Special Relativity

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Albert Einstein revolutionized modern physics. He explained the random movements of pollen grains, which proved the existence of atoms, and the photoelectric effect, which showed that light was a particle as well as a wave. His theory of special relativity made clear the foundations of space and time, and his theory of gravitation—general relativity—is the most accurate theory in physics today. He was also deeply concerned with the social impact of scientific discovery.

CHAPTER

26

Relativity

OUTLINE

Most of our everyday experiences and observations have to do with objects that move at speeds much less than the speed of light. Newtonian mechanics was formulated to describe the motion of such objects, and its formalism is quite successful in describing a wide range of phenomena that occur at low speeds. It fails, however, when applied to particles having speeds approaching that of light.

This chapter introduces Einstein's theory of special relativity and includes a section on general relativity. The concepts of special relativity often violate our common sense. Moving clocks run slow, and the length of a moving meter stick is contracted. Nonetheless, the theory has been rigorously tested, correctly predicting the results of experiments involving speeds near the speed of light. The theory is verified daily in particle accelerators around the world.

26.1 INTRODUCTION

Experimentally, the predictions of Newtonian theory can be tested at high speeds by accelerating electrons or other charged particles through a large electric potential difference. For example, it's possible to accelerate an electron to a speed of $0.99c$ (where c is the speed of light) by using a potential difference of several million volts. According to Newtonian mechanics, if the potential difference is increased by a factor of 4, the electron's kinetic energy is four times greater and its speed should double to $1.98c$. However, experiments show that the speed of the electron—as well as the speed of any other particle that has mass—always remains *less* than the speed of light, regardless of the size of the accelerating voltage.

The existence of a universal speed limit has far-reaching consequences. It means that the usual concepts of force, momentum, and energy no longer apply for rapidly moving objects. Less obvious consequences include the fact that observers moving at different speeds will measure different time intervals and displacements between the same two events. Newtonian mechanics was contradicted by experimental observations, so it was necessary to replace it with another theory.

In 1905, at the age of 26, Einstein published his special theory of relativity. Regarding the theory, Einstein wrote:

> The relativity theory arose from necessity, from serious and deep contradictions in the old theory from which there seemed no escape. The strength of the new theory lies in the consistency and simplicity with which it solves all these difficulties, using only a few very convincing assumptions.[1]

Although Einstein made many other important contributions to science, his theory of relativity alone represents one of the greatest intellectual achievements of all time. With this theory, experimental observations can be correctly predicted over the range of speeds from $v = 0$ to speeds approaching the speed of light. Newtonian mechanics, which was accepted for more than 200 years, remains valid, but only for speeds much smaller than the speed of light.

At the foundation of special relativity is reconciling the measurements of two observers moving relative to each other. Normally, two such observers will measure different outcomes for the same event. If the measurement is the speed of a car, for example, an observer standing on the road will measure a different speed for the car than an observer in a truck traveling at speed v relative the stationary observer. Special relativity is all about relating two such measurements—and this rather innocuous relating of measurements leads to some of the most bizarre consequences in physics!

26.2 THE PRINCIPLE OF GALILEAN RELATIVITY

In order to describe a physical event, it's necessary to choose a *frame of reference.* For example, when you perform an experiment in a laboratory, you select a coordinate system, or frame of reference, that is at rest with respect to the laboratory. However, suppose an observer in a passing car moving at a constant velocity with respect to the lab were to observe your experiment. Would the observations made by the moving observer differ dramatically from yours? That is, if you found Newton's first law to be valid in your frame of reference, would the moving observer agree with you?

According to the principle of Galilean relativity, **the laws of mechanics must be the same in all inertial frames of reference**. Inertial frames of reference are those reference frames in which Newton's laws are valid. Practically, such frames are those in which objects subjected to no forces move in straight lines at constant speed—thus the name "inertial frame" because objects observed from these frames obey Newton's first law, the law of inertia. For the situation described in the previous paragraph, the laboratory coordinate system and the coordinate system of the moving car are both inertial frames of reference. Consequently, if the laws of mechanics are found to be true in the laboratory, then the person in the car must also observe the same laws.[2]

Consider an airplane in flight, moving with a constant velocity, as in Figure 26.1a. If a passenger in the airplane throws a ball straight up in the air, the passenger observes that the ball moves in a vertical path. The motion of the ball is precisely the same as it would be if the ball were thrown while at rest on Earth. The law of gravity and the equations of motion under constant acceleration are obeyed whether the airplane is at rest or in uniform motion.

Now consider the same experiment when viewed by another observer at rest on Earth. This stationary observer views the path of the ball in the plane to be a parabola, as in Figure 26.1b. Further, according to this observer, the ball has a velocity to the right equal to the velocity of the plane. Although the two observers disagree on the shape of the ball's path, both agree that the motion of the ball obeys the law of gravity and Newton's laws of motion, and even agree on how long

[1]A. Einstein and L. Infeld, *The Evolution of Physics* (New York: Simon and Schuster, 1961).

[2]What is an example of a *non*inertial frame? A frame undergoing translational acceleration or a frame rotating with respect to the two inertial frames just mentioned.

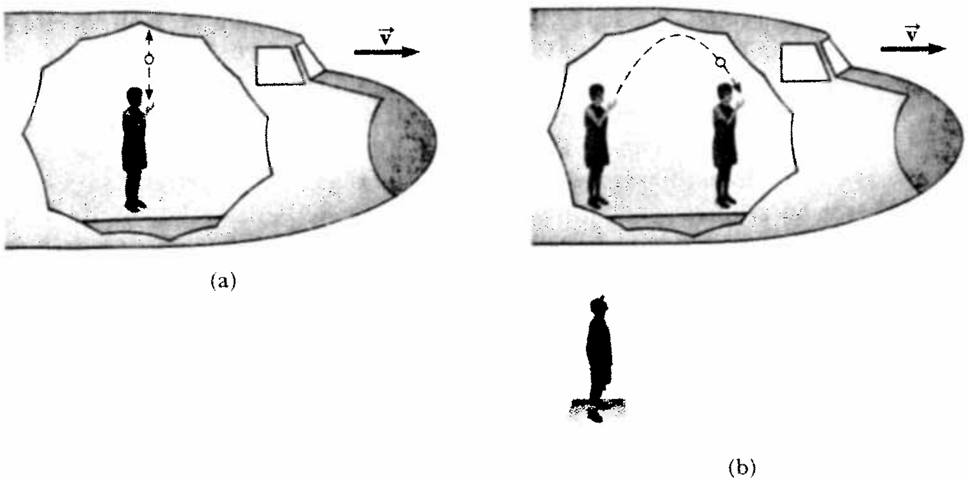


Figure 26.1 (a) The observer on the airplane sees the ball move in a vertical path when thrown upward. (b) The observer on Earth views the path of the ball to be a parabola.

the ball is in the air. We draw the following important conclusion: **There is no preferred frame of reference for describing the laws of mechanics.**

26.3 THE SPEED OF LIGHT

It's natural to ask whether the concept of Galilean relativity in mechanics also applies to experiments in electricity, magnetism, optics, and other areas. Experiments indicate the answer is no. For example, if we assume that the laws of electricity and magnetism are the same in all inertial frames, a paradox concerning the speed of light immediately arises. This can be understood by recalling that, according to electromagnetic theory, the speed of light always has the fixed value of $2.997\,924\,58 \times 10^8$ m/s in free space. But this is in direct contradiction to common sense. For example, suppose a light pulse is sent out by an observer in a boxcar moving with a velocity $\vec{v}$ (Fig. 26.2). The light pulse has a velocity $\vec{c}$ relative to observer S' in the boxcar. According to Galilean relativity, the speed of the pulse relative to the stationary observer S outside the boxcar should be $c + v$. This obviously contradicts Einstein's theory, which postulates that the velocity of the light pulse is the same for all observers.

In order to resolve this paradox, we must conclude that either (1) the addition law for velocities is incorrect or (2) the laws of electricity and magnetism are not the same in all inertial frames. Assume that the second conclusion is true; then a preferred reference frame must exist in which the speed of light has the value c, but in any other reference frame the speed of light must have a value that is greater or less than c. It's useful to draw an analogy with sound waves, which propagate through a medium such as air. The speed of sound in air is about 330 m/s when measured in a reference frame in which the air is stationary. However, the speed of sound is greater or less than this value when measured from a reference frame that is moving with respect to the air.

In the case of light signals (electromagnetic waves), recall that electromagnetic theory predicted that such waves must propagate through free space with a speed

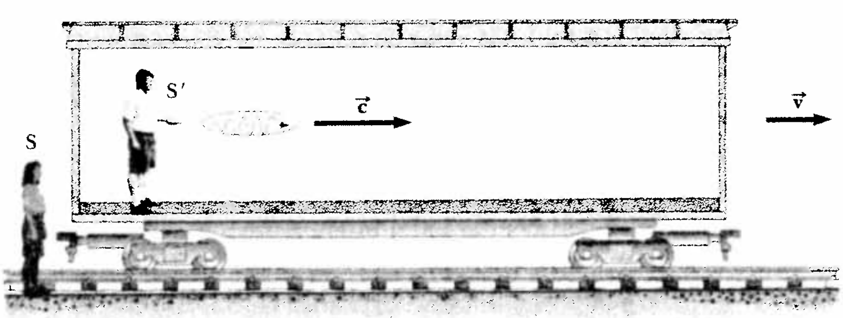


Figure 26.2 A pulse of light is sent out by a person in a moving boxcar. According to Newtonian relativity, the speed of the pulse should be $\vec{c} + \vec{v}$ relative to a stationary observer.

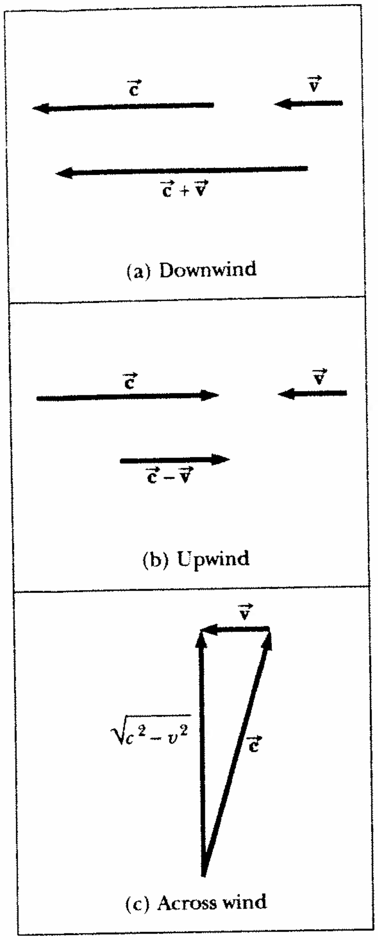


Figure 26.3 If the speed of the ether wind relative to Earth is v, and c is the speed of light relative to the ether, the speed of light relative to Earth is (a) $c + v$ in the downwind direction, (b) $c - v$ in the upwind direction, and (c) $\sqrt{c^2 - v^2}$ in the direction perpendicular to the wind.

equal to the speed of light. However, the theory doesn't require the presence of a medium for wave propagation. This is in contrast to other types of waves, such as water and sound waves, that do require a medium to support the disturbances. In the 19th century, physicists thought that electromagnetic waves also required a medium in order to propagate. They proposed that such a medium existed and gave it the name **luminiferous ether**. The ether was assumed to be present everywhere, even in empty space, and light waves were viewed as ether oscillations. Further, the ether would have to be a massless but rigid medium with no effect on the motion of planets or other objects. These are strange concepts indeed. In addition, it was found that the troublesome laws of electricity and magnetism would take on their simplest forms in a special frame of reference at *rest* with respect to the ether. This frame was called the *absolute frame*. The laws of electricity and magnetism would be valid in this absolute frame, but they would have to be modified in any reference frame moving with respect to the absolute frame.

As a result of the importance attached to the ether and the absolute frame, it became of considerable interest in physics to prove by experiment that they existed. Since it was considered likely that Earth was in motion through the ether, from the view of an experimenter on Earth, there was an "ether wind" blowing through his laboratory. A direct method for detecting the ether wind would use an apparatus fixed to Earth to measure the wind's influence on the speed of light. If v is the speed of the ether relative to Earth, then the speed of light should have its maximum value, $c + v$, when propagating downwind, as shown in Figure 26.3a. Likewise, the speed of light should have its minimum value, $c - v$, when propagating upwind, as in Figure 26.3b, and an intermediate value, $(c^2 - v^2)^{1/2}$, in the direction perpendicular to the ether wind, as in Figure 26.3c. If the Sun were assumed to be at rest in the ether, then the velocity of the ether wind would be equal to the orbital velocity of Earth around the Sun, which has a magnitude of approximately 3×10^4 m/s. Because $c = 3 \times 10^8$ m/s, it should be possible to detect a change in speed of about 1 part in 10^4 for measurements in the upwind or downwind directions. However, as we will see in the next section, all attempts to detect such changes and establish the existence of the ether (and hence the absolute frame) proved futile.

In conclusion, we see that the second hypothesis in our introduction to this section is false—and we now believe that **the laws of electricity and magnetism are the same in all inertial frames**. It is the simple classical addition laws for velocities that are incorrect and must be modified, as shown in Section 26.8.

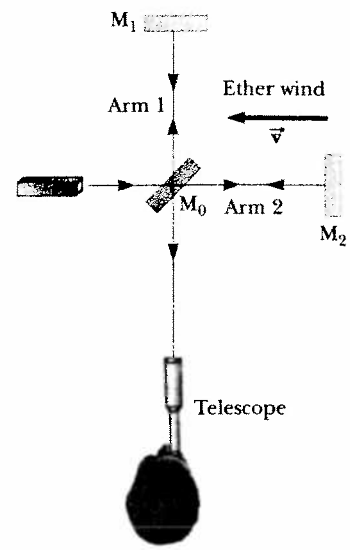
26.4 THE MICHELSON–MORLEY EXPERIMENT

The most famous experiment designed to detect small changes in the speed of light was first performed in 1881 by Albert A. Michelson (1852–1931) and later repeated under various conditions by Michelson and Edward W. Morley (1838–1923). We state at the outset that the outcome of the experiment contradicted the ether hypothesis.

The experiment was designed to determine the velocity of Earth relative to the hypothetical ether. The experimental tool used was the Michelson interferometer, which was discussed in Section 25.7 and is shown again in Active Figure 26.4. Arm 2 is aligned along the direction of Earth's motion through space. Earth's moving through the ether at speed v is equivalent to the ether's flowing past Earth in the opposite direction with speed v. This ether wind blowing in the direction opposite the direction of Earth's motion should cause the speed of light measured in Earth frame to be $c - v$ as the light approaches mirror M_2 and $c + v$ after reflection, where c is the speed of light in the ether frame.

The two beams reflected from M_1 and M_2 recombine, and an interference pattern consisting of alternating dark and bright fringes is formed. The interference pattern was observed while the interferometer was rotated through an angle of 90°. This rotation supposedly would change the speed of the ether wind along the direction of arm 1. The effect of such rotation should have been to cause the fringe pattern to shift slightly but measurably; however, measurements failed to show any change in the interference pattern! The Michelson–Morley experiment was repeated at different times of the year when the ether wind was expected to change direction, but the results were always the same: **no fringe shift of the magnitude required was ever observed.**

The negative results of the Michelson–Morley experiment not only contradicted the ether hypothesis, but also showed that it was impossible to measure the absolute velocity of Earth with respect to the ether frame. However, as we will see in the next section, Einstein suggested a postulate in the special theory of relativity that places quite a different interpretation on these negative results. In later years, when more was known about the nature of light, the idea of an ether that permeates all of space was relegated to the theoretical graveyard. **Light is now understood to be an electromagnetic wave, which requires no medium for its propagation.** As a result, the idea of an ether in which these waves could travel became unnecessary.



ACTIVE FIGURE 26.4
According to the ether wind theory, the speed of light should be $c - v$ as the beam approaches mirror M_2 and $c + v$ after reflection.

Physics Now™
Log into PhysicsNow at **www.cp7e.com** and go to Active Figure 26.4, where you can adjust the speed of a fictitious ether wind and observe the effect on beams of light.

Details of the Michelson–Morley Experiment

As we mentioned earlier, the Michelson–Morley experiment was designed to detect the motion of Earth with respect to the ether. Before we examine the details of this historical experiment, it is instructive to consider a race between two airplanes, as shown in Figure 26.5a. One airplane flies from point O to point A perpendicular to the direction of the wind, and the second airplane flies from point O to point B parallel to the wind. We will assume that they start at O at the same time, travel the same distance L with the same cruising speed c with respect to the wind, and return to O. Which airplane will win the race? In order to answer this question, we calculate the time of flight for both airplanes.

First, consider the airplane that moves along path I parallel to the wind. As it moves to the right, its speed is enhanced by the wind, and its speed with respect to Earth is $c + v$. As it moves to the left on its return journey, it must fly opposite the wind; hence, its speed with respect to Earth is $c - v$. The times of flight to the right and to the left are, respectively,

$$t_R = \frac{L}{c+v} \quad \text{and} \quad t_L = \frac{L}{c-v}$$

and the total time of flight for the airplane moving along path I is

$$t_1 = t_R + t_L = \frac{L}{c+v} + \frac{L}{c-v} = \frac{2Lc}{c^2 - v^2}$$

$$= \frac{2L}{c\left(1 - \dfrac{v^2}{c^2}\right)} \qquad [26.1]$$

Now consider the airplane flying along path II. If the pilot aims the airplane directly toward point A, it will be blown off course by the wind and won't reach its

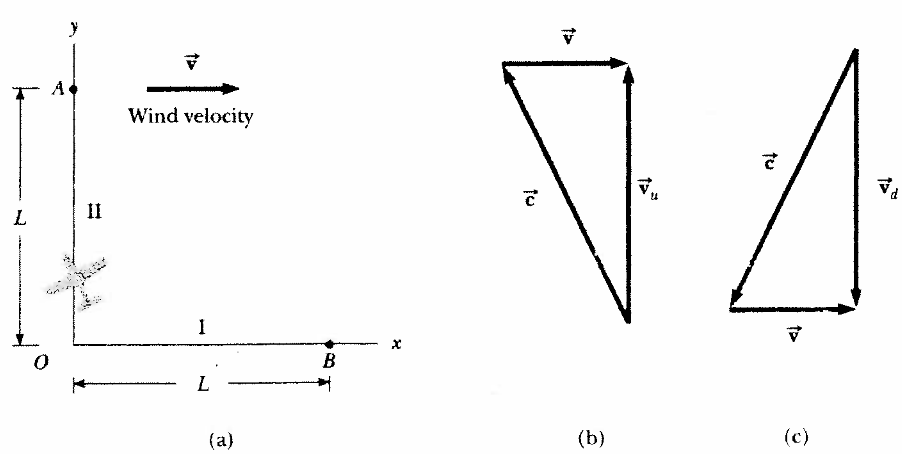


Figure 26.5 (a) If an airplane travels from O to A with a wind blowing to the right, it must head into the wind at some angle. (b) Vector diagram for determining the airplane's direction for the trip from O to A. (c) Vector diagram for determining its direction for the trip from A to O.

destination. To compensate for the wind, the pilot must point the airplane into the wind at some angle, as shown in Figure 26.5a. This angle must be selected so that the vector sum of $\vec{c}$ and $\vec{v}$ leads to a velocity vector pointed directly toward *A*. The resultant vector diagram is shown in Figure 26.5b, where $\vec{v}_u$ is the velocity of the airplane with respect to the ground as it moves from *O* to *A*. From the Pythagorean theorem, the magnitude of the vector $\vec{v}_u$ is

$$v_u = \sqrt{c^2 - v^2} = c\sqrt{1 - \frac{v^2}{c^2}}$$

Likewise, on the return trip from *A* to *O*, the pilot must again head into the wind so that the airplane's velocity $\vec{v}_d$ with respect to Earth will be directed toward *O*, as shown in Figure 26.5c. From this figure, we see that

$$v_d = \sqrt{c^2 - v^2} = c\sqrt{1 - \frac{v^2}{c^2}}$$

The total time of flight for the trip along path II is therefore

$$t_2 = \frac{L}{v_u} + \frac{L}{v_d} = \frac{L}{c\sqrt{1 - \frac{v^2}{c^2}}} + \frac{L}{c\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{2L}{c\sqrt{1 - \frac{v^2}{c^2}}} \qquad [26.2]$$

Comparing Equations 26.1 and 26.2, we see that the airplane flying along path II wins the race. The difference in flight times is given by

$$\Delta t = t_1 - t_2 = \frac{2L}{c}\left[\frac{1}{\left(1 - \frac{v^2}{c^2}\right)} - \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\right]$$

This expression can be simplified by noting that the ratio of wind speed to plane speed, v/c, is usually much smaller than 1, and by using the following binomial expansions in v/c after dropping all terms higher than second order:

$$\left(1 - \frac{v^2}{c^2}\right)^{-1} \approx 1 + \frac{v^2}{c^2}$$

and

$$\left(1 - \frac{v^2}{c^2}\right)^{-1/2} \approx 1 + \frac{1}{2}\frac{v^2}{c^2}$$

The difference in flight times is therefore

$$\Delta t \approx \frac{Lv^2}{c^3} \quad \text{for} \quad v/c \ll 1 \qquad [26.3]$$

The analogy between this airplane race and the Michelson–Morley experiment is shown in Figure 26.6a. Two beams of light travel along two arms of an interferometer. In this case, the "wind" is the ether blowing across Earth from left to right as Earth moves through the ether from right to left. Because the speed of Earth in its orbital path is approximately 3×10^4 m/s, it is reasonable to use that value for the speed of the ether wind. Notice in this case that $v/c \approx 1 \times 10^{-4} \ll 1$. The two light beams start out in phase and return to form an interference pattern. We assume that the interferometer is adjusted for parallel fringes and that a telescope is focused on one of these fringes. The time difference between the two light beams gives rise to a phase difference between the beams, producing an interference pattern when they combine at the position of the telescope. The difference in the pattern is detected by rotating the interferometer through 90° in a horizontal plane, so that the two beams exchange roles (Fig. 26.6b). This results in a net

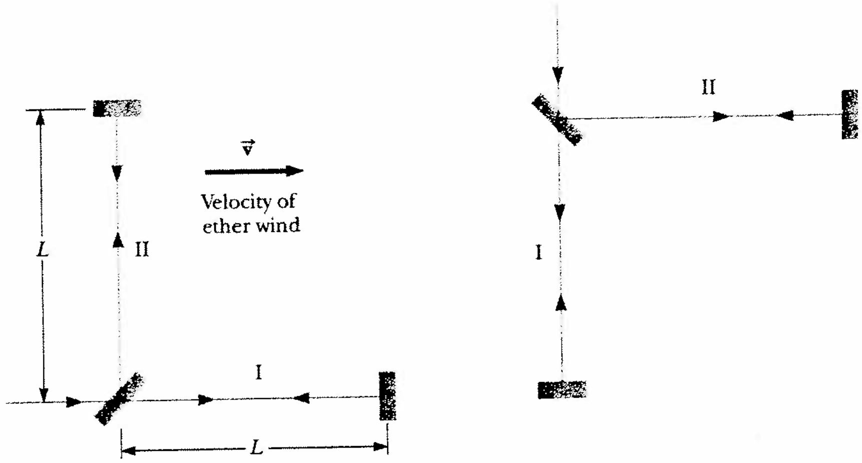


Figure 26.6 (a) Top view of the Michelson–Morley interferometer, where $\vec{v}$ is the velocity of the ether and L is the length of each arm. (b) When the interferometer is rotated by 90°, the role of each arm is reversed.

time shift of twice the time difference given by Equation 26.3. The net time difference is therefore

$$\Delta t_{\text{net}} = 2\,\Delta t = \frac{2Lv^2}{c^3} \qquad [26.4]$$

The corresponding path difference is

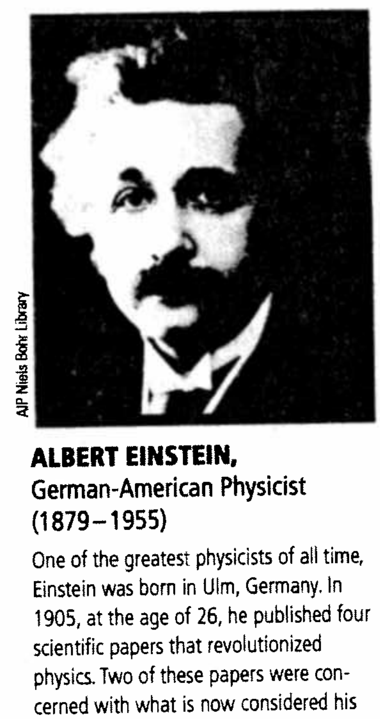
$$\Delta d = c\,\Delta t_{\text{net}} = \frac{2Lv^2}{c^2} \qquad [26.5]$$

In the first experiments by Michelson and Morley, each light beam was reflected by the mirrors many times to give an increased effective path length L of about 11 meters. Using this value and taking v to be equal to 3×10^4 m/s gives a path difference of

$$\Delta d = \frac{2(11\text{ m})(3.0 \times 10^4\text{ m/s})^2}{(3.0 \times 10^8\text{ m/s})^2} = 2.2 \times 10^{-7}\text{ m}$$

This extra travel distance should produce a noticeable shift in the fringe pattern. Specifically, calculations show that if the pattern is viewed while the interferometer is rotated through 90°, a shift of about 0.4 fringe should be observed. The instrument used by Michelson and Morley was capable of detecting a shift in the fringe pattern as small as 0.01 fringe. However, *it detected no shift whatsoever in the fringe pattern.* Since then, the experiment has been repeated many times by different scientists under a wide variety of conditions and no fringe shift has ever been detected. The inescapable conclusion is that motion of Earth with respect to the ether can't be detected.

Many efforts were made to explain the null results of the Michelson–Morley experiment and to save the ether frame concept and the Galilean addition law for the velocity of light. All proposals resulting from these efforts have been shown to be wrong. No experiment in the history of physics has received such valiant efforts to explain the absence of an expected result as was the Michelson–Morley experiment. The stage was set for Einstein, who, at the age of only 26, solved the problem in 1905 with his special theory of relativity.



ALBERT EINSTEIN,
German-American Physicist
(1879–1955)

One of the greatest physicists of all time, Einstein was born in Ulm, Germany. In 1905, at the age of 26, he published four scientific papers that revolutionized physics. Two of these papers were concerned with what is now considered his most important contribution: the special theory of relativity. In 1916, Einstein published his work on the general theory of relativity. The most dramatic prediction of this theory is the degree to which light is deflected by a gravitational field. Measurements made by astronomers on bright stars in the vicinity of the eclipsed Sun in 1919 confirmed Einstein's prediction, and as a result, Einstein became a world celebrity. Einstein was deeply disturbed by the development of quantum mechanics in the 1920s despite his own role as a scientific revolutionary. In particular, he could never accept the probabilistic view of events in nature that is a central feature of quantum theory. The last few decades of his life were devoted to an unsuccessful search for a unified theory that would combine gravitation and electromagnetism.

26.5 EINSTEIN'S PRINCIPLE OF RELATIVITY

In the previous section we noted the serious contradiction between the Galilean addition law for velocities and the fact that the speed of light is the same for all observers. In 1905 Albert Einstein proposed a theory that resolved this contradiction but at the same time completely altered our notions of space and time. He based his special theory of relativity on two postulates:

Postulates of relativity ▶

1. **The principle of relativity:** All the laws of physics are the same in all inertial frames.
2. **The constancy of the speed of light:** The speed of light in a vacuum has the same value, $c = 2.997\,924\,58 \times 10^8$ m/s, in all inertial reference frames, regardless of the velocity of the observer or the velocity of the source emitting the light.

The first postulate asserts that *all* the laws of physics are the same in all reference frames moving with constant velocity relative to each other. This postulate is a sweeping generalization of the principle of Galilean relativity, which refers only to the laws of mechanics. From an experimental point of view, Einstein's principle of relativity means that *any* kind of experiment—mechanical, thermal, optical, or electrical—performed in a laboratory at rest, must give the same result when performed in a laboratory moving at a constant speed past the first one. Hence, no preferred inertial reference frame exists, and it is impossible to detect absolute motion.

Although postulate 2 was a brilliant theoretical insight on Einstein's part in 1905, it has since been confirmed experimentally in many ways. Perhaps the most direct demonstration involves measuring the speed of photons emitted by particles traveling at 99.99% of the speed of light. The measured photon speed in this case agrees to five significant figures with the speed of light in empty space.

The null result of the Michelson–Morley experiment can be readily understood within the framework of Einstein's theory. According to his principle of relativity, the premises of the Michelson–Morley experiment were incorrect. In the process of trying to explain the expected results, we stated that when light traveled against the ether wind its speed was $c - v$. However, if the state of motion of the observer or of the source has no influence on the value found for the speed of light, the measured value must always be c. Likewise, the light makes the return trip after reflection from the mirror at a speed of c, not at a speed of $c + v$. Thus, the motion of Earth does not influence the fringe pattern observed in the Michelson–Morley experiment, and a null result should be expected.

If we accept Einstein's theory of relativity, we must conclude that uniform relative motion is unimportant when measuring the speed of light. At the same time, we have to adjust our commonsense notions of space and time and be prepared for some rather bizarre consequences.

26.6 CONSEQUENCES OF SPECIAL RELATIVITY

Almost everyone who has dabbled even superficially in science is aware of some of the startling predictions that arise because of Einstein's approach to relative motion. As we examine some of the consequences of relativity in this section, we'll find that they conflict with some of our basic notions of space and time. We will restrict our discussion to the concepts of length, time, and simultaneity, which are quite different in relativistic mechanics from what they are in Newtonian mechanics. For example, in relativistic mechanics, the distance between two points and the time interval between two events depend on the frame of reference in which they are measured. **In relativistic mechanics, there is no such thing as absolute length or absolute time**. Further, **events at different locations that are observed to occur simultaneously in one frame are not observed to be simultaneous in another frame moving uniformly past the first.**

Absolute length and absolute time intervals are meaningless in relativity. ▶

Simultaneity and the Relativity of Time

A basic premise of Newtonian mechanics is that a universal time scale exists that is the same for all observers. In fact, Newton wrote, "Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external." Newton and his followers simply took simultaneity for granted. In his special theory of relativity, Einstein abandoned that assumption.

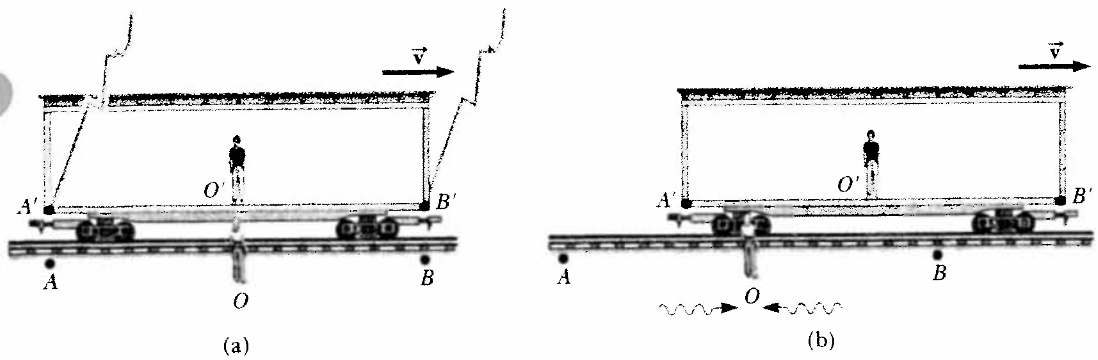


Figure 26.7 Two lightning bolts strike the ends of a moving boxcar. (a) The events appear to be simultaneous to the stationary observer at O, who is midway between A and B. (b) The events don't appear to be simultaneous to the observer at O', who claims that the front of the train is struck *before* the rear.

Einstein devised the following thought experiment to illustrate this point: a boxcar moves with uniform velocity, and two lightning bolts strike its ends, as in Figure 26.7a, leaving marks on the boxcar and the ground. The marks on the boxcar are labeled A' and B', and those on the ground are labeled A and B. An observer at O′ moving with the boxcar is midway between A' and B', and an observer on the ground at O is midway between A and B. The events recorded by the observers are the striking of the boxcar by the two lightning bolts.

The light signals recording the instant at which the two bolts struck reach observer O at the same time, as indicated in Figure 26.7b. This observer realizes that the signals have traveled at the same speed over equal distances, and so rightly concludes that the events at A and B occurred simultaneously. Now consider the same events as viewed by observer O'. By the time the signals have reached observer O, observer O' has moved as indicated in Figure 26.7b. Thus, the signal from B' has already swept past O', but the signal from A' has not yet reached O'. In other words, O' sees the signal from B' before seeing the signal from A'. According to Einstein, *the two observers must find that light travels at the same speed.* Therefore, observer O' concludes that the lightning struck the front of the boxcar before it struck the back.

This thought experiment clearly demonstrates that the two events which appear to be simultaneous to observer O do not appear to be simultaneous to observer O'. In other words,

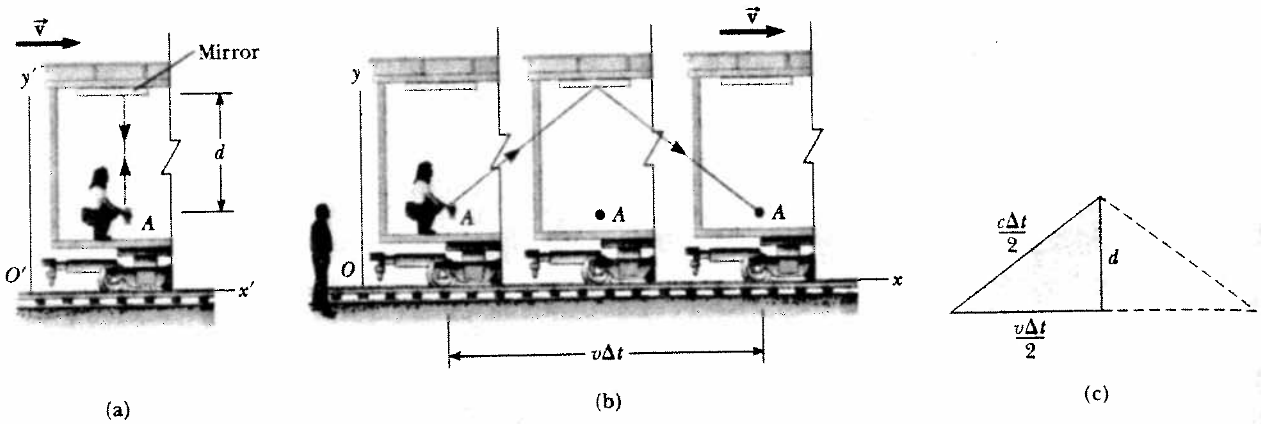
> Two events that are simultaneous in one reference frame are in general not simultaneous in a second frame moving relative to the first. Simultaneity depends on the state of motion of the observer, and is therefore not an absolute concept.

At this point, you might wonder which observer is right concerning the two events. The answer is that *both* are correct, because the principle of relativity states that **there is no preferred inertial frame of reference**. Although the two observers reach different conclusions, both are correct in their own reference frames because the concept of simultaneity is not absolute. In fact, this is the central point of relativity. Any inertial frame of reference can be used to describe events and do physics.

TIP 26.1 Who's Right?
Which person is correct concerning the simultaneity of the two events? Both are correct, because the principle of relativity states that no inertial frame of reference is preferred. Although the two observers may reach different conclusions, both are correct in their own reference frame. Any uniformly moving frame of reference can be used to describe events and do physics.

Time Dilation

We can illustrate the fact that observers in different inertial frames may measure different time intervals between a pair of events by considering a vehicle moving to the right with a speed v as in Active Figure 26.8a (page 852). A mirror is fixed to the ceiling of the vehicle, and an observer O' at rest in this system holds a laser a distance d below the mirror. At some instant, the laser emits a pulse of light



ACTIVE FIGURE 26.8
(a) A mirror is fixed to a moving vehicle, and a light pulse leaves O' at rest in the vehicle. (b) Relative to a stationary observer on Earth, the mirror and O' move with a speed v. Note that the distance the pulse travels is greater than $2d$ as measured by the stationary observer. (c) The right triangle for calculating the relationship between Δt and Δt_p.

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directed toward the mirror (event 1), and at some later time after reflecting from the mirror, the pulse arrives back at the laser (event 2). Observer O' carries a clock and uses it to measure the time interval Δt_p between these two events which she views as occurring at the same place. (The subscript p stands for *proper*, as we'll see in a moment.) Because the light pulse has a speed c, the time it takes it to travel from point A to the mirror and back to point A is

$$\Delta t_p = \frac{\text{Distance traveled}}{\text{Speed}} = \frac{2d}{c} \qquad [26.6]$$

The time interval Δt_p measured by O' requires only a single clock located at the same place as the laser in this frame.

Now consider the same set of events as viewed by O in a second frame, as shown in Active Figure 26.8b. According to this observer, the mirror and laser are moving to the right with a speed v, and as a result, the sequence of events appears different. By the time the light from the laser reaches the mirror, the mirror has moved to the right a distance $v\,\Delta t/2$, where Δt is the time it takes the light pulse to travel from point A to the mirror and back to point A as measured by O. In other words, O concludes that, because of the motion of the vehicle, if the light is to hit the mirror, it must leave the laser at an angle with respect to the vertical direction. Comparing Active Figures 26.8a and 26.8b, we see that the light must travel farther in (b) than in (a). (Note that neither observer "knows" that he or she is moving. Each is at rest in his or her own inertial frame.)

According to the second postulate of the special theory of relativity, both observers must measure c for the speed of light. Because the light travels farther in the frame of O, it follows that the time interval Δt measured by O is longer than the time interval Δt_p measured by O'. To obtain a relationship between these two time intervals, it is convenient to examine the right triangle shown in Active Figure 26.8c. The Pythagorean theorem gives

$$\left(\frac{c\Delta t}{2}\right)^2 = \left(\frac{v\Delta t}{2}\right)^2 + d^2$$

Solving for Δt yields

$$\Delta t = \frac{2d}{\sqrt{c^2 - v^2}} = \frac{2d}{c\sqrt{1 - v^2/c^2}}$$

Because $\Delta t_p = 2d/c$, we can express this result as

$$\Delta t = \frac{\Delta t_p}{\sqrt{1 - v^2/c^2}} = \gamma \Delta t_p \qquad [26.7]$$

◀ Time dilation

where

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \qquad [26.8]$$

Because γ is always greater than one, Equation 26.7 says that **the time interval Δt between two events measured by an observer moving with respect to a clock**[3] **is longer than the time interval Δt_p between the same two events measured by an observer at rest with respect to the clock**. Consequently, $\Delta t > \Delta t_p$, and the proper time interval is expanded or dilated by the factor γ. Hence, this effect is known as **time dilation**.

For example, suppose the observer at rest with respect to the clock measures the time required for the light flash to leave the laser and return. We assume that the measured time interval in this frame of reference, Δt_p, is one second. (This would require a very tall vehicle.) Now we find the time interval as measured by observer O moving with respect to the same clock. If observer O is traveling at half the speed of light ($v = 0.500c$), then $\gamma = 1.15$, and according to Equation 26.7, $\Delta t = \gamma \Delta t_p = 1.15(1.00 \text{ s}) = 1.15 \text{ s}$. Therefore, when observer O' claims that 1.00 s has passed, observer O claims that 1.15 s has passed. Observer O considers the clock of O' to be reading too low a value for the elapsed time between the two events and says that the clock of O' is "running slow." From this phenomenon, we may conclude the following:

A clock moving past an observer at speed v runs more slowly than an identical clock at rest with respect to the observer by a factor of γ^{-1}.

◀ A clock in motion runs more slowly than an identical stationary clock.

The time interval Δt_p in Equations 26.6 and 26.7 is called the **proper time**. In general, **proper time is the time interval between two events as measured by an observer who sees the events occur at the same position**.

Although you may have realized it by now, it's important to spell out that relativity is a scientific democracy: the view of O' that O is really the one moving with speed v to the left and that O's clock is running more slowly is just as valid as the view of O. The principle of relativity requires that the views of two observers in uniform relative motion be equally valid and capable of being checked experimentally.

TIP 26.2 Proper Time Interval

You must be able to correctly identify the observer who measures the proper time interval. The proper time interval between two events is the time interval measured by an observer for whom the two events take place at the same position.

We have seen that moving clocks run slow by a factor of γ^{-1}. This is true for ordinary mechanical clocks as well as for the light clock just described. In fact, we can generalize these results by stating that all physical processes, including chemical and biological ones, slow down relative to a clock when those processes occur in a frame moving with respect to the clock. For example, the heartbeat of an astronaut moving through space would keep time with a clock inside the spaceship. Both the astronaut's clock and heartbeat would be slowed down relative to a clock back on Earth (although the astronaut would have no sensation of life slowing down in the spaceship).

Time dilation is a very real phenomenon that has been verified by various experiments involving the ticking of natural clocks. An interesting example of time dilation involves the observation of *muons*—unstable elementary particles that are very similar to electrons, having the same charge, but 207 times the mass. Muons can be produced by the collision of cosmic radiation with atoms high in the atmosphere. These particles have a lifetime of 2.2 μs when measured in a reference frame at rest with respect to them. If we take 2.2 μs as the average lifetime of a muon and assume that their speed is close to the speed of light, we find that

[3] Actually, Figure 26.8 shows the clock moving and not the observer, but this is equivalent to observer O moving to the left with velocity $\vec{v}$ with respect to the clock.

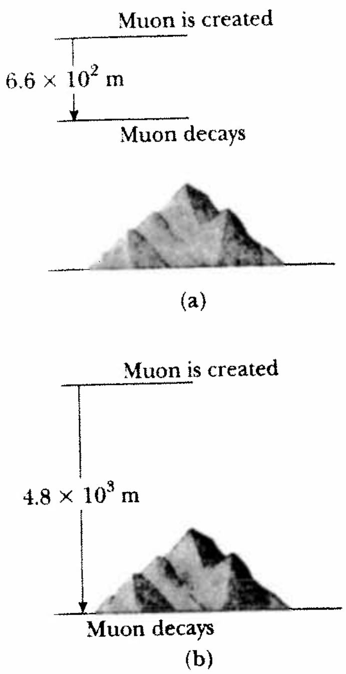


Figure 26.9 (a) The muons travel only about 6.6×10^2 m as measured in the muons' reference frame, in which their lifetime is about 2.2 μs. Because of time dilation, the muons' lifetime is longer as measured by the observer on Earth. (b) Muons traveling with a speed of $0.99c$ travel a distance of about 4.80×10^3 m as measured by an observer on Earth.

these particles can travel only about 600 m before they decay (Fig. 26.9a). Hence, they could never reach Earth from the upper atmosphere where they are produced. However, experiments show that a large number of muons *do* reach Earth, and the phenomenon of time dilation explains how. Relative to an observer on Earth, the muons have a lifetime equal to $\gamma\tau_p$, where $\tau_p = 2.2\ \mu$s is the lifetime in a frame of reference traveling with the muons. For example, for $v = 0.99c$, $\gamma \approx 7.1$ and $\gamma\tau_p \approx 16\ \mu$s. Hence, the average distance muons travel as measured by an observer on Earth is $\gamma v \tau_p \approx 4\,800$ m, as indicated in Figure 26.9b. Consequently, muons can reach Earth's surface.

In 1976 experiments with muons were conducted at the laboratory of the European Council for Nuclear Research (CERN) in Geneva. Muons were injected into a large storage ring, reaching speeds of about $0.9994c$. Electrons produced by the decaying muons were detected by counters around the ring, enabling scientists to measure the decay rate, and hence the lifetime of the muons. The lifetime of the moving muons was measured to be about 30 times as long as that of stationary muons to within two parts in a thousand, in agreement with the prediction of relativity.

Quick Quiz 26.1

Suppose you're an astronaut being paid according to the time you spend traveling in space. You take a long voyage traveling at a speed near that of light. Upon your return to Earth, you're asked how you'd like to be paid: according to the time elapsed on a clock on Earth or according to your ship's clock. Which should you choose in order to maximize your paycheck? (a) the Earth clock (b) the ship's clock (c) Either clock, it doesn't make a difference.

EXAMPLE 26.1 Pendulum Periods

Goal Apply the concept of time dilation.

Problem The period of a pendulum is measured to be 3.00 s in the inertial frame of the pendulum. What is the period as measured by an observer moving at a speed of $0.950c$ with respect to the pendulum?

Strategy Here, we're given the period of the clock as measured by an observer in the rest frame of the clock, so that's a proper time interval Δt_p. We want to know how much time passes as measured by an observer in a frame moving relative to the clock, which is Δt. Substitution into Equation 26.7 then solves the problem.

Solution

Substitute the proper time and relative speed into Equation 26.7:

$$\Delta t = \frac{\Delta t_p}{\sqrt{1 - v^2/c^2}} = \frac{3.00\text{ s}}{\sqrt{1 - \dfrac{(0.950c)^2}{c^2}}} = 9.61\text{ s}$$

Remarks The moving observer considers the *pendulum* to be moving, and moving clocks are observed to run more slowly: while the pendulum oscillates once in 3 s for an observer in the rest frame of the clock, it takes nearly 10 s to oscillate once according the moving observer.

Exercise 26.1

What pendulum period does a third observer moving at $0.900c$ measure?

Answer 6.88 s

The confusion that arises in problems like Example 26.1 lies in the fact that movement is relative: from the point of view of someone in the pendulum's rest frame, the pendulum is standing still (except, of course, for the swinging motion), whereas to someone in a frame that is moving with respect to the pendulum, it's the pendulum that's doing the moving. To keep this straight, always focus on the observer who is doing the measurement, and ask yourself whether the clock being measured is moving with respect to that observer. If the answer is no, then the observer is in the rest frame of the clock and measures the clock's proper time. If the answer is yes, then the time measured by the observer will be dilated—larger than the clock's proper time.

This confusion of perspectives led to the famous "twin paradox."

The Twin Paradox

An intriguing consequence of time dilation is the so-called twin paradox (Fig. 26.10). Consider an experiment involving a set of twins named Speedo and Goslo. When they are 20 years old, Speedo, the more adventuresome of the two, sets out on an epic journey to Planet X, located 20 lightyears from Earth. Further, his spaceship is capable of reaching a speed of $0.95c$ relative to the inertial frame of his twin brother back home. After reaching Planet X, Speedo becomes homesick and immediately returns to Earth at the same speed of $0.95c$. Upon his return, Speedo is shocked to discover that Goslo has aged $2D/v = 2(20\text{ ly})/(0.95\text{ ly/y}) =$ 42 years and is now 62 years old. Speedo, on the other hand, has aged only 13 years.

◀ The space traveler ages more slowly than his twin who remains on Earth.

Some wrongly consider *this* the paradox; that twins could age at different rates and end up after a period of time having very different ages. While contrary to our common sense, this isn't the paradox at all. The paradox lies in the fact that from Speedo's point of view, *he* was at rest while Goslo (on Earth) sped away from *him* at $0.95c$ and returned later. So Goslo's clock was moving relative to Speedo and hence running slow compared with Speedo's clock. The conclusion: Speedo, not Goslo, should be the older of the twins!

To resolve this apparent paradox, consider a third observer moving at a constant speed of $0.5c$ relative to Goslo. To the third observer, Goslo never changes inertial frames: His speed relative to the third observer is always the same. The third observer notes, however, that Speedo accelerates during his journey, *changing reference frames in the process.* From the third observer's perspective, it's clear that there is something very different about the motion of Goslo when compared to Speedo. The roles played by Goslo and Speedo are not symmetric, so it isn't surprising that time flows differently for each. Further, because Speedo accelerates, he is in a noninertial frame of reference—technically outside the bounds of special relativity (though there are methods for dealing with accelerated motion in relativity). Only Goslo, who is in a single inertial frame, can apply the simple time-dilation formula to Speedo's trip. Goslo finds that instead of aging 42 years,

(a) (b)

Figure 26.10 (a) As the twins depart, they're the same age. (b) When Speedo returns from his journey to Planet X, he's younger than his twin Goslo, who remained on Earth.

Speedo ages only $(1 - v^2/c^2)^{1/2}(42 \text{ years}) = 13$ years. Of these 13 years, Speedo spends 6.5 years traveling to Planet X and 6.5 years returning, for a total travel time of 13 years, in agreement with our earlier statement.

Length Contraction

The measured distance between two points depends on the frame of reference of the observer. The **proper length** L_p of an object is **the length of the object as measured by an observer at rest relative to the object.** The length of an object measured in a reference frame that is moving with respect to the object is always less than the proper length. This effect is known as **length contraction.**

TIP 26.3 The Proper Length

You must be able to correctly identify the observer who measures the proper length. The proper length between two points in space is the length measured by an observer at rest with respect to the length. Very often, the proper time interval and the proper length are not measured by the same observer.

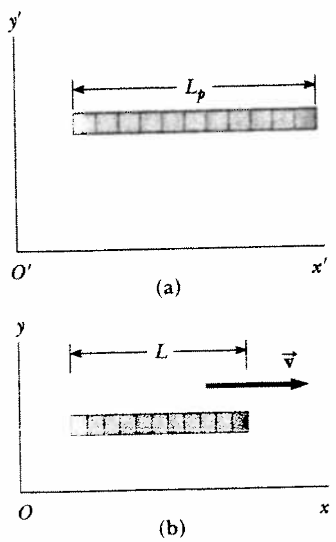
To understand length contraction quantitatively, consider a spaceship traveling with a speed v from one star to another, as seen by two observers, one on Earth and the other in the spaceship. The observer at rest on Earth (and also assumed to be at rest with respect to the two stars) measures the distance between the stars to be L_p. According to this observer, the time it takes the spaceship to complete the voyage is $\Delta t = L_p/v$. Because of time dilation, the space traveler, using his spaceship clock, measures a smaller time of travel: $\Delta t_p = \Delta t/\gamma$. The space traveler claims to be at rest and sees the destination star moving toward the spaceship with speed v. Because the space traveler reaches the star in time Δt_p, he concludes that the distance L between the stars is shorter than L_p. The distance measured by the space traveler is

$$L = v\,\Delta t_p = v\frac{\Delta t}{\gamma}$$

Because $L_p = v\,\Delta t$, it follows that

Length contraction ►

$$L = \frac{L_p}{\gamma} = L_p\sqrt{1 - v^2/c^2} \qquad [26.9]$$



ACTIVE FIGURE 26.11
A meter stick moves to the right with a speed v. (a) The meter stick as viewed by an observer at rest with respect to the meter stick. (b) The meter stick as seen by an observer moving with a speed v with respect to it. The moving meter stick is always measured to be *shorter* than in its own rest frame by a factor of $\sqrt{1 - v^2/c^2}$.

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According to this result, illustrated in Active Figure 26.11, if an observer at rest with respect to an object measures its length to be L_p, an observer moving at a speed v relative to the object will find it to be shorter than its proper length by the factor $\sqrt{1 - v^2/c^2}$. Note that **length contraction takes place only along the direction of motion.**

Time-dilation and length contraction effects have interesting applications for future space travel to distant stars. In order for the star to be reached in a fraction of a human lifetime, the trip must be taken at very high speeds. According to an Earth-bound observer, the time for a spacecraft to reach the destination star will be dilated compared with the time interval measured by travelers. As was discussed in the treatment of the twin paradox, the travelers will be younger than their twins when they return to Earth. Therefore, by the time the travelers reach the star, they will have aged by some number of years, while their partners back on Earth will have aged a larger number of years, the exact ratio depending on the speed of the spacecraft. At a spacecraft speed of $0.94c$, this ratio is about 3:1.

Quick Quiz 26.2

You are packing for a trip to another star, and on your journey you will be traveling at a speed of $0.99c$. Can you sleep in a smaller cabin than usual, because you will be shorter when you lie down? Explain your answer.

Quick Quiz 26.3

You observe a rocket moving away from you. Compared to its length when it was at rest on the ground, you will measure its length to be (a) shorter, (b) longer, or (c) the same. Compared to the passage of time measured by the watch on your wrist, the passage of time on the rocket's clock is (d) faster, (e) slower, or (f) the same. Answer the same questions if the rocket turns around and comes toward you.

EXAMPLE 26.2 Starship Contraction

Goal Apply the concept of length contraction to a moving object.

Problem A starship is measured to be 125 m long while it is at rest with respect to an observer. If this starship now flies past the observer at a speed of $0.99c$, what length will the observer measure for the starship?

Strategy Moving objects are observed to be contracted, or shorter. Substitute into Equation 26.9.

Solution

Substitute into Equation 26.9 to find the length as measured by the observer:

$$L = L_p\sqrt{1 - v^2/c^2} = (125\text{ m})\sqrt{1 - (0.99\,c)^2/c^2} = \mathbf{17.6\text{ m}}$$

Exercise 26.2

If the ship moves past the observer with a speed of $0.80c$, what length will the observer measure?

Answer 75.0 m

EXAMPLE 26.3 Speedy Plunge

Goal Apply the concept of length contraction to a distance.

Problem **(a)** An observer on Earth sees a spaceship at an altitude of 4 350 km moving downward toward Earth with a speed of $0.970c$. What is the distance from the spaceship to Earth as measured by the spaceship's captain? **(b)** After firing his engines, the captain measures her ship's altitude as 267 km, while the observer on Earth measures it to be 625 km. What is the speed of the spaceship at this instant?

Strategy To the captain, the Earth is rushing toward her ship at $0.970c$; hence the distance between her ship and the Earth is contracted. Substitution into Equation 26.9 yields the answer. In part (b) use the same equation, substituting the distances and solving for the speed.

Solution

(a) Find the distance from the ship to Earth as measured by the captain.

Substitute into Equation 26.9, getting the altitude as measured by the captain in the ship.

$$L = L_p\sqrt{1 - v^2/c^2} = (4\ 350\text{ km})\sqrt{1 - (0.970c)^2/c^2}$$
$$= \mathbf{1.06 \times 10^3\text{ km}}$$

(b) What is the subsequent speed of the spaceship if the Earth observer measures the distance from the ship to Earth as 625 km and the captain measures it as 267 km?

Apply the length-contraction equation:

$$L = L_p\sqrt{1 - v^2/c^2}$$

Square both sides of this equation and solve for v:

$$L^2 = L_p^2(1 - v^2/c^2) \quad\rightarrow\quad 1 - v^2/c^2 = \left(\frac{L}{L_p}\right)^2$$

$$v = c\sqrt{1 - (L/L_p)^2} = c\sqrt{1 - (267\text{ km}/625\text{ km})^2}$$

$$v = \mathbf{0.904c}$$

Remarks The proper length is always the length measured by an observer at rest with respect to that length.

Exercise 26.3

Suppose the observer on the ship measures the distance from Earth as 50.0 km, while the observer on Earth measures the distance as 125 km. At what speed is the spacecraft approaching Earth?

Answer $0.917c$

Length contraction occurs only in the direction of the observer's motion. No contraction occurs perpendicular to that direction. For example, a spaceship at rest relative to an observer may have the shape of an equilateral triangle, but if it passes the observer at relativistic speed in a direction parallel to its base, the base will shorten while the height remains the same. Hence, the craft will be observed to be isosceles. An observer traveling with the ship will still observe it to be equilateral.

26.7 RELATIVISTIC MOMENTUM

Properly describing the motion of particles within the framework of special relativity requires generalizing Newton's laws of motion and the definitions of momentum and energy. These generalized definitions reduce to the classical (nonrelativistic) definitions when v is much less than c.

First, recall that conservation of momentum states that when two objects collide, the total momentum of the system remains constant, assuming that the objects are isolated, reacting only with each other. However, analyzing such collisions from rapidly moving inertial frames, it is found that momentum is not conserved if the classical definition of momentum, $p = mv$, is used. In order to have momentum conservation in all inertial frames—even those moving at an appreciable fraction of c—the definition of momentum must be modified to read

Momentum ▶

$$p \equiv \frac{mv}{\sqrt{1 - v^2/c^2}} = \gamma mv \qquad [26.10]$$

where v is the speed of the particle and m is its mass as measured by an observer at rest with respect to the particle. Note that when v is much less than c, the denominator of Equation 26.10 approaches one, so that p approaches mv. Therefore, the relativistic equation for momentum reduces to the classical expression when v is small compared with c.

EXAMPLE 26.4 The Relativistic Momentum of an Electron

Goal Contrast the classical and relativistic definitions of momentum.

Problem An electron, which has a mass of 9.11×10^{-31} kg, moves with a speed of $0.750c$. Find the classical (nonrelativistic) momentum and compare it to its relativistic counterpart p_{rel}.

Strategy Substitute into the classical definition to get the classical momentum, then multiply by the gamma factor to obtain the relativistic version.

Solution

First, compute the classical (nonrelativistic) momentum with $v = 0.750c$:

$$p = mv = (9.11 \times 10^{-31}\text{ kg})(0.750 \times 3.00 \times 10^{8}\text{ m/s})$$
$$= 2.05 \times 10^{-22}\text{ kg}\cdot\text{m/s}$$

Multiply this result by γ to obtain the relativistic momentum:

$$p_{\text{rel}} = \frac{mv}{\sqrt{1 - v^2/c^2}} = \frac{2.05 \times 10^{-22}\text{ kg}\cdot\text{m/s}}{\sqrt{1 - (0.750c/c)^2}}$$
$$= 3.10 \times 10^{-22}\text{ kg}\cdot\text{m/s}$$

Remark The (correct) relativistic result is 50% greater than the classical result. In subsequent calculations, no notational distinction will be made between classical and relativistic momentum. For problems involving relative speeds of $0.2c$, the answer using the classical expression is about 2% below the correct answer.

Exercise 26.4

Repeat the calculation for a proton traveling at $0.600c$.

Answers $p = 3.01 \times 10^{-19}\text{ kg}\cdot\text{m/s}$, $p_{\text{rel}} = 3.76 \times 10^{-19}\text{ kg}\cdot\text{m/s}$

26.8 RELATIVISTIC ADDITION OF VELOCITIES

Imagine a motorcycle rider moving with a speed of $0.80c$ past a stationary observer, as shown in Figure 26.12. If the rider tosses a ball in the forward direction with a speed of $0.70c$ relative to himself, what is the speed of the ball as seen by the stationary observer at the side of the road? Common sense and the ideas of Newtonian relativity say that the speed should be the sum of the two speeds, or $1.50c$. This answer must be incorrect because it contradicts the assertion that no material object can travel faster than the speed of light.

Einstein resolved this dilemma by deriving an equation for the relativistic addition of velocities. Here, only one dimension of motion will be considered. Let two frames or reference be labeled b and d, and suppose that frame d is moving at velocity v_{db} in the position x-direction relative frame b. If the velocity of an object a as measured in frame d is called v_{ad}, then the velocity of a as measured in frame b, v_{ab}, is given by

$$v_{ab} = \frac{v_{ad} + v_{db}}{1 + \frac{v_{ad}v_{db}}{c^2}} \qquad [26.11]$$

◀ Relativistic velocity addition

The left side of this equation and the numerator on the right are like the equations of Galilean relativity discussed in Chapter 3, and the evaluation of subscripts is applied in the same way as discussed in Section 3.6. The denominator of Equation 26.11 is a correction to Galilean relativity based on length contraction and time dilation.

We apply Equation 26.11 to Figure 26.13, which shows a motorcyclist, his ball, and a stationary observer. We are given

v_{bm} = the velocity of the ball with respect to the motorcycle = $0.70c$

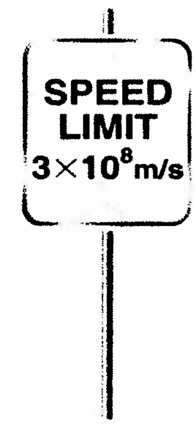
v_{mo} = the velocity of the motorcycle with respect to the stationary observer = $0.80c$,

and we want to find

v_{bo} = the velocity of the ball with respect to the stationary observer.

Thus,

$$v_{bo} = \frac{v_{bm} + v_{mo}}{1 + \frac{v_{bm}v_{mo}}{c^2}} = \frac{0.70c + 0.80c}{1 + \frac{(0.70c)(0.80c)}{c^2}} = 0.96c$$



The speed of light is the speed limit of the Universe.

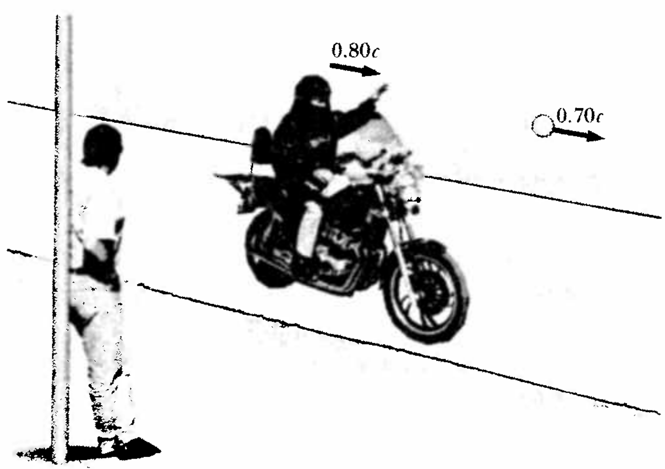


Figure 26.12 A motorcycle moves past a stationary observer with a speed of $0.80c$; the motorcyclist throws a ball in the direction of motion with a speed of $0.70c$ relative to himself.

EXAMPLE 26.5 Urgent Course Correction Needed!

Goal Apply the concept of the relativistic addition of velocities.

Problem Suppose that Bob's spacecraft is traveling at $0.600c$ in the positive x-direction, as measured by a nearby observer, while Mike is traveling in his own vehicle directly toward Bob in the negative x-direction at $-0.800c$ relative the nearby observer. What's the velocity of Bob relative to Mike?

Strategy This problem requires correctly identifying the quantities that go into Equation 26.11, followed by substitution. The measurement of Bob's velocity as determined in the observer's frame O is given, and the measurement of Bob's velocity in Mike's frame is desired.

Solution

Identify the velocity terms in Equation 26.11.

v_{BM} = the velocity of Bob with respect to Mike. This will be substituted for v_{ad} in Equation 26.11.

v_{MO} = the velocity of the Mike with respect to the stationary observer = $-0.800c$. This will be substituted for v_{db} in Equation 26.11.

v_{BO} = the velocity of the Bob with respect to the stationary observer = $0.600c$. This will be substituted for v_{ab} in Equation 26.11.

Substitute the velocity expressions into Equation 26.11. Examining the form of Equation 26.11, we can see intuitively that v_{BM} and v_{MO} belong on the right hand side (the letter M appears in both a first and a second position), so our previous choices are verified.

$$v_{BO} = \frac{v_{BM} + v_{MO}}{1 + \frac{v_{BM}v_{MO}}{c^2}}$$

Substitute given quantities and solve for v_{BM}:

$$0.600c = \frac{v_{BM} - 0.800c}{1 + \frac{v_{BM}(-0.800c)}{c^2}}$$

$$\left(1 - \frac{0.800v_{BM}}{c}\right)0.600c = v_{BM} - 0.800c$$

$$0.600c - 0.480\,v_{BM} = v_{BM} - 0.800c$$

$$v_{BM} = 0.946c$$

Remarks Notice how much care had to be taken in identifying quantities and their proper signs. Common sense might lead us to believe that Mike would measure Bob's velocity as $1.40c$, but as the calculation shows, Mike measures Bob's velocity as less than that of light.

Exercise 26.5

Suppose Bob shines a laser beam in the direction of his ship's motion. What speed would the nearby observer measure for the beam? Don't guess: do the calculation that proves the answer.

Answer c

26.9 RELATIVISTIC ENERGY AND THE EQUIVALENCE OF MASS AND ENERGY

We have seen that the definition of momentum required generalization to make it compatible with the principle of relativity. Likewise, the definition of kinetic energy requires modification in relativistic mechanics. Einstein found that the correct expression for the **kinetic energy** of an object is

Kinetic energy ▶

$$KE = \gamma mc^2 - mc^2 \qquad [26.12]$$

The constant term mc^2 in Equation 26.12, which is independent of the speed of the object, is called the **rest energy** of the object, E_R:

$$E_R = mc^2 \quad [26.13]$$

◀ Rest energy

The term γmc^2 in Equation 26.12 depends on the object's speed and is the sum of the kinetic and rest energies. We define γmc^2 to be the **total energy** E, so that

total energy = kinetic energy + rest energy

or, using Equation 26.12,

$$E = KE + mc^2 = \gamma mc^2 \quad [26.14]$$

Because $\gamma = (1 - v^2/c^2)^{-1/2}$, we can also express E as

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \quad [26.15]$$

◀ Total energy

This is Einstein's famous mass–energy equivalence equation.[4]

The relation $E = \gamma mc^2 = KE + mc^2$ shows the amazing result that **a stationary particle with zero kinetic energy has an energy proportional to its mass**. Further, a small mass corresponds to an enormous amount of energy because the proportionality constant between mass and energy is large: $c^2 = 9 \times 10^{16}\ m^2/s^2$. The equation $E_R = mc^2$, as Einstein first suggested, indirectly implies that the mass of a particle may be completely convertible to energy and that pure energy—for example, electromagnetic energy—may be converted to particles having mass. This is indeed the case, as has been shown in the laboratory many times. For example, the coming together of a slowly moving electron and its antiparticle, the positron, a particle with the same mass m_e as the electron, but opposite charge, results in the disappearance of both particles and the appearance of a burst of electromagnetic energy in the amount $2m_ec^2$. The reverse process is also fairly easily observed in the laboratory: A high-energy pulse of electromagnetic energy, a gamma ray—disappears near an atom and an electron–positron pair is created with nearly 100% conversion of the gamma ray's energy into mass. Such a pair-production process is shown in the bubble chamber photo of Figure 26.13. We will discuss pair production and annihilation in more detail in Section 26.10.

On a larger scale, nuclear power plants produce energy by the fission of uranium, which involves the conversion of a small amount of the mass of the uranium into energy. The Sun, too, converts mass into energy, and continually loses mass in pouring out a tremendous amount of electromagnetic energy in all directions.

It's extremely interesting that while we have been talking about the interconversion of mass and energy for particles, the expression $E = mc^2$ is universal and applies to all objects, processes, and systems: a hot object has slightly more mass and is slightly more difficult to accelerate than an identical cold object because it has more thermal energy, and a stretched spring has more elastic potential energy and more mass than an identical unstretched spring. A key point, however, is that these changes in mass are often far too small to measure. Our best bet for measuring mass changes is in nuclear transformations, where a measurable fraction of the mass is converted into energy.

Figure 26.13 Bubble-chamber photograph of electron (green) and positron (red) tracks produced by energetic gamma rays. The highly curved tracks at the top are due to the electron and positron in an electron–positron pair bending in opposite directions in the magnetic field.

EXAMPLE 26.6 Pool Heater

Goal Combine the concepts of density, rest mass, and heat capacity.

Problem Suppose some mechanism allowed the conversion of the rest mass of water completely into energy. **(a)** How much rest energy is contained in 0.500 mm^3 of water? **(b)** If all this energy is used to heat an Olympic swim-

[4]Although this doesn't look exactly like the famous equation $E = mc^2$, it used to be common to write $m = \gamma m_0$ (Einstein himself wrote it that way), where m is the effective mass of an object moving at speed v and m_0 is the mass of that object as measured by an observer at rest with respect to the object. Then our $E = \gamma mc^2$ becomes the familiar $E = mc^2$. It is currently unfashionable to use $m = \gamma m_0$.

ming pool with dimensions 2.00 m deep, 25.0 m wide, and 50.0 m long, what is the change in temperature of the water?

Strategy Use the density of water to find the mass in the given volume of water, and multiply by c^2 to get the energy. The heat capacity equation then yields the temperature change.

Solution

(a) How much rest energy is contained in 0.500 mm^3 of water?

Use the density to find the mass of this volume of water:

$$\rho = \frac{m}{V} \quad \rightarrow \quad m = \rho V$$

$$m = (1.00 \times 10^3 \text{ kg/m}^3)(0.500 \text{ mm}^3)\left(\frac{1.00 \text{ m}}{1.00 \times 10^3 \text{ mm}}\right)^3$$

$$= 5.00 \times 10^{-7} \text{ kg}$$

The energy equivalent of the water is found from Equation 26.13:

$$E_R = mc^2 = (5.00 \times 10^{-7} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2$$

$$= 4.50 \times 10^{10} \text{ J}$$

(b) Find the change in temperature of the pool water.

First find the volume of water in the pool:

$$V = L \times W \times H = (50.0 \text{ m})(25.0 \text{ m})(2.00 \text{ m})$$

$$= 2.50 \times 10^3 \text{ m}^3$$

Using the definition of density, calculate the mass of the water in the pool:

$$m = \rho V = (1.00 \times 10^3 \text{ kg/m}^3)(2.50 \times 10^3 \text{ m}^3)$$

$$= 2.50 \times 10^6 \text{ kg}$$

Use the heat capacity equation and the result of part (a) to calculate the temperature change of the water in the pool:

$$Q = mc\Delta T$$

$$\Delta T = \frac{Q}{mc} = \frac{4.50 \times 10^{10} \text{ J}}{(2.50 \times 10^6 \text{ kg})(4.19 \times 10^3 \text{ J/kg}\cdot\text{K})}$$

$$= 4.30 \text{ K}$$

Remarks Only 12 mm^3 of water, completely converted to energy, could raise the water temperature of an Olympic-sized pool by 100 K! However, it's generally impossible to achieve the complete conversion of mass to energy. Nuclear power plants convert only a tiny percentage of the mass of uranium. An exception is the interaction of matter with antimatter.

Exercise 26.6

(a) What mass, when completely converted into energy, would provide the annual energy needs of the entire world (about 4×10^{20} J) (b) What volume of water contains that much energy?

Answers (a) 4×10^3 kg (b) 4 m^3

Energy and Relativistic Momentum

Often the momentum or energy of a particle is measured rather than its speed, so it's useful to have an expression relating the total energy E to the relativistic momentum p. This is accomplished by using the expressions $E = \gamma mc^2$ and $p = \gamma mv$. By squaring these equations and subtracting, we can eliminate v. The result, after some algebra, is

$$E^2 = p^2c^2 + (mc^2)^2 \qquad [26.16]$$

When the particle is at rest, $p = 0$, so $E = E_R = mc^2$. In this special case, the total energy equals the rest energy. For the case of particles that have zero mass, such as photons (massless, chargeless particles of light), we set $m = 0$ in Equation 26.16 and find that

$$E = pc \qquad [26.17]$$

This equation is an exact expression relating energy and momentum for photons, which always travel at the speed of light.

In dealing with subatomic particles, it's convenient to express their energy in electron volts (eV), because the particles are given energy when accelerated through an electrostatic potential difference. The conversion factor is

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$$

For example, the mass of an electron is 9.11×10^{-31} kg. Hence, the rest energy of the electron is

$$m_e c^2 = (9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 = 8.20 \times 10^{-14} \text{ J}$$

Converting to eV, we have

$$m_e c^2 = (8.20 \times 10^{-14} \text{ J})(1 \text{ eV}/1.60 \times 10^{-19} \text{ J}) = 0.511 \text{ MeV}$$

Because we frequently use the expression $E = \gamma mc^2$ in nuclear physics, and because m is usually in atomic mass units, u, it is useful to have the conversion factor $1 \text{ u} = 931.494 \text{ MeV}/c^2$. Using this factor makes it easy, for example, to find the rest energy in MeV of the nucleus of a uranium atom with a mass of 235.043 924 u:

$$E_R = mc^2 = (235.043\,924 \text{ u})(931.494 \text{ MeV/u}\cdot c^2)(c^2) = 2.189\,42 \times 10^5 \text{ MeV}$$

Quick Quiz 26.4

A photon is reflected from a mirror. **True or false**: (a) Because a photon has zero mass, it does not exert a force on the mirror. (b) Although the photon has energy, it can't transfer any energy to the surface because it has zero mass. (c) The photon carries momentum, and when it reflects off the mirror, it undergoes a change in momentum and exerts a force on the mirror. (d) Although the photon carries momentum, its change in momentum is zero when it reflects from the mirror, so it can't exert a force on the mirror.

EXAMPLE 26.7 A Speedy Electron

Goal Compute a total energy and a relativistic kinetic energy.

Problem An electron moves with a speed $v = 0.850c$. Find its total energy and kinetic energy in mega electron volts (MeV), and compare the latter to the classical kinetic energy (10^6 eV = 1 MeV).

Strategy Substitute into Equation 26.15 to get the total energy, and subtract the rest mass energy to obtain the kinetic energy.

Solution

Substitute values into Equation 26.15 to obtain the total energy:

$$E = \frac{m_e c^2}{\sqrt{1 - v^2/c^2}} = \frac{(9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2}{\sqrt{1 - (0.850c/c)^2}}$$

$$= 1.56 \times 10^{-13} \text{ J} = (1.56 \times 10^{-13} \text{ J})\left(\frac{1.00 \text{ eV}}{1.60 \times 10^{-19} \text{ J}}\right)$$

$$= 0.975 \text{ MeV}$$

The kinetic energy is obtained by subtracting the rest energy from the total energy:

$$KE = E - m_e c^2 = 0.975 \text{ MeV} - 0.511 \text{ MeV} = 0.464 \text{ MeV}$$

Calculate the classical kinetic energy:

$$KE_{\text{classical}} = \tfrac{1}{2} m_e v^2$$

$$= \tfrac{1}{2}(9.11 \times 10^{-31} \text{ kg})(0.850 \times 3.00 \times 10^8 \text{ m/s})^2$$

$$= 2.96 \times 10^{-14} \text{ J} = 0.185 \text{ MeV}$$

Remarks Notice the large discrepancy between the relativistic kinetic energy and the classical kinetic energy.

Exercise 26.7
Calculate the total energy and the kinetic energy in MeV of a proton traveling at $0.600c$. (The rest energy of a proton is approximately 938 MeV.)

Answers $E = 1.17 \times 10^3$ MeV, $KE = 232$ MeV

EXAMPLE 26.8 The Conversion of Mass to Kinetic Energy in Uranium Fission

Goal Understand the production of energy from nuclear sources.

Problem The fission, or splitting, of uranium was discovered in 1938 by Lise Meitner, who successfully interpreted some curious experimental results found by Otto Hahn as due to fission. (Hahn received the Nobel prize.) The fission of $^{235}_{92}U$ begins with the absorption of a slow-moving neutron that produces an unstable nucleus of ^{236}U. The ^{236}U nucleus then quickly decays into two heavy fragments moving at high speed, as well as several neutrons. Most of the kinetic energy released in such a fission is carried off by the two large fragments. **(a)** For the typical fission process

$$^1_0n + ^{235}_{92}U \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3^1_0n$$

calculate the kinetic energy in MeV carried off by the fission fragments. **(b)** What percentage of the initial energy is converted into kinetic energy? The atomic masses involved are given below in atomic mass units.

$^1_0n = 1.008\,665$ u $^{235}_{92}U = 235.043\,924$ u $^{141}_{56}Ba = 140.903\,496$ u $^{92}_{36}Kr = 91.907\,936$ u

Strategy This is an application of the conservation of relativistic energy. Write the conservation law as a sum of kinetic energy and rest energy, and solve for the final kinetic energy. Equation 26.15, solved for v, then yields the speeds.

Solution
(a) Calculate the final kinetic energy for the given process.

Apply the conservation of relativistic energy equation, assuming that $KE_{\text{initial}} = 0$:

$$(KE + mc^2)_{\text{initial}} = (KE + mc^2)_{\text{final}}$$

$$0 + m_nc^2 + m_Uc^2 = m_{Ba}c^2 + m_{Kr}c^2 + 3m_nc^2 + KE_{\text{final}}$$

Solve for KE_{final} and substitute, converting to MeV in the last step:

$$KE_{\text{final}} = [(m_n + m_U) - (m_{Ba} + m_{Kr} + 3m_n)]c^2$$

$$KE_{\text{final}} = (1.008\,665\text{u} + 235.043\,924\text{ u})c^2$$
$$-[140.903\,496\text{ u} + 91.907\,936\text{ u} + 3(1.008\,665\text{ u})]c^2$$
$$= (0.215\,162\text{ u})(931.494\text{ MeV/u}\cdot c^2)(c^2)$$
$$= 200.422\text{ MeV}$$

(b) What percentage of the initial energy is converted into kinetic energy?

Compute the total energy, which is the initial energy:

$$E_{\text{initial}} = 0 + m_nc^2 + m_Uc^2$$
$$= (1.008\,665\text{u} + 235.043\,924\text{ u})c^2$$
$$= (236.052\,59\text{ u})(931.494\text{ MeV/u}\cdot c^2)(c^2)$$
$$= 2.198\,82 \times 10^5\text{ MeV}$$

Divide the kinetic energy by the total energy and multiply by 100%:

$$\frac{200.422\text{ MeV}}{2.198\,82 \times 10^5\text{ MeV}} \times 100\% = 9.115 \times 10^{-2}\%$$

Remarks This calculation shows that nuclear reactions liberate only about a tenth of one percent of the rest energy of the constituent particles. Some fusion reactions better that number by several times.

Exercise 26.8
In a fusion reaction, light elements combine to form a heavier element. Deuterium, which is also called heavy hydrogen, has an extra neutron in its nucleus. Two such particles can fuse into a heavier form of hydrogen, called tritium, plus an ordinary hydrogen atom. The reaction is

$${}^{2}_{1}\mathrm{D} + {}^{2}_{1}\mathrm{D} \rightarrow {}^{3}_{1}\mathrm{T} + {}^{1}_{1}\mathrm{H}$$

(a) Calculate the energy released in the form of kinetic energy, assuming for simplicity that the initial kinetic energy is zero. (b) What percentage of the rest mass was converted to energy? The atomic masses involved are as follows:

$${}^{2}_{1}\mathrm{D} = 2.014\,102\text{ u} \qquad {}^{3}_{1}\mathrm{T} = 3.016\,049\text{ u} \qquad {}^{1}_{1}\mathrm{H} = 1.007\,825\text{ u}$$

Answers (a) 4.033 37 MeV (b) 0.1075%

26.10 PAIR PRODUCTION AND ANNIHILATION

In general, converting mass into energy is a low-yield process. Burning wood or coal, or even the fission or fusion processes presented in Example 26.8, convert only a very small percentage of the available energy. An exception is the reaction of matter with antimatter.

A common process in which a photon creates matter is called **pair production**, illustrated in Figure 26.14. In this process, an electron and a positron are simultaneously produced, while the photon disappears. (Note that the positron is a positively charged particle having the same mass as an electron. The positron is often called the *antiparticle* of the electron.) In order for pair production to occur, energy, momentum, and charge must all be conserved during the process. It's impossible for a photon to produce a single electron because the photon has zero charge and charge would not be conserved in the process.

As we explain in more detail in Chapter 27, the energy of a photon having a frequency f is given by $E = hf$, where h is Planck's constant. The *minimum* energy that a photon must have to produce an electron–positron pair can be found using conservation of energy by equating the photon energy hf_{min} to the total rest energy of the pair. That is,

$$hf_{\text{min}} = 2m_ec^2 \qquad [26.18]$$

Because the energy of an electron is $m_ec^2 = 0.51$ MeV, the minimum energy required for pair production is 1.02 MeV.

Pair production can't occur in a vacuum, but can only take place in the presence of a massive particle such as an atomic nucleus. The massive particle must participate in the interaction in order that energy and momentum be conserved simultaneously.

Pair annihilation is a process in which an electron–positron pair produces two photons—the inverse of pair production. Figure 26.15 is one example of pair annihilation in which an electron and positron initially at rest combine with each other, disappear, and create two photons. Because the initial momentum of the pair is zero, it's impossible to produce a single photon. Momentum can be conserved only if two photons moving in opposite directions, both with the same energy and magnitude of momentum, are produced. We will discuss particles and their antiparticles further in Chapter 30.

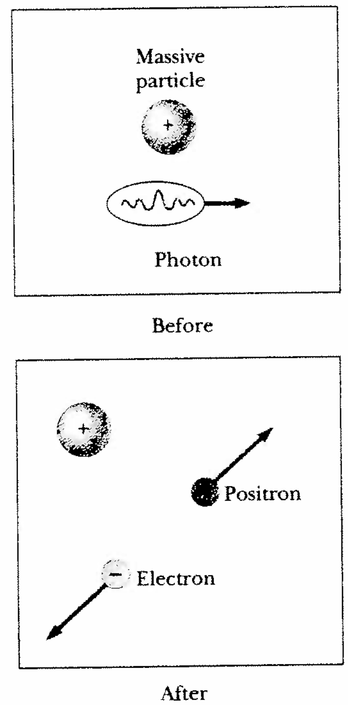


Figure 26.14 Representation of the process of pair production.

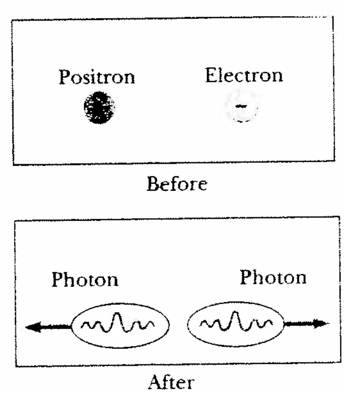


Figure 26.15 Representation of the process of pair annihilation.

26.11 GENERAL RELATIVITY

Special relativity relates observations of inertial observers. Einstein sought a more general theory that would address accelerating systems. His search was motivated in part by the following curious fact: mass determines the inertia of an object and also the strength of the gravitational field. The mass involved in inertia is called inertial mass, m_i, whereas the mass responsible for the gravitational field is called the gravitational mass, m_g. They appear in Newton's law of gravitation and in the second law of motion:

$$\text{Gravitational property} \qquad F_g = G\frac{m_g m_g'}{r^2}$$

$$\text{Inertial property} \qquad F_i = m_i a$$

The value for the gravitational constant G was chosen to make the magnitudes of m_g and m_i numerically equal. Regardless of how G is chosen, however, the strict proportionality of m_g and m_i has been established experimentally to an extremely high degree: a few parts in 10^{12}. It appears that gravitational mass and inertial mass may indeed be exactly equal: $m_i = m_g$.

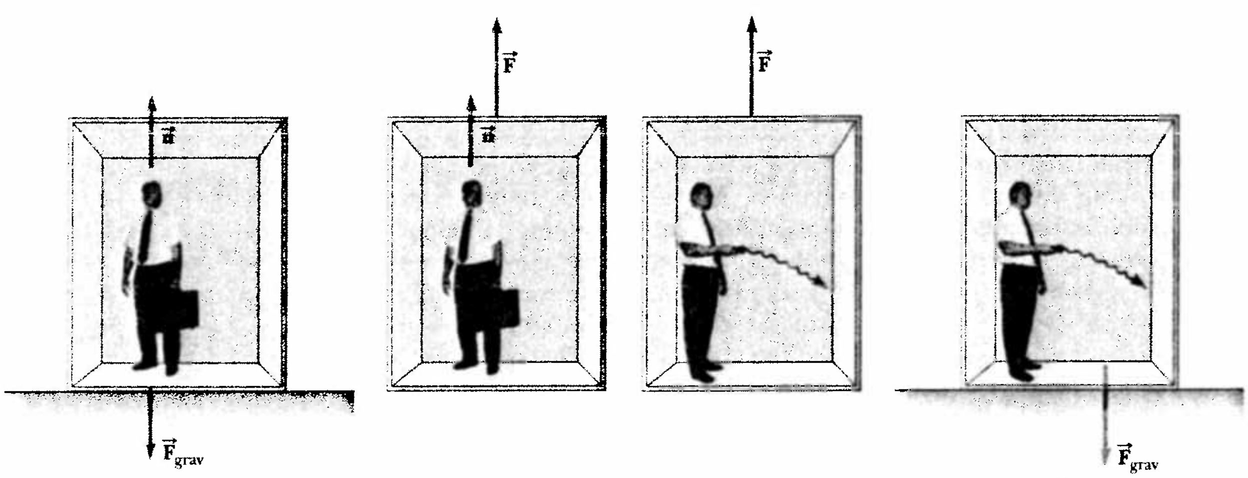
There is no reason a priori, however, why these two very different quantities should be equal. They seem to involve two entirely different concepts: a force of mutual gravitational attraction between two masses and the resistance of a single mass to being accelerated. This question puzzled Newton and many other physicists over the years and was finally incorporated as a fundamental principle of Einstein's remarkable theory of gravitation, known as *general relativity*, in 1916.

In Einstein's view, the remarkable coincidence that m_g and m_i were exactly equal was evidence for an intimate connection between the two concepts. He pointed out that no mechanical experiment (such as releasing a mass) could distinguish between the two situations illustrated in Figures 26.16a and 26.16b. In each case, a mass released by the observer undergoes a downward acceleration of g relative to the floor.

Einstein carried this idea further and proposed that *no* experiment, mechanical or otherwise, could distinguish between the two cases. This extension to include all phenomena (not just mechanical ones) has interesting consequences. For example, suppose that a light pulse is sent horizontally across the box, as in Figure 26.16c. The trajectory of the light pulse bends downward as the box accelerates upward to meet it. Einstein proposed that a beam of light should also be bent downward by a gravitational field (Fig. 26.16d).

The two postulates of Einstein's **general relativity** are as follows:

1. All the laws of nature have the same form for observers in any frame of reference, accelerated or not.
2. In the vicinity of any given point, a gravitational field is equivalent to an accelerated frame of reference without a gravitational field. (This is the *principle of equivalence.*)



(a) (b) (c) (d)

Figure 26.16 (a) The observer in the cubicle is at rest in a uniform gravitational field $\vec{g}$. He experiences a normal force $\vec{n}$. (b) Now the observer is in a region where gravity is negligible, but an external force $\vec{F}$ acts on the frame of reference, producing an acceleration with magnitude g. Again, the man experiences a normal force $\vec{n}$ that accelerates him along with the cubicle. According to Einstein, the frames of reference in parts (a) and (b) are equivalent in every way. No local experiment could distinguish between them. (c) The observer turns on his pocket flashlight. Because of the acceleration of the cubicle, the beam would appear to bend toward the floor, just as a tossed ball would. (d) Given the equivalence of the frames, the same phenomenon should be observed in the presence of a gravity field.

The second postulate implies that gravitational mass and inertial mass are completely equivalent, not just proportional. What were thought to be two different types of mass are actually identical.

One interesting effect predicted by general relativity is that time scales are altered by gravity. A clock in the presence of gravity runs more slowly than one in which gravity is negligible. As a consequence, light emitted from atoms in a strong gravity field, such as the Sun's, is observed to have a lower frequency than the same light emitted by atoms in the laboratory. This gravitational shift has been detected in spectral lines emitted by atoms in massive stars. It has also been verified on Earth by comparing the frequencies of gamma rays emitted from nuclei separated vertically by about 20 m.

Quick Quiz 26.5

Two identical clocks are in the same house, one upstairs in a bedroom and the other downstairs in the kitchen. Which statement is correct? (a) The clock in the kitchen runs more slowly than the clock in the bedroom. (b) The clock in the bedroom runs more slowly than the clock in the kitchen. (c) Both clocks keep the same time.

The second postulate suggests that a gravitational field may be "transformed away" at any point if we choose an appropriate accelerated frame of reference—a freely falling one. Einstein developed an ingenious method of describing the acceleration necessary to make the gravitational field "disappear." He specified a certain quantity, the *curvature of spacetime*, that describes the gravitational effect at every point. In fact, the curvature of spacetime completely replaces Newton's gravitational theory. According to Einstein, there is no such thing as a gravitational force. Rather, the presence of a mass causes a curvature of spacetime in the vicinity of the mass. Planets going around the Sun follow the natural contours of the spacetime, much as marbles roll around inside a bowl. In 1979, John Wheeler summarized Einstein's general theory of relativity in a single sentence: "Mass one tells spacetime how to curve; curved spacetime tells mass two how to move." The fundamental equation of general relativity can be roughly stated as a proportion as follows:

$$\text{Average curvature of spacetime} \propto \text{energy density}$$

The equation corresponding to this proportion is solved for a mathematical quantity called the *metric*, which can be used to measure the lengths of vectors and to compute trajectories of bodies through space. The metric looks something like a matrix, with different entries at each point of space and time. (There are a few important differences, beyond the level of this course.)

Einstein pursued a new theory of gravity in large part because of a discrepancy in the orbit of Mercury as calculated from Newton's second law. The closest approach of the Mercury to the Sun, called the perihelion, changes position slowly over time. Newton's theory accounted for all but 43 seconds of arc per century; Einstein's general relativity explained the discrepancy.

The most dramatic test of general relativity came shortly after the end of World War I. The theory predicts that a star would bend a light ray by a certain precise amount. Sir Arthur Eddington mounted an expedition to Africa and, during a solar eclipse, confirmed that starlight bent on passing the Sun in an amount matching the prediction of general relativity (Fig. 26.17). When this discovery was announced, Einstein became an international celebrity.

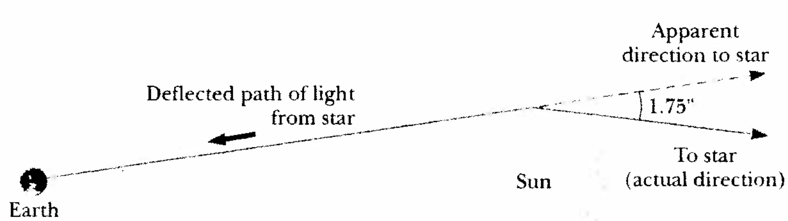


Figure 26.17 Deflection of starlight passing near the Sun. Because of this effect, the Sun and other remote objects can act as a *gravitational lens*. In his general theory of relativity, Einstein calculated that starlight just grazing the Sun's surface should be deflected by an angle of 1.75″.

Other tests were proposed and verified long after Einstein's death, including the time delay of radar bounced off Venus, and the gradual lengthening of the period of binary pulsars due to the emission of gravitational radiation. The latter has been verified with such precision that general relativity can lay claim to being the most accurate theory in physics.

General relativity also predicts extreme states of matter created by gravitational collapse. If the concentration of mass becomes very great, as is believed to occur when a large star exhausts its nuclear fuel and collapses to a very small volume, a **black hole** may form. Here the curvature of spacetime is so extreme that all matter and light within a certain radius becomes trapped. This radius, called the *Schwarzschild radius* or *event horizon*, is about 3 km for a black hole with the mass of our Sun. At the black hole's center may lurk a *singularity*—a point of infinite density and curvature where spacetime comes to an end.

There is strong evidence for the existence of a black hole having a mass of millions of Suns at the center of our galaxy.

Applying Physics 26.1 Faster Clocks in a "Mile High City"

Atomic clocks are extremely accurate; in fact, an error of 1 second in 3 million years is typical. This error can be described as about one part in 10^{14}. On the other hand, the atomic clock in Boulder, Colorado, is often 15 ns faster than the one in Washington after only one day. This is an error of about one part in 6×10^{12}, which is about 17 times larger than the typical error. If atomic clocks are so accurate, why does a clock in Boulder not remain synchronous with one in Washington?

Explanation According to the general theory of relativity, the passage of time depends on gravity—clocks run more slowly in strong gravitational fields. Washington is at an elevation very close to sea level, whereas Boulder is about a mile higher in altitude. Hence, the gravitational field at Boulder is weaker than at Washington. As a result, an atomic clock runs more rapidly in Boulder than in Washington. (This effect has been verified by experiment.)

SUMMARY

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26.5 Einstein's Principle of Relativity

The two basic postulates of the **special theory of relativity** are as follows:

1. The laws of physics are the same in all inertial frames of reference.
2. The speed of light is the same for all inertial observers, independently of their motion or of the motion of the source of light.

26.6 Consequences of Special Relativity

Some of the consequences of the special theory of relativity are as follows:

1. Clocks in motion relative to an observer slow down, a phenomenon known as **time dilation**. The relationship between time intervals in the moving and at-rest systems is

$$\Delta t = \gamma \Delta t_p \quad [26.7]$$

where Δt is the time interval measured in the system in relative motion with respect to the clock,

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \quad [26.8]$$

and Δt_p is the proper time interval measured in the system moving with the clock.

2. The length of an object in motion is *contracted* in the direction of motion. The equation for **length contraction** is

$$L = L_p\sqrt{1 - v^2/c^2} \quad [26.9]$$

where L is the length measured by an observer in motion relative to the object and L_p is the proper length measured by an observer for whom the object is at rest.

3. Events that are simultaneous for one observer are not simultaneous for another observer in motion relative to the first.

26.7 Relativistic Momentum

The relativistic expression for the **momentum** of a particle moving with velocity v is

$$p \equiv \frac{mv}{\sqrt{1 - v^2/c^2}} = \gamma mv \quad [26.10]$$

26.8 Relativistic Addition of Velocities

The relativistic expression for the addition of velocities is

$$v_{ab} = \frac{v_{ad} + v_{db}}{1 + \frac{v_{ad} v_{db}}{c^2}} \quad [26.11]$$

where v_{ab} is the velocity of object a with as measured in frame b, v_{ad} is the velocity of object a as measured in frame d, and v_{db} is the velocity of frame d as measured in frame b.

26.9 Relativistic Energy and the Equivalence of Mass and Energy

The relativistic expression for the **kinetic energy** of an object is

$$KE = \gamma mc^2 - mc^2 \qquad [26.12]$$

where mc^2 is the **rest energy** of the object, E_R.

The **total energy** of a particle is

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \qquad [26.15]$$

This is Einstein's famous mass–energy equivalence equation.

The relativistic momentum is related to the total energy through the equation

$$E^2 = p^2c^2 + (mc^2)^2 \qquad [26.16]$$

26.10 Pair Production and Annihilation

Pair production is a process in which the energy of a photon is converted into mass. In this process, the photon disappears as an electron–positron pair is created. Likewise, the energy of an electron–positron pair can be converted into electromagnetic radiation by the process of **pair annihilation**.

CONCEPTUAL QUESTIONS

1. A spacecraft with the shape of a sphere of diameter D moves past an observer on Earth with a speed $0.5c$. What shape does the observer measure for the spacecraft as it moves past?
2. The equation $E = mc^2$ is often given in popular descriptions of Einstein's theory of relativity. Is this expression strictly correct? For example, does it accurately account for the kinetic energy of a moving mass?
3. You are in a speedboat on a lake. You see ahead of you a wave front, caused by the previous passage of another boat, moving away from you. You accelerate, catch up with, and pass the wave front. Is this scenario possible if you are in a rocket and you detect a wave front of light ahead of you?
4. What two speed measurements will two observers in relative motion always agree upon?
5. The speed of light in water is 2.30×10^8 m/s. Suppose an electron is moving through water at 2.50×10^8 m/s. Does this particle speed violate the principle of relativity?
6. With regard to reference frames, how does general relativity differ from special relativity?
7. Some distant starlike objects, called quasars, are receding from us at half the speed of light (or greater). What is the speed of the light we receive from these quasars?
8. It is said that Einstein, in his teenage years, asked the question, "What would I see in a mirror if I carried it in my hands and ran at a speed near that of light?" How would you answer this question?
9. List some ways our day-to-day lives would change if the speed of light were only 50 m/s.
10. Two identically constructed clocks are synchronized. One is put into orbit around Earth while the other remains on Earth. Which clock runs more slowly? When the moving clock returns to Earth, will the two clocks still be synchronized.
11. Photons of light have zero mass. How is it possible that they have momentum?
12. Imagine an astronaut on a trip to Sirius, which lies 8 lightyears from Earth. Upon arrival at Sirius, the astronaut finds that the trip lasted 6 years. If the trip was made at a constant speed of $0.8c$, how can the 8-lightyear distance be reconciled with the 6-year duration?
13. Explain why it is necessary, when defining length, to specify that the positions of the ends of a rod are to be measured simultaneously.

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging ☐ = full solution available in *Student Solutions Manual/Study Guide*

PhysicsNow™ = coached problem with hints available at **www.cp7e.com** = biomedical application

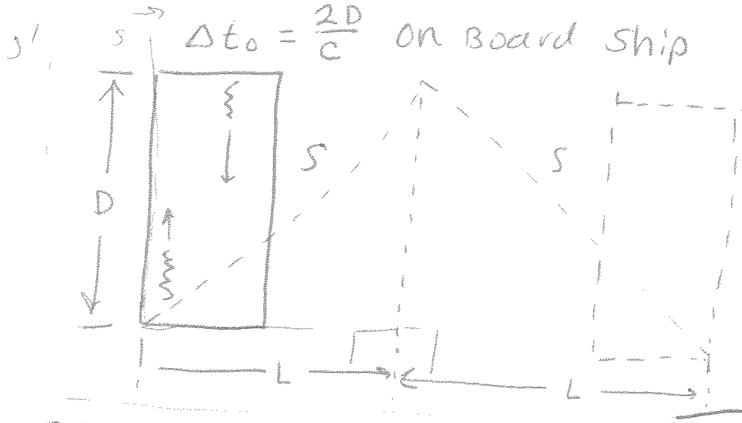
Section 26.4 The Michelson–Morley Experiment

1. Two airplanes fly paths I and II specified in Figure 26.5a. Both planes have air speeds of 100 m/s and fly a distance $L = 200$ km. The wind blows at 20.0 m/s in the direction shown in the figure. Find (a) the time of flight to each city, (b) the time to return, and (c) the difference in total flight times.
2. In one version of the Michelson–Morley experiment, the lengths L in Figure 26.6 were 28 m. Take v to be 3.0×10^4 m/s, and find the time difference caused by rotation of the interferometer and (b) the expected fringe shift, assuming that the light used has a wavelength of 550 nm.

Section 26.6 Consequences of Special Relativity

3. A deep-space probe moves away from Earth with a speed of $0.80c$. An antenna on the probe requires 3.0 s, in probe time, to rotate through 1.0 rev. How much time is required for 1.0 rev according to an observer on Earth?
4. If astronauts could travel at $v = 0.950c$, we on Earth would say it takes $(4.20/0.950) = 4.42$ years to reach Alpha Centauri, 4.20 lightyears away. The astronauts disagree. (a) How much time passes on the astronauts' clocks? (b) What is the distance to Alpha Centauri as measured by the astronauts?

Time Dilation



$$\Delta t_0 = \frac{2D}{c}$$

$2L = v\,\Delta t$ on earth $\Rightarrow$ $$L = \frac{v\,\Delta t}{2}$$

$$2S = c\,\Delta t$$

$$2S = 2\sqrt{D^2 + L^2}$$

$$2S = 2\sqrt{D^2 + \left(\frac{v\,\Delta t}{2}\right)^2}$$

$$2S = 2\sqrt{D^2 + \left(\frac{v\,\Delta t}{2}\right)^2} = c\,\Delta T$$

From Lorentz TRANSFORMATION

Rest frame

$$t_1' = \gamma\left(t_1 - \frac{v x_0}{c^2}\right)$$

$$t_2' = \gamma\left(t_2 - \frac{v x_0}{c^2}\right)$$

$$t_2' - t_1' = \gamma(t_2 - t_1)$$

$$t = \gamma\, t_0$$

$$t = \frac{t_0}{\sqrt{1 - v^2/c^2}}$$

$$c\,\Delta t = 2\sqrt{D^2 + \left(\frac{v\,\Delta t}{2}\right)^2}$$

$$c^2\,\Delta t^2 = 4\left(D^2 + \left(\frac{v\,\Delta t}{2}\right)^2\right)$$

$$\Delta t^2 = \frac{4}{c^2}\left(D^2 + \left(\frac{v\,\Delta t}{2}\right)^2\right)$$

$$\Delta t^2 = \frac{4D^2}{c^2} + \frac{\Delta t^2 v^2}{c^2}$$

$$\Delta t^2 - \frac{v^2}{c^2}\Delta t^2 = \frac{4D^2}{c^2}$$

$$\Delta t^2\left(1 - \frac{v^2}{c^2}\right) = \frac{4D^2}{c^2}$$

$$\Delta t^2 = \frac{4D^2}{c^2} \cdot \frac{1}{\left(1 - \frac{v^2}{c^2}\right)}$$

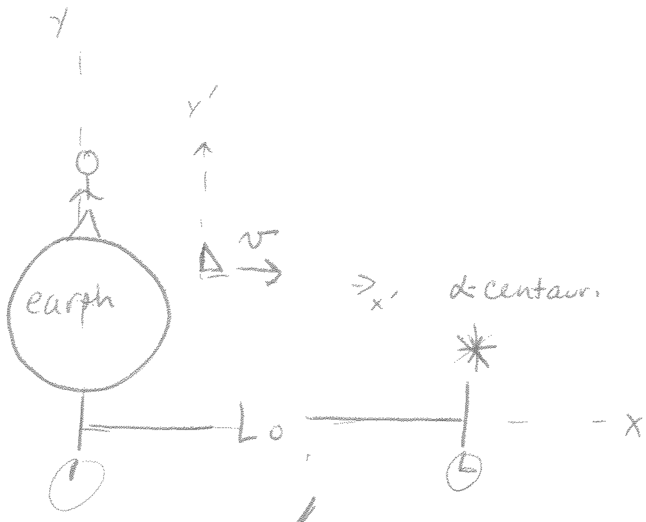
$$\Delta t = \frac{2D}{c} \cdot \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

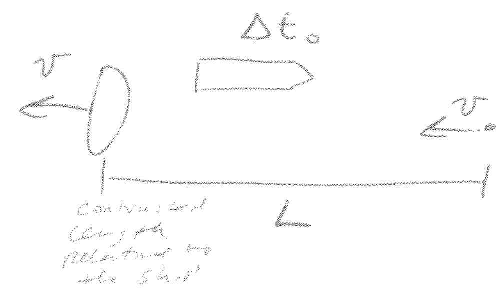
$$\Delta t = \Delta t_0 \cdot \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\boxed{\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

Proper time (Δt_0)

Length Contraction





$$v = L_0 / (\Delta t)$$

$$v = \frac{L}{\Delta t_0}$$

$$L_0 / \Delta t = \frac{L}{\Delta t_0}$$

$$L_0 \left(\frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \right)^{-1} = \frac{L}{\Delta t_0}$$

$$L_0 \frac{\sqrt{1 - v^2/c^2}}{\Delta t_0} = \frac{L}{\Delta t_0}$$

$$\boxed{L = L_0 \sqrt{1 - v^2/c^2}}$$

↑ Proper length

$$L_0 = x_2 - x_1$$

$$L = x_2' - x_1'$$

Rest frame: $x_1 = \gamma (x_1' + v t')$

$x_2 = \gamma (x_2' + v t')$ rest

if t' is same

$$x_2 - x_1 = \gamma (x_2' - x_1')$$

$$L_0 = \gamma L$$

$$L = \frac{L_0}{\gamma}$$

$$L = \frac{L_0}{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

$$\boxed{L = L_0 \sqrt{1 - \frac{v^2}{c^2}}}$$

Relativistic Energy

Rest mass energy $E_0 = m_0 c^2$ $\qquad m = \dfrac{m_0}{\sqrt{1-v^2/c^2}}$

TOTAL Energy

$$\boxed{E = \frac{m_0 c^2}{\sqrt{1-v^2/c^2}}}$$

TOTAL ENERGY

Kinetic Energy $\quad KE = (E - E_0)$

$$KE = \frac{m_0 c^2}{\sqrt{1-v^2/c^2}} - m_0 c^2$$

$$\boxed{KE = \left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right) m_0 c^2}$$

Note:

$$\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)$$

$$\frac{1}{(1-x)^{1/2}} = 1 + \frac{x}{2} + \frac{3}{8}x^2 + \cdots$$

sub $\quad KE = m_0 c^2\left(1 - \dfrac{v^2}{2c^2} + \dfrac{3}{8}\dfrac{v^4}{c^4} + \cdots\right)$

$$= \tfrac{1}{2}mv^2 + 3/8\, m v^4/c^2 + \cdots$$

if $\frac{v}{c} \ll 1$ then $\boxed{= 1/2\, m v^2}$

Relativistic Momentum + Mass

26.9

$\gamma = \frac{1}{\sqrt{1-\frac{V^2}{C^2}}}$

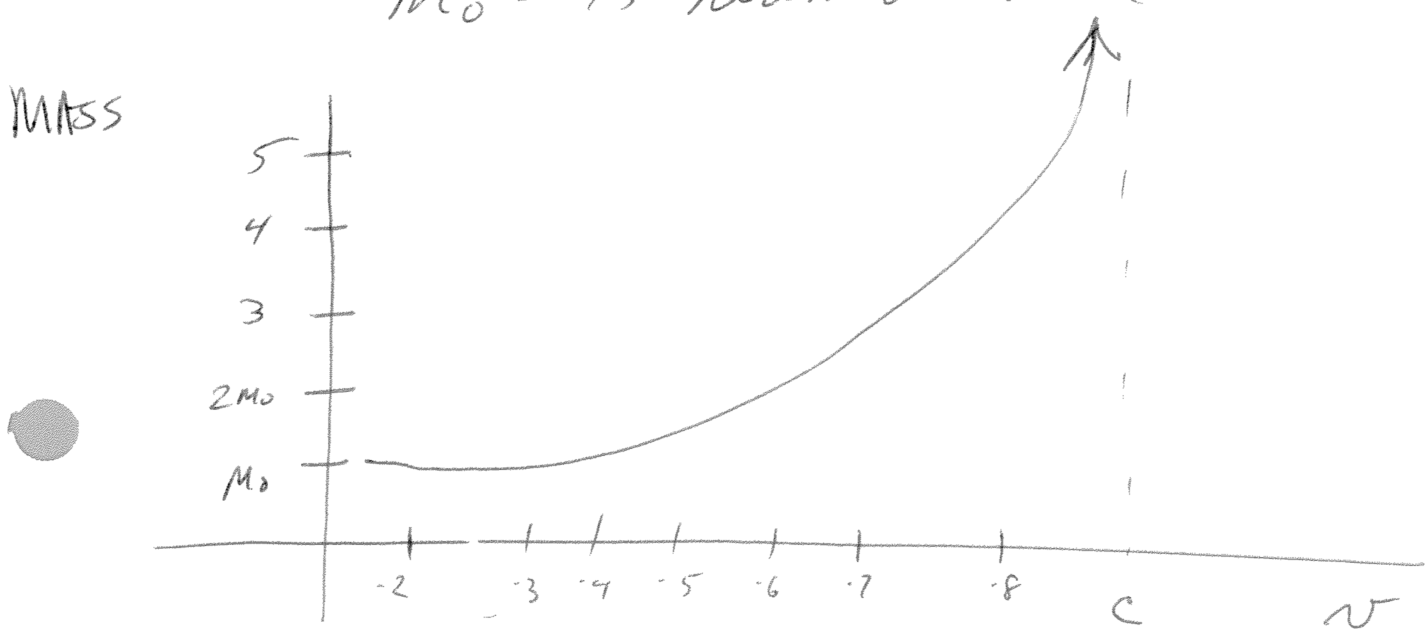
$P_{\text{Slow Speeds}} = m_0 V \;\therefore\; \boxed{P = \gamma m_0 V}$

$m = \gamma m_0$ is Relativistic mass.

*As the speed Increases so does the mass.

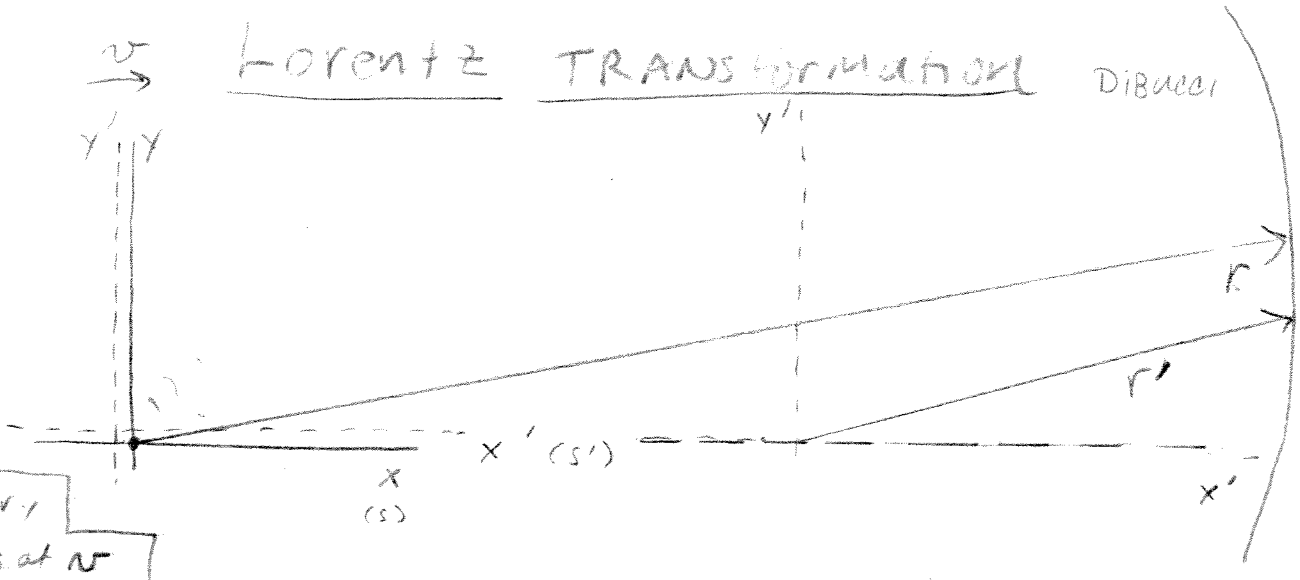
Relativistic mass } $m = \gamma m_0 = \frac{m_0}{\sqrt{1-\frac{V^2}{c^2}}}$

m_0 = is mass at rest (Rest Mass)



Lorentz Transformation

DiBucci



S is stationary
S' is moving at v

* Let t AND t' = 0 when a spherical light pulse is emitted

Spherical pulse at t + t'

* if the speed of light is constant in both frames S + S' then

$r = r'$

$$r^2 = x^2 + y^2 + z^2 = (ct)^2$$
$$r'^2 = x'^2 + y'^2 + z'^2 = (ct')^2$$

* let $y' = y$, $z' = z$ — *motion is along x axis only

$$\begin{array}{rl} & x^2 + y^2 + z^2 = (ct)^2 \\ - & x'^2 + y^2 + z'^2 = (ct')^2 \\ \hline & x^2 - x'^2 = (ct)^2 - (ct')^2 \end{array}$$

$$x^2 - c^2t^2 = x'^2 - c^2t'^2$$

* $(x = vt$ at $x' = 0)$

$$\boxed{(vt)^2 - c^2t^2 = x'^2 - c^2t'^2}$$

$$v^2t^2 - c^2t^2 = 0^2 - c^2t'^2$$

$$\frac{t^2(v^2 - c^2)}{c^2} = \frac{-c^2t'^2}{c^2}$$

$$t^2\left(\frac{v^2}{c^2} - 1\right) = -t'^2$$

$$t^2\left(1 - \frac{v^2}{c^2}\right) = t'^2 \Rightarrow t = \frac{t'}{\sqrt{1 - v^2/c^2}}$$

AP Physics B
DiBucci

Relativistic mass AND Energy

1. A space ship at rest on a launching pad has a mass of 1.00×10^5 kg. How much will its energy have increased when the ship is moving at 0.600*c* ?

A) 1.12×10^{21} J
B) 1.62×10^{21} J
C) 2.25×10^{21} J
D) 6.00×10^{21} J
E) 9.00×10^{21} J

2. The temperature of a 5.00-kg lead brick is increased by 225 C°. If the specific heat capacity of lead is 128 J/(kg • C°), what is the *increase* in the mass of the lead brick when it has reached its final temperature?

A) 4.33×10^{-11} kg
B) 9.66×10^{-11} kg
C) 1.60×10^{-12} kg
D) 2.40×10^{-12} kg
E) 4.80×10^{-4} kg

3. How much energy would be released if 1.0 g of material were completely converted into energy?

A) 9×10^8 J B) 9×10^9 J C) 9×10^{11} J D) 9×10^{13} J E) 9×10^{16} J

4. A particle travels at 0.60*c*. Determine the ratio of its kinetic energy to its rest energy.

A) 0.25 B) 0.50 C) 0.60 D) 0.64 E) 0.80

5. The average power output of a nuclear power plant is 500 MW. In 1 minute, what is the change in the mass of the nuclear fuel due to the energy being taken from the reactor? *Assume 100% efficiency.*

A) 9.3×10^{-17} kg B) 9.3×10^{-11} kg C) 3.3×10^{-13} kg D) 3.3×10^{-7} kg E) 9.3 kg

$$E_0 = m_0 c^2 \qquad \Delta m_0 = \frac{\Delta E_0}{c^2}$$

$$E = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}}$$

$$\therefore E = \gamma m_0 c^2$$

Answer Key

1. C 2.25×10^{21} J
2. C 1.60×10^{-12} kg
3. D 9×10^{13} J
4. A 0.25
5. D 3.3×10^{-7} kg

Relativistic Momentum

AP Physics - B
DiBucci

1. A proton has a mass of 1.673×10^{-27} kg. If the proton is accelerated to a speed of $0.93c$, what is the magnitude of the relativistic momentum of the proton?

A) 6.2×10^{-17} kg • m/s
B) 1.3×10^{-18} kg • m/s
C) 4.7×10^{-19} kg • m/s
D) 5.9×10^{-24} kg • m/s
E) 1.6×10^{-27} kg • m/s

2. The momentum of an electron is 1.60 times larger than the value computed non-relativistically. What is the speed of the electron?

A) 2.94×10^{8} m/s
B) 2.76×10^{8} m/s
C) 2.61×10^{8} m/s
D) 2.34×10^{8} m/s
E) 1.83×10^{8} m/s

3. An electron gun inside a computer monitor sends an electron toward the screen at a speed of 1.20×10^{8} m/s. If the mass of the electron is 9.109×10^{-31} kg, what is the magnitude of its relativistic momentum?

A) 9.88×10^{-23} kg • m/s
B) 1.09×10^{-22} kg • m/s
C) 1.20×10^{-22} kg • m/s
D) 1.41×10^{-22} kg • m/s
E) 3.25×10^{-22} kg • m/s

4. At what speed is a particle traveling if its kinetic energy is three times its rest energy?

A) $0.879c$ B) $0.918c$ C) $0.943c$ D) $0.968c$ E) $0.989c$

Answer Key

1. B 1.3×10^{-18} kg • m/s
2. D 2.34×10^{8} m/s
3. C 1.20×10^{-22} kg • m/s
4. D $0.968c$

MATTER AND RADIATION: THE INERTIA OF ENERGY

Are not gross Bodies and Light convertible into one another, and may not Bodies receive much of their Activity from the Particles of Light which enter their Composition?

Newton, *Opticks* (4th ed., 1730)

It would be quite wrong to suggest that Newton had really anticipated 20th-century physics to the extent that the above quotation might imply, but his provocative query is superbly appropriate as an introduction to the discussion that we shall now undertake. For we shall consider the intimate connection between the inertia of ordinary matter and the energy of radiation, and in so doing we shall develop some dynamical results that apply equally to photons and "gross bodies." We shall obtain, as one of the consequences, a full account of the relation between speed and kinetic energy for the electrons in the ultimate-speed experiment.

Our starting point will be a *gedanken experiment* (literally a "thought experiment," i.e., a fictitious, not really feasible experiment) which was invented by Einstein himself in 1906.[1] The purpose of it is to suggest that energy must have associated with it a certain inertial mass equivalent.[2] We suppose that an amount E of radiant energy (a burst of photons) is emitted from one end of a box of mass M and length L that is isolated from its surroundings and is initially stationary [Fig. 1–4(a)]. The radiation carries momentum E/c. Since the total momentum of the system remains equal to zero, the box must acquire a momentum equal

[1] A. Einstein, *Ann. Phys.*, **20**, 627–633 (1906).

[2] By inertial mass we mean the ratio of linear momentum to velocity.

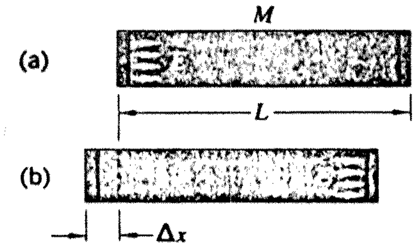


Fig. 1–4 Einstein's box—a hypothetical experiment in which a box recoils from its initial position (a) to a final position (b) as a result of a burst of radiant energy traveling from one end of the box to the other.

to $-E/c$. Hence the box recoils with a speed v, given by

$$v = -\frac{E}{Mc} \tag{1–5}$$

After traveling freely for a time Δt ($= L/c$ very nearly, provided $v \ll c$), the radiation hits the other end of the box and conveys an impulse, equal and opposite to the one it gave initially, which brings the box to rest again.[1] Thus the result of this process is to move the box through a distance Δx:

$$\Delta x = v\,\Delta t = -\frac{EL}{Mc^2} \tag{1–6}$$

But this being an isolated system, we are reluctant to believe that the center of mass of the box plus its contents has moved. We therefore postulate that the radiation has carried with it the equivalent of a mass m, such that

$$mL + M\,\Delta x = 0 \tag{1–7}$$

Putting the last two equations together, we have

$$m = \frac{E}{c^2} \quad \text{or} \quad E = mc^2 \tag{1–8}$$

For the man on the street, Einstein and relativity are probably epitomized by this result. For the physicist, its importance is not lessened by its becoming hackneyed; it asserts a fundamental inertia of energy. Although the calculation as we have presented it (which differs somewhat from Einstein's original version) points in the first instance to the mass associated with radiant energy, one quickly recognizes that the implications are much wider than this. When the radiation is emitted from one end of Einstein's box, that end must surely suffer a decrease, by

[1] If you feel that more careful account should be taken of the recoil of the box and its effect on the time and distance of transit of the radiation, see Problem 1–13.

the amount E/c^2, in its inertial mass. Likewise, the absorption of the radiation at the other end means an addition to the mass of that portion. Once the energy has been absorbed, it loses its identification as the energy of photons and ultimately becomes just an addition to the thermal energy. And we are quickly led to the idea that energy in any form has the mass equivalent defined by Eq. (1–8)—a general principle of the inertia of energy.[1]

The prime example of the mass-energy equivalence, to which we owe our continuing existence, is provided by thermonuclear reactions occurring in stars such as the sun. Observation tells us that radiant energy is reaching us from the sun at the rate of 1.35×10^3 watts/m^2. Given this figure and Eq. (1–8), we can infer that the mass of the sun is decreasing at the rate of about 4.5×10^6 tons/sec—an impressively rapid loss, even though it is only about 1 part in 10^{13} of the sun's mass per year. This comes about through chains of nuclear reactions, chief among which is the sequence by which hydrogen (^{1}H) is converted to helium (^{4}He). One must, of course, have four hydrogen atoms to end up with one helium atom, and the process takes place in several separate steps. One of these steps is particularly worth mentioning here, because it is a simple and remarkably direct example of the equivalence of the mass of ordinary matter and the energy of photons. It is this:

$$p + \mathrm{D} \rightarrow {}^3\mathrm{He} + \gamma \qquad (1\text{–}9)$$

A proton fuses with a deuteron D (the nucleus of hydrogen-2, containing one proton and one neutron), making a system of two protons and one neutron, which is the nuclear composition of ^{3}He. But, as mass-spectrometer measurements show us, the mass of this combination is greater than the mass of ^{3}He in its normal state. Here are the approximate values:

Proton	1.6724×10^{-27} kg
Deuteron	3.3432
p + D	5.0156
^{3}He nucleus	5.0058
Mass excess	9.8×10^{-30} kg

This amount of mass is carried off by a photon (a γ ray) as indicated by Eq. (1–9). The energy of that photon is given by Eq. (1–8):

$$E = mc^2 = 9.8 \times 10^{-30} \times 9.0 \times 10^{16} = 8.8 \times 10^{-13}\ \text{joule} = 5.5\ \text{MeV}$$

This process has been studied in the laboratory, and γ rays of the expected energy have been observed.[1] It should perhaps be added that such reactions, when they occur as thermonuclear reactions in the sun, require temperatures of the order of 10^7 °K and thus take place only in the inner regions. Gamma rays, such as those just considered, are completely absorbed before reaching the sun's surface, and their energy finally escapes in photons with individual energies of the order of only 1 eV—infrared, visible, and ultraviolet—that constitute the familiar solar spectrum.

The equation $E = mc^2$ has (at least in popular accounts) been so exclusively linked to nuclear transformations as to divert attention from its universality. But the message of Einstein's equation is that *any* change ΔE in the energy of a body implies a corresponding change Δm in its inertial mass:

$$\Delta E = c^2 \Delta m \qquad (1\text{–}10)$$

A golf ball in motion has more mass than the same golf ball at rest. The heated filament of a lamp has more mass than the same filament when cold. A charged capacitor has more mass than the same capacitor uncharged. And so on. Because, in terms of familiar magnitudes, the mass associated with a given amount of energy is exceedingly small (e.g., the energy used per day for domestic purposes in a city of a million people has a mass equivalent of only about 1 g), this intimate connection between the two was long unrecognized. Einstein regarded the discovery of this connection as being extremely important. To quote his own words[2]:

> The most important result of a general character to which the special theory has led is concerned with the conception of mass. Before the advent of relativity, physics recognized two conservation laws of fundamental importance, namely, the law of the conservation of energy and the law of the conservation of mass; these two fundamental laws appeared to be quite inde-

[1] For a fine discussion of this question, see M. von Laue's article "Inertia and Energy" in *Albert Einstein: Philosopher-Scientist*, Vol. II, (P. A. Schilpp, ed.), Harper Torchbook, Harper and Row, New York, 1959.

[1] W. A. Fowler, C. C. Lauritsen, and A. V. Tollestrup, *Phys. Rev.*, **76**, 1767 (1949).

[2] A. Einstein, *Relativity*, Crown, New York, 1961.

pendent of each other. By means of the theory of relativity they have been united into one law.

Perhaps one of the best ways to appreciate the pervasive character of the mass-energy equivalence is to consider a single, neutral atom in a piece of ordinary matter. From one point of view it is just one of a collection of what Newton called "solid, massy, hard, impenetrable, movable Particles."[1] The question of any inner structure does not arise, and it seems almost obvious that the atom's inertial property should be described by a single quantity that we call the mass. But now consider this same atom from the standpoint of present-day knowledge. It is a complicated assembly of electrons, neutrons, and protons (and if we want to probe more deeply, there is finer structure yet). The mass of the atom as a whole contains positive contributions from the kinetic energies of its swiftly moving constituents, and contributions of both signs (predominantly negative) from the potential energy of their electrical and nuclear interactions. (Note that a force of attraction between two particles automatically represents a *negative* contribution to the total mass of the system.[2]) Any change in the internal state of the atom is accompanied by a flow of energy into or out of it, with an associated increase or decrease in its mass. The ability of the constituents to cohere depends on the fact that their total energy in this configuration is less than if they were all separated from one another. In these terms, then, the mass of an atom is the result of a remarkable and subtle synthesis. Yet it serves to characterize the whole atom in every dynamical context—including gravitation—in which it moves as a single unit.

1-9 An eccentric billionaire decides to sterilize his 10^6-liter swimming pool by boiling the water in it. For heating purposes he uses the fusion reaction

$$^1H + {}^3H \rightarrow {}^4He + \text{radiant energy}$$

Assuming the heating system is 20% efficient, how much does he pay for the tritium (^3H) to raise the pool temperature from 20 to 100°C? It takes 4.2 joules to raise 1 g of water through 1°C. Tritium costs about $5 per cm³ of gas at STP.

Atomic masses:	^1H	1.0081 amu
	^3H	3.0170 amu
	^4He	4.0039 amu

11. Determine the mass defect of the nucleus for cobalt $^{59}_{27}Co$, which has an atomic mass of 58.933 198 u. Express your answer in (a) atomic mass units and (b) kilograms.

12. Find the binding energy (in MeV) for aluminum $^{27}_{13}Al$ (atomic mass = 26.981 539 u).

13. Find the binding energy (in MeV) for lithium ^7_3Li (atomic mass = 7.016 003 u).

14. What is the binding energy (in MeV) for oxygen $^{16}_8O$ (atomic mass = 15.994 915 u)?

Table 31.1 **Properties of Particles in the Atom**

		Mass	
Particle	Electric Charge (C)	Kilograms (kg)	Atomic Mass Units (u)
Electron	-1.60×10^{-19}	$9.109\,390 \times 10^{-31}$	$5.485\,799 \times 10^{-4}$
Proton	$+1.60 \times 10^{-19}$	$1.672\,623 \times 10^{-27}$	1.007 276
Neutron	0	$1.674\,929 \times 10^{-27}$	1.008 665

(3