LECTURE 1
The Birth of Quantum Physics

- *Historical Introduction*
- *Light and Maxwell's Equations*

Introduction

By the end of the nineteenth century, classical physics was well developed; the body of knowledge that comprised theoretical physics was thought to be complete. In the seventeenth century, Sir Isaac Newton had generalized the laws of mechanics and gravitation in terms of four equations from which all of theoretical mechanics could be derived and explained. In the middle of the nineteenth century, James Clerk Maxwell had generalized all of electricity and magnetism into a cohesive theory of electromagnetism that not only explained all the previously observed electric and magnetic phenomena, but also predicted the existence of electromagnetic waves. Statistical mechanics had been able to explain the macroscopic behavior of thermodynamic systems composed of many microscopic particles. The ideas of classical physics progressed toward greater simplicity and unity in the explanation of natural phenomena. It was smugly felt by most physicists that any future progress in physics would involve refining measurements in the seventh decimal place.

The basic notions of classical physics were deeply rooted in Newtonian particle dynamics and Maxwell's theory of electromagnetism. According to classical theory, all physical phenomena could be described in terms of *matter* and *radiation*, and their interactions. Inherent in these ideas is that matter is composed of *particles* that can be perfectly localized in space, regardless of their state of motion. The location and motion of a particle could be described in terms of orbits or trajectories. It was firmly believed that a measurement of the particle's position, velocity, inertia or acceleration could be made, at least in principle, to any desired precision. Furthermore, it was believed that such properties of a particle were well separated from such properties of the measuring device. In other words, *the role of the observer could be made negligible.* Classically, the *state* of a particle is completely determined by specifying the *position* and *velocity* of the particle. If the initial position and velocity of a particle are known, then the position and velocity at any later time can be unequivocally determined from knowledge of the forces that act on the particle and the application of Newton's laws. In other words, *the classical motion of particles is completely deterministic.*

Radiation, on the other hand, was mathematically described in the language of wave motion. The classical electromagnetic field, by definition, has spatial extension. It is spread out in space, not localized like a particle. The most "spread out" case of electric and magnetic fields occurs in unbounded free space where the solutions to Maxwell's equations are plane waves of infinite extent. Because electromagnetic waves have spatial distribution, they can be diffracted and two coherent electromagnetic waves can interfere with each other.

At the end of the nineteenth century, classical physics was marked by impressive success in its ability to explain most known physical phenomena in terms of a general theory of "particulate" matter and "wave-like" radiation. When the twentieth century began, there were a few puzzles that were not explainable within the framework of classical physics. Physicists were still unable to measure the earth's progress through the aether; the radiation from hot bodies was still quite mystifying, and the properties of the recently discovered electron were all but unknown. It was confidently felt, however, that these problems would be solved within the classical framework provided by Newtonian mechanics and Maxwell's electromagnetic theory. Although the explanation of these puzzles required a new, "non-classical" theory, little damage was done to the content of classical physics; we still find classical physics useful in a variety of situations. However, the philosophical foundations of classical physics were reduced to rubble. One consequence of this was the birth of quantum physics; the other, the birth of relativity.

A Brief History of Light

Since many of the problems that led to the development of modern quantum theory dealt with light and its interaction with matter, it is appropriate to briefly examine the history of light. Light has a rich history that dates back to the thirteenth century. Historical records indicate that the Franciscan Roger Bacon (1215 - 1294) initiated the idea of using lenses to correct vision problems. Later records in the archives at the Hague indicate that Hans Lippershey (1587 - 1619), a Dutch lens maker, applied for the first patent on the telescope. Galileo Galilei (1564 - 1642) built his own telescope and used it in 1610 to make the first observations of the moons of Jupiter. Later that year, he observed Saturn's rings and sunspots. Though unsuccessful, Galileo is credited with one of the earliest attempts to measure the speed of light. Diffraction effects of light were observed by both Francesco Maria Grimaldi (1618 - 1663) and Robert Hooke (1635 - 1703). Apart from the practical use of lenses, early investigators of light were concerned with answering two basic questions:

1. Does light travel with a finite speed, and if so, what is this speed?

2. What is the exact nature of light -- does it represent the propagation of a stream of particles or does it represent a kind of wave motion? If light is a wave, what precisely is "waving"?

A modern understanding of light begins with the work of Galileo Galilei and Sir Isaac Newton. The first recorded experiment to measure the speed of light was carried out by Galileo Galilei in the early 1600's. As the story goes, Galileo stood on a hilltop while an assistant stood on a distant hilltop. Both men held shuttered lanterns. First, Galileo opened the shutter of his lantern. When the assistant saw the light from Galileo's lantern, he opened the shutter of his own lantern. Galileo attempted to measure the time between the opening of his shutter and seeing the light from his assistant's lantern. By knowing the distance between them, he could then, in principle, calculate the speed of light (*speed = distance/time*). Unfortunately, Galileo used his pulse beats to measure the time. He noticed that no matter how much distance was placed between him and his assistant, there was no noticeable increase in the measured time. His only conclusion was that the speed of light was too great to be measured in this unsophisticated way. More accurate experimental methods were required.

The first accurate measurement of the speed of light had to wait until 1675. A Danish astronomer named Olaus Roemer had been studying the moons of Jupiter (ironically, one of Galileo's telescope discoveries). He noticed, much to his surprise, that as the Earth moved closer to Jupiter, the satellites seemed to move faster, and that, as the Earth moved away, they seemed to slow down slightly. The only explanation for these observations was that light was taking a finite time to travel from Jupiter's moons to the Earth. Roemer concluded that it took light sixteen and one-half minutes to travel the diameter of the Earth's orbit about the sun. The size of the Earth's orbit was not accurately known in Roemer's day, however, when the modern value of the average Earth-sun distance is used, a value close to 3×10^8 m/s is obtained.

The first successful determination of the speed of light in a laboratory occurred around 1850 in France by two French scientists, Fizeau and Foucault. In their celebrated experiment, they built an apparatus that consisted of a light source, a rotating mirror and a stationary mirror. Light from the source was reflected from the rotating mirror to the stationary mirror and back to the observer. Because of the rotating mirror, the light reaching the observer was slightly deflected from its path in the original beam as shown in Figure 1-1. By knowing the dimensions of the apparatus and the angle of deflection, Fizeau and Foucault were able to deduce the speed with which the light was traveling. The value they obtained was very close to 3.0 x 10^8 m/s, a result that is consistent with Roemer's observations if the modern value of the earth-sun distance is used in the calculation.

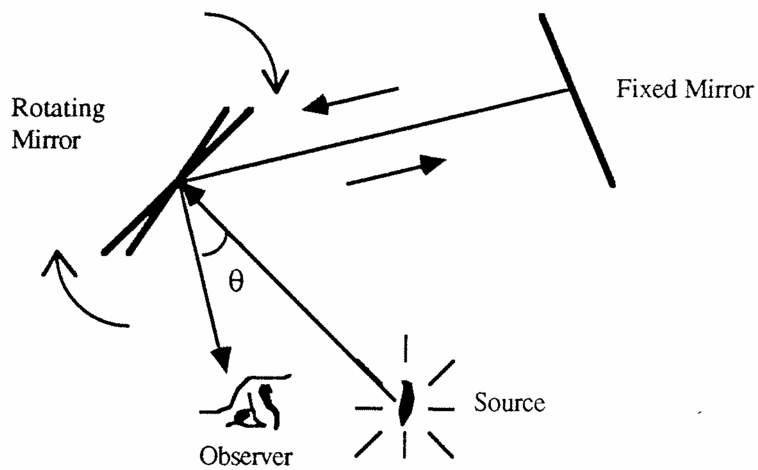

Figure 1-1

The most contemporary measurements of the speed of light through vacuum yield the value 299, 792,458 m/s (see for example, *Physics Today*, August 1990). This value, when rounded to two significant digits agrees with both Roemer and Fizeau. This is the speed of light in vacuum and is one of the most important physical constants of nature. It is customary to represent this constant with the symbol c. When light travels through transparent materials such as glass or transparent liquids, it travels with a speed that is less than c (in air, the speed of light is extremely close to c, so a distinction between air and vacuum is rarely made except for precision work).

Newton proposed a corpuscular theory of light. Another breakthrough in the nature of light came in the late 1600's. This time the breakthrough was due to the work of Sir Isaac Newton with what he called the celebrated "Phenomenon of Colours." A beam of sunlight was passed through a glass prism and was observed to spread into the colors of the rainbow as suggested in Figure 1-2. This rainbow is called a **spectrum.**

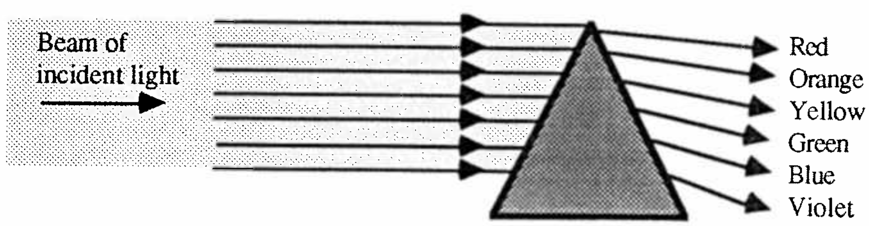

Figure 1-2

From his experiment, Newton concluded that:

1. White light is a mixture of many colors.
2. A prism does not create colors; it merely separates the colors that already existed as a mixture of white light.

The last point is important, for earlier observers of this phenomenon believed that the colors were somehow added to the light by the prisms. Newton confirmed his conclusions by positioning a second prism so that the spectrum of colors would recombine to form a beam of white light.

Newton "erroneously" concluded, however, that light was composed of tiny particles or corpuscles. Although a corpuscular theory of light could account for his "celebrated phenomenon of colours," it could not account for other later observed phenomena.

A Dutch astronomer Christian Huygens suggested that light traveled in the form of waves not particles. The wavelike nature of light was convincingly demonstrated in 1801 in a landmark experiment performed by an English physicist named Thomas Young. In his experiment, Young passed a beam of white light through two narrow slits in an opaque screen. On another distant screen beyond the slits, Young observed a series of alternating light and dark bands as shown in Figure 1-3a. *Such a pattern is characteristic of wave* ***interference*** and is called **an interference pattern.**

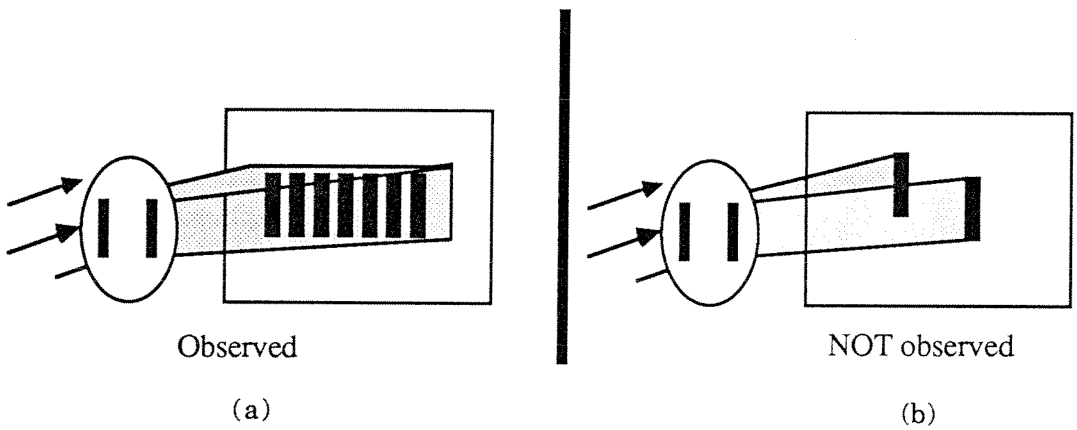

Figure 1-3

Young reasoned that if light were composed of corpuscles or particles, the distant screen would contain only two spots, each of which contained particles that passed through one or the other of the two slits as shown in Figure 1-3b. In other words, if light was composed of particles, there would be *no interference pattern*. The study of mechanical waves was well known in Young's time; interference was known to be strictly associated with wave phenomena. Young's observations could only be made if light possessed wavelike properties.

Question 1-1: When light passes through two adjacent windows, we generally do not notice any interference pattern, but rather, two bright spots with the same general shape as the windows. Does this imply, then, that light is made up of particles? Why or why not?

Once it was established that light was a wave phenomenon, the crucial question to be answered was: *If light is a wave, what is waving?*

Maxwell's Theory of Electromagnetic Radiation

The early nineteenth century saw tremendous growth in the understanding of electric and magnetic phenomena. By 1850, the laws of electrostatics, current electricity, magnetism and electromagnetic induction were well established. A Scottish mathematician and theoretical physicist named James Clerk Maxwell set out to generalize these ideas in terms of a few simple equations from which all of electricity and magnetism could be derived (much as Newton had done in mechanics two centuries earlier).

Everything that was known concerning electricity and magnetism in 1850 can be summarized by the following four equations (which hold in vacuum or free space):

$$\oint \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0} \qquad \oint \mathbf{E} \cdot d\ell = -\frac{d\Phi_B}{dt} \tag{1.1}$$

$$\oint \mathbf{B} \cdot d\mathbf{S} = 0 \qquad \oint \mathbf{B} \cdot d\ell = \mu_0 i$$

Question 1-2: (a) Identify each of the above equations. (b) What is the physical significance of each?

Notice that there is a certain symmetry associated with the left hand sides of each of the above equations. Two equations involve the flux of a field through a closed surface, and two of them involve the line integral of a field over a closed curve in the respective field. The right hand sides, display asymmetries.

The first asymmetry deals with the fact that while there are point sources for the electric field, there are no observed point sources for the magnetic field. Thus, $\oint \mathbf{B} \cdot d\mathbf{S}$ is always equal to zero, while in general, $\oint \mathbf{E} \cdot d\mathbf{S} \neq 0$.

Question 1-3: (a) When will $\oint \mathbf{E} \cdot d\mathbf{S} = 0$? (b) Give an example that illustrates this situation.

This asymmetry has led physicists to search for *magnetic monopoles*, or point sources of the magnetic field; however, none have ever been observed. Until we find magnetic monopoles, we must be content with this asymmetry.

The second asymmetry deals with the fact that the line integral of **E** around a closed loop is equal to a time derivative of the magnetic flux through the loop, yet, the line integral of **B** around a closed loop is not equal, nor even proportional, to the time derivative of electric flux through the loop. Maxwell was bothered by this asymmetry, and concluded that the laws of electromagnetism would be more esthetically pleasing if the last equation were written as

$$\oint \mathbf{B} \cdot d\ell = \mu_0 i + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}.$$

where the constants in front of $d\Phi_E/dt$ have been chosen for consistent dimensionally.

Question 1-4: Isn't there still an asymmetry? What about the $\mu_0 i$ term?

Confident in nature's inherent symmetry, Maxwell boldly assumed that the term involving the time derivative of electric flux should be included in the last equation. With its inclusion, the set of equations listed in (1.1) are called *Maxwell's equations*. In the absence of sources, that is, in regions of space where there are no sources of electric or magnetic fields, these equations, in free space, reduce to

$$\oint \mathbf{E}\cdot d\mathbf{S} = 0 \qquad \oint \mathbf{E}\cdot d\ell = -\frac{d\Phi_B}{dt} \tag{1.2}$$

$$\oint \mathbf{B}\cdot d\mathbf{S} = 0 \qquad \oint \mathbf{B}\cdot d\ell = \mu_0\varepsilon_0\frac{d\Phi_E}{dt}$$

Question 1-5: Show that this is true.

Since these equations are written in terms of integrals, they are frequently referred to as the *integral form of Maxwell's equations in free space.* The divergence theorem and Stoke's theorem can be used to change these equations into *differential form.*

$$\nabla\cdot\mathbf{E} = 0 \qquad \nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \tag{1.3}$$

$$\nabla\cdot\mathbf{B} = 0 \qquad \nabla\times\mathbf{B} = \mu_0\varepsilon_0\frac{\partial\mathbf{E}}{\partial t}$$

Question 1-6: (a) What is the physical interpretation of each of the above equations? (b) Describe the symmetries in these equations.

These equations can then be combined to give the following equations for **E** and **B**:

$$\nabla^2\mathbf{E} = \mu_0\varepsilon_0\frac{\partial^2\mathbf{E}}{\partial t^2}$$

and

$$\nabla^2\mathbf{B} = \mu_0\varepsilon_0\frac{\partial^2\mathbf{B}}{\partial t^2}.$$

Notice that these equations are of the same form as the equation that describes classical wave motion:

$$\nabla^2\Psi = \frac{1}{v^2}\frac{\partial^2\Psi}{\partial t^2}.$$

Thus, Maxwell's theory predicts the existence of coupled electric and magnetic fields that propagate through space as *waves.* In 1856, in his famous paper, Maxwell proposed the existence of these *electromagnetic waves* -- disturbances that consist of changing electric and magnetic fields that propagate through space much as a physical disturbance propagates down a rope or a string. Furthermore, Maxwell's theory predicted that these waves could be reflected from a conductor or dielectric, refracted in a dielectric, polarized and could be made to exhibit interference and diffraction.

> **Question 1-7**: What is the speed of propagation of these electromagnetic waves?
> **Hint**: Compare the classical wave equation with those for **E** and **B**. Then evaluate the resulting expression.

This result was a tremendous break through in the understanding of the nature of light. Remember that the speed of light had been accurately measured by 1856 and was known to be close to 2.998×10^8 m/s. Maxwell's theory seemed to suggest that light consists of changing electric and magnetic fields that propagate through space. That is, light was one type of Maxwell's waves.

Direct experimental verification of this theory would be difficult. How does one make an electromagnetic wave? According to Maxwell's theory, an oscillating current in a wire would set up fluctuating electric and magnetic fields in the region surrounding the wire. This can be understood by considering the following thought experiment. Imagine a point charge that is somehow accelerated from rest. When the charge is at rest, it gives rise to a stationary radial electric field. At the instant that the charge begins to move, the electric field in the vicinity of the charge is altered or distorted from its radial configuration. This distortion then propagates away from the charge at some finite speed. Mathematically, the distortion in the electric field means that $\partial E/\partial t \neq 0$. From Eqns. (1.3), this implies that there will be an induced magnetic field. In other words, the changing electric field induces a magnetic field. However, since the charge is accelerated, the quantity $\partial E/\partial t$ is not constant, but rather, it is itself a time-dependent quantity. This implies that the induced magnetic field will be time dependent. But a time dependent magnetic field induces an electric field. And so we have an electromagnetic wave. A time dependent electric field that induces a time dependent magnetic field that induces a time dependent electric field and so on. As one field changes, it generates a new field that extends further on and the disturbance moves outward through space. These disturbances would then propagate through space by transferring energy from the changing electric field back to the magnetic field and so on. In the case of an oscillating current, the charges oscillate with simple harmonic motion thereby producing electromagnetic waves with sinusoidal dependencies in both space and time. The problem with generating light in this manner is that the highest electrical frequencies attainable in Maxwell's day were about 10^9 Hz; the frequency of visible light was known to be a million times higher.

> **Question 1-8**: How could they know the frequency of visible light in Maxwell's time?

Heinrich Hertz's Experimental Verification of Maxwell's Waves

Heinrich Hertz (1857-1984) was an extraordinarily gifted German experimental physicist. In an exhaustive series of experiments in 1886, Hertz demonstrated that an oscillating electric current does, in fact, produce an electromagnetic wave that possesses the same physical characteristics of light (except that it has a different wavelength). A greatly simplified version of Hertz's experiment is shown in Figure 1-4. This set-up shows two loops of wire of identical size and shape. Each loop is open and the ends are covered with brass knobs. One loop is placed in series with a switch a high voltage source. When the switch is closed, a spark jumps between the two brass knobs. Such a spark consists of a series of high-frequency surges of electric charge. The oscillations in the surge occur while the air gap remains conducting and the charge oscillates back and forth until electrical equilibrium is established. The frequency depends on the inductance and capacitance of the loop.

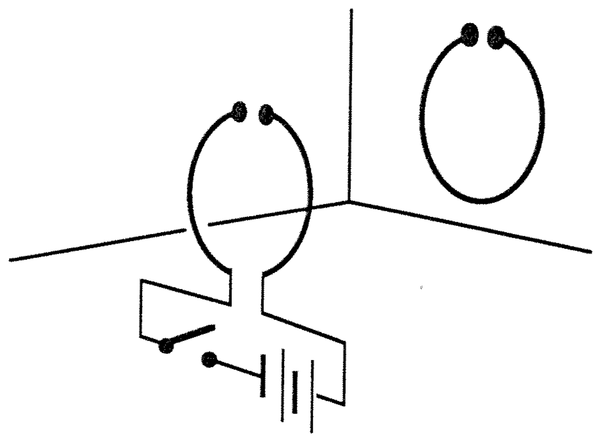

Figure 1-4

Question 1-9: (a) Explain how the loop has both inductance and capacitance. (b) What is the expression that determines the frequency of the charge oscillations?

In other words, during the "ringing" or charge oscillations, the loop acts like a resonant LC circuit. Since the charges are accelerated while they are oscillating, Maxwell's theory predicts that they should produce an electromagnetic wave at the resonant frequency of the loop. If Maxwell's theory is correct, when this electromagnetic wave reaches the second loop, it should "drive" the second loop at this same frequency (since the second loop, being identical to the first has the same natural oscillation frequency). The oscillations in the second loop would be detected as a weak spark at the gap (The first loop acts as the transmitter, the second loop acts as the receiver).

Hertz quickly succeeded in detecting the predicted sparks even at distances of several hundred meters with an oscillation frequency of 5×10^8 Hz.

Question 1-10: (a) What was the wavelength of the electromagnetic wave? (b) Consult a chart of the electromagnetic spectrum. To which type of waves do Hertz's waves correspond?

Hertz then proceeded to carry out a series of exhaustive experiments in which he showed that these waves could be reflected from metallic surfaces, reflected from and refracted through paraffin, focused by paraffin "lenses," polarized, and made to interfere. In short, Hertz had provided the experimental evidence that demonstrated Maxwell's predictions -- the existence of electromagnetic waves that possess the same properties of light. Not only did these observations give empirical evidence of the existence of electromagnetic radiation, but they provided the scientific community with a more complete "picture" of light. Together, Maxwell and Hertz had succeeded in uniting electricity, magnetism and optics. **Light is an electromagnetic wave.**

In the course of Hertz's studies, he noted that "it is essential that the pole surfaces of the spark gap should be frequently repolished" to ensure the production of the spark (H. Hertz, *Ann. Physik* (Leipzig), **33:** 983, 1887). In further investigating this phenomenon, Hertz concluded that ultraviolet radiation falling on the clean metallic knobs facilitated the surge of charge in the spark. Hertz had discovered the *photoelectric effect*, a phenomenon that *could not be explained* *by treating light simply as an electromagnetic wave.* As we shall see, a complete understanding of the photoelectric effect required the abandonment of the classical wave description of light and the adoption of a new model. In verifying the existence of electromagnetic radiation and establishing the "true" nature of light, Hertz had discovered a phenomenon that would ultimately discredit this theory and provide a firm cornerstone for the birth of quantum physics!