LECTURE 2
An Introduction to the Old Quantum Theory

- *Blackbody Radiation*
- *The Photoelectric Effect*

2.1 Introduction

One of the tremendous successes of Maxwell's theory of electromagnetism was its prediction of the existence of electromagnetic waves that move through vacuum with the same speed as that of light. As we discussed in the previous lecture, Maxwell's theory not only gave conclusive evidence that light was in fact a type of wave motion, but it answered the question, "what is waving?" According to Maxwell's theory, light consists of a changing electric field that generates a changing magnetic field, that in turn, generates a changing electric field, and so on indefinitely. The wave travels by transferring energy from the electric field to the magnetic field and back again, in much the same way that a wave in a spring travels by transferring energy from the potential energy of compressions and rarefactions to the kinetic energy of the spring's mass and back again.

In particular, Maxwell's theory provides a mathematical description of the propagation of light through vacuum, as well as transparent materials.

Question 2-1: In what way(s) does the propagation of light through vacuum differ from that through a transparent material (say glass)?

Much of the behavior of light as it encounters materials, i.e, its reflection, refraction, dispersion and scattering, can be explained by assuming that the light sets the electrons of the material into vibration. According to classical theory, these vibrating electrons then act as the source of the light wave that is observed to be reflected, refracted, dispersed or scattered. Encouraged by the success of Maxwell's theory of light, physicists immediately attempted to apply it to a long-standing puzzle of classical physics -- the so called "blackbody problem". The problem is to predict the intensity of radiation emitted at a given wavelength (or frequency) by a hot glowing solid at a specified temperature. As we shall see, for all of its strengths, Maxwell's theory was unable to provide a reasonable explanation of the blackbody problem that was consistent with observed experimental results.

2.2 Thermal Radiation

Any object that is warmer than its environment emits electromagnetic radiation. Objects that are cooler than their environments *absorb* radiation. They type of electromagnetic radiation that is emitted from, or absorbed by an object depends on the temperature of the object. Imagine heating a tungsten wire by allowing an electric current to flow through it. If you place your hand near the filament, you will soon feel the "heat" radiated from the wire. Energy is being emitted from the filament in the form of electromagnetic waves in the *infrared* region of the spectrum. If we continue to heat the wire to higher temperatures we will see that the wire eventually begins to glow a faint red color. Now energy is being emitted from the filament in the form of electromagnetic waves in the *visible* region of the spectrum -- that is, in the form of *light*. Experience tells us that the amount of light emitted from an object, as well as the color of the emitted light depends on the temperature of the object. Our language reflects such ideas; when a piece of metal is heated until it just begins to glow, we say that the metal is "red hot". If we continue to heat the metal to higher and higher temperatures, we observe that the light emitted becomes *more intense* as the metal becomes "white hot".

In 1792, Thomas Wedgewood, a renowned maker of china and a relative of Charles Darwin, first observed that all the china in his ovens became red at the same temperature regardless of their size, shape or chemical composition. In fact, it is possible to get a rough idea of the temperature of a heated solid by observing its color. The table below gives the temperature ranges associated with the observed color for an object that is heated from 500 °C to 1550 °C.

TABLE 2-1

COLOR AND TEMPERATURE OF A HEATED SOLID

Color	Temperature, ° C
incipient red	500 - 550
dark red	650 - 750
bright red	850 - 950
yellowish red	1050 - 1150
yellowish white	1250 - 1350
white	1450 - 1550

Information taken from Table 19-1, *Foundations of Physics,* Robert L. Lehrman and Clifford Swartz. Copyright Holt Rinehart and Winston, Inc., 1969.

The colors listed in table 2-1 correspond to those of a nearly perfect absorber; that is, the object absorbs (nearly) all the radiation that falls upon it. Such objects appear to be black when cold, and therefore, they are called *blackbodies*. Ideal blackbodies are perfect absorbers, absorbing all the electromagnetic radiation that falls upon them regardless of the frequency of the radiation. Furthermore, since blackbodies are perfect absorbers, it follows from thermodynamic arguments that they are also ideal emitters. They are of special interest to physicists because the radiation that they emit at a given temperature depends *only on the temperature* and is *independent of the composition and geometric features of the body*. Blackbodies are of special interest to astronomers and astrophysicists because most stars, including our own Sun, act as nearly perfect blackbodies.

Question 2-2: How would the temperatures in Table 2-1 compare with those for an object that is *not* a blackbody (that is, for an object that is *not* black when cold)?

Most ordinary objects do not act like blackbodies. We see most objects because of the light that they reflect. The best approximation in nature to blackbodies are stars. They absorb almost all of the radiation that is incident upon them. They appear bright because of the light that they emit due to their surface temperatures. The best man-made approximation to a blackbody is a small hole made on the side of a large closed box as shown in Figure 2-1. Any light which enters the hole will bounce around inside the box. Some light will be absorbed with each reflection, until finally all of the light is converted into the internal energy of the matter that makes up the inner surface of the box. Only a negligibly small amount of the incident light will leave the box. Since the best man-made black body is a single hole that enters a cavity, blackbody radiation is frequently called *cavity radiation*.

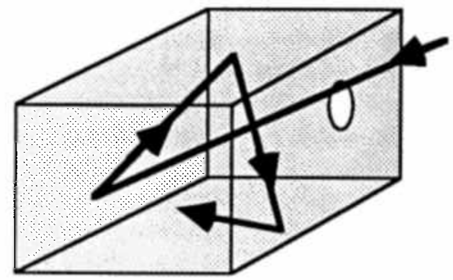

Figure 2-1

Question 2-3: True or false? No matter how black you paint a bird house, the hole in the front will always appear to be blacker. Briefly support your choice.

The Spectral Distribution of Blackbody Radiation

If such a cavity is maintained at a specific temperature, so that the radiation inside the cavity is in thermal equilibrium with the cavity walls, it can be experimentally demonstrated that the intensity of the radiation emitted from the cavity is, in fact, *dependent only the temperature*. It does not matter whether the cavity walls are made of copper, brass, iron or any other material that will tolerate the temperature. The radiation will be emitted with a range of wavelengths. Figure 2-2 shows plots of the intensity radiated at all wavelengths for a black body at three different temperatures. Such plots are frequently called *blackbody spectra* or more simply *blackbody curves.*

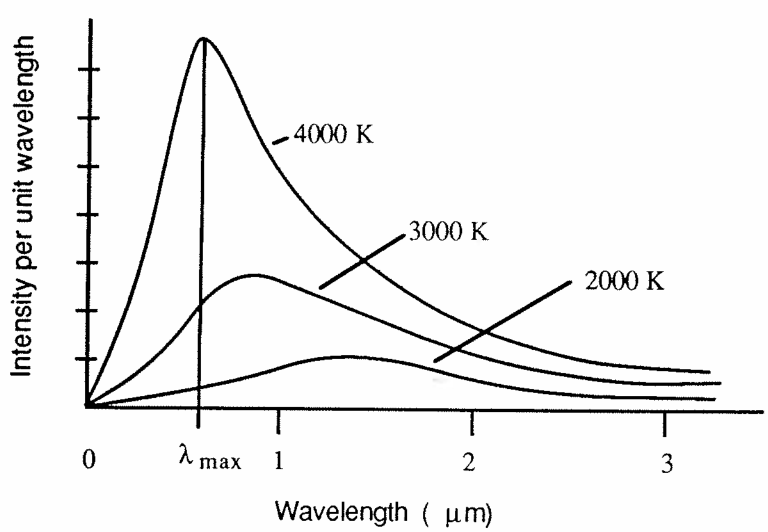

Figure 2-2

All blackbody curves display the following important features:

1. **The radiation has a continuous spectrum.** The intensity is not the same at all wavelengths, but rather, each curve has a clearly visible maximum (the maximum occurs at the wavelength labeled λ_{max} for the case $T = 4000$ K).

2. **For a given temperature, the curve (and therefore the energy distribution) is the same, regardless of the nature and composition of the cavity walls.**

3. **The area under any of the curves represents the total intensity radiated by the blackbody.**

4. **There is a simple relationship between the absolute temperature and the wavelength at which the maximum intensity occurs.** It is called *the Wein Displacement Law* and is quantitatively expressed as

$$\lambda_{max} T = \text{constant} = 0.29 \text{ cm K.}$$

   This relationship was empirically established by Wilhelm Wien in 1893.

5. **The total intensity radiated is a simple function of the temperature.** It is called the *Stefan-Boltzmann law* and is mathematically expressed as

$$I = \sigma T^4$$

   where σ is called the Stefan-Boltzmann constant and equal to

$$\sigma = 5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}.$$

   This relationship was found experimentally in 1879 by physicist J. Stefan. Ludwig Boltzmann later showed that it followed from thermodynamic arguments. An object that is not a blackbody will follow a law of the same general form: $I = a\,\sigma T^4$, where the constant a is less than unity.

Question 2-4: Use Figure 2-2 to explain the color changes indicated in Table 2-1 as the blackbody is heated from 500 °C to 1550 °C.

Question 2-5: What is the predominant type of electromagnetic radiation that is emitted by a blackbody at 4000 K?

Question 2-6: What is the predominant type of electromagnetic radiation that is emitted by your body at this moment? Assume blackbody arguments.

Question 2-7: Use Wein's law to estimate the surface temperature of the Sun.

Question 2-8: How do the areas under the three blackbody curves shown in Figure 2-2 compare with each other? Justify your answers.

Question 2-9: Betelgeuse, a supergiant star of radius 3×10^{11} m, emits radiant energy at an average rate of 4×10^{30} W. How does its surface temperature and power output compare with those of our Sun? The radius of the Sun is 7×10^{8} m. (You may use your result from Question 2-7.)

There appears to be nothing obviously unusual about blackbody radiation or blackbody curves. The qualitative aspects of blackbody curves seem to be compatible with sound physical intuition. The quantitative aspects expressed through Wein's law and the Stefan-Boltzmann law can be experimentally verified. Questions 2-4 through 2-9 suggest that blackbody theory has a wide variety of practical applications in both science and industry. Yet classical physics was unable to provide a simple explanation of the radiation emitted from glowing objects and the general features of blackbody curves. Many attempts were made using a combination of classical thermodynamics, statistical mechanics and Maxwell's theory of electromagnetism. The best attempts displayed flawless reasoning and careful mathematical derivations. The only thing wrong with them was that they failed to stand the test of experiment!

The Classical Theory of Blackbody Radiation

Although the mathematical derivation for the energy density of cavity radiation using classical arguments is not beyond the scope of this discussion, it is standard and can be found in most introductory texts on modern physics and quantum theory (see for example the derivation of the Rayleigh-Jeans formula, *Eisberg and Resnick*, section 1-3). For our purposes, a detailed discussion of the theory and why it failed will be more useful.

According to classical theory, when light (or any other form of electromagnetic radiation) interacts with matter, the electric field drives the electrons in the material. The electrons, in turn oscillate with the field. The oscillating electrons produce electromagnetic waves at their vibrational frequencies. It is worth noting at this point, that this model works well to describe many phenomena including reflection, refraction, dispersion, light scattering and polarization.

Consider a cavity that is maintained at a certain temperature T. The electrons in the walls of the cavity will oscillate because of their thermal motion. Their oscillations will produce electromagnetic radiation at their vibrational frequencies. This electromagnetic radiation fills the cavity. The oscillation frequency of the electrons will depend on the actual location of the electron and in general will vary with time. Any given electron will lose energy in the form of electromagnetic radiation, but it will also gain energy from the radiation in the cavity. The energy of a given electron also depends on its location and will vary with time. In general, the energy and frequency will vary greatly among electrons, but from statistical mechanics arguments, the average energy of the electrons is $3kT$, where k is Boltzmann's constant and T is the absolute temperature.

Question 2-10: Why is this so? Hints: Consider the equipartition theorem. Also note that the electrons can oscillate in three dimensions and that their average kinetic energy is equal to their average potential energy.

If the system is maintained at a certain temperature T, the entire system will be in thermal equilibrium at this temperature. On average the energy that the oscillating electrons lose in radiation will be gained by the energy that they absorb from the radiation. From statistical considerations, however, the average energy per electron is $3kT$ regardless of the oscillation frequency. But there are restrictions on the permitted frequencies. In order for electromagnetic waves to exist in equilibrium inside the cavity, they must satisfy certain boundary conditions at the cavity walls. In particular, only waves with the precise wavelength to form standing waves can exist in the cavity. Since the wavelength of an electromagnetic wave determines its frequency, the cavity will contain electromagnetic waves of only certain frequencies.

Question 2-11: Why do the boundary conditions imply that the electromagnetic radiation must set up standing waves?

The situation is somewhat similar that of a string that is clamped at both ends. If the string is plucked, it must vibrate so that there is a node at each end. If the string has length L, it can vibrate with wavelengths $2L$, L, $2L/3$, $L/2$, $2L/5$, . . . etc. The standing waves are restricted to integer multiples of the fundamental frequency. If the fundamental frequency is 10 Hz, the string can in principle, simultaneously vibrate at 20 Hz, 30 Hz, 40 Hz, and so on, without any upper bound. Since the string is clamped at both ends, the amplitude of vibration must decrease as the frequency increases. Since the energy of a wave depends on the square of the amplitude, modes with short wavelengths and high frequencies will have smaller energies than long wavelength high frequency modes. In other words, the energy will drop off at higher frequencies and at some upper limit, the energy will be vanishingly small.

The standing waves formed by the radiation in the cavity are somewhat different from the standing waves on the string. Since the standing waves in the cavity are three dimensional there are many more modes of vibration, particularly at high frequencies. More importantly, however, there is a very crucial difference. *The energy per mode does not drop off at higher frequencies as it does for standing waves on a string.* The cavity radiation is in thermal equilibrium with the oscillating electrons in the cavity walls. But since the average energy of the electrons is $3kT$ regardless of their frequency, the average energy per electron is $3kT$ for all possible modes of vibration. In other words, *classical theory predicts that every mode of standing waves of radiation in the cavity has the same amount of energy.* Since there are more possible vibrational modes at higher frequencies, most of the energy will be in high-frequency radiation. If radiation from the cavity is analyzed, high frequency radiation should be emitted with the highest intensity. Instead of the behavior observed in Figure 2-2, a typical blackbody curve would look like the darkened line shown in Figure 2-3 (an actual blackbody curve is shown in that figure for comparison). Note the absurdity of these predictions. These results imply that an opened oven should be an excellent source of high frequency radiation such as UV, X-rays and gamma rays!

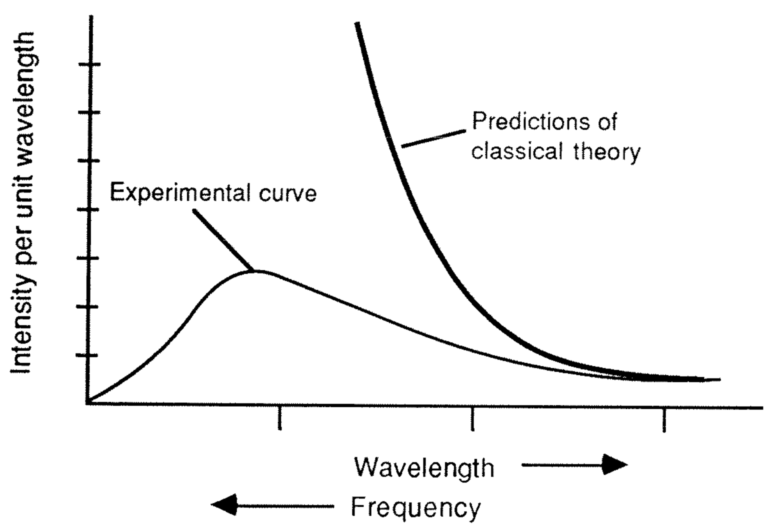

Figure 2-3

Near the end of the nineteenth century, Lord Rayleigh and Sir James Jean used the classical model of cavity radiation discussed above to derive an expression for the energy density of blackbody radiation. The famous *Rayleigh-Jeans formula for blackbody radiation* is

$$\rho(\nu) = \frac{8\pi\nu^2 kT}{c^3} \tag{2.1}$$

where $\rho(\nu)$ is the energy per unit volume of cavity radiation of frequency ν. As suggested in Figure 2-3, the classical theory fits the experimental observations in the low frequency, long wavelength limit. At fixed T, equation (2.1) agrees with the experimentally predicted values for low frequency (long wavelength). It is evident, however, that the energy density will increase without bound as ν^2 with increasing frequency. Since the discrepancy between theory and experiment usually occurs in the ultraviolet region of the frequency spectrum, the unrealistic behavior of equation (2.1) is usually referred to by physicists as the "ultraviolet catastrophe."

Using only thermodynamics and Maxwell's equations it is possible to derive Wein's displacement law using an energy density of the form

$$\rho(\nu) = A\nu^3 e^{-\beta\nu/T} \tag{2.2}$$

where A and β are constants. This expression for the energy density of cavity radiation is known as *Wein's exponential law* and resembles Maxwell's velocity distribution from kinetic theory. Equation (2.2) fits the experimental blackbody curves reasonably well at high frequencies but fails in the low frequency, long wavelength limit where it predicts that the intensity drops to zero rapidly. Figure 2-4 compares the experimental curve with the theoretical results predicted by equations (2.1) and (2.2).

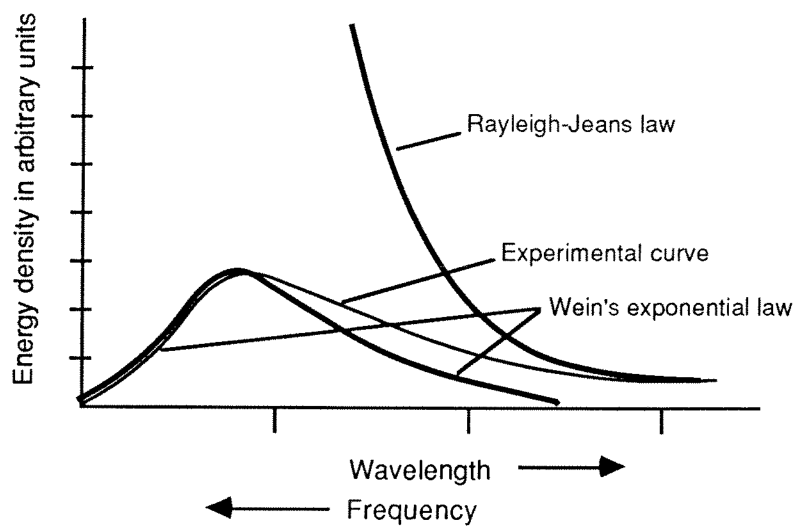

Figure 2-4

Max Planck and the Discovery of Energy Quantization

Max Planck (1858 - 1947) was a purely classical physicist. Throughout most of his life, his professional interests were almost exclusively in the field of thermodynamics. In October of 1900, Max Planck discovered the famous blackbody formula that gave birth to quantum theory. Planck knew that the Rayleigh-Jeans formula agreed with the experimental blackbody curves in the low-frequency, long-wavelength limit. Guided by the agreement of Wein's exponential law with the data in the high-frequency, short-wavelength limit, Planck interpolated between the two to obtain the formula

$$\rho(\lambda, T) = \frac{A}{\lambda^5}\left[\frac{1}{e^{B/\lambda T} - 1}\right]$$

where $A = 8\pi ch$ and $B = hc/k$. The constants c and k are the speed of light in vacuum and Boltzmann's constant respectively; the constant h was determined as a "best fit value" to the experimental curve. The "best fit" value of h was found to be 6.55 x 10^{-34} J·s. The constant h is known as *Planck's constant* and *sets the scale for the quantum regime.* The smallness of h is if no practical importance in large-scale phenomena in the macroscopic world; it is, however, of the utmost importance on the atomic scale. Planck's formula is usually expressed in "frequency" language:

$$\rho(\nu, T) = \frac{8\pi h\nu^3}{c^3}\left[\frac{1}{e^{h\nu/kT} - 1}\right]. \qquad (2.3)$$

Question 2-12: Show that Planck's radiation law, equation (2.3), reduces to the Rayleigh-Jeans law, and to the Wein exponential law in the appropriate limits. Why is this significant?

In his Nobel Prize acceptance speech in 1920 Planck explained:

> But even if the radiation formula proved to be perfectly correct, it would after all have been only an interpolation formula found by lucky guess-work and thus, would have left us rather unsatisfied. I therefore strived from the day of its discovery, to give it a real physical interpretation and this led me to consider the relations between entropy and probability according to Boltzmann's ideas. After some weeks of the most intense work of my life, light began to appear to me and unexpected views revealed themselves in the distance.

Planck found that he was able to derive his blackbody radiation formula from physical principles, but only if he assumed that *the electrons in the cavity wall, oscillating with frequency* ν *could possess total energies that satisfy the relation*

$$E = nh\nu \qquad \text{where} \qquad n = 1, 2, 3, \ldots \qquad (2.4)$$

where h is Planck's constant. Furthermore, such an electron could not gain or lose any fraction of its total energy. An oscillating electron in the cavity wall could change its energy only by an amount ΔE given by

$$\Delta E = h\nu. \qquad (2.5)$$

In other words, the energy of an oscillating electron could change only by discrete *quantized* amounts. Not only were these ideas bold and revolutionary, but they contradicted classical mechanics. According to classical physics, the energy of an oscillator is *independent* of its frequency. An oscillator of frequency ν can have *any value for its total energy* and can change its amplitude in a *continuous fashion* as it loses or gains any fraction of its total energy. Any oscillator observed in the macroscopic world certainly seems to behave in a classical way. Such observations do not contradict Planck's assumptions. If the physical world obeyed classical physics exactly, then $h = 0$. According to Planck's results, h is very close to zero, so that on a macroscopic scale, energy quantization is not evident (see *Eisberg and Resnick*, Example 1-6). On an atomic scale, however, the fact that h is not zero has ramifying implications - *energy quantization is apparent.*

It is interesting to note that Planck's work was not well received in the scientific community. Even Planck was not pleased with the required assumption of energy quantization. He spent the next fifteen years trying to derive his radiation formula from purely classical arguments. In a journal article in volume 72 of *Nature* (1905), James Jeans wrote, comparing his work to that of Planck:

> The methods of both are in effect the methods of statistical mechanics and the theorem of equipartition of energy, but I carry the method further than Planck, since Planck stops short of the step of putting $h = 0$. I venture to express the opinion that it is not legitimate to stop short at this point, as the hypotheses upon which Planck has worked lead to the relation $h = 0$ as a necessary consequence.
>
> Of course, I am aware that Planck's law is in good agreement with experiment if h is given a value different from zero, while my own law, obtained by putting $h = 0$, cannot possibly agree with experiment. This does not alter my belief that the value $h = 0$ is the only value which it is possible to take, my view being that the supposition that the energy of the ether is in equilibrium with that of matter is utterly erroneous in the case of ether vibrations of short wavelength under experimental conditions.

The quantization of the energy of the oscillating electrons had even further implications that Planck did not consider. If the electrons in a blackbody cavity can exchange energy with the radiation only in discrete amounts, then the radiation itself must also be quantized in discrete amounts. In 1905, Albert Einstein proposed the existence of *light quanta* (now called *photons*) which he described as concentrated "bundles" or packets of electromagnetic energy. Einstein was proposing a "corpuscular" or "particle" description for electromagnetic radiation. We will now see how this led to further developments in the history of quantum physics.

2.3 The Photoelectric Effect

In the years 1886 and 1887 Heinrich Hertz performed a series of experiments that confirmed the existence of Maxwell's electromagnetic waves. It is almost ironic that in the course of those experiments, Hertz discovered a phenomenon that would eventually discredit the classical theory of electromagnetic radiation. During the course of his experiments, Hertz observed that clean metal surfaces emit charged particles when they are exposed to ultraviolet radiation. In 1888, Hallwachs determined that the emitted particles were negatively charged, and one year later, J. J. Thompson showed that the particles were electrons. The process whereby electrons are emitted from a metal by the action of incident radiation is called the *photoelectric effect*.

Figure 2-5 shows a typical experimental set-up for studying the photoelectric effect. It consists of a metallic surface and a collector that are enclosed within a vacuum tube. Placing the system in a vacuum eliminates any collisions that emitted electrons might make with air molecules. A potential difference is maintained between the metallic surface and the collector by placing them in a circuit. Thus, the metallic surface and the collector will carry charges of opposite sign. The circuit contains a variable resistor (so that the potential difference between the plates can be systematically varied), and an ammeter and a voltmeter so that the current and potential difference or "applied voltage" can be measured. The circuit is usually equipped with a polarity reversing switch (not shown) so that the sign of the charges on the metal and the collector can be reversed. When incident radiation falls on the metallic surface, electrons, called *photoelectrons*, are emitted.

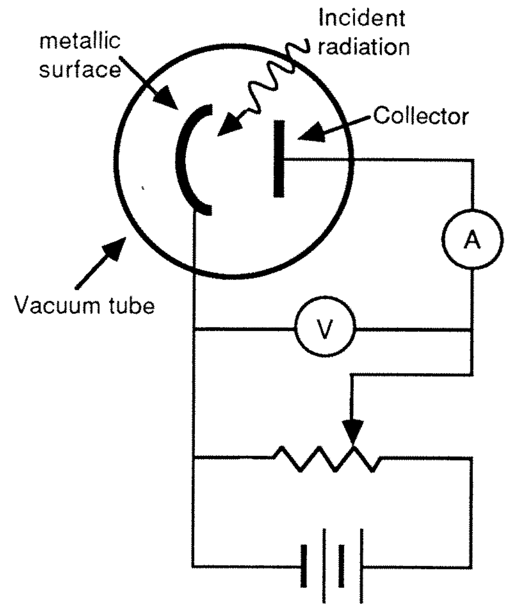

Figure 2-5

The photoelectrons are emitted from the metal with a range of kinetic energies. If the experiment is set up so that the metallic surface is negatively charged and the collector is positively charged, photoelectrons will be accelerated toward the collector thereby establishing a current, called a *photocurrent*, that can be measured with the ammeter. If the applied voltage is steadily increased from zero, the photocurrent will increase from some initial value I_o. When the applied voltage reaches a certain limiting value (the "saturation" voltage), all of the photoelectrons reach the collector; the photocurrent will reach a maximum value and remain constant for any further increase in the applied voltage (Figure 2-6). If, however, the polarity of the circuit is reversed so that the metallic plate is positively charged, and the collector is negatively charged, the photoelectrons will experience a *retarding potential*. In this case, the photoelectrons will *decelerate* as they approach the collector. Only the most energetic photoelectrons will reach the collector.

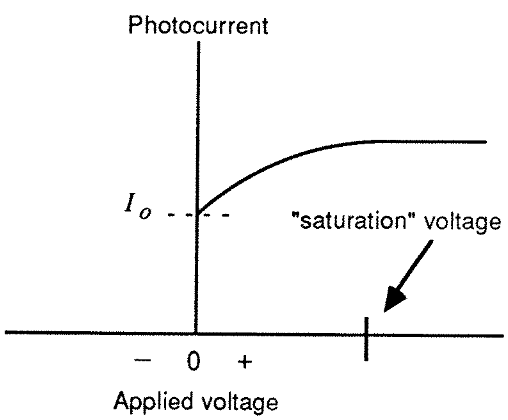

Figure 2-6

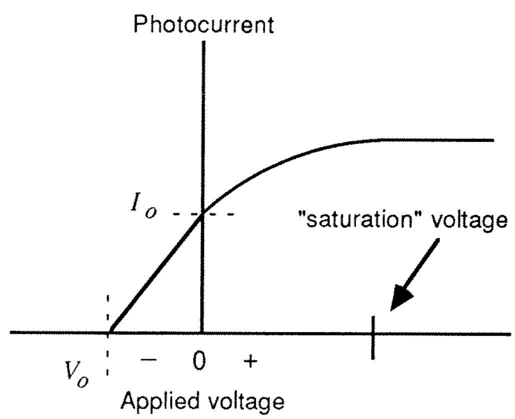

Figure 2-7

As the applied *retarding* potential is increased from zero (that is, as the applied voltage is made more *negative*), fewer and fewer photoelectrons reach the collector and the photocurrent decreases from the initial value I_o (refer to the negative voltage axis in Figure 2-7). When the applied retarding potential reaches a limiting value V_o called the *stopping potential*, the photocurrent drops to zero. None of the photoelectrons reach the collector. Since the stopping potential just prevents the most energetic electrons from reaching the collector, it is related to the kinetic energy of the most energetic electrons, K_{max}, by

$$K_{max} = \frac{1}{2} m_e v_{max}^2 = e V_o \tag{2.6}$$

where m_e is the mass of the electron, v_{max} is the speed of the most energetic photoelectrons, and e is the electronic charge.

In 1902, Philip Lenard studied the photoelectric effect using carbon arc sources. His experimental results and those of later investigators are summarized below:

1. When electromagnetic radiation falls on a metallic surface, the photoelectrons are emitted almost instantaneously. There was no appreciable time lag even when the intensity was very small (10^{-10} W/m^2).

2. For any fixed values of the frequency and retarding potential, the photocurrent is directly proportional to the intensity of the radiation. In other words, *the number of photoelectrons emitted per unit time is proportional to the radiation intensity.*

3. The stopping potential *does not* depend on the *intensity* of the radiation. Figure 2-8 shows the photocurrent as a function of the applied voltage for two cases in which the intensity is varied. For the higher intensity, the "saturated" photocurrent is higher as predicted from result **2** above, but the stopping potential (and therefore K_{max}) is the same at both intensities.

4. For any given metal, the stopping potential *depends on the frequency of the radiation* and is *independent of the intensity*. Figure 2-9 shows the dependence of V_o on the frequency ν of the radiation for a certain metallic surface. The stopping potential increases with increasing frequency. Furthermore, there is a *threshold frequency*, ν_o, below which no photoelectrons are emitted no matter how high the intenisity of the incident radiation.

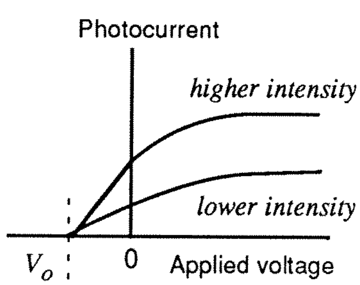

Figure 2-8

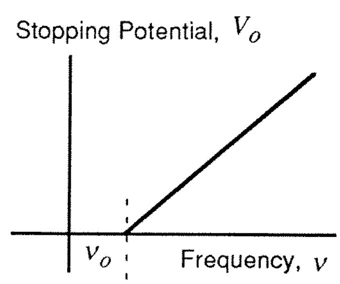

Figure 2-9

The Classical Analysis of the Photoelectric Effect

For the most part, the results of the photoelectric experiments cannot be understood on the basis of Maxwell's electromagnetic theory. With reference to the results previously stated:

1. Electrons are bound to a metallic surface. Any given electron will be emitted if it is given enough energy to overcome the attractive forces that bind it to the metal. According to classical electromagnetic theory, the energy of an electromagnetic wave is spread out over the wavefronts. The energy absorbed on the metallic surface should be proportional to the intensity of the radiation. All electrons bound to the metal with the same energy are equivalent; thus when any one of these electrons has absorbed enough energy to escape, so should all the others with the same binding energy. At low intensities it should take a finite amount of time for the surface to absorb enough energy to free the most weakly bound electrons. With an intensity of 10^{-10} W/m^2, it should take several hundred hours before photoelectrons are observed. No such time delay has ever been observed. Classical theory cannot account for the nearly instantaneous emission of photoelectrons.

2. As the intensity of the radiation is increased, classical theory predicts that more energy should be absorbed by the electrons on the metallic surface. It is reasonable that more photoelectrons should be emitted thereby increasing the photocurrent. The predictions of classical physics do agree with the observed results in this case.

3. Classically, radiation of higher intensity should distribute more energy per unit time over the metallic surface. This would imply that the most energetic electrons would have more energy than those for incident radiation of lower intensity. Photoelectrons produced by more intense radiation should require a larger retarding potential so stop them. This is in complete disagreement with the experimental observations.

4. According to classical physics, the energy of an electromagnetic wave depends only on the square of the amplitude of the wave and has nothing to do with the frequency. The existence of a threshold frequency is completely inexplicable from a classical perspective.

Einstein to the Rescue

As we have seen, the most reasonable explanation of blackbody radiation requires that the energy of the oscillating electrons in the walls of the blackbody cavity is quantized. By 1905, most members of the physics community had not heard of Planck's theory; those who had heard of it did not believe it. Planck himself probably believed that the electromagnetic radiation, once emitted by the oscillators spread out through space in a wave-like fashion. Einstein, however, realized, that if the oscillators in the walls of a blackbody cavity can possess energy only in quantized amounts, they must radiate the energy in quantized amounts. In his famous article on the photoelectric effect [A. Einstein, *Ann. Phys.* (*Leipzig*) **17**, 132-148 (1905)], he proposes:

> According to the assumption considered here, the spreading of a light beam emanating from a point source does not cause the energy to be distributed continuously over larger and larger volumes, but rather the energy consists of a finite number of energy quanta, localized at space points, which move without breaking up and which can be absorbed or emitted only as wholes.

Einstein assumed that each energy quanta or *photon* remains localized as it moves away from its source with speed c. Since the oscillators in the cavity walls could exchange energy only by discrete amounts of $h\nu$, Einstein reasoned that the emitted energy must equal $h\nu$. Thus, the energy of a photon depends on the frequency, ν, of the emitted radiation, and is given by the equation

$$E = h\nu. \tag{2.7}$$

When an oscillator emits a photon of energy $h\nu$, its energy *decreases* by an amount equal to E. Likewise, when a photon of energy $h\nu$ is absorbed by such an oscillator, its energy *increases* by an amount E.

Einstein's idea was truly revolutionary - Einstein was proposing a model of light in which light behaves in a manner *normally attributed to particles*. Yet, in 1801, Thomas Young showed that light can be made to produce the interference pattern normally attributed to waves. Einstein contended that this posed no contradiction. Maxwell's classical theory of radiation is successful in describing the propagation of electromagnetic radiation through space *over long time intervals*. However, this model has its limitations. A different model of radiation is needed to describe the *momentary interaction* of radiation and matter such as the light emission in a blackbody cavity and the interaction of light and electrons in the photoelectric effect.

Einstein's photon theory of radiation immediately explains the seemingly anomalous results of the photoelectric effect. Each metal possesses a characteristic amount of work required to remove an electron from its surface. This is the energy one must expend to overcome the attractive forces that normally bind the electron to the metal. The minimum amount of work required to remove an electron from a given metal is called the *work function* of the metal, and is denoted by the symbol w_o. Table 2-2 lists the work functions for some typical metals. According to Einstein, when a photon is incident on a metallic surface, it gives *all* of its energy, $h\nu$, to a given electron. If that amount of energy is sufficient to remove the electron, the electron will be freed. If, it is not, the electron remains bound in the metal.

TABLE 2-2

WORK FUNCTIONS OF SOME COMMON METALS

Metal	Work Function, eV
iron	4.50
lead	4.14
zinc	4.31
copper	4.70
aluminum	4.08
sodium	2.28

Information taken from Table 2.1, *Modern Physics*, Serway, Moses, and Moyer. Copyright Saunders College Publishing, 1989.

When a photon of energy $h\nu$ is absorbed by an electron, some of the energy, w_o, is used to free the electron. The remaining energy appears as kinetic energy, some of which is lost due to collisions with other electrons. The most energetic electrons, undergo no such collisions, and from energy conservation we have

$$h\nu = w_o + K_{max}. \tag{2.8}$$

Question 2-13: (a) Use equations (2.8) and (2.6) to explain Figure 2.9.
(b) What is the physical significance of the slope of the graph in Figure 2.9?
(c) What is the physical significance of the intercept on the frequency axis?

We are now in a position to use Einstein's arguments to explain the experimental results of the photoelectric effect stated on page 2-11.

1. The electron interacts with the photon as if the photon were a colliding particle with energy $h\nu$. The energy is *not* spread out over a wavefront as classical physics would predict. No time delay is to be expected.

2. As the intensity of the radiation is increased, more photons per unit area strike the metallic surface. This results in the emission of more electrons and the photocurrent will increase.

3. The stopping potential depends on the kinetic energy of the most energetic photoelectrons. From equation (2.8), K_{max} depends on the *frequency*, ν, of the radiation, *not its intensity*. Changing the intensity will change the number of photoelectrons emitted, but the energy required to stop any one of them will be the same.

4. Since the stopping potential depends on K_{max}, it follows from equation (2.8) that the stopping potential will depend on the *frequency* of the radiation. The existence of a threshold frequency can be explained as follows. The energy of any photon is given by $h\nu$. Photons of lower frequency have lower energy. If the energy $h\nu$ of a single photon is not at least equal to the work function, the colliding photon cannot supply enough energy to remove the electron. This will be true regardless of the intensity. Increasing the intensity of low frequency radiation simply increases the number of low energy photons that strike the metal. If the energy of the photons is less than the work function of the metal, no photoelectrons will be emitted no matter how many photons strike the surface.

In 1916, Robert A. Millikan performed a series of experiments with alkali metals that confirmed equation (2.8) and demonstrated the linear relationship between the stopping potential and the frequency of the radiation (as shown in Figure 2-9). From his results, Millikan was able to experimentally determine Planck's constant h to with 0.5 per cent.

Question 2-14: How do you think he did this (assuming that he had experimental curves like Figure 2-9)?

In 1921, Einstein received the Nobel Prize for his theoretical explanation for the photoelectric effect. In 1923, Millikan received the Nobel prize for his experimental confirmation of Einstein's predictions (and for his oil-drop experiment to measure the charge to mass ratio for the electron). The quantum hypothesis was now on solid ground - it had stood the test of experiment.

2.4 The Nature of Electromagnetic Radiation - Is it a Particle or a Wave?

A reasonable question at this point might be: "What is light? Is it composed of particles or is it composed of waves?" The best answer that we can give at this point is "neither." Light is neither a particle nor a wave phenomena. It is something more complex than our descriptions of "particles" or "waves" can explain. It possesses properties of both, and in certain situations (diffraction, interference, polarization) it behaves as if it were strictly a wave phenomenon. In other situations (blackbody radiation, photoelectric effect) electromagnetic radiation behaves as if it were composed of discrete particles. No single experiment can simultaneously display both wave and particle behaviors for any form of electromagnetic radiation. Apparently the two behaviors are mutually exclusive. The best that we can say is that electromagnetic radiation is *dualistic* and exhibits a *wave-particle duality.*

2.5 Remarks on the size of h

As we discussed earlier, the quantization of energy is not apparent in the macroscopic world because the magnitude of h is so small. On the atomic scale, however, h is not small and energy quantization is apparent. Actually, the exact value of h is crucial to the operation of the universe as we know it.

If the value of h were reduced by a factor of 2, that is if it somehow took on the value $h/2$, we would be in lots of trouble. It can be shown that if h were somehow reduced by a factor of two, the average radius of atoms would decrease by a factor of 4. This may not seem alarming at first, but the consequences would be devastating. The density of all matter in the universe would increase drastically (remember that density depends is inversely proportional to volume which depends on r^3). All matter in the universe would contract giving off energy in the process. The balance of gravity, and chemical and nuclear processes in stars would change. Nuclear reactions in stars would change and stellar evolution would be suddenly altered. The earth would be bombarded with massive amounts of radiation from the Sun thereby stripping the atmosphere and destroying life on earth.

References and Suggested Reading

Much of the history in the development of the early quantum theory can be found in
Gamow's *Thirty Years That Shook Physics: The Story of Quantum Theory*

Complete discussions of thermal radiation and Planck's developments including the mathematical derivations of the classical and quantum radiation formulas are given in
Eisberg and Resnick, Chapter 1.

The photoelectric effect is discussed in
Eisberg and Resnick, Chapter 2, sections 1 through 3.