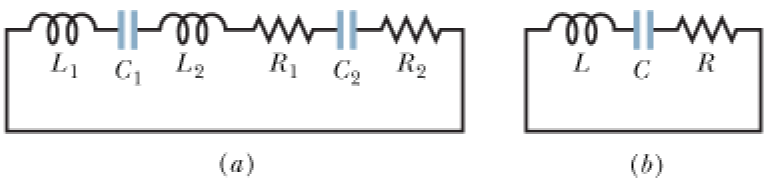


Figure 31-26 Problem 8.

•9 ILW In an oscillating *LC* circuit with L = 50 mH and C = 4.0 μF, the current is initially a maximum. How long will it take before the capacitor is fully charged for the first time?

Answer:

7.0×10^{-4} s

•10 *LC* oscillators have been used in circuits connected to loudspeakers to create some of the sounds of electronic music. What inductance must be used with a 6.7 μF capacitor to produce a frequency of 10 kHz, which is near the middle of the audible range of frequencies?

••11 SSM WWW A variable capacitor with a range from 10 to 365 pF is used with a coil to form a variable-frequency *LC* circuit to tune the input to a radio. (a) What is the ratio of maximum frequency to minimum frequency that can be obtained with such a capacitor? If this circuit is to obtain frequencies from 0.54 MHz to 1.60 MHz, the ratio computed in (a) is too large. By adding a capacitor in parallel to the variable capacitor, this range can be adjusted. To obtain the desired frequency range, (b) what capacitance should be added and (c) what inductance should the coil have?

Answer:

(a) 6.0; (b) 36 pF; (c) 0.22 mH

••12 In an oscillating LC circuit, when 75.0% of the total energy is stored in the inductor's magnetic field, (a) what multiple of the maximum charge is on the capacitor and (b) what multiple of the maximum current is in the inductor?

••13 In an oscillating LC circuit, $L = 3.00$ mH and $C = 2.70\ \mu$F. At $t = 0$ the charge on the capacitor is zero and the current is 2.00 A. (a) What is the maximum charge that will appear on the capacitor? (b) At what earliest time $t > 0$ is the rate at which energy is stored in the capacitor greatest, and (c) what is that greatest rate?

Answer:

(a) 0.180 mC; (b) 70.7 μs; (c) 66.7 W

••14 To construct an oscillating LC system, you can choose from a 10 mH inductor, a 5.0 μF capacitor, and a 2.0 μF capacitor. What are the (a) smallest, (b) second smallest, (c) second largest, and (d) largest oscillation frequency that can be set up by these elements in various combinations?

••15 ILW An oscillating LC circuit consisting of a 1.0 nF capacitor and a 3.0 mH coil has a maximum voltage of 3.0 V. What are (a) the maximum charge on the capacitor, (b) the maximum current through the circuit, and (c) the maximum energy stored in the magnetic field of the coil?

Answer:

(a) 3.0 nC; (b) 1.7 mA; (c) 4.5 nJ

••16 An inductor is connected across a capacitor whose capacitance can be varied by turning a knob. We wish to make the frequency of oscillation of this LC circuit vary linearly with the angle of rotation of the knob, going from 2×10^5 to 4×10^5 Hz as the knob turns through 180°. If $L = 1.0$ mH, plot the required capacitance C as a function of the angle of rotation of the knob.

••17 GO ILW In Fig. 31-27, $R = 14.0\ \Omega$, $C = 6.20\ \mu$F, and $L = 54.0$ mH, and the ideal battery has emf $\mathscr{E} = 34.0$ V. The switch is kept at a for a long time and then thrown to position b. What are the (a) frequency and (b) current amplitude of the resulting oscillations?

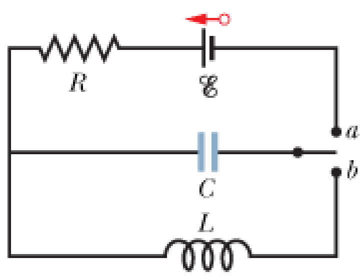


Figure 31-27 Problem 17.

Answer:

(a) 275 Hz; (b) 365 mA

••18 An oscillating LC circuit has a current amplitude of 7.50 mA, a potential amplitude of 250 mV, and a capacitance of 220 nF. What are (a) the period of oscillation, (b) the maximum energy stored in the capacitor, (c) the maximum energy stored in the inductor, (d) the maximum rate at which the current changes, and (e) the maximum rate at which the inductor gains energy?

••19 Using the loop rule, derive the differential equation for an *LC* circuit (Eq. 31-11).

••20 GO In an oscillating *LC* circuit in which $C = 4.00\ \mu$F, the maximum potential difference across the capacitor during the oscillations is 1.50 V and the maximum current through the inductor is 50.0 mA. What are (a) the inductance *L* and (b) the frequency of the oscillations? (c) How much time is required for the charge on the capacitor to rise from zero to its maximum value?

••21 ILW In an oscillating *LC* circuit with $C = 64.0\ \mu$F, the current is given by $i = (1.60)\sin(2500t + 0.680)$, where *t* is in seconds, *i* in amperes, and the phase constant in radians. (a) How soon after $t = 0$ will the current reach its maximum value? What are (b) the inductance *L* and (c) the total energy?

Answer:

(a) 356 μs; (b) 2.50 mH; (c) 3.20 mJ

••22 A series circuit containing inductance L_1 and capacitance C_1 oscillates at angular frequency ω. A second series circuit, containing inductance L_2 and capacitance C_2, oscillates at the same angular frequency. In terms of ω, what is the angular frequency of oscillation of a series circuit containing all four of these elements? Neglect resistance. (*Hint:* Use the formulas for equivalent capacitance and equivalent inductance; see Section 25-4 and Problem 47 in Chapter 30.)

••23 In an oscillating *LC* circuit, $L = 25.0$ mH and $C = 7.80\ \mu$F. At time $t = 0$ the current is 9.20 mA, the charge on the capacitor is 3.80 μC, and the capacitor is charging. What are (a) the total energy in the circuit, (b) the maximum charge on the capacitor, and (c) the maximum current? (d) If the charge on the capacitor is given by $q = Q\cos(\omega t +)$, what is the phase angle ? (e) Suppose the data are the same, except that the capacitor is discharging at $t = 0$. What then is ?

Answer:

(a) 1.98 μJ; (b) 5.56 μC; (c) 12.6 mA; (d) -46.9°; (e) +46.9°

sec. 31-5 Damped Oscillations in an *RLC* Circuit

••24 GO A single-loop circuit consists of a 7.20 Ω resistor, a 12.0 H inductor, and a 3.20 μF capacitor. Initially the capacitor has a charge of 6.20 μC and the current is zero. Calculate the charge on the capacitor *N* complete cycles later for (a) $N = 5$, (b) $N = 10$, and (c) $N = 100$.

••25 ILW What resistance *R* should be connected in series with an inductance $L = 220$ mH and capacitance $C = 12.0\ \mu$F for the maximum charge on the capacitor to decay to 99.0% of its initial value in 50.0 cycles? (Assume $\omega' \approx \omega$.)

Answer:

8.66 m Ω

••26 In an oscillating series *RLC* circuit, find the time required for the maximum energy present in the capacitor during an oscillation to fall to half its initial value. Assume $q = Q$ at $t = 0$.

•••27 SSM In an oscillating series *RLC* circuit, show that $\Delta U/U$, the fraction of the energy lost per cycle of oscillation, is given to a close approximation by $2\pi R/\omega L$. The quantity $\omega L/R$ is often called the *Q* of the circuit (for *quality*). A high-*Q* circuit has low resistance and a low fractional energy loss ($= 2\pi/Q$) per cycle.

sec. 31-8 Three Simple Circuits

•28 A 1.50 μF capacitor is connected as in Fig. 31-10 to an ac generator with $\mathscr{E}_m = 30.0$ V. What is the amplitude of the resulting alternating current if the frequency of the emf is (a) 1.00 kHz and (b) 8.00 kHz?

•29 ILW A 50.0 mH inductor is connected as in Fig. 31-12 to an ac generator with $\mathscr{E}_m = 30.0$ V. What is the amplitude of the resulting alternating current if the frequency of the emf is (a) 1.00 kHz and (b) 8.00 kHz?

Answer:

(a) 95.5 mA; (b) 11.9 mA

•30 A 50.0 Ω resistor is connected as in Fig. 31-8 to an ac generator with $\mathscr{E}_m = 30.0$ V. What is the amplitude of the resulting alternating current if the frequency of the emf is (a) 1.00 kHz and (b) 8.00 kHz?

•31 (a) At what frequency would a 6.0 mH inductor and a 10 μF capacitor have the same reactance? (b) What would the reactance be? (c) Show that this frequency would be the natural frequency of an oscillating circuit with the same L and C.

Answer:

(a) 0.65 kHz; (b) 24 Ω

••32 GO An ac generator has emf $\mathscr{E} = \mathscr{E}_m \sin \omega_d t$, with $\mathscr{E}_m = 25.0$ V and $\omega_d = 377$ rad/s. It is connected to a 12.7 H inductor. (a) What is the maximum value of the current? (b) When the current is a maximum, what is the emf of the generator? (c) When the emf of the generator is -12.5 V and increasing in magnitude, what is the current?

••33 SSM An ac generator has emf $\mathscr{E} = \mathscr{E}_m \sin(\omega_d t - \pi/4)$, where $\mathscr{E}_m = 30.0$ V and $\omega_d = 350$ rad/s. The current produced in a connected circuit is $i(t) = I \sin(\omega_d t - 3\pi/4)$, where $I = 620$ mA. At what time after $t = 0$ does (a) the generator emf first reach a maximum and (b) the current first reach a maximum? (c) The circuit contains a single element other than the generator. Is it a capacitor, an inductor, or a resistor? Justify your answer. (d) What is the value of the capacitance, inductance, or resistance, as the case may be?

Answer:

(a) 6.73 ms; (b) 11.2 ms; (c) inductor; (d) 138 mH

••34 GO An ac generator with emf $\mathscr{E} = \mathscr{E}_m \sin \omega_d t$, where $\mathscr{E}_m = 25.0$ V and $\omega_d = 377$ rad/s, is connected to a 4.15 μF capacitor. (a) What is the maximum value of the current? (b) When the current is a maximum, what is the emf of the generator? (c) When the emf of the generator is -12.5 V and increasing in magnitude, what is the current?

sec. 31-9 The Series *RLC* Circuit

•35 ILW A coil of inductance 88 mH and unknown resistance and a 0.94 μF capacitor are connected in series with an alternating emf of frequency 930 Hz. If the phase constant between the applied voltage and the current is 75°, what is the resistance of the coil?

Answer:

89 Ω

•36 An alternating source with a variable frequency, a capacitor with capacitance C, and a resistor with resistance R are connected in series. Figure 31-28 gives the impedance Z of the circuit versus the driving angular frequency ω_d; the curve reaches an asymptote of 500 Ω, and the horizontal scale is set by ω_{ds} = 300 rad/s. The figure also gives the reactance X_C for the capacitor versus ω_d. What are (a) R and (b) C?

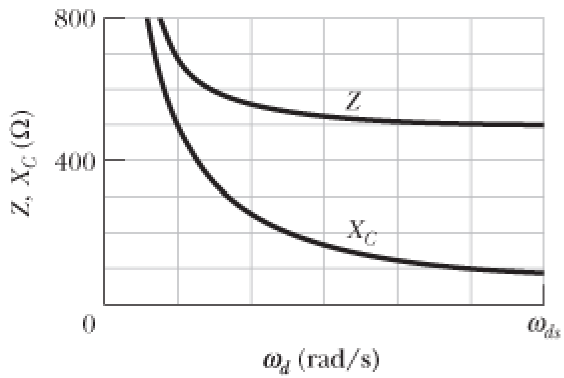


Figure 31-28 Problem 36.

•**37**An electric motor has an effective resistance of 32.0 Ω and an inductive reactance of 45.0 Ω when working under load. The rms voltage across the alternating source is 420 V. Calculate the rms current.

Answer:

7.61 A

•**38**The current amplitude I versus driving angular frequency ω_d for a driven RLC circuit is given in Fig. 31-29, where the vertical axis scale is set by I_s = 4.00 A. The inductance is 200 μH, and the emf amplitude is 8.0 V. What are (a) C and (b) R?

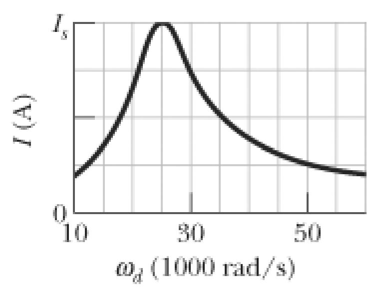


Figure 31-29 Problem 38.

•**39**Remove the inductor from the circuit in Fig. 31-7 and set R = 200 Ω, C = 15.0 μF, f_d = 60.0 Hz, and $\mathscr{E}_m$ = 36.0 V. What are (a) Z, (b) , and (c) I? (d) Draw a phasor diagram.

Answer:

(a) 267 Ω; (b) -41.5°; (c) 135 mA

•**40**An alternating source drives a series RLC circuit with an emf amplitude of 6.00 V, at a phase angle of +30.0°. When the potential difference across the capacitor reaches its maximum positive value of +5.00 V, what is the potential difference across the inductor (sign included)?

•**41** SSM In Fig. 31-7, set R = 200 Ω, C = 70.0 μF, L = 230 mH, f_d = 60.0 Hz, and $\mathscr{E}_m$ = 36.0 V.

What are (a) Z, (b) , and (c) I? (d) Draw a phasor diagram.

Answer:

(a) 206 Ω; (b) 13.7°; (c) 175 mA

•42 An alternating source with a variable frequency, an inductor with inductance L, and a resistor with resistance R are connected in series. Figure 31-30 gives the impedance Z of the circuit versus the driving angular frequency ω_d, with the horizontal axis scale set by ω_{ds} = 1600 rad/s. The figure also gives the reactance X_L for the inductor versus ω_d. What are (a) R and (b) L?

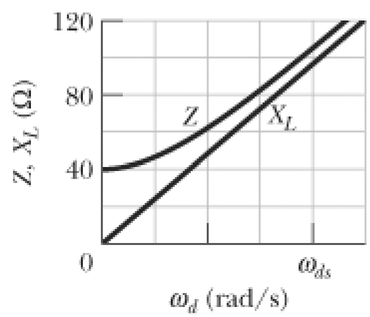


Figure 31-30 Problem 42.

•43 Remove the capacitor from the circuit in Fig. 31-7 and set R = 200 Ω, L = 230 mH, f_d = 60.0 Hz, and $\mathscr{E}_m$ = 36.0 V. What are (a) Z, (b) , and (c) I? (d) Draw a phasor diagram.

Answer:

(a) 218 Ω; (b) 23.4°; (c) 165 mA

••44 GO An ac generator with $\mathscr{E}_m$ = 220 V and operating at 400 Hz causes oscillations in a series RLC circuit having R = 220 Ω, L = 150 mH, and C = 24.0 μF. Find (a) the capacitive reactance X_C, (b) the impedance Z, and (c) the current amplitude I. A second capacitor of the same capacitance is then connected in series with the other components. Determine whether the values of (d) X_C, (e) Z, and (f) I increase, decrease, or remain the same.

••45 GO ILW (a) In an RLC circuit, can the amplitude of the voltage across an inductor be greater than the amplitude of the generator emf? (b) Consider an RLC circuit with $\mathscr{E}_m$ = 10 V, R = 10 Ω, L = 1.0 H, and C = 1.0 μF. Find the amplitude of the voltage across the inductor at resonance.

Answer:

(a) yes; (b) 1.0 kV

••46 An alternating emf source with a variable frequency f_d is connected in series with a 50.0 Ω resistor and a 20.0 μF capacitor. The emf amplitude is 12.0 V. (a) Draw a phasor diagram for phasor V_R (the potential across the resistor) and phasor V_C (the potential across the capacitor). (b) At what driving frequency f_d do the two phasors have the same length? At that driving frequency, what are (c) the phase angle in degrees, (d) the angular speed at which the phasors rotate, and (e) the current amplitude?

••47 SSM WWW An RLC circuit such as that of Fig. 31-7 has R = 5.00 Ω, C = 20.0 μF, L = 1.00 H, and $\mathscr{E}_m$ = 30.0 V. (a) At what angular frequency ω_d will the current amplitude have its maximum value, as in the resonance curves of Fig. 31-16? (b) What is this maximum value? At what (c) lower angular frequency ω_{d1} and (d) higher angular frequency ω_{d2} will the current amplitude be half this maximum value? (e) For the resonance curve for this circuit, what is the fractional half-width $(\omega_{d1} - \omega_{d2})/\omega$?

Answer:

(a) 224 rad/s; (b) 6.00 A; (c) 219 rad/s; (d) 228 rad/s; (e) 0.040

••48 Figure 31-31 shows a driven *RLC* circuit that contains two identical capacitors and two switches. The emf amplitude is set at 12.0 V, and the driving frequency is set at 60.0 Hz. With both switches open, the current leads the emf by 30.9°. With switch S_1 closed and switch S_2 still open, the emf leads the current by 15.0°. With both switches closed, the current amplitude is 447 mA. What are (a) *R*, (b) *C*, and (c) *L*?

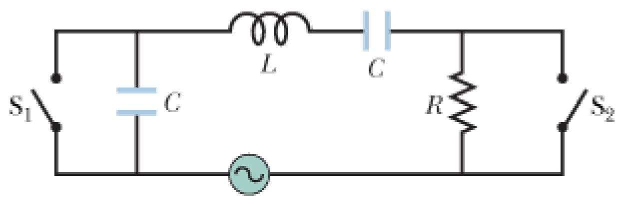


Figure 31-31 Problem 48.

••49 In Fig. 31-32, a generator with an adjustable frequency of oscillation is connected to resistance $R = 100\ \Omega$, inductances $L_1 = 1.70$ mH and $L_2 = 2.30$ mH, and capacitances $C_1 = 4.00\ \mu$F, $C_2 = 2.50\ \mu$F, and $C_3 = 3.50\ \mu$F. (a) What is the resonant frequency of the circuit? (*Hint:* See Problem 47 in Chapter 30.) What happens to the resonant frequency if (b) *R* is increased, (c) L_1 is increased, and (d) C_3 is removed from the circuit?

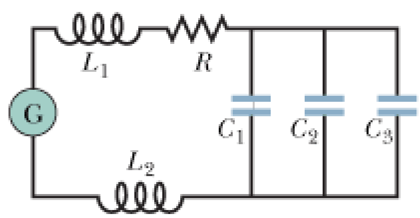


Figure 31-32 Problem 49.

Answer:

(a) 796 Hz; (b) no change; (c) decreased; (d) increased

••50 An alternating emf source with a variable frequency f_d is connected in series with an 80.0 Ω resistor and a 40.0 mH inductor. The emf amplitude is 6.00 V. (a) Draw a phasor diagram for phasor V_R (the potential across the resistor) and phasor V_L (the potential across the inductor). (b) At what driving frequency f_d do the two phasors have the same length? At that driving frequency, what are (c) the phase angle in degrees, (d) the angular speed at which the phasors rotate, and (e) the current amplitude?

••51 SSM The fractional half-width $\Delta\omega_d$ of a resonance curve, such as the ones in Fig. 31-16, is the width of the curve at half the maximum value of *I*. Show that $\Delta\omega_d/\omega = R(3C/L)^{1/2}$, where ω is the angular frequency at resonance. Note that the ratio $\Delta\omega_d/\omega$ increases with *R*, as Fig. 31-16 shows.

sec. 31-10 Power in Alternating-Current Circuits

•52 An ac voltmeter with large impedance is connected in turn across the inductor, the capacitor, and the resistor in a series circuit having an alternating emf of 100 V (rms); the meter gives the same reading in volts in each case. What is this reading?

•53 SSM An air conditioner connected to a 120 V rms ac line is equivalent to a 12.0 Ω resistance and a 1.30 Ω inductive reactance in series. Calculate (a) the impedance of the air conditioner and (b) the average rate at which energy is supplied to the appliance.

Answer:

(a) 12.1 Ω; (b) 1.19 kW

•54 What is the maximum value of an ac voltage whose rms value is 100 V?

•55 What direct current will produce the same amount of thermal energy, in a particular resistor, as an alternating current that has a maximum value of 2.60 A?

Answer:

1.84 A

••56 A typical light dimmer used to dim the stage lights in a theater consists of a variable inductor L (whose inductance is adjustable between zero and L_{max}) connected in series with a lightbulb B, as shown in Fig. 31-33. The electrical supply is 120 V (rms) at 60.0 Hz; the light-bulb is rated at 120 V, 1000 W. (a) What L_{max} is required if the rate of energy dissipation in the lightbulb is to be varied by a factor of 5 from its upper limit of 1000 W? Assume that the resistance of the lightbulb is independent of its temperature. (b) Could one use a variable resistor (adjustable between zero and R_{max}) instead of an inductor? (c) If so, what R_{max} is required? (d) Why isn't this done?

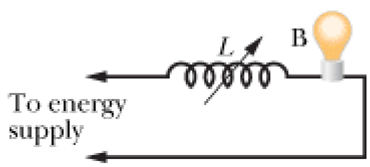


Figure 31-33 Problem 56.

••57 In an *RLC* circuit such as that of Fig. 31-7 assume that $R = 5.00\ \Omega$, $L = 60.0$ mH, $f_d = 60.0$ Hz, and $\mathscr{E}_m = 30.0$ V. For what values of the capacitance would the average rate at which energy is dissipated in the resistance be (a) a maximum and (b) a minimum? What are (c) the maximum dissipation rate and the corresponding (d) phase angle and (e) power factor? What are (f) the minimum dissipation rate and the corresponding (g) phase angle and (h) power factor?

Answer:

(a) 117 μF; (b) 0; (c) 90.0 W; (d) 0°; (e) 1; (f) 0; (g) -90°; (h) 0

••58 For Fig. 31-34, show that the average rate at which energy is dissipated in resistance R is a maximum when R is equal to the internal resistance r of the ac generator. (In the text discussion we tacitly assumed that $r = 0$.)

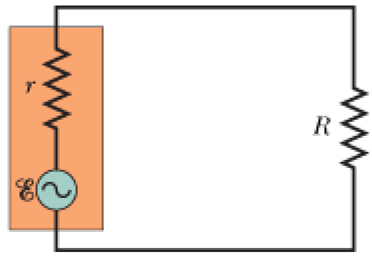


Figure 31-34 Problems 58 and 66.

••59 In Fig. 31-7, $R = 15.0\ \Omega$, $C = 4.70\ \mu F$, and $L = 25.0$ mH. The generator provides an emf with rms voltage 75.0 V and frequency 550 Hz. (a) What is the rms current? What is the rms voltage across (b) R, (c) C, (d) L, (e) C and L together, and (f) R, C, and L together? At what average rate is energy dissipated by (g) R, (h) C, and (i) L?

Answer:

(a) 2.59 A; (b) 38.8 V; (c) 159 V; (d) 224 V; (e) 64.2 V; (f) 75.0 V; (g) 100 W; (h) 0; (i) 0

••60 GO In a series oscillating RLC circuit, $R = 16.0\ \Omega$, $C = 31.2\ \mu F$, $L = 9.20$ mH, and $\mathscr{E} = \mathscr{E}_m \sin \omega_d t$ with $\mathscr{E}_m = 45.0$ V and $\omega_d = 3000$ rad/s. For time $t = 0.442$ ms find (a) the rate P_g at which energy is being supplied by the generator, (b) the rate P_C at which the energy in the capacitor is changing, (c) the rate P_L at which the energy in the inductor is changing, and (d) the rate P_R at which energy is being dissipated in the resistor. (e) Is the sum of P_C, P_L, and P_R greater than, less than, or equal to P_g?

••61 SSM WWW Figure 31-35 shows an ac generator connected to a "black box" through a pair of terminals. The box contains an RLC circuit, possibly even a multiloop circuit, whose elements and connections we do not know. Measurements outside the box reveal that

$$\mathscr{E}(t) = (75.0\ \text{V}) \sin \omega_d t$$

and

$$i(t) = (1.20\ \text{A}) \sin(\omega_d t + 42.0°).$$

(a) What is the power factor? (b) Does the current lead or lag the emf? (c) Is the circuit in the box largely inductive or largely capacitive? (d) Is the circuit in the box in resonance? (e) Must there be a capacitor in the box? (f) An inductor? (g) A resistor? (h) At what average rate is energy delivered to the box by the generator? (i) Why don't you need to know ω_d to answer all these questions?

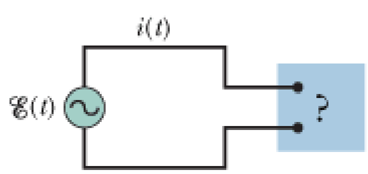


Figure 31-35 Problem 61.

Answer:

(a) 0.743; (b) lead; (c) capacitive; (d) no; (e) yes; (f) no; (g) yes; (h) 33.4 W

sec. 31-11 Transformers

•62 A generator supplies 100 V to a transformer's primary coil, which has 50 turns. If the secondary coil has 500 turns, what is the secondary voltage?

•63 SSM ILW A transformer has 500 primary turns and 10 secondary turns. (a) If V_p is 120 V (rms), what is V_s with an open circuit? If the secondary now has a resistive load of 15 Ω, what is the current in the (b) primary and (c) secondary?

Answer:

(a) 2.4 V; (b) 3.2 mA; (c) 0.16 A

•64 Figure 31-36 shows an "autotransformer." It consists of a single coil (with an iron core). Three taps T_i are provided. Between taps T_1 and T_2 there are 200 turns, and between taps T_2 and T_3 there are 800 turns. Any two taps can be chosen as the primary terminals, and any two taps can be chosen as the secondary terminals. For choices producing a step-up transformer, what are the (a) smallest, (b) second smallest, and (c) largest values of the ratio V_s/V_p? For a step-down transformer, what are the (d) smallest, (e) second smallest, and (f) largest values of V_s/V_p?

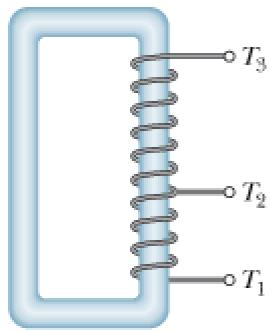


Figure 31-36 Problem 64.

••65 An ac generator provides emf to a resistive load in a remote factory over a two-cable transmission line. At the factory a step-down transformer reduces the voltage from its (rms) transmission value V_t to a much lower value that is safe and convenient for use in the factory. The transmission line resistance is 0.30 Ω/cable, and the power of the generator is 250 kW. If V_t = 80 kV, what are (a) the voltage decrease ΔV along the transmission line and (b) the rate P_d at which energy is dissipated in the line as thermal energy? If V_t = 8.0 kV, what are (c) ΔV and (d) P_d? If V_t = 0.80 kV, what are (e) ΔV and (f) P_d?

Answer:

(a) 1.9 V; (b) 5.9 W; (c) 19 V; (d) 5.9×10^2 W; (e) 0.19 kV; (f) 59 kW

Additional Problems

66 In Fig. 31-34, let the rectangular box on the left represent the (high-impedance) output of an audio amplifier, with r = 1000 Ω. Let R = 10 Ω represent the (low-impedance) coil of a loudspeaker. For maximum transfer of energy to the load R we must have $R = r$, and that is not true in this case. However, a transformer can be used to "transform" resistances, making them behave electrically as if they were larger or smaller than they actually are. (a) Sketch the primary and secondary coils of a transformer that can be introduced between the amplifier and the speaker in Fig. 31-34 to match the impedances. (b) What must be the turns ratio?

67 An ac generator produces emf $\mathscr{E} = \mathscr{E}_m \sin(\omega_d t - \pi/4)$, where $\mathscr{E}_m$ = 30.0 V and ω_d = 350 rad/s. The current in the circuit attached to the generator is $i(t) = I \sin(\omega_d t + \pi/4)$, where I = 620 mA. (a) At what time after t = 0 does the generator emf first reach a maximum? (b) At what time after t = 0 does the current first reach a maximum? (c) The circuit contains a single element other than the

generator. Is it a capacitor, an inductor, or a resistor? Justify your answer. (d) What is the value of the capacitance, inductance, or resistance, as the case may be?

Answer:

(a) 6.73 ms; (b) 2.24 ms; (c) capacitor; (d) 59.0 μF

68 A series *RLC* circuit is driven by a generator at a frequency of 2000 Hz and an emf amplitude of 170 V. The inductance is 60.0 mH, the capacitance is 0.400 μF, and the resistance is 200 Ω. (a) What is the phase constant in radians? (b) What is the current amplitude?

69 A generator of frequency 3000 Hz drives a series *RLC* circuit with an emf amplitude of 120 V. The resistance is 40.0 Ω, the capacitance is 1.60 μF, and the inductance is 850 μH. What are (a) the phase constant in radians and (b) the current amplitude? (c) Is the circuit capacitive, inductive, or in resonance?

Answer:

(a) -0.405 rad; (b) 2.76 A; (c) capacitive

70 A 45.0 mH inductor has a reactance of 1.30 kΩ. (a) What is its operating frequency? (b) What is the capacitance of a capacitor with the same reactance at that frequency? If the frequency is doubled, what is the new reactance of (c) the inductor and (d) the capacitor?

71 An *RLC* circuit is driven by a generator with an emf amplitude of 80.0 V and a current amplitude of 1.25 A. The current leads the emf by 0.650 rad. What are the (a) impedance and (b) resistance of the circuit? (c) Is the circuit inductive, capacitive, or in resonance?

Answer:

(a) 64.0 Ω; (b) 50.9 Ω; (c) capacitive

72 A series *RLC* circuit is driven in such a way that the maximum voltage across the inductor is 1.50 times the maximum voltage across the capacitor and 2.00 times the maximum voltage across the resistor. (a) What is for the circuit? (b) Is the circuit inductive, capacitive, or in resonance? The resistance is 49.9 Ω, and the current amplitude is 200 mA. (c) What is the amplitude of the driving emf?

73 A capacitor of capacitance 158 μF and an inductor form an *LC* circuit that oscillates at 8.15 kHz, with a current amplitude of 4.21 mA. What are (a) the inductance, (b) the total energy in the circuit, and (c) the maximum charge on the capacitor?

Answer:

(a) 2.41 μH; (b) 21.4 pJ; (c) 82.2 nC

74 An oscillating *LC* circuit has an inductance of 3.00 mH and a capacitance of 10.0 μF. Calculate the (a) angular frequency and (b) period of the oscillation. (c) At time $t = 0$, the capacitor is charged to 200 μC and the current is zero. Roughly sketch the charge on the capacitor as a function of time.

75 For a certain driven series *RLC* circuit, the maximum generator emf is 125 V and the maximum current is 3.20 A. If the current leads the generator emf by 0.982 rad, what are the (a) impedance and (b) resistance of the circuit? (c) Is the circuit predominantly capacitive or inductive?

Answer:

(a) 39.1 Ω; (b) 21.7 Ω; (c) capacitive

76 A 1.50 μF capacitor has a capacitive reactance of 12.0 Ω. (a) What must be its operating frequency? (b) What will be the capacitive reactance if the frequency is doubled?

77 SSM In Fig. 31-37, a three-phase generator G produces electrical power that is transmitted by means of three wires. The electric potentials (each relative to a common reference level) are $V_1 = A \sin \omega_d t$ for wire 1, $V_2 = A \sin(\omega_d t - 120°)$ for wire 2, and $V_3 = A \sin(\omega_d t - 240°)$ for wire 3. Some types of industrial equipment (for example, motors) have three terminals and are designed to be connected directly to these three wires. To use a more conventional two-terminal device (for example, a lightbulb), one connects it to any two of the three wires. Show that the potential difference between *any two* of the wires (a) oscillates sinusoidally with angular frequency ω_d and (b) has an amplitude of $A\sqrt{3}$.

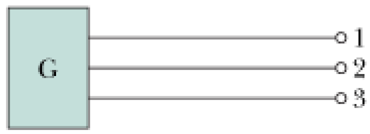


Figure 31-37 Problem 77.

78 An electric motor connected to a 120 V, 60.0 Hz ac outlet does mechanical work at the rate of 0.100 hp (1 hp = 746 W). (a) If the motor draws an rms current of 0.650 A, what is its effective resistance, relative to power transfer? (b) Is this the same as the resistance of the motor's coils, as measured with an ohmmeter with the motor disconnected from the outlet?

79 SSM (a) In an oscillating *LC* circuit, in terms of the maximum charge *Q* on the capacitor, what is the charge there when the energy in the electric field is 50.0% of that in the magnetic field? (b) What fraction of a period must elapse following the time the capacitor is fully charged for this condition to occur?

Answer:

(a) 0.577*Q*; (b) 0.152

80 A series *RLC* circuit is driven by an alternating source at a frequency of 400 Hz and an emf amplitude of 90.0 V. The resistance is 20.0 Ω, the capacitance is 12.1 μF, and the inductance is 24.2 mH. What is the rms potential difference across (a) the resistor, (b) the capacitor, and (c) the inductor? (d) What is the average rate at which energy is dissipated?

81 SSM In a certain series *RLC* circuit being driven at a frequency of 60.0 Hz, the maximum voltage across the inductor is 2.00 times the maximum voltage across the resistor and 2.00 times the maximum voltage across the capacitor. (a) By what angle does the current lag the generator emf? (b) If the maximum generator emf is 30.0 V, what should be the resistance of the circuit to obtain a maximum current of 300 mA?

Answer:

(a)45.0°; (b)70.7 Ω

82 A 1.50 mH inductor in an oscillating *LC* circuit stores a maximum energy of 10.0 μJ. What is the maximum current?

83 A generator with an adjustable frequency of oscillation is wired in series to an inductor of L = 2.50 mH and a capacitor of C = 3.00 μF. At what frequency does the generator produce the largest possible current amplitude in the circuit?

Answer:

1.84 kHz

84 A series *RLC* circuit has a resonant frequency of 6.00 kHz. When it is driven at 8.00 kHz, it has an impedance of 1.00 kΩ and a phase constant of 45°. What are (a) *R*, (b) *L*, and (c) *C* for this circuit?

85 SSM An *LC* circuit oscillates at a frequency of 10.4 kHz. (a) If the capacitance is 340 μF, what is the inductance? (b) If the maximum current is 7.20 mA, what is the total energy in the circuit? (c) What is the maximum charge on the capacitor?

Answer:

(a) 0.689 μH; (b) 17.9 pJ; (c) 0.110 μC

86 When under load and operating at an rms voltage of 220 V, a certain electric motor draws an rms current of 3.00 A. It has a resistance of 24.0 Ω and no capacitive reactance. What is its inductive reactance?

87 The ac generator in Fig. 31-38 supplies 120 V at 60.0 Hz. With the switch open as in the diagram, the current leads the generator emf by 20.0°. With the switch in position 1, the current lags the generator emf by 10.0°. When the switch is in position 2, the current amplitude is 2.00 A. What are (a) *R*, (b) *L*, and (c) *C*?

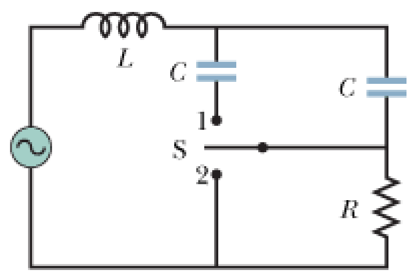


Figure 31-38 Problem 87.

Answer:

(a) 165 Ω; (b) 313 mH; (c) 14.9 μF

88 In an oscillating *LC* circuit, L = 8.00 mH and C = 1.40 μF. At time t = 0, the current is maximum at 12.0 mA. (a) What is the maximum charge on the capacitor during the oscillations? (b) At what earliest time $t > 0$ is the rate of change of energy in the capacitor maximum? (c) What is that maximum rate of change?

89 SSM For a sinusoidally driven series *RLC* circuit, show that over one complete cycle with period *T* (a) the energy stored in the capacitor does not change; (b) the energy stored in the inductor does not change; (c) the driving emf device supplies energy $\left(\frac{1}{2}T\right)\mathcal{E}_m I\cos\phi$ and (d) the resistor dissipates energy $\left(\frac{1}{2}T\right)RI^2$. (e) Show that the quantities found in (c) and (d) are equal.

90 What capacitance would you connect across a 1.30 mH inductor to make the resulting oscillator resonate at 3.50 kHz?

91 A series circuit with resistor–inductor–capacitor combination R_1, L_1, C_1 has the same resonant

frequency as a second circuit with a different combination R_2, L_2, C_2. You now connect the two combinations in series. Show that this new circuit has the same resonant frequency as the separate circuits.

92 Consider the circuit shown in Fig. 31-39. With switch S_1 closed and the other two switches open, the circuit has a time constant τ_C. With switch S_2 closed and the other two switches open, the circuit has a time constant τ_L. With switch S_3 closed and the other two switches open, the circuit oscillates with a period T. Show that $T = 2\pi\sqrt{\tau_C \tau_L}$.

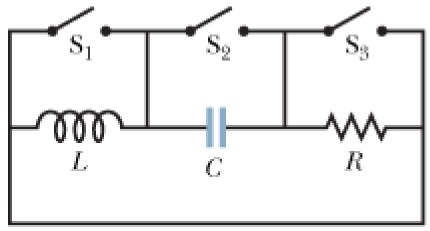


Figure 31-39 Problem 92.

sec. 32-2 Gauss' Law for Magnetic Fields

•1 The magnetic flux through each of five faces of a die (singular of "dice") is given by $\Phi_B = \pm N$ Wb, where N (= 1 to 5) is the number of spots on the face. The flux is positive (outward) for N even and negative (inward) for N odd. What is the flux through the sixth face of the die?

Answer:

+ 3 Wb

•2 Figure 32-26 shows a closed surface. Along the flat top face, which has a radius of 2.0 cm, a perpendicular magnetic field $\vec{B}$ of magnitude 0.30 T is directed outward. Along the flat bottom face, a magnetic flux of 0.70 mWb is directed outward. What are the (a) magnitude and (b) direction (inward or outward) of the magnetic flux through the curved part of the surface?

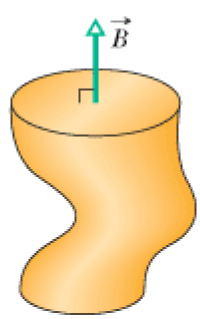


Figure 32-26 Problem 2.

••3 SSM ILW A Gaussian surface in the shape of a right circular cylinder with end caps has a radius of 12.0 cm and a length of 80.0 cm. Through one end there is an inward magnetic flux of 25.0 μWb. At the other end there is a uniform magnetic field of 1.60 mT, normal to the surface and directed outward. What are the (a) magnitude and (b) direction (inward or outward) of the net magnetic flux through the curved surface?

Answer:

(a) 47.4 μWb; (b) inward

•••4 Two wires, parallel to a *z* axis and a distance 4*r* apart, carry equal currents *i* in opposite directions, as shown in Fig. 32-27. A circular cylinder of radius *r* and length *L* has its axis on the *z* axis, midway between the wires. Use Gauss' law for magnetism to derive an expression for the net outward magnetic flux through the half of the cylindrical surface above the *x* axis. (*Hint:* Find the flux through the portion of the *xz* plane that lies within the cylinder.)

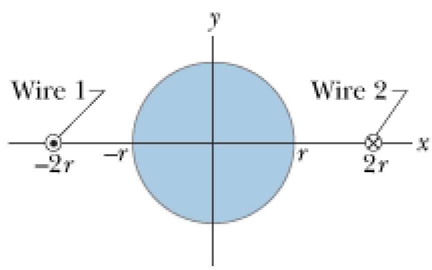


Figure 32-27 Problem 4.

sec. 32-3 Induced Magnetic Fields

•5 SSM The induced magnetic field at radial distance 6.0 mm from the central axis of a circular parallel-plate capacitor is 2.0×10^{-7} T. The plates have radius 3.0 mm. At what rate $d\vec{E}/dt$ is the electric field between the plates changing?

Answer:

2.4×10^{13} V/m · s

•6 A capacitor with square plates of edge length *L* is being discharged by a current of 0.75 A. Figure 32-28 is a head-on view of one of the plates from inside the capacitor. A dashed rectangular path is shown. If *L* = 12 cm, *W* = 4.0 cm, and *H* = 2.0 cm, what is the value of $\oint \vec{B} \cdot d\vec{s}$ around the dashed path?

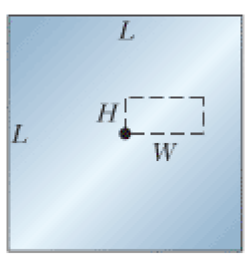


Figure 32-28 Problem 6.

••7 GO *Uniform electric flux.* Figure 32-29 shows a circular region of radius *R* = 3.00 cm in which a uniform electric flux is directed out of the plane of the page. The total electric flux through the region is given by $\Phi_E = (3.00 \text{mV} \cdot \text{m/s})t$, where t is in seconds. What is the magnitude of the magnetic field that is induced at radial distances (a) 2.00 cm and (b) 5.00 cm?

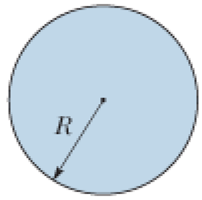


Figure 32-29 Problems 7 to 9 and 19 to 21.

Answer:

(a) 1.18×10^{-19} T; (b) 1.06×10^{-19} T

••8 GO *Nonuniform electric flux.* Figure 32-29 shows a circular region of radius R = 3.00 cm in which an electric flux is directed out of the plane of the page. The flux encircled by a concentric circle of radius r is given by $\Phi_{E,\text{enc}} = (0.600 \text{ V} \cdot \text{m/s})(\text{r/R})t$, where $r \le R$ and t is in seconds. What is the magnitude of the induced magnetic field at radial distances (a) 2.00 cm and (b) 5.00 cm?

••9 GO *Uniform electric field.* In Fig. 32-29, a uniform electric field is directed out of the page within a circular region of radius R = 3.00 cm. The field magnitude is given by $E = (4.50 \times 10^{-3} \text{ V/m} \cdot \text{s})t$, where t is in seconds. What is the magnitude of the induced magnetic field at radial distances (a) 2.00 cm and (b) 5.00 cm?

Answer:

(a) 5.01×10^{-22} T; (b) 4.51×10^{-22} T

••10 GO *Nonuniform electric field.* In Fig. 32-29, an electric field is directed out of the page within a circular region of radius R = 3.00 cm. The field magnitude is $E = (0.500 \text{ V/m} \cdot \text{s})(1 - r/R)t$, where t is in seconds and r is the radial distance ($r \le R$). What is the magnitude of the induced magnetic field at radial distances (a) 2.00 cm and (b) 5.00 cm?

••11 Suppose that a parallel-plate capacitor has circular plates with radius R = 30 mm and a plate separation of 5.0 mm. Suppose also that a sinusoidal potential difference with a maximum value of 150 V and a frequency of 60 Hz is applied across the plates; that is,

$$v = (150\ v)\sin[2\pi(60 \text{ Hz})t].$$

(a) Find $B_{max}(R)$, the maximum value of the induced magnetic field that occurs at $r = R$. (b) Plot $B_{max}(r)$ for $0 < r < 10$ cm.

Answer:

(a) 1.9 pT

••12 A parallel-plate capacitor with circular plates of radius 40 mm is being discharged by a current of 6.0 A. At what radius (a) inside and (b) outside the capacitor gap is the magnitude of the induced magnetic field equal to 75% of its maximum value? (c) What is that maximum value?

sec. 32-4 Displacement Current

•13 At what rate must the potential difference between the plates of a parallel-plate capacitor with a 2.0 μF capacitance be changed to produce a displacement current of 1.5 A?

Answer:

7.5×10^5 V/s

•14 A parallel-plate capacitor with circular plates of radius R is being charged. Show that the magnitude of the current density of the displacement current is $J_d = \varepsilon_0(dE/dt)$ for $r \leq R$.

•15 SSM Prove that the displacement current in a parallel-plate capacitor of capacitance C can be written as $i_d = C(dV/dt)$, where V is the potential difference between the plates.

•16 A parallel-plate capacitor with circular plates of radius 0.10 m is being discharged. A circular loop of radius 0.20 m is concentric with the capacitor and halfway between the plates. The displacement current through the loop is 2.0 A. At what rate is the electric field between the plates changing?

••17 A silver wire has resistivity $\rho = 1.62 \times 10^{-8}\ \Omega \cdot$ m and a cross-sectional area of 5.00 mm^2. The current in the wire is uniform and changing at the rate of 2000 A/s when the current is 100 A. (a) What is the magnitude of the (uniform) electric field in the wire when the current in the wire is 100 A? (b) What is the displacement current in the wire at that time? (c) What is the ratio of the magnitude of the magnetic field due to the displacement current to that due to the current at a distance r from the wire?

Answer:

(a) 0.324 V/m; (b) 2.87×10^{-16}A; (c) 2.87 10^{-18}

••18 The circuit in Fig. 32-30 consists of switch S, a 12.0 V ideal battery, a 20.0 MΩ resistor, and an air-filled capacitor. The capacitor has parallel circular plates of radius 5.00 cm, separated by 3.00 mm. At time $t = 0$, switch S is closed to begin charging the capacitor. The electric field between the plates is uniform. At $t = 250\ \mu$s, what is the magnitude of the magnetic field within the capacitor, at radial distance 3.00 cm?

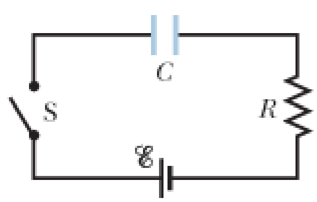


Figure 32-30 Problem 18.

••19 *Uniform displacement-current density.* Figure 32-29 shows a circular region of radius $R = 3.00$ cm in which a displacement current is directed out of the page. The displacement current has a uniform density of magnitude $J_d = 6.00$ A/m^2 What is the magnitude of the magnetic field due to the displacement current at radial distances (a) 2.00 cm and (b) 5.00 cm?

Answer:

(a) 75.4 nT; (b) 67.9 nT

••20 *Uniform displacement current.* Figure 32-29 shows a circular region of radius $R = 3.00$ cm in which a uniform displacement current $i_d = 0.500$ A is out of the page. What is the magnitude of the magnetic field due to the displacement current at radial distances (a) 2.00 cm and (b) 5.00 cm?

••21 GO *Nonuniform displacement-current density.* Figure 32-29 shows a circular region of radius R =

3.00 cm in which a displacement current is directed out of the page. The magnitude of the density of this displacement current is $J_d = (4.00\ \text{A/m}^2)(1 - r/R)$, where r is the radial distance ($r \le R$). What is the magnitude of the magnetic field due to the displacement current at (a) r = 2.00 cm and (b) r = 5.00 cm?

Answer:

27.9 nT; (b) 15.1 nT

••22 GO *Nonuniform displacement current.* Figure 32-29 shows a circular region of radius R = 3.00 cm in which a displacement current i_d is directed out of the page. The magnitude of the displacement current is given by $i_d = (3.00\ \text{A})(r/R)$, where r is the radial distance ($r \le R$). What is the magnitude of the magnetic field due to i_d at radial distances (a) 2.00 cm and (b) 5.00 cm?

••23 SSM ILW In Fig. 32-31, a parallel-plate capacitor has square plates of edge length L = 1.0 m. A current of 2.0 A charges the capacitor, producing a uniform electric field $\vec{E}$ between the plates, with $\vec{E}$ perpendicular to the plates. (a) What is the displacement current i_d through the region between the plates? (b) What is dE/dt in this region? (c) What is the displacement current encircled by the square dashed path of edge length d = 0.50 m? (d) What $\oint \vec{B} \cdot d\vec{s}$ around this square dashed path?

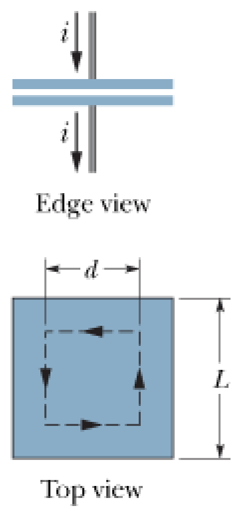


Figure 32-31 Problem 23.

Answer:

(a) 2.0 A; (b) 2.3×10^{11} V/m · s; (c) 0.50 A; (d) 0.63 μT · m

••24 The magnitude of the electric field between the two circular parallel plates in Fig. 32-32 is $E = (4.0 \times 10^5) - (6.0 \times 10^4 t)$, with E in volts per meter and t in seconds. At $t = 0$, $\vec{E}$ is upward. The plate area is $4.0 \times 10^{-2}\ \text{m}^2$. For $t \ge 0$, what are the (a) magnitude and (b) direction (up or down) of the displacement current between the plates and (c) is the direction of the induced magnetic field clockwise or counterclockwise in the figure?

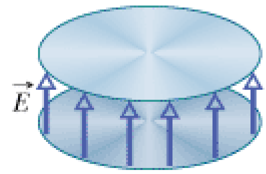


Figure 32-32 Problem 24.

••25 ILW As a parallel-plate capacitor with circular plates 20 cm in diameter is being charged, the current density of the displacement current in the region between the plates is uniform and has a magnitude of 20 A/m^2. (a) Calculate the magnitude B of the magnetic field at a distance r = 50 mm from the axis of symmetry of this region. (b) Calculate dE/dt in this region.

Answer:

(a) 0.63 μT; (b) 2.3 × 10^{12} V/m · s

••26 A capacitor with parallel circular plates of radius R = 1.20 cm is discharging via a current of 12.0 A. Consider a loop of radius R/3 that is centered on the central axis between the plates. (a) How much displacement current is encircled by the loop? The maximum induced magnetic field has a magnitude of 12.0 mT. At what radius (b) inside and (c) outside the capacitor gap is the magnitude of the induced magnetic field 3.00 mT?

••27 ILW In Fig. 32-33, a uniform electric field $\vec{E}$ collapses. The vertical axis scale is set by E_s = 6.0 × 10^5 N/C, and the horizontal axis scale is set by t_s = 12.0 μs. Calculate the magnitude of the displacement current through a 1.6 m^2 area perpendicular to the field during each of the time intervals a, b, and c shown on the graph. (Ignore the behavior at the ends of the intervals.)

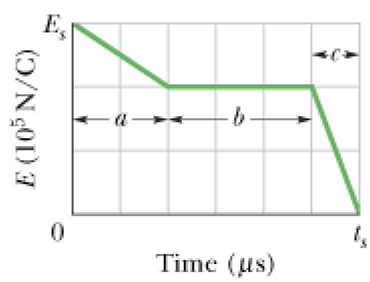


Figure 32-33 Problem 27.

Answer:

(a) 0.71 A; (b) 0; (c) 2.8 A

••28 GO Figure 32-34a shows the current i that is produced in a wire of resistivity 1.62 × 10^{-8} Ω. The magnitude of the current versus time t is shown in Fig. 32-34b. The vertical axis scale is set by i_s = 10.0 A, and the horizontal axis scale is set by t_s = 50.0 ms. Point P is at radial distance 9.00 mm from the wire's center. Determine the magnitude of the magnetic field $\vec{B}_i$ at point P due to the actual current i in the wire at (a) t = 20 ms, (b) t = 40 ms, and (c) t = 60 ms. Next, assume that the electric field driving the current is confined to the wire. Then determine the magnitude of the magnetic field $\vec{B}_{id}$ at point P due to the displacement current i_d in the wire at (d) t = 20 ms, (e) t = 40 ms, and (f) t = 60 ms. At point P at t = 20 s, what is the direction (into or out of the page) of (g) $\vec{B}_i$ and (h) $\vec{B}_{id}$?

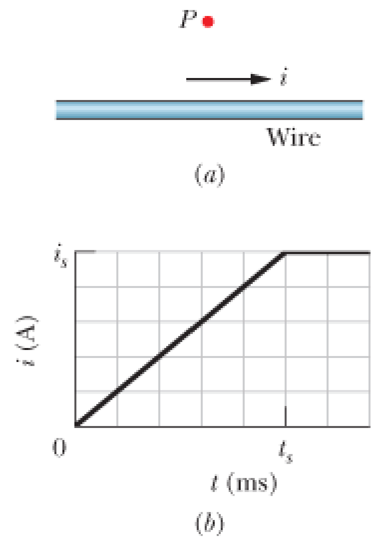


Figure 32-34 Problem 28.

•••29 In Fig. 32-35, a capacitor with circular plates of radius R = 18.0 cm is connected to a source of emf $\mathscr{E} = \mathscr{E}_m \sin \omega t$, where $\mathscr{E}_m$ = 220 V and ω = 130 rad/s. The maximum value of the displacement current is i_d = 7.60 μA. Neglect fringing of the electric field at the edges of the plates. (a) What is the maximum value of the current i in the circuit? (b) What is the maximum value of $d\Phi_E/dt$, where Φ_E is the electric flux through the region between the plates? (c) What is the separation d between the plates? (d) Find the maximum value of the magnitude of $\vec{B}$ between the plates at a distance r = 11.0 cm from the center.

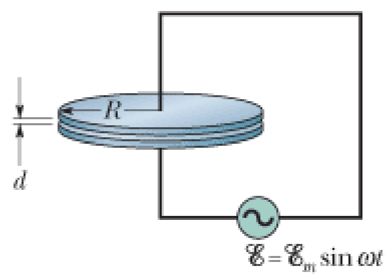


Figure 32-35 Problem 29.

Answer:

(a) 7.60 μA; (b) 859 kV m/s; (c) 3.39 mm; (d) 5.16 pT

sec. 32-6 Magnets

•30 Assume the average value of the vertical component of Earth's magnetic field is 43 μT (downward) for all of Arizona, which has an area of 2.95×10^5 km^2. What then are the (a) magnitude and (b) direction (inward or outward) of the net magnetic flux through the rest of Earth's surface (the entire surface excluding Arizona)?

•31 In New Hampshire the average horizontal component of Earth's magnetic field in 1912 was 16 μT, and the average inclination or “dip” was 73°. What was the corresponding magnitude of Earth's magnetic field?

Answer:

55 μT

sec. 32-7 Magnetism and Electrons

•**32** Figure 32-36*a* is a one-axis graph along which two of the allowed energy values (*levels*) of an atom are plotted. When the atom is placed in a magnetic field of 0.500 T, the graph changes to that of Fig. 32-36*b* because of the energy associated with $\vec{\mu}_{orb} \cdot \vec{B}$. (We neglect $\vec{\mu}_s$.) Level E_1 is unchanged, but level E_2 splits into a (closely spaced) triplet of levels. What are the allowed values of m_{ell} associated with (a) energy level E_1 and (b) energy level E_2? (c) In joules, what amount of energy is represented by the spacing between the triplet levels?

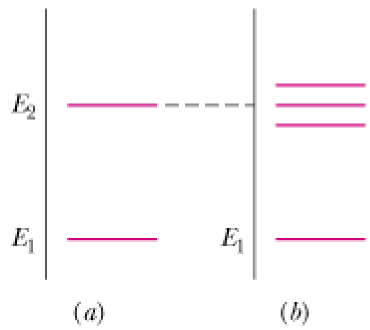


Figure 32-36 Problem 32.

•**33** SSM WWW If an electron in an atom has an orbital angular momentum with m = 0, what are the components (a) $L_{orb,z}$ and (b) $\mu_{orb,z}$? If the atom is in an external magnetic field $\vec{B}$ that has magnitude 35 mT and is directed along the z axis, what are (c) the energy U_{orb} associated with $\vec{\mu}_{orb}$ and (d) the energy U_{spin} associated with $\vec{\mu}_s$? If, instead, the electron has m = -3, what are (e) $L_{orb,z}$, (f) $\mu_{orb,z}$, (g) U_{orb}, and (h) U_{spin}?

Answer:

(a) 0; (b) 0; (c) 0; (d) ± 3.2 10^{-25} J; (e) - 3.2 10^{-34} J · s; (f) 2.8 10^{-23} J/T; (g) 9.7 10^{-25} J; (h) ± 3.2 10^{-25} J

•**34** What is the energy difference between parallel and antiparallel alignment of the z component of an electron's spin magnetic dipole moment with an external magnetic field of magnitude 0.25 T, directed parallel to the z axis?

•**35** What is the measured component of the orbital magnetic dipole moment of an electron with (a) m_ℓ = 1 and (b) m_ℓ = -2?

Answer:

(a) - 9.3 × 10^{-24}J/T; (b) 1.9 × 10^{-23} J/T

•**36** An electron is placed in a magnetic field $\vec{B}$ that is directed along a z axis. The energy difference between parallel and antiparallel alignments of the z component of the electron's spin magnetic moment with $\vec{B}$ is 6.00 × 10^{-25} J. What is the magnitude of $\vec{B}$?

sec. 32-9 Diamagnetism

•37 Figure 32-37 shows a loop model (loop L) for a diamagnetic material. (a) Sketch the magnetic field lines within and about the material due to the bar magnet. What is the direction of (b) the loop's net magnetic dipole moment $\vec{\mu}$ (c) the conventional current i in the loop (clockwise or counterclockwise in the figure), and (d) the magnetic force on the loop?

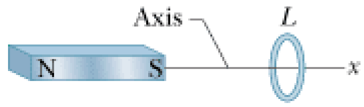


Figure 32-37 Problems 37 and 71.

Answer:

(b) x; (c) clockwise; (d) x

•••38 Assume that an electron of mass m and charge magnitude e moves in a circular orbit of radius r about a nucleus. A uniform magnetic field $\vec{B}$ is then established perpendicular to the plane of the orbit. Assuming also that the radius of the orbit does not change and that the change in the speed of the electron due to field $\vec{B}$ is small, find an expression for the change in the orbital magnetic dipole moment of the electron due to the field.

sec. 32-10 Paramagnetism

•39 A sample of the paramagnetic salt to which the magnetization curve of Fig. 32-14 applies is to be tested to see whether it obeys Curie's law. The sample is placed in a uniform 0.50 T magnetic field that remains constant throughout the experiment. The magnetization M is then measured at temperatures ranging from 10 to 300 K. Will it be found that Curie's law is valid under these conditions?

Answer:

yes

•40 A sample of the paramagnetic salt to which the magnetization curve of Fig. 32-14 applies is held at room temperature (300 K). At what applied magnetic field will the degree of magnetic saturation of the sample be (a) 50% and (b) 90%? (c) Are these fields attainable in the laboratory?

•41 SSM ILW A magnet in the form of a cylindrical rod has a length of 5.00 cm and a diameter of 1.00 cm. It has a uniform magnetization of 5.30×10^{3} A/m. What is its magnetic dipole moment?

Answer:

20.8 mJ/T

•42 A 0.50 T magnetic field is applied to a paramagnetic gas whose atoms have an intrinsic magnetic dipole moment of 1.0×10^{-23} J/T. At what temperature will the mean kinetic energy of translation of the atoms equal the energy required to reverse such a dipole end for end in this magnetic field?

••43 An electron with kinetic energy K_e travels in a circular path that is perpendicular to a uniform magnetic field, which is in the positive direction of a z axis. The electron's motion is subject only to the force due to the field. (a) Show that the magnetic dipole moment of the electron due to its orbital motion has magnitude $\mu = K_e/B$ and that it is in the direction opposite that of $\vec{B}$. What are the (b) magnitude and (c) direction of the magnetic dipole moment of a positive ion with kinetic energy K_i under the same circumstances? (d) An ionized gas consists of 5.3×10^{21} electrons/m^3

and the same number density of ions. Take the average electron kinetic energy to be 6.2×10^{-20} J and the average ion kinetic energy to be 7.6×10^{-21} J. Calculate the magnetization of the gas when it is in a magnetic field of 1.2 T.

Answer:

(b) K_i/B; (c) $-z$; (d) 0.31 kA/m

••44 Figure 32-38 gives the magnetization curve for a paramagnetic material. The vertical axis scale is set by $a = 0.15$, and the horizontal axis scale is set by $b = 0.2$ T/K. Let μ_{sam} be the measured net magnetic moment of a sample of the material and μ_{max} be the maximum possible net magnetic moment of that sample. According to Curie's law, what would be the ratio μ_{sam}/μ_{max} were the sample placed in a uniform magnetic field of magnitude 0.800 T, at a temperature of 2.00 K?

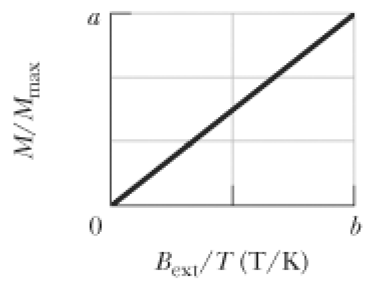


Figure 32-38 Problem 44.

•••45 SSM Consider a solid containing N atoms per unit volume, each atom having a magnetic dipole moment $\vec{\mu}$. Suppose the direction of $\vec{\mu}$ can be only parallel or antiparallel to an externally applied magnetic field $\vec{B}$ (this will be the case if $\vec{\mu}$ is due to the spin of a single electron). According to statistical mechanics, the probability of an atom being in a state with energy U is proportional to $e^{-U/kT}$, where T is the temperature and k is Boltzmann's constant. Thus, because energy U is $-\vec{\mu} \cdot \vec{B}$, the fraction of atoms whose dipole moment is parallel to $\vec{B}$ is proportional to $e^{\mu}B/kT$ and the fraction of atoms whose dipole moment is antiparallel to $\vec{B}$ is proportional to $e^{-B/kT}$. (a) Show that the magnitude of the magnetization of this solid is $M = N_{\mu} \tanh(\mu B/kT)$. Here tanh is the hyperbolic tangent function: $\tanh(x)\ (e^x - e^{-x})/(e^x + e^{-x})$. (b) Show that the result given in (a) reduces to $M = N\mu^2 B/kT$ for $\mu B \ll kT$. (c) Show that the result of (a) reduces to $M = N\mu$ for $\mu B \gg kT$. (d) Show that both (b) and (c) agree qualitatively with Fig. 32-14.

sec. 32-11 Ferromagnetism

•46 GO You place a magnetic compass on a horizontal surface, allow the needle to settle, and then give the compass a gentle wiggle to cause the needle to oscillate about its equilibrium position. The oscillation frequency is 0.312 Hz. Earth's magnetic field at the location of the compass has a horizontal component of 18.0 μT. The needle has a magnetic moment of 0.680 μJ/T. What is the needle's rotational inertia about its (vertical) axis of rotation?

••47 SSM WWW ILW The magnitude of the magnetic dipole moment of Earth is 8.0×10^{22} J/T. (a) If the origin of this magnetism were a magnetized iron sphere at the center of Earth, what would be its radius? (b) What fraction of the volume of Earth would such a sphere occupy? Assume complete alignment of the dipoles. The density of Earth's inner core is 14 g/cm^3. The magnetic dipole moment of an iron atom is 2.1×10^{-23} J/T. (*Note:* Earth's inner core is in fact thought to be in both liquid and solid forms and partly iron, but a permanent magnet as the source of Earth's magnetism has been ruled out by several considerations. For one, the temperature is certainly above the Curie point.)

Answer:

(a) 1.8×10^2 km; (b) 2.3×10^{-5}

••48 The magnitude of the dipole moment associated with an atom of iron in an iron bar is 2.1×10^{-23} J/T. Assume that all the atoms in the bar, which is 5.0 cm long and has a cross-sectional area of 1.0 cm^2, have their dipole moments aligned. (a) What is the dipole moment of the bar? (b) What torque must be exerted to hold this magnet perpendicular to an external field of magnitude 1.5 T? (The density of iron is 7.9 g/cm^3.)

••49 SSM The exchange coupling mentioned in Section 32-11 as being responsible for ferromagnetism is *not* the mutual magnetic interaction between two elementary magnetic dipoles. To show this, calculate (a) the magnitude of the magnetic field a distance of 10 nm away, along the dipole axis, from an atom with magnetic dipole moment 1.5×10^{-23} J/T (cobalt), and (b) the minimum energy required to turn a second identical dipole end for end in this field. (c) By comparing the latter with the mean translational kinetic energy of 0.040 eV, what can you conclude?

Answer:

(a) 3.0 μT; (b) 5.6×10^{-10} eV

••50 A magnetic rod with length 6.00 cm, radius 3.00 mm, and (uniform) magnetization 2.70×10^3 A/m can turn about its center like a compass needle. It is placed in a uniform magnetic field $\vec{B}$ of magnitude 35.0 mT, such that the directions of its dipole moment and $\vec{B}$ make an angle of 68.0°. (a) What is the magnitude of the torque on the rod due to $\vec{B}$? (b) What is the change in the orientation energy of the rod if the angle changes to 34.0°?

••51 The saturation magnetization M_{max} of the ferromagnetic metal nickel is 4.70×10^5 A/m. Calculate the magnetic dipole moment of a single nickel atom. (The density of nickel is 8.90 g/cm^3, and its molar mass is 58.71 g/mol.)

Answer:

5.15×10^{-24} A · m^2

••52 Measurements in mines and boreholes indicate that Earth's interior temperature increases with depth at the average rate of 30 C°/km. Assuming a surface temperature of 10°C, at what depth does iron cease to be ferromagnetic? (The Curie temperature of iron varies very little with pressure.)

••53 A Rowland ring is formed of ferromagnetic material. It is circular in cross section, with an inner radius of 5.0 cm and an outer radius of 6.0 cm, and is wound with 400 turns of wire. (a) What current must be set up in the windings to attain a toroidal field of magnitude $B_0 = 0.20$ mT? (b) A secondary coil wound around the toroid has 50 turns and resistance 8.0 Ω. If, for this value of B_0, we have $B_M = 800B_0$, how much charge moves through the secondary coil when the current in the toroid windings is turned on?

Answer:

(a) 0.14 A; (b) 79 μC

Additional Problems

54 Using the approximations given in Problem 61, find (a) the altitude above Earth's surface where the magnitude of its magnetic field is 50.0% of the surface value at the same latitude; (b) the maximum magnitude of the magnetic field at the core–mantle boundary, 2900 km below Earth's surface; and the (c) magnitude and (d) inclination of Earth's magnetic field at the north geographic pole. (e) Suggest why the values you calculated for (c) and (d) differ from measured values.

55 Earth has a magnetic dipole moment of 8.0×10^{22} J/T. (a) What current would have to be produced in a single turn of wire extending around Earth at its geomagnetic equator if we wished to set up such a dipole? Could such an arrangement be used to cancel out Earth's magnetism (b) at points in space well above Earth's surface or (c) on Earth's surface?

Answer:

(a) 6.3×10^8 A; (b) yes; (c) no

56 A charge q is distributed uniformly around a thin ring of radius r. The ring is rotating about an axis through its center and perpendicular to its plane, at an angular speed ω. (a) Show that the magnetic moment due to the rotating charge has magnitude $\mu = \frac{1}{2} q \omega r^2$. (b) What is the direction of this magnetic moment if the charge is positive?

57 A magnetic compass has its needle, of mass 0.050 kg and length 4.0 cm, aligned with the horizontal component of Earth's magnetic field at a place where that component has the value B_h = 16 μT. After the compass is given a momentary gentle shake, the needle oscillates with angular frequency ω = 45 rad/s. Assuming that the needle is a uniform thin rod mounted at its center, find the magnitude of its magnetic dipole moment.

Answer:

0.84 kJ/T

58 The capacitor in Fig. 32-7 is being charged with a 2.50 A current. The wire radius is 1.50 mm, and the plate radius is 2.00 cm. Assume that the current i in the wire and the displacement current i_d in the capacitor gap are both uniformly distributed. What is the magnitude of the magnetic field due to i at the following radial distances from the wire's center: (a) 1.00 mm (inside the wire), (b) 3.00 mm (outside the wire), and (c) 2.20 cm (outside the wire)? What is the magnitude of the magnetic field due to i_d at the following radial distances from the central axis between the plates: (d) 1.00 mm (inside the gap), (e) 3.00 mm (inside the gap), and (f) 2.20 cm (outside the gap)? (g) Explain why the fields at the two smaller radii are so different for the wire and the gap but the fields at the largest radius are not.

59 A parallel-plate capacitor with circular plates of radius R = 16 mm and gap width d = 5.0 mm has a uniform electric field between the plates. Starting at time t = 0, the potential difference between the two plates is $V = (100\text{ V})e^{-t/\tau}$, where the time constant τ = 12 ms. At radial distance $r = 0.80R$ from the central axis, what is the magnetic field magnitude (a) as a function of time for $t \geq 0$ and (b) at time $t = 3\tau$?

Answer:

(a) $(1.2 \times 10^{-13}\text{ T})\exp[-t/(0.012\text{ s})]$; (b) 5.9×10^{-15}T

60 A magnetic flux of 7.0 mWb is directed outward through the flat bottom face of the closed surface shown in Fig. 32-39. Along the flat top face (which has a radius of 4.2 cm) there is a 0.40 T magnetic field $\vec{B}$ directed perpendicular to the face. What are the (a) magnitude and (b) direction (inward or outward) of the magnetic flux through the curved part of the surface?

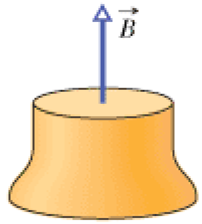


Figure 32-39 Problem 60.

61 SSM The magnetic field of Earth can be approximated as the magnetic field of a dipole. The horizontal and vertical components of this field at any distance r from Earth's center are given by

$$B_h = \frac{\mu_0 \mu}{4\pi r^3} \cos \lambda_m, \quad B_v = \frac{\mu_0 \mu}{2\pi r^3} \sin \lambda_m,$$

where λ_m is the *magnetic latitude* (this type of latitude is measured from the geomagnetic equator toward the north or south geomagnetic pole). Assume that Earth's magnetic dipole moment has magnitude $\mu = 8.00 \times 10^{22}$ A · m^2. (a) Show that the magnitude of Earth's field at latitude λ_m is given by

$$B = \frac{\mu_0 \mu}{4\pi r^3} \sqrt{1 + 3 \sin^2 \lambda_m}.$$

(b) Show that the inclination ${}_i$ of the magnetic field is related to the magnetic latitude λ_m by tan ${}_i$ = 2 tan λ_m.

62 Use the results displayed in Problem 61 to predict the (a) magnitude and (b) inclination of Earth's magnetic field at the geomagnetic equator, the (c) magnitude and (d) inclination at geo-magnetic latitude 60.0°, and the (e) magnitude and (f) inclination at the north geomagnetic pole.

63 A parallel-plate capacitor with circular plates of radius 55.0 mm is being charged. At what radius (a) inside and (b) outside the capacitor gap is the magnitude of the induced magnetic field equal to 50.0% of its maximum value?

Answer:

(a) 27.5 mm; (b) 110 mm

64 A sample of the paramagnetic salt to which the magnetization curve of Fig. 32-14 applies is immersed in a uniform magnetic field of 2.0 T. At what temperature will the degree of magnetic saturation of the sample be (a) 50% and (b) 90%?

65 A parallel-plate capacitor with circular plates of radius R is being discharged. The displacement current through a central circular area, parallel to the plates and with radius $R/2$, is 2.0 A. What is the discharging current?

Answer:

8.0 A

66 Figure 32-40 gives the variation of an electric field that is perpendicular to a circular area of 2.0 m^2. During the time period shown, what is the greatest displacement current through the area?

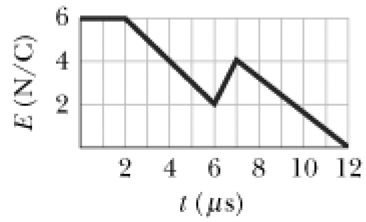


Figure 32-40 Problem 66.

67In Fig. 32-41, a parallel-plate capacitor is being discharged by a current $i = 5.0$ A. The plates are square with edge length $L = 8.0$ mm. (a) What is the rate at which the electric field between the plates is changing? (b) What is the value of $\oint \vec{B} \cdot d\vec{s}$ around the dashed path, where $H = 2.0$ mm and $W = 3.0$ mm?

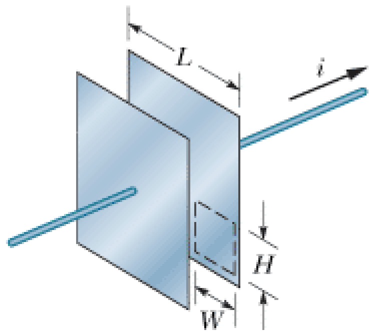


Figure 32-41 Problem 67.

Answer:

(a) 8.8×10^{15} V/m · s; (b) 5.9×10^{-7} T · m

68What is the measured component of the orbital magnetic dipole moment of an electron with the values (a) $m_\ell = 3$ and (b)$m_\ell = -4$?

69In Fig. 32-42, a bar magnet lies near a paper cylinder. (a) Sketch the magnetic field lines that pass through the surface of the cylinder. (b) What is the sign of $\vec{B} \cdot d\vec{A}$ for every area $d\vec{A}$ on the surface? (c) Does this contradict Gauss' law for magnetism? Explain.

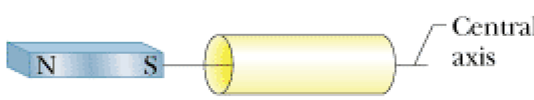


Figure 32-42 Problem 69.

Answer:

(b) sign is minus; (c) no, because there is compensating positive flux through open end nearer to magnet

70In the lowest energy state of the hydrogen atom, the most probable distance of the single electron from the central proton (the nucleus) is 5.2×10^{-11} m. (a) Compute the magnitude of the proton's electric field at that distance. The component $\mu_{s,z}$ of the proton's spin magnetic dipole moment measured on a z axis is 1.4×10^{-26} J/T. (b) Compute the magnitude of the proton's magnetic field at the distance 5.2×10^{-11} m on the z axis. (*Hint:* Use Eq. 29-27.) (c) What is the ratio of the spin magnetic dipole moment of the electron to that of the proton?

71 Figure 32-37 shows a loop model (loop L) for a paramagnetic material. (a) Sketch the field lines through and about the material due to the magnet. What is the direction of (b) the loop's net magnetic dipole moment $\vec{\mu}$, (c) the conventional current i in the loop (clockwise or counterclockwise in the figure), and (d) the magnetic force acting on the loop?

Answer:

(b) $-x$; (c) counterclockwise; (d) $-x$

72 Two plates (as in Fig. 32-7) are being discharged by a constant current. Each plate has a radius of 4.00 cm. During the discharging, at a point between the plates at radial distance 2.00 cm from the central axis, the magnetic field has a magnitude of 12.5 nT. (a) What is the magnitude of the magnetic field at radial distance 6.00 cm? (b) What is the current in the wires attached to the plates?

73 SSM If an electron in an atom has orbital angular momentum with m_ℓ values limited by ±3, how many values of (a) $L_{orb,z}$ and (b) $\mu_{orb,z}$ can the electron have? In terms of h, m, and e, what is the greatest allowed magnitude for (c) $L_{orb,z}$ and (d) $\mu_{orb,z}$? (e) What is the greatest allowed magnitude for the z component of the electron's *net* angular momentum (orbital plus spin)? (f) How many values (signs included) are allowed for the z component of its net angular momentum?

Answer:

(a) 7; (b) 7; (c) $3h/2\pi$; (d) $3eh/4\pi m$; (e) $3.5h/2\pi$; (f) 8

74 A parallel-plate capacitor with circular plates is being charged. Consider a circular loop centered on the central axis and located between the plates. If the loop radius of 3.00 cm is greater than the plate radius, what is the displacement current between the plates when the magnetic field along the loop has magnitude 2.00 μT?

75 Suppose that ±4 are the limits to the values of for an electron in an atom. (a) How many different values of the electron's $\mu_{orb,z}$ are possible? (b) What is the greatest magnitude of those possible values? Next, if the atom is in a magnetic field of magnitude 0.250 T, in the positive direction of the z axis, what are (c) the maximum energy and (d) the minimum energy associated with those possible values of $\mu_{orb,z}$?

Answer:

(a) 9; (b) 3.71×10^{-23} J/T; (c) $+9.27 \times 10^{-24}$ J; (d) -9.27×10^{-24} J