Introduction to Quantum Physics of Atoms and Nuclei

Advanced Placement Physics B

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Introduction to Quantum Theory

AP Physics B

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LECTURE 2
An Introduction to the Old Quantum Theory

- *Blackbody Radiation*
- *The Photoelectric Effect*

2.1 Introduction

One of the tremendous successes of Maxwell's theory of electromagnetism was its prediction of the existence of electromagnetic waves that move through vacuum with the same speed as that of light. As we discussed in the previous lecture, Maxwell's theory not only gave conclusive evidence that light was in fact a type of wave motion, but it answered the question, "what is waving?" According to Maxwell's theory, light consists of a changing electric field that generates a changing magnetic field, that in turn, generates a changing electric field, and so on indefinitely. The wave travels by transferring energy from the electric field to the magnetic field and back again, in much the same way that a wave in a spring travels by transferring energy from the potential energy of compressions and rarefactions to the kinetic energy of the spring's mass and back again.

In particular, Maxwell's theory provides a mathematical description of the propagation of light through vacuum, as well as transparent materials.

Question 2-1: In what way(s) does the propagation of light through vacuum differ from that through a transparent material (say glass)?

Much of the behavior of light as it encounters materials, i.e, its reflection, refraction, dispersion and scattering, can be explained by assuming that the light sets the electrons of the material into vibration. According to classical theory, these vibrating electrons then act as the source of the light wave that is observed to be reflected, refracted, dispersed or scattered. Encouraged by the success of Maxwell's theory of light, physicists immediately attempted to apply it to a long-standing puzzle of classical physics -- the so called "blackbody problem". The problem is to predict the intensity of radiation emitted at a given wavelength (or frequency) by a hot glowing solid at a specified temperature. As we shall see, for all of its strengths, Maxwell's theory was unable to provide a reasonable explanation of the blackbody problem that was consistent with observed experimental results.

2.2 Thermal Radiation

Any object that is warmer than its environment emits electromagnetic radiation. Objects that are cooler than their environments *absorb* radiation. They type of electromagnetic radiation that is emitted from, or absorbed by an object depends on the temperature of the object. Imagine heating a tungsten wire by allowing an electric current to flow through it. If you place your hand near the filament, you will soon feel the "heat" radiated from the wire. Energy is being emitted from the filament in the form of electromagnetic waves in the *infrared* region of the spectrum. If we continue to heat the wire to higher temperatures we will see that the wire eventually begins to glow a faint red color. Now energy is being emitted from the filament in the form of electromagnetic waves in the *visible* region of the spectrum -- that is, in the form of *light*. Experience tells us that the amount of light emitted from an object, as well as the color of the emitted light depends on the temperature of the object. Our language reflects such ideas; when a piece of metal is heated until it just begins to glow, we say that the metal is "red hot". If we continue to heat the metal to higher and higher temperatures, we observe that the light emitted becomes *more intense* as the metal becomes "white hot".

In 1792, Thomas Wedgewood, a renowned maker of china and a relative of Charles Darwin, first observed that all the china in his ovens became red at the same temperature regardless of their size, shape or chemical composition. In fact, it is possible to get a rough idea of the temperature of a heated solid by observing its color. The table below gives the temperature ranges associated with the observed color for an object that is heated from 500 °C to 1550 °C.

TABLE 2-1

COLOR AND TEMPERATURE OF A HEATED SOLID

Color	Temperature, ° C
incipient red	500 - 550
dark red	650 - 750
bright red	850 - 950
yellowish red	1050 - 1150
yellowish white	1250 - 1350
white	1450 - 1550

Information taken from Table 19-1, *Foundations of Physics*, Robert L. Lehrman and Clifford Swartz. Copyright Holt Rinehart and Winston, Inc., 1969.

The colors listed in table 2-1 correspond to those of a nearly perfect absorber; that is, the object absorbs (nearly) all the radiation that falls upon it. Such objects appear to be black when cold, and therefore, they are called *blackbodies*. Ideal blackbodies are perfect absorbers, absorbing all the electromagnetic radiation that falls upon them regardless of the frequency of the radiation. Furthermore, since blackbodies are perfect absorbers, it follows from thermodynamic arguments that they are also ideal emitters. They are of special interest to physicists because the radiation that they emit at a given temperature depends *only on the temperature* and is *independent of the composition and geometric features of the body.* Blackbodies are of special interest to astronomers and astrophysicists because most stars, including our own Sun, act as nearly perfect blackbodies.

Question 2-2: How would the temperatures in Table 2-1 compare with those for an object that is *not* a blackbody (that is, for an object that is *not* black when cold)?

Most ordinary objects do not act like blackbodies. We see most objects because of the light that they reflect. The best approximation in nature to blackbodies are stars. They absorb almost all of the radiation that is incident upon them. They appear bright because of the light that they emit due to their surface temperatures. The best man-made approximation to a blackbody is a small hole made on the side of a large closed box as shown in Figure 2-1. Any light which enters the hole will bounce around inside the box. Some light will be absorbed with each reflection, until finally all of the light is converted into the internal energy of the matter that makes up the inner surface of the box. Only a negligibly small amount of the incident light will leave the box. Since the best man-made black body is a single hole that enters a cavity, blackbody radiation is frequently called *cavity radiation*.

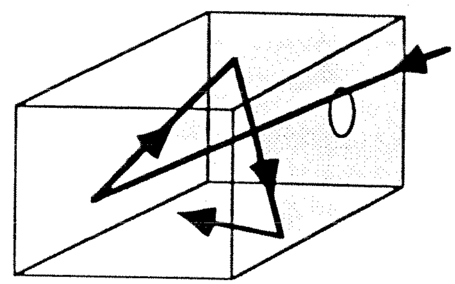

Figure 2-1

Question 2-3: True or false? No matter how black you paint a bird house, the hole in the front will always appear to be blacker. Briefly support your choice.

The Spectral Distribution of Blackbody Radiation

If such a cavity is maintained at a specific temperature, so that the radiation inside the cavity is in thermal equilibrium with the cavity walls, it can be experimentally demonstrated that the intensity of the radiation emitted from the cavity is, in fact, *dependent only the temperature*. It does not matter whether the cavity walls are made of copper, brass, iron or any other material that will tolerate the temperature. The radiation will be emitted with a range of wavelengths. Figure 2-2 shows plots of the intensity radiated at all wavelengths for a black body at three different temperatures. Such plots are frequently called *blackbody spectra* or more simply *blackbody curves*.

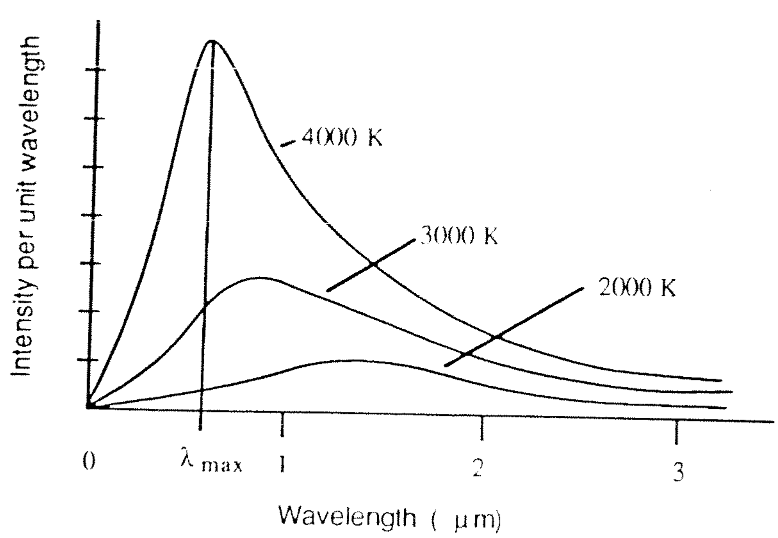

Figure 2-2

All blackbody curves display the following important features:

1. **The radiation has a continuous spectrum.** The intensity is not the same at all wavelengths, but rather, each curve has a clearly visible maximum (the maximum occurs at the wavelength labeled λ_{max} for the case T = 4000 K).

2. **For a given temperature, the curve (and therefore the energy distribution) is the same, regardless of the nature and composition of the cavity walls.**

3. **The area under any of the curves represents the total intensity radiated by the blackbody.**

4. **There is a simple relationship between the absolute temperature and the wavelength at which the maximum intensity occurs.** It is called *the Wein Displacement Law* and is quantitatively expressed as

$$\lambda_{max}\, T = \text{constant} = 0.29 \text{ cm K}.$$

This relationship was empirically established by Wilhelm Wien in 1893.

5. **The total intensity radiated is a simple function of the temperature.** It is called the *Stefan-Boltzmann law* and is mathematically expressed as

$$I = \sigma T^4$$

where σ is called the Stefan-Boltzmann constant and equal to

$$\sigma = 5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}.$$

This relationship was found experimentally in 1879 by physicist J. Stefan. Ludwig Boltzmann later showed that it followed from thermodynamic arguments. An object that is not a blackbody will follow a law of the same general form: $I = a\,\sigma T^4$, where the constant a is less than unity.

Question 2-4: Use Figure 2-2 to explain the color changes indicated in Table 2-1 as the blackbody is heated from 500 °C to 1550 °C.

Question 2-5: What is the predominant type of electromagnetic radiation that is emitted by a blackbody at 4000 K?

Question 2-6: What is the predominant type of electromagnetic radiation that is emitted by your body at this moment? Assume blackbody arguments.

Question 2-7: Use Wein's law to estimate the surface temperature of the Sun.

Question 2-8: How do the areas under the three blackbody curves shown in Figure 2-2 compare with each other? Justify your answers.

Question 2-9: Betelgeuse, a supergiant star of radius 3×10^{11} m, emits radiant energy at an average rate of 4×10^{30} W. How does its surface temperature and power output compare with those of our Sun? The radius of the Sun is 7×10^{8} m. (You may use your result from Question 2-7.)

There appears to be nothing obviously unusual about blackbody radiation or blackbody curves. The qualitative aspects of blackbody curves seem to be compatible with sound physical intuition. The quantitative aspects expressed through Wein's law and the Stefan-Boltzmann law can be experimentally verified. Questions 2-4 through 2-9 suggest that blackbody theory has a wide variety of practical applications in both science and industry. Yet classical physics was unable to provide a simple explanation of the radiation emitted from glowing objects and the general features of blackbody curves. Many attempts were made using a combination of classical thermodynamics, statistical mechanics and Maxwell's theory of electromagnetism. The best attempts displayed flawless reasoning and careful mathematical derivations. The only thing wrong with them was that they failed to stand the test of experiment!

The Classical Theory of Blackbody Radiation

Although the mathematical derivation for the energy density of cavity radiation using classical arguments is not beyond the scope of this discussion, it is standard and can be found in most introductory texts on modern physics and quantum theory (see for example the derivation of the Rayleigh-Jeans formula, *Eisberg and Resnick*, section 1-3). For our purposes, a detailed discussion of the theory and why it failed will be more useful.

According to classical theory, when light (or any other form of electromagnetic radiation) interacts with matter, the electric field drives the electrons in the material. The electrons, in turn oscillate with the field. The oscillating electrons produce electromagnetic waves at their vibrational frequencies. It is worth noting at this point, that this model works well to describe many phenomena including reflection, refraction, dispersion, light scattering and polarization.

Consider a cavity that is maintained at a certain temperature T. The electrons in the walls of the cavity will oscillate because of their thermal motion. Their oscillations will produce electromagnetic radiation at their vibrational frequencies. This electromagnetic radiation fills the cavity. The oscillation frequency of the electrons will depend on the actual location of the electron and in general will vary with time. Any given electron will lose energy in the form of electromagnetic radiation, but it will also gain energy from the radiation in the cavity. The energy of a given electron also depends on its location and will vary with time. In general, the energy and frequency will vary greatly among electrons, but from statistical mechanics arguments, the average energy of the electrons is $3kT$, where k is Boltzmann's constant and T is the absolute temperature.

Question 2-10: Why is this so? Hints: Consider the equipartition theorem. Also note that the electrons can oscillate in three dimensions and that their average kinetic energy is equal to their average potential energy.

If the system is maintained at a certain temperature T, the entire system will be in thermal equilibrium at this temperature. On average the energy that the oscillating electrons lose in radiation will be gained by the energy that they absorb from the radiation. From statistical considerations, however, the average energy per electron is $3kT$ regardless of the oscillation frequency. But there are restrictions on the permitted frequencies. In order for electromagnetic waves to exist in equilibrium inside the cavity, they must satisfy certain boundary conditions at the cavity walls. In particular, only waves with the precise wavelength to form standing waves can exist in the cavity. Since the wavelength of an electromagnetic wave determines its frequency, the cavity will contain electromagnetic waves of only certain frequencies.

Question 2-11: Why do the boundary conditions imply that the electromagnetic radiation must set up standing waves?

The situation is somewhat similar that of a string that is clamped at both ends. If the string is plucked, it must vibrate so that there is a node at each end. If the string has length L, it can vibrate with wavelengths $2L$, L, $2L/3$, $L/2$, $2L/5$, . . . etc. The standing waves are restricted to integer multiples of the fundamental frequency. If the fundamental frequency is 10 Hz, the string can in principle, simultaneously vibrate at 20 Hz, 30 Hz, 40 Hz, and so on, without any upper bound. Since the string is clamped at both ends, the amplitude of vibration must decrease as the frequency increases. Since the energy of a wave depends on the square of the amplitude, modes with short wavelengths and high frequencies will have smaller energies than long wavelength high frequency modes. In other words, the energy will drop off at higher frequencies and at some upper limit, the energy will be vanishingly small.

The standing waves formed by the radiation in the cavity are somewhat different from the standing waves on the string. Since the standing waves in the cavity are three dimensional there are many more modes of vibration, particularly at high frequencies. More importantly, however, there is a very crucial difference. *The energy per mode does not drop off at higher frequencies as it does for standing waves on a string.* The cavity radiation is in thermal equilibrium with the oscillating electrons in the cavity walls. But since the average energy of the electrons is $3kT$ regardless of their frequency, the average energy per electron is $3kT$ for all possible modes of vibration. In other words, *classical theory predicts that every mode of standing waves of radiation in the cavity has the same amount of energy.* Since there are more possible vibrational modes at higher frequencies, most of the energy will be in high-frequency radiation. If radiation from the cavity is analyzed, high frequency radiation should be emitted with the highest intensity. Instead of the behavior observed in Figure 2-2, a typical blackbody curve would look like the darkened line shown in Figure 2-3 (an actual blackbody curve is shown in that figure for comparison). Note the absurdity of these predictions. These results imply that an opened oven should be an excellent source of high frequency radiation such as UV, X-rays and gamma rays!

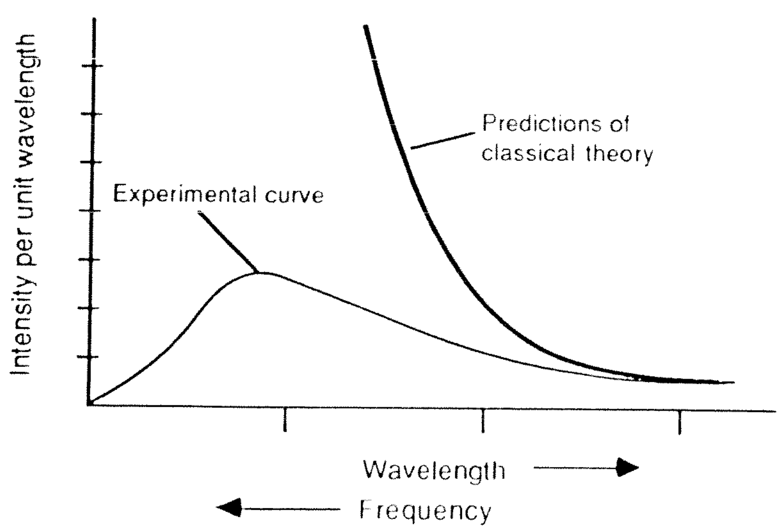

Figure 2-3

Near the end of the nineteenth century, Lord Rayleigh and Sir James Jean used the classical model of cavity radiation discussed above to derive an expression for the energy density of blackbody radiation. The famous *Rayleigh-Jeans formula for blackbody radiation* is

$$\rho(\nu) = \frac{8\pi\nu^2 kT}{c^3} \tag{2.1}$$

where $\rho(\nu)$ is the energy per unit volume of cavity radiation of frequency ν. As suggested in Figure 2-3, the classical theory fits the experimental observations in the low frequency, long wavelength limit. At fixed T, equation (2.1) agrees with the experimentally predicted values for low frequency (long wavelength). It is evident, however, that the energy density will increase without bound as ν^2 with increasing frequency. Since the discrepancy between theory and experiment usually occurs in the ultraviolet region of the frequency spectrum, the unrealistic behavior of equation (2.1) is usually referred to by physicists as the "ultraviolet catastrophe."

Using only thermodynamics and Maxwell's equations it is possible to derive Wein's displacement law using an energy density of the form

$$\rho(\nu) = A\nu^3 e^{-\beta\nu/T} \tag{2.2}$$

where A and β are constants. This expression for the energy density of cavity radiation is known as *Wein's exponential law* and resembles Maxwell's velocity distribution from kinetic theory. Equation (2.2) fits the experimental blackbody curves reasonably well at high frequencies but fails in the low frequency, long wavelength limit where it predicts that the intensity drops to zero rapidly. Figure 2-4 compares the experimental curve with the theoretical results predicted by equations (2.1) and (2.2).

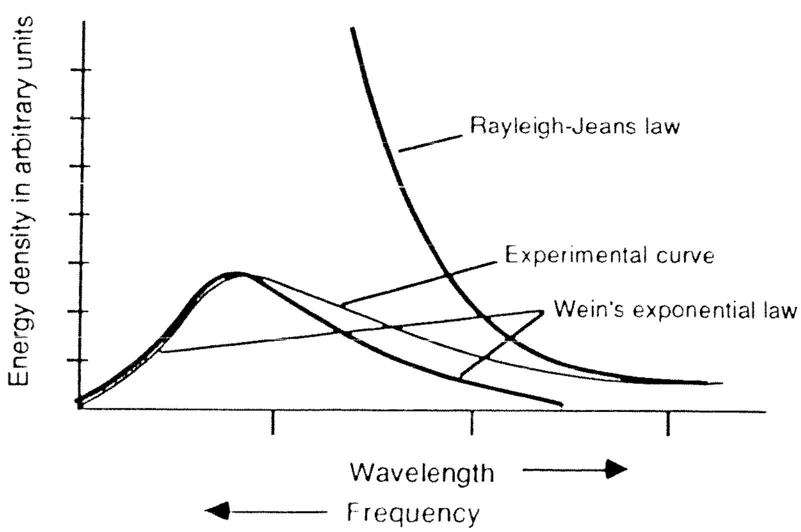

Figure 2-4

Max Planck and the Discovery of Energy Quantization

Max Planck (1858 - 1947) was a purely classical physicist. Throughout most of his life, his professional interests were almost exclusively in the field of thermodynamics. In October of 1900, Max Planck discovered the famous blackbody formula that gave birth to quantum theory. Planck knew that the Rayleigh-Jeans formula agreed with the experimental blackbody curves in the low-frequency, long-wavelength limit. Guided by the agreement of Wein's exponential law with the data in the high-frequency, short-wavelength limit, Planck interpolated between the two to obtain the formula

$$\rho(\lambda, T) = \frac{A}{\lambda^5}\left[\frac{1}{e^{B/\lambda T} - 1}\right]$$

where $A = 8\pi ch$ and $B = hc/k$. The constants c and k are the speed of light in vacuum and Boltzmann's constant respectively; the constant h was determined as a "best fit value" to the experimental curve. The "best fit" value of h was found to be 6.55×10^{-34} J·s. The constant h is known as *Planck's constant* and *sets the scale for the quantum regime*. The smallness of h is if no practical importance in large-scale phenomena in the macroscopic world; it is, however, of the utmost importance on the atomic scale. Planck's formula is usually expressed in "frequency" language:

$$\rho(\nu, T) = \frac{8\pi h\nu^3}{c^3}\left[\frac{1}{e^{h\nu/kT} - 1}\right]. \tag{2.3}$$

Question 2-12: Show that Planck's radiation law, equation (2.3), reduces to the Rayleigh-Jeans law, and to the Wein exponential law in the appropriate limits. Why is this significant?

In his Nobel Prize acceptance speech in 1920 Planck explained:

> But even if the radiation formula proved to be perfectly correct, it would after all have been only an interpolation formula found by lucky guess-work and thus, would have left us rather unsatisfied. I therefore strived from the day of its discovery, to give it a real physical interpretation and this led me to consider the relations between entropy and probability according to Boltzmann's ideas. After some weeks of the most intense work of my life, light began to appear to me and unexpected views revealed themselves in the distance.

Planck found that he was able to derive his blackbody radiation formula from physical principles, but only if he assumed that *the electrons in the cavity wall, oscillating with frequency ν could possess total energies that satisfy the relation*

$$E = nh\nu \qquad \text{where} \qquad n = 1, 2, 3, \ldots \tag{2.4}$$

where h is Planck's constant. Furthermore, such an electron could not gain or lose any fraction of its total energy. An oscillating electron in the cavity wall could change its energy only by an amount ΔE given by

$$\Delta E = h\nu. \tag{2.5}$$

In other words, the energy of an oscillating electron could change only by discrete *quantized* amounts. Not only were these ideas bold and revolutionary, but they contradicted classical mechanics. According to classical physics, the energy of an oscillator is *independent* of its frequency. An oscillator of frequency v can have *any value for its total energy* and can change its amplitude in a *continuous fashion* as it loses or gains any fraction of its total energy. Any oscillator observed in the macroscopic world certainly seems to behave in a classical way. Such observations do not contradict Planck's assumptions. If the physical world obeyed classical physics exactly, then $h = 0$. According to Planck's results, h is very close to zero, so that on a macroscopic scale, energy quantization is not evident (see *Eisberg and Resnick*, Example 1-6). On an atomic scale, however, the fact that h is not zero has ramifying implications - *energy quantization is apparent.*

It is interesting to note that Planck's work was not well received in the scientific community. Even Planck was not pleased with the required assumption of energy quantization. He spent the next fifteen years trying to derive his radiation formula from purely classical arguments. In a journal article in volume 72 of *Nature* (1905), James Jeans wrote, comparing his work to that of Planck:

> The methods of both are in effect the methods of statistical mechanics and the theorem of equipartition of energy, but I carry the method further than Planck, since Planck stops short of the step of putting $h = 0$. I venture to express the opinion that it is not legitimate to stop short at this point, as the hypotheses upon which Planck has worked lead to the relation $h = 0$ as a necessary consequence.
>
> Of course, I am aware that Planck's law is in good agreement with experiment if h is given a value different from zero, while my own law, obtained by putting $h = 0$, cannot possibly agree with experiment. This does not alter my belief that the value $h = 0$ is the only value which it is possible to take, my view being that the supposition that the energy of the ether is in equilibrium with that of matter is utterly erroneous in the case of ether vibrations of short wavelength under experimental conditions.

The quantization of the energy of the oscillating electrons had even further implications that Planck did not consider. If the electrons in a blackbody cavity can exchange energy with the radiation only in discrete amounts, then the radiation itself must also be quantized in discrete amounts. In 1905, Albert Einstein proposed the existence of *light quanta* (now called *photons*) which he described as concentrated "bundles" or packets of electromagnetic energy. Einstein was proposing a "corpuscular" or "particle" description for electromagnetic radiation. We will now see how this led to further developments in the history of quantum physics.

2.3 The Photoelectric Effect

In the years 1886 and 1887 Heinrich Hertz performed a series of experiments that confirmed the existence of Maxwell's electromagnetic waves. It is almost ironic that in the course of those experiments, Hertz discovered a phenomenon that would eventually discredit the classical theory of electromagnetic radiation. During the course of his experiments, Hertz observed that clean metal surfaces emit charged particles when they are exposed to ultraviolet radiation. In 1888, Hallwachs determined that the emitted particles were negatively charged, and one year later, J. J. Thompson showed that the particles were electrons. The process whereby electrons are emitted from a metal by the action of incident radiation is called the *photoelectric effect*.

Figure 2-5 shows a typical experimental set-up for studying the photoelectric effect. It consists of a metallic surface and a collector that are enclosed within a vacuum tube. Placing the system in a vacuum eliminates any collisions that emitted electrons might make with air molecules. A potential difference is maintained between the metallic surface and the collector by placing them in a circuit. Thus, the metallic surface and the collector will carry charges of opposite sign. The circuit contains a variable resistor (so that the potential difference between the plates can be systematically varied), and an ammeter and a voltmeter so that the current and potential difference or "applied voltage" can be measured. The circuit is usually equipped with a polarity reversing switch (not shown) so that the sign of the charges on the metal and the collector can be reversed. When incident radiation falls on the metallic surface, electrons, called *photoelectrons*, are emitted.

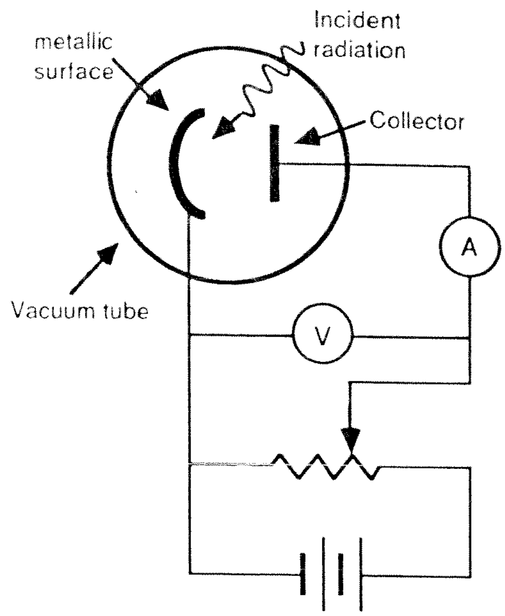

Figure 2-5

The photoelectrons are emitted from the metal with a range of kinetic energies. If the experiment is set up so that the metallic surface is negatively charged and the collector is positively charged, photoelectrons will be accelerated toward the collector thereby establishing a current, called a *photocurrent*, that can be measured with the ammeter. If the applied voltage is steadily increased from zero, the photocurrent will increase from some initial value I_o. When the applied voltage reaches a certain limiting value (the "saturation" voltage), all of the photoelectrons reach the collector; the photocurrent will reach a maximum value and remain constant for any further increase in the applied voltage (Figure 2-6). If, however, the polarity of the circuit is reversed so that the metallic plate is positively charged, and the collector is negatively charged, the photoelectrons will experience a *retarding potential*. In this case, the photoelectrons will *decelerate* as they approach the collector. Only the most energetic photoelectrons will reach the collector.

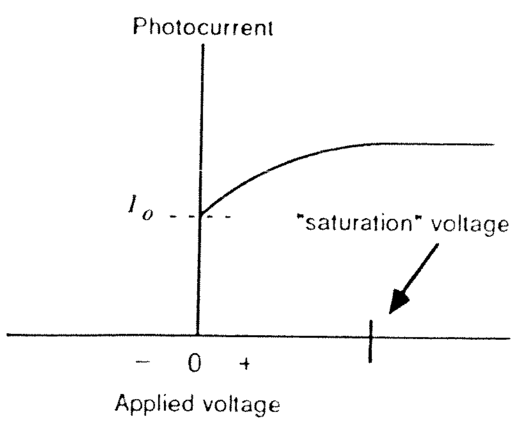

Figure 2-6

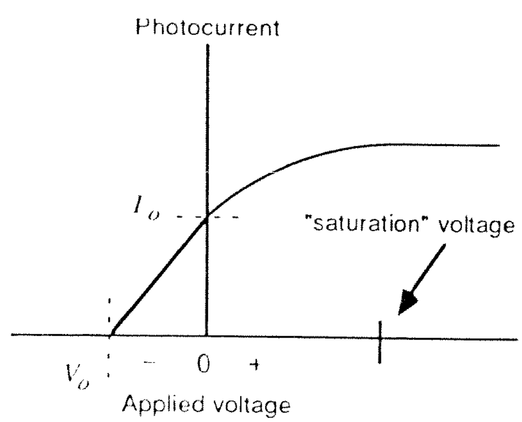

Figure 2-7

As the applied *retarding* potential is increased from zero (that is, as the applied voltage is made more *negative*), fewer and fewer photoelectrons reach the collector and the photocurrent decreases from the initial value I_o (refer to the negative voltage axis in Figure 2-7). When the applied retarding potential reaches a limiting value V_o called the *stopping potential*, the photocurrent drops to zero. None of the photoelectrons reach the collector. Since the stopping potential just prevents the most energetic electrons from reaching the collector, it is related to the kinetic energy of the most energetic electrons, K_{max}, by

$$K_{max} = \frac{1}{2} m_e v_{max}^2 = e V_o \qquad (2.6)$$

where m_e is the mass of the electron, v_{max} is the speed of the most energetic photoelectrons, and e is the electronic charge.

In 1902, Philip Lenard studied the photoelectric effect using carbon arc sources. His experimental results and those of later investigators are summarized below:

1. When electromagnetic radiation falls on a metallic surface, the photoelectrons are emitted almost instantaneously. There was no appreciable time lag even when the intensity was very small (10^{-10} W/m^2).

2. For any fixed values of the frequency and retarding potential, the photocurrent is directly proportional to the intensity of the radiation. In other words, *the number of photoelectrons emitted per unit time is proportional to the radiation intensity*.

3. The stopping potential *does not* depend on the *intensity* of the radiation. Figure 2-8 shows the photocurrent as a function of the applied voltage for two cases in which the intensity is varied. For the higher intensity, the "saturated" photocurrent is higher as predicted from result 2 above, but the stopping potential (and therefore K_{max}) is the same at both intensities.

4. For any given metal, the stopping potential *depends on the frequency of the radiation* and is *independent of the intensity*. Figure 2-9 shows the dependence of V_o on the frequency ν of the radiation for a certain metallic surface. The stopping potential increases with increasing frequency. Furthermore, there is a *threshold frequency*, ν_o, below which no photoelectrons are emitted no matter how high the intensity of the incident radiation.

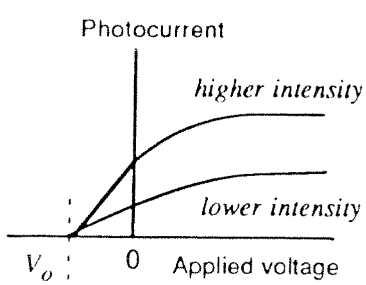

Figure 2-8

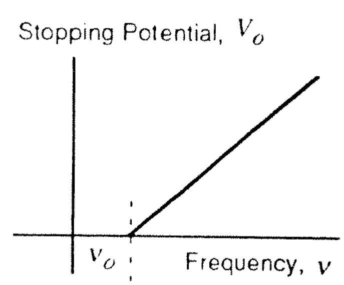

Figure 2-9

The Classical Analysis of the Photoelectric Effect

For the most part, the results of the photoelectric experiments cannot be understood on the basis of Maxwell's electromagnetic theory. With reference to the results previously stated:

1. Electrons are bound to a metallic surface. Any given electron will be emitted if it is given enough energy to overcome the attractive forces that bind it to the metal. According to classical electromagnetic theory, the energy of an electromagnetic wave is spread out over the wavefronts. The energy absorbed on the metallic surface should be proportional to the intensity of the radiation. All electrons bound to the metal with the same energy are equivalent; thus when any one of these electrons has absorbed enough energy to escape, so should all the others with the same binding energy. At low intensities it should take a finite amount of time for the surface to absorb enough energy to free the most weakly bound electrons. With an intensity of 10^{-10} W/m^2, it should take several hundred hours before photoelectrons are observed. No such time delay has ever been observed. Classical theory cannot account for the nearly instantaneous emission of photoelectrons.

2. As the intensity of the radiation is increased, classical theory predicts that more energy should be absorbed by the electrons on the metallic surface. It is reasonable that more photoelectrons should be emitted thereby increasing the photocurrent. The predictions of classical physics do agree with the observed results in this case.

3. Classically, radiation of higher intensity should distribute more energy per unit time over the metallic surface. This would imply that the most energetic electrons would have more energy than those for incident radiation of lower intensity. Photoelectrons produced by more intense radiation should require a larger retarding potential so stop them. This is in complete disagreement with the experimental observations.

4. According to classical physics, the energy of an electromagnetic wave depends only on the square of the amplitude of the wave and has nothing to do with the frequency. The existence of a threshold frequency is completely inexplicable from a classical perspective.

Einstein to the Rescue

As we have seen, the most reasonable explanation of blackbody radiation requires that the energy of the oscillating electrons in the walls of the blackbody cavity is quantized. By 1905, most members of the physics community had not heard of Planck's theory; those who had heard of it did not believe it. Planck himself probably believed that the electromagnetic radiation, once emitted by the oscillators spread out through space in a wave-like fashion. Einstein, however, realized, that if the oscillators in the walls of a blackbody cavity can possess energy only in quantized amounts, they must radiate the energy in quantized amounts. In his famous article on the photoelectric effect [A. Einstein, *Ann. Phys. (Leipzig)* **17**, 132-148 (1905)], he proposes:

> According to the assumption considered here, the spreading of a light beam emanating from a point source does not cause the energy to be distributed continuously over larger and larger volumes, but rather the energy consists of a finite number of energy quanta, localized at space points, which move without breaking up and which can be absorbed or emitted only as wholes.

Einstein assumed that each energy quanta or *photon* remains localized as it moves away from its source with speed c. Since the oscillators in the cavity walls could exchange energy only by discrete amounts of $h\nu$, Einstein reasoned that the emitted energy must equal $h\nu$. Thus, the energy of a photon depends on the frequency, ν, of the emitted radiation, and is given by the equation

$$E = h\nu. \qquad (2.7)$$

When an oscillator emits a photon of energy $h\nu$, its energy *decreases* by an amount equal to E. Likewise, when a photon of energy $h\nu$ is absorbed by such an oscillator, its energy *increases* by an amount E.

Einstein's idea was truly revolutionary - Einstein was proposing a model of light in which light behaves in a manner *normally attributed to particles*. Yet, in 1801, Thomas Young showed that light can be made to produce the interference pattern normally attributed to waves. Einstein contended that this posed no contradiction. Maxwell's classical theory of radiation is successful in describing the propagation of electromagnetic radiation through space *over long time intervals*. However, this model has its limitations. A different model of radiation is needed to describe the *momentary interaction* of radiation and matter such as the light emission in a blackbody cavity and the interaction of light and electrons in the photoelectric effect.

Einstein's photon theory of radiation immediately explains the seemingly anomalous results of the photoelectric effect. Each metal possesses a characteristic amount of work required to remove an electron from its surface. This is the energy one must expend to overcome the attractive forces that normally bind the electron to the metal. The minimum amount of work required to remove an electron from a given metal is called the *work function* of the metal, and is denoted by the symbol w_o. Table 2-2 lists the work functions for some typical metals. According to Einstein, when a photon is incident on a metallic surface, it gives *all* of its energy, $h\nu$, to a given electron. If that amount of energy is sufficient to remove the electron, the electron will be freed. If, it is not, the electron remains bound in the metal.

TABLE 2-2

WORK FUNCTIONS OF SOME COMMON METALS

Metal	Work Function, eV
iron	4.50
lead	4.14
zinc	4.31
copper	4.70
aluminum	4.08
sodium	2.28

Information taken from Table 2.1, *Modern Physics*, Serway, Moses, and Moyer.

When a photon of energy $h\nu$ is absorbed by an electron, some of the energy, w_o, is used to free the electron. The remaining energy appears as kinetic energy, some of which is lost due to collisions with other electrons. The most energetic electrons, undergo no such collisions, and from energy conservation we have

$$h\nu = w_o + K_{max} . \tag{2.8}$$

Question 2-13: (a) Use equations (2.8) and (2.6) to explain Figure 2.9.
(b) What is the physical significance of the slope of the graph in Figure 2.9?
(c) What is the physical significance of the intercept on the frequency axis?

We are now in a position to use Einstein's arguments to explain the experimental results of the photoelectric effect stated on page 2-11.

1. The electron interacts with the photon as if the photon were a colliding particle with energy $h\nu$. The energy is *not* spread out over a wavefront as classical physics would predict. No time delay is to be expected.

2. As the intensity of the radiation is increased, more photons per unit area strike the metallic surface. This results in the emission of more electrons and the photocurrent will increase.

3. The stopping potential depends on the kinetic energy of the most energetic photoelectrons. From equation (2.8), K_{max} depends on the *frequency*, ν, of the radiation, *not its intensity*. Changing the intensity will change the number of photoelectrons emitted, but the energy required to stop any one of them will be the same.

4. Since the stopping potential depends on K_{max}, it follows from equation (2.8) that the stopping potential will depend on the *frequency* of the radiation. The existence of a threshold frequency can be explained as follows. The energy of any photon is given by $h\nu$. Photons of lower frequency have lower energy. If the energy $h\nu$ of a single photon is not at least equal to the work function, the colliding photon cannot supply enough energy to remove the electron. This will be true regardless of the intensity. Increasing the intensity of low frequency radiation simply increases the number of low energy photons that strike the metal. If the energy of the photons is less than the work function of the metal, no photoelectrons will be emitted no matter how many photons strike the surface.

In 1916, Robert A. Millikan performed a series of experiments with alkali metals that confirmed equation (2.8) and demonstrated the linear relationship between the stopping potential and the frequency of the radiation (as shown in Figure 2-9). From his results, Millikan was able to experimentally determine Planck's constant h to with 0.5 per cent.

Question 2-14: How do you think he did this (assuming that he had experimental curves like Figure 2-9)?

In 1921, Einstein received the Nobel Prize for his theoretical explanation for the photoelectric effect. In 1923, Millikan received the Nobel prize for his experimental confirmation of Einstein's predictions (and for his oil-drop experiment to measure the charge to mass ratio for the electron). The quantum hypothesis was now on solid ground - it had stood the test of experiment.

2.4 The Nature of Electromagnetic Radiation - Is it a Particle or a Wave?

A reasonable question at this point might be: "What is light? Is it composed of particles or is it composed of waves?" The best answer that we can give at this point is "neither." Light is neither a particle nor a wave phenomena. It is something more complex than our descriptions of "particles" or "waves" can explain. It possesses properties of both, and in certain situations (diffraction, interference, polarization) it behaves as if it were strictly a wave phenomenon. In other situations (blackbody radiation, photoelectric effect) electromagnetic radiation behaves as if it were composed of discrete particles. No single experiment can simultaneously display both wave and particle behaviors for any form of electromagnetic radiation. Apparently the two behaviors are mutually exclusive. The best that we can say is that electromagnetic radiation is *dualistic* and exhibits a *wave-particle duality*.

2.5 Remarks on the size of h

As we discussed earlier, the quantization of energy is not apparent in the macroscopic world because the magnitude of h is so small. On the atomic scale, however, h is not small and energy quantization is apparent. Actually, the exact value of h is crucial to the operation of the universe as we know it.

If the value of h were reduced by a factor of 2, that is if it somehow took on the value $h/2$, we would be in lots of trouble. It can be shown that if h were somehow reduced by a factor of two, the average radius of atoms would decrease by a factor of 4. This may not seem alarming at first, but the consequences would be devastating. The density of all matter in the universe would increase drastically (remember that density depends is inversely proportional to volume which depends on r^3). All matter in the universe would contract giving off energy in the process. The balance of gravity, and chemical and nuclear processes in stars would change. Nuclear reactions in stars would change and stellar evolution would be suddenly altered. The earth would be bombarded with massive amounts of radiation from the Sun thereby stripping the atmosphere and destroying life on earth.

References and Suggested Reading

Much of the history in the development of the early quantum theory can be found in
Gamow's *Thirty Years That Shook Physics: The Story of Quantum Theory*

Complete discussions of thermal radiation and Planck's developments including the mathematical derivations of the classical and quantum radiation formulas are given in
Eisberg and Resnick, Chapter 1.

The photoelectric effect is discussed in
Eisberg and Resnick, Chapter 2, sections 1 through 3.

Particles and Waves

PREVIEW

This chapter begins the study of a branch of what is often called modern physics. Modern physics was developed mainly during this century and shows a new way of looking at nature which is both intriguing and often strange. One branch of modern physics called quantum mechanics views light as having both wave and particle properties and views matter as also having both particle and wave properties. Relativity is also part and parcel to modern physics and was discussed in an earlier chapter.

The main ideas of quantum mechanics grew out of the failure of classical physics to explain the results of two experiments: blackbody radiation and the photoelectric effect. Albert Einstein's explanation of the photoelectric effect led to the concept of light as being packets of energy called photons. Louis de Broglie, noticing that light appeared to behave both as a wave and a particle, suggested that electrons, protons, etc, which were normally thought of as particles may also behave like waves.

One of the foundation principles of quantum mechanics is the Heisenberg uncertainty principle which asserts that we cannot know some quantities such as the position and the momentum of an object accurately at the same time. In fact, the best we can do is to know the probability of the object having a certain momentum and a certain position.

QUICK REFERENCE

Important Terms

Wave-particle duality
The idea that waves can exhibit particle-like characteristics and particles can exhibit wave-like characteristics.

Blackbody radiation
The light emitted by a perfect emitter and absorber of light (a blackbody) due to its temperature.

Photoelectric effect
The phenomenon in which light incident on a surface such as metal ejects electrons from the surface.

Photon
A discrete packet of light energy postulated to exist by Albert Einstein to explain the photoelectric effect.

Work function
The minimum energy needed to eject an electron from a metal surface.

Compton effect
The phenomenon in which an x-ray photon is scattered from an electron with a change in frequency of the photon.

Compton wavelength of an electron
Half the maximum wavelength change of a photon in a Compton scattering with an electron.

de Broglie wavelength
The wavelength of the wave associated with a material particle. The de Broglie wavelength of a particle is inversely proportional to the momentum of the particle.

Wave function (Ψ)
A function which contains all of the information which can be known about a particle or system of particles. The square of the wavefunction $(\Psi)^2$ is interpreted as being related to the probability of finding the particle at a certain position.

Quantum mechanics
The theoretical framework for determining the wavefunction of a particle or system of particles.

Uncertainty principle
A principle of quantum mechanics which asserts that it is impossible to know with 100% certainty the position and momentum of a particle at the same time.

Equations

Blackbody Radiation

The possible **energies** of the **atomic vibrators** which produce **blackbody radiation**.

$$E = nhf \qquad n = 0, 1, 2, 3, \ldots \tag{29.1}$$

Photoelectric Effect

Energy of a photon

$$E = hf \tag{29.2}$$

The **photoelectric** equation

$$hf = KE_{max} + W_0 \tag{29.3}$$

Compton Effect

Conservation of energy for the **Compton effect**

$$hf = hf' + KE \tag{29.4}$$

The **momentum of a photon**

$$p = \frac{hf}{c} = \frac{h}{\lambda} \tag{29.6}$$

The **wavelength change** of the **scattered photon**

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta) \tag{29.7}$$

The **deBroglie wavelength** of a particle

$$\lambda = \frac{h}{p} \tag{29.8}$$

The **Heisenberg uncertainty principle**

$$(\Delta p_y)(\Delta y) \geq \frac{h}{2\pi} \tag{29.10}$$

$$(\Delta E)(\Delta t) \geq \frac{h}{2\pi} \tag{29.11}$$

Planck's constant

$$h = 6.63 \times 10^{-34}\ \text{J s}$$

$$h = 4.14 \times 10^{-15}\ (\text{eV})\ \text{s}$$

DISCUSSION OF SELECTED SECTIONS

29.3 Photons and the Photoelectric Effect

The explanation of the photoelectric effect by Albert Einstein led to the concept of the photon as a "particle" or quantum of light. The main results of the photoelectric effect experiment are:

1. Light incident on a clean metal surface can cause electrons to be ejected from the surface.
2. The time between the introduction of the light and the ejection of electrons is short.
3. Whether or not electrons were actually ejected from the metal depends on the frequency of the light. Above a certain frequency electrons are ejected, below this frequency no electrons are ejected.
4. If electrons are ejected, a higher frequency of light causes the electrons to have more kinetic energy (higher speeds).

Classical physics was useless in explaining the details of the photoelectric experiment because electromagnetic waves should:

1. give electrons in the metal energy a little at a time since the energy of a wave is spread out. Indeed it should take a long time (even days) for an electron to get enough energy to be ejected from the metal.
2. impart energy to the electron independent of the frequency of the wave. The energy carried by an electromagnetic wave depends on the square of its electric field.

Einstein assumed that light consisted of photons each of energy E = hf. This frequency dependence of the energy carried by light allowed Einstein to completely explain the photoelectric effect by an application of the conservation of energy (equation 29.3).

Example 1
What is the frequency of light which will cause electrons to be emitted from a magnesium surface whose work function is W_0 = 3.68 eV with a kinetic energy of 2.52 eV?
Equation (29.3) gives

$$f = \frac{KE_{max} + W_0}{h} = \frac{(1.60 \text{ X } 10^{-19} \text{ J/ 1 eV})(2.52 \text{ eV} + 3.68 \text{ eV})}{6.63 \text{ X } 10^{-34} \text{ J s}}$$

$$f = 1.50 \text{ X } 10^{15} \text{ Hz}$$

Example 2
A laser beam has an average intensity of 1.5 W/m^2 and a wavelength of 515 nm. What is the density of photons in the beam?

The energy density of the beam is

$$u = \frac{\bar{S}}{c} = \frac{1.5 \text{ W/m}^2}{3.00 \text{ X } 10^8 \text{ m/s}} = 5.0 \text{ X } 10^{-9} \text{ J/m}^3$$

A single photon has energy

$$E = hf = hc/\lambda = (6.63 \text{ X } 10^{-34} \text{ J s})(3.00 \text{ X } 10^8 \text{ m/s})/(515 \text{ X } 10^{-9} \text{ m}) = 3.86 \text{ X } 10^{-19} \text{ J.}$$

The density of photons in the beam is then

$$\text{density} = u/E = (5.0 \text{ X } 10^{-9} \text{ J/m}^3)/(3.86 \text{ X } 10^{-19} \text{ J}) = 1.3 \text{ X } 10^{10} \text{ photons/m}^3.$$

29.4 The Momentum of a Photon and the Compton Effect

Evidence that light has particle properties comes from the collision of a photon with an electron. In this collision it can be demonstrated that BOTH energy and momentum are conserved if the momentum of the photon can be considered to be $p = h/\lambda$. The change in the wavelength of the scattered photons is given by equation (29.7).

Example 3
What is the momentum of a 0.055 nm photon? How fast must an electron travel to have this momentum (Assume the speed is much less than the speed of light)?

The momentum of the photon is

$$p = h/\lambda = (6.63 \times 10^{-34}\ \text{J s})/(0.055 \times 10^{-9}\ \text{m}) = 1.21 \times 10^{-23}\ \text{kg m/s}.$$

An electron would have to travel with a speed of

$$v = p/m = (1.21 \times 10^{-23}\ \text{kg m/s})/(9.11 \times 10^{-31}\ \text{kg}) = 1.33 \times 10^{7}\ \text{m/s}$$

to have the same momentum.

Example 4
A 3.10 MeV photon collides with a stationary electron and is scattered backwards. How much energy does the photon impart to the electron?

The photon gives the electron the same amount of energy that it loses since the collision is elastic. The change in the photon energy is

$$\Delta E = hf' - hf = hc(1/\lambda' - 1/\lambda)$$

The initial wavelength of the electron is

$$\lambda = c/f = hc/E$$
$$= (4.14 \times 10^{-15}\ \text{eV s})(3.00 \times 10^{8}\ \text{m/s})/(3.10 \times 10^{6}\ \text{eV}) = 4.01 \times 10^{-13}\ \text{m}.$$

The final wavelength of the photon is given by (29.7)

$$\lambda' = \lambda + (h/mc)(1 - \cos\theta) = 4.01 \times 10^{-13}\ \text{m} + (2.43 \times 10^{-12}\ \text{m})(1 - \cos 180°)$$

$$= 4.01 \times 10^{-13}\ \text{m} + 4.86 \times 10^{-12}\ \text{m} = 5.26 \times 10^{-12}\ \text{m}.$$

Now

$$\Delta E = hc\left(\frac{1}{\lambda'} - \frac{1}{\lambda}\right) = (6.63 \times 10^{-34}\ \text{J s})(3.00 \times 10^{8}\ \text{m/s})\left(\frac{1}{5.26 \times 10^{-12}\ \text{m}} - \frac{1}{4.01 \times 10^{-13}\ \text{m}}\right)$$

$$\Delta E = -4.58 \times 10^{-13}\ \text{J}$$

The energy imparted to the electron is then 4.58×10^{-13} J.

29.5 The de Broglie Wavelength and the Wave Nature of Matter

Much of what you have already learned about waves applies to the wave nature of particles. The connection between the wave and particle nature of matter is in the de Broglie relationship (equation 29.8) which gives the wavelength of a particle. In principle, all matter has a wave nature, hence a de Broglie wavelength.

Example 5
A 75 kg person is running with a speed of 2.0 m/s. What is the de Broglie wavelength of the person? Comment on the potential importance of the wave nature of common objects. Equation (29.8) gives

$$\lambda = \frac{h}{mv} = \frac{6.63 \text{ X } 10^{-34} \text{ J s}}{(75 \text{ kg})(2.0 \text{ m/s})} = 4.4 \text{ X } 10^{-36} \text{ m}$$

This is an incredibly short wavelength compared to the size of objects in our everyday world. It's little wonder that we do not see the wave nature of people.

Example 6
An electron has been accelerated from rest through a potential difference of $2.5 \text{ X } 10^3$ V. What is the deBroglie wavelength of the electron? Comment on the potential importance of the wave nature of electrons.

The speed of the electron can be found by applying the conservation of mechanical energy

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2(1.6 \text{ X } 10^{-19} \text{ C})((2.5 \text{ X } 10^3 \text{ V})}{9.11 \text{ X } 10^{-31} \text{ kg}}} = 3.0 \text{ X } 10^7 \text{ m/s}$$

The de Broglie wavelength is then

$$\lambda = \frac{h}{mv} = \frac{6.63 \text{ X } 10^{-34} \text{ J s}}{(9.11 \text{ X } 10^{-31} \text{ kg})(3.0 \text{ X } 10^7 \text{ m/s})} = 2.4 \text{ X } 10^{-11} \text{ m}$$

The wavelength of the electron is small but comparable to the size of atoms and molecules. The wave nature of electrons is important, and we should expect them to interfere, diffract and generally behave like the other kinds of waves that you have studied.

29.6 The Heisenberg Uncertainty Principle

Since particles can behave like waves, it becomes difficult to tell exactly where a particle is located. It is very much like generating a wave on a pond and asking "Where is it?" Since the wave is spread out on the surface of the pond, its position is ambiguous. How much a water wave is spread out depends on the wavelength of the wave.

The Heisenberg uncertainty principle (equation 29.10) addresses this ambiguity for matter waves. The uncertainty of the knowledge of a particle's position is related to the uncertainty in the simultaneous knowledge of the particle's momentum. The more accurately you know the position, the less accurately you can know the momentum, and vice versa. Uncertainty also applies to the knowledge of the particle's energy and the time available for measuring the energy (equation 29.11). If you try to measure the energy in a short time the measurement will be very uncertain. However, if your measurement of the energy is made over a long time interval it can be quite accurate.

Example 7

A measurement of the position of an electron shows it to be within a region which is 0.15 nm wide. How much uncertainty is there in the electron's momentum? Compare this uncertainty with the momentum of an electron traveling at 5.5 X 10^6 m/s.

$$\Delta p \geq \frac{\frac{h}{2\pi}}{\Delta x} = \frac{\frac{6.63 \text{ X } 10^{-34} \text{ J s}}{2\pi}}{0.15 \text{ X } 10^{-9} \text{ m}} = 7.0 \text{ X } 10^{-25} \text{ kg m/s}$$

The momentum of an electron traveling at 5.5 X 10^6 m/s is

$$p = mv = (9.11 \text{ X } 10^{-31} \text{ kg})(5.5 \text{ X } 10^{6} \text{ m/s}) = 5.0 \text{ X } 10^{-24} \text{ kg m/s.}$$

$$\Delta p/p = (7.0 \text{ X } 10^{-25} \text{ kg m/s})/(5.0 \text{ X } 10^{-24} \text{ kg m/s}) = 0.14.$$

The uncertainty is 14 % of the momentum.

Example 8

An atom loses energy by radiating away a photon. The process takes 1.5 X 10^{-8} s. What is the uncertainty in the energy of the photon? In its frequency?

Equation (29.11) gives the smallest uncertainty in the energy when the time interval is Δt = 1.5 X 10^{-8} s.

$$\Delta E = \frac{\frac{h}{2\pi}}{\Delta t} = \frac{\frac{6.63 \text{ X } 10^{-34} \text{ J s}}{2\pi}}{1.5 \text{ X } 10^{-8} \text{ s}} = 7.0 \text{ X } 10^{-27} \text{ J}$$

Now the frequency is f = E/h. If the energy of the photon is uncertain by at least ΔE, then the frequency of the photon is uncertain by at least

$$\Delta f = \Delta E/h = (7.0 \text{ X } 10^{-27} \text{ J})/(6.63 \text{ X } 10^{-34} \text{ J s}) = 1.1 \text{ X } 10^{7} \text{ Hz.}$$

PRACTICE PROBLEMS

1. At night approximately 530 photons per second must enter an unaided human eye for an object to be seen, assuming the light is green. The light bulb in Example 1 in the text emits green light uniformly in all directions, and the diameter of the pupil of the eye is 7.0 mm. What is the minimum distance from which the bulb could be seen?

2. A photon scattered from an electron suffers a decrease in its wavelength of 25 % of the Compton wavelength of the electron. Through what angle was the electron scattered?

3. The repeating crystal structure of a solid can act like a diffraction grating for electrons. The atoms in the crystal correspond to the lines on a grating. Electrons traveling at a speed of 4.6×10^7 m/s are diffracted by a crystal whose atomic spacing is 150 nm. About how many orders of electron diffraction are produced?

4. Electrons traveling at a speed of 2.4×10^5 m/s are diffracted through a single slit and fall on a screen located 25 cm away. How wide must the slit be to produce a central fringe 0.50 mm wide?

5. Can any object ever be considered to be truly at rest? Explain why or why not on the basis of the Heisenberg uncertainty principle.

6. A vacuum in classical physics is considered to be empty space devoid of energy and matter. Is there such a thing as empty space devoid of energy or matter according to the uncertainty principle? HINT: Consider what could happen in a short time period, say 10^{-20} s.

7. In "Mr. Thompkins in Wonderland", by George Gamow, Mr. Thompkins, while listening to a stimulating lecture on quantum mechanics, doses off. He dreams that he is in a land where Planck's constant is a large value, say h = 100.0 J s. In this land a herd of giraffes move through a row of evenly spaced trees with a speed of 0.12 m/s. If each giraffe has a mass of 360 kg, what must be the spacing of the trees for the giraffes to produce 3 complete orders of diffraction?

HELPFUL SUGGESTIONS

1. The wave particle duality means that matter and light can have both wave and particle properties. Matter usually shows us the particle side of its nature when the objects in question are large and slow (baseballs, cars, etc.). In contrast light shows its particle side when it interacts with small objects like electrons. Matter manifests its wave characteristics when the objects are small and fast (electrons, protons, etc.), while light behaves as waves when it interacts with objects such as slits.

2. Often you are given the wavelength of light and need to calculate the energy of a photon of the light in electron volts. The procedure is $E = hf = h(c/\lambda) = hc/\lambda$.
   The constant $hc = (6.63 \times 10^{-34}\text{ J s})(3.00 \times 10^{8}\text{ m/s}) = 1.99 \times 10^{-25}\text{ J m}$. Now $1\text{ J} = 1/(1.60 \times 10^{-19})\text{ eV}$, so

   $$E = (1.24 \times 10^{-6}\text{ eV m})/\lambda.$$

   This is a handy expression to keep in mind so that you don't have to do the calculation repeatedly.

3. Remember that the uncertainty principle places limits on what we can know. This is fundamental in nature and does not depend on how well we design our experiments. Even a "perfect" experiment will yield an uncertainty in the position and momentum of an object.

EVERYDAY PHYSICS

1. The concepts presented in this chapter do not lend themselves to direct observation in everyday life. Many devices, principally electronic devices, have been designed using quantum mechanical principles and work as planned. This is indirect evidence of the validity of quantum mechanics. Two of these devices are:

a. *Tunnel diodes*
A tunnel diode is an electronic device which is useful in various electronic circuits. The tunnel diode operates only because of the wave/probability effect called "tunneling" of electrons. Tunneling can be understood by thinking about throwing a ball against a building. It is quite obvious that if you do not give the ball enough energy to rise at least to the top of the building, it will not appear on the other side. Instead, it will hit the wall and bounce off. Imagine that the ball is a wave and has a wavelength about the size of the building. The wave may extend to the top of the building even if the main part of it does not go that high. Since the wave represents a probability of finding the ball at a particular location, there is a small probability that the ball will be at the top of the building. Hence it may make it over even though the ball was not given the required amount of energy.
The tunnel diode conducts a current through an electrical potential barrier even though the electrons do not have enough energy to make it over the barrier. This is due solely to the wave nature of the electron.

b. *Photocells*
Photocells convert light energy into electrical energy. They work on the same principles as the photoelectric emission studied in this chapter, except that they use semiconductor material instead of metals. Photons are absorbed by electrons which gain enough energy to move to another region of the semiconductor causing a charge separation, hence an emf. The emf can be used to produce currents in an external circuit as long as light shines on the photocell to replenish the energy lost by the currents. Photocells would not work at all if light did not have its energy concentrated as photons.

2. The wave function which represents an object, such as yourself, usually has a value everywhere in space although its value may be quite small. Hence, Ψ^2 has a value everywhere in space. Since Ψ^2 represents a probability of locating you, there is a some probability of you being anywhere in space. When I try to find you, there is some probability, albeit small, that you are in another solar system. Can you relate this to the uncertainty principle? Could this be used to explain some missing persons reports?

CHAPTER QUIZ

1. A photon has an energy of 5.42 eV. What is the frequency of the photon?
   a. 1.31×10^{15} Hz b. 6.63×10^{14} Hz c. 5.42×10^{10} Hz d. 8.17 X 1033 Hz

2. Light falling on a metal whose work function is 4.08 eV ejects electrons from the surface with a kinetic energy of 1.29 eV. What is the frequency of the light?
   a. 4.45×10^{19} Hz b. 6.73×10^{14} Hz c. 1.30×10^{15} Hz d. 8.95×10^{-19} H

3. The Compton wavelength of an electron is 2.43×10^{-12} m. A 0.056 nm photon is scattered from an electron at an angle of 60.0°. What is the wavelength of the scattered photon?
   a. 2.43×10^{-12} m b. 1.21×10^{-12} m c. 5.72×10^{-11} m d. 5.47×10^{-11} m

4. What is the de Broglie wavelength of a 0.50 g bullet traveling with a speed of 320 m/s?
   a. 4.14×10^{-36} m b. 2.4×10^{33} m c. 4.1×10^{-33} m d. 0 m

5. An electron traveling with a speed of 6.5×10^{6} m/s passes through a slit of width, W. Which of the following widths would you expect to produce the most diffraction?
   a. 1.0 cm b. 1.0 mm c. 1.0 μm d. 1.0 nm

6. The Heisenberg uncertainty principle implies that the momentum and position cannot be accurately determined at the same time. What is the truth of this statement?
   a. It is true as stated.
   b. It is true only if your experiments are not perfect.
   c. It is true if particles are NOT waves.
   d. It is definitely not true.

7. A particle of mass 0.35 μg is trapped in a box of length 1.0 μm. What is the minimum uncertainty in the velocity of the particle along the length of the box ?
   a. zero b. 3.0×10^{-19} m/s c. 1.5 m/s d. 3.2×10^{-7} m/s

8. How much mass could possibly "appear" and" disappear" in a vacuum during the time period 1.0×10^{-15} s.
   a. 3.5×10^{-28} kg b. 1.1×10^{-19} kg c. 1.2×10^{-36} kg d. none

9. A wavefunction, Ψ, describes a particle. What describes the particle's probability?
   a. Ψ b. Ψ^2 c. Ψ^3 d. Δp

10. The *wave-particle duality* refers to the fact that ________ can exhibit both wave and particle characteristics.
    a. light b. matter c. both light and matter d. nothing

SOLUTIONS AND ANSWERS

Practice Problems

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1. According to Example 1, 3.6×10^8 photons/s are emitted by the bulb. These photons spread out over the surface of a sphere as they leave the bulb. After traveling a distance, R, to reach the eye, the light has an intensity

$$I = (3.6 \times 10^8 \text{ photons/s})/(4\pi R^2).$$

In order to the eye to detect the light the intensity entering the eye must be

$$I = (530 \text{ photons/s})/\pi r^2$$

The maximum distance that the light can travel and still be seen is

$$R = r\sqrt{(3.6 \times 10^8 \text{photons / s})/(2120\text{photons / s})} = \mathbf{1.4 \times 10^5\ m}$$

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2. We must have $\lambda' - \lambda = (h/mc)(0.250)$. Comparing this with equation (29.7) reveals that

$$1 - \cos\theta = 0.250 \Rightarrow \theta = \mathbf{41.4\ ^\circ}.$$

---

3. The order number of the last order produced is

$$m = (d/\lambda)\sin 90° = d/\lambda.$$

The wavelength of the electron is

$$\lambda = h/p = (6.63 \times 10^{-34}\text{ J s})/[(9.11 \times 10^{-31}\text{ kg})(4.6 \times 10^7\text{ m/s})] = 1.6 \times 10^{-11}\text{ m}.$$

The number of orders produced is then

$$m = \mathbf{9400}.$$

---

4. Using the small angle approximation and the same notation as used in the text, we find that the angle of diffraction is $\theta = \lambda/W$ and $\theta = y/L$ so

$$W = \lambda(L/y).$$

The de Broglie wavelength of the electrons is

$$\lambda = h/p = (6.63 \times 10^{-34}\text{ J s})/[(9.11 \times 10^{-31}\text{ kg})(2.4 \times 10^5\text{ m/s})] = 3.0 \times 10^{-9}\text{ m}.$$

The slit width must be

$$W = (3.0 \times 10^{-9}\text{ m})[(2.5 \times 10^{-2}\text{ m})/(0.25 \times 10^{-3}\text{ m})] = \mathbf{3.0 \times 10^{-6}\ m}.$$

---

5. **No**. The Heisenberg uncertainty principle requires that the minimum uncertainty in the position of an object be

$$\Delta x = (h/2\pi)/\Delta p.$$

If an object is truly at rest, then Δp is zero and the object's position is completely uncertain. If an object is truly at rest though, it should be quite easy to find with great certainty. The contradiction leads us to the conclusion that the object cannot be at rest.

---

6. **No**. A measurement of the energy in the "vacuum" in a short time period will yield at least

$$\Delta E = (h/2\pi)/\Delta t.$$

For $\Delta t = 10^{-20}$ s the energy will be more than $\Delta E = 1.1 \times 10^{-14}$ J = 0.069 MeV.

---

7. The highest order of diffraction produced is m = 3. The separation needed between the trees is

$$d = (3\lambda/m) \sin 90° = 3\lambda.$$

The wavelength is

$$\lambda = h/p = (100.0 \text{ J s})/[(360 \text{ kg})(0.12 \text{ m/s})] = 2.3 \text{ m}.$$

Hence,

$$d = 3(2.3 \text{ m}) = \mathbf{6.9 \text{ m}}.$$

Quiz answers

1. a	3. c	5. d	7. b	9. b
2. c	4. c	6. a	8. c	10. c

Section 29.3 Photons and the Photoelectric Effect

1. **ssm** Ultraviolet light is responsible for sun tanning. Find the wavelength (in nm) of an ultraviolet photon whose energy is 6.4×10^{-19} J.

2. The dissociation energy of a molecule is the energy required to break apart the molecule into its separate atoms. The dissociation energy for the cyanogen molecule is 1.22×10^{-18} J. Suppose that this energy is provided by a single photon. Determine the (a) wavelength and (b) frequency of the photon. (c) In what region of the electromagnetic spectrum does this photon lie (see Figure 24.9)?

3. An FM radio station broadcasts at a frequency of 98.1 MHz. The power radiated from the antenna is 5.0×10^4 W. How many photons per second does the antenna emit?

4. The work function for a sodium surface is 2.28 eV. What is the maximum wavelength (in nm) that an electromagnetic wave can have and still eject electrons from this surface?

5. **ssm** Ultraviolet light with a frequency of 3.00×10^{15} Hz strikes a metal surface and ejects electrons that have a maximum kinetic energy of 6.1 eV. What is the work function (in eV) of the metal?

6. White light, whose wavelengths range from 380 to 750 nm, is incident on the surface of sodium (W_0 = 2.28 eV). (a) What is the maximum kinetic energy (in joules) of the photoelectrons emitted from the surface? (b) For what range of wavelengths will *no* photoelectrons be emitted?

7. Radiation of a certain wavelength causes electrons with a maximum kinetic energy of 0.68 eV to be ejected from a metal whose work function is 2.75 eV. What will be the maximum kinetic energy (in eV) with which this same radiation ejects electrons from another metal whose work function is 2.17 eV?

8. An AM radio station broadcasts an electromagnetic wave at a frequency of 665 kHz, while an FM station broadcasts at 91.9 MHz. How many AM photons are needed to have a total energy equal to that of one FM photon?

*9. **ssm** An owl has good night vision because its eyes can detect a light intensity as small as 5.0×10^{-13} W/m^2. What is the minimum number of photons per second that an owl eye can detect if its pupil has a diameter of 8.5 mm and the light has a wavelength of 510 nm?

*10. Radiation with a wavelength of 238 nm shines on a metal surface and ejects electrons that have a maximum speed of 3.75×10^5 m/s. Which one of the following metals is it, the values in parentheses being the work functions: potassium (2.24 eV), calcium (2.71 eV), uranium (3.63 eV), aluminum (4.08 eV), or gold (4.82 eV)?

*11. Example 1 in the text calculates the number of photons per second given off by a sixty-watt incandescent light bulb. The photons are emitted uniformly in all directions. From a distance of 3.1 m you glance at this bulb for 0.10 s. The light from the bulb travels directly to your eye and does not reflect from anything. The pupil of the eye has a diameter of 2.0 mm. How many photons enter your eye?

*12. A proton is located at a distance of 0.420 m from a point charge of +8.30 μC. The repulsive electric force moves the proton until it is at a distance of 1.58 m from the charge. Suppose that the electric potential energy lost by the system is carried off by a photon that is emitted during the process. What is its wavelength?

13. **ssm **www** (a) How many photons (wavelength = 620 nm) must be absorbed to melt a 2.0-kg block of ice at 0 °C into water at 0 °C? (b) On the average, how many H_2O molecules does one photon convert from the ice phase to the water phase?

**14. A laser emits 1.30×10^{18} photons per second in a beam of light that has a diameter of 2.00 mm and a wavelength of 514.5 nm. Determine (a) the average electric field strength and (b) the average magnetic field strength for the electromagnetic wave that constitutes the beam.

1) 310 nm

2) 1.63×10^{-7} m, 1.84×10^{15} Hz, 1.84×10^{15} Hz
ultraviolet Region

3) 7.7×10^{29} photons/second

4) 5.5×10^{14} Hz, 545 nm

5) 6.3 eV

6) 1.6×10^{-19} J, 545 → 750 nm

7) 1.26 eV

8) 138

9) 73 Photons/second

10) gold

11) 9.4×10^{9} photons

12) 9.56×10^{-12} m

13) 2.1×10^{24} photons

14) 1.6×10^{5} W/m^2, 7760 N/C, 2.59×10^{-5} T

Section 29.4 The Momentum of a Photon and the Compton Effect

15. The microwaves used in a microwave oven have a wavelength of about 0.13 m. What is the momentum of a microwave photon?

16. A photon of red light has a wavelength of 730 nm, while a photon of violet light has a wavelength of 380 nm. Find the ratio p_{violet}/p_{red} of the photon momenta.

17. **ssm** Incident X-rays have a wavelength of 0.3120 nm and are scattered by the "free" electrons in graphite. The scattering angle in Figure 29.9 is $\theta = 135.0°$. What is the magnitude of the momentum of (a) the incident photon and (b) the scattered photon? (For accuracy, use $h = 6.626 \times 10^{-34}$ J · s and $c = 2.998 \times 10^8$ m/s.)

18. Determine the *change* in the photon's wavelength that occurs when an electron scatters an X-ray photon (a) straight back at an angle of $\theta = 180.0°$ and (b) at an angle of $\theta = 30.0°$. All angles are measured as in Figure 29.9.

19. An incident X-ray photon of wavelength 0.2800 nm is scattered from an electron that is initially at rest. The photon is scattered at an angle of $\theta = 180.0°$ in Figure 29.9 and has a wavelength of 0.2849 nm. Use the conservation of linear momentum to find the momentum gained by the electron.

***20.** The X-rays detected at a scattering angle of $\theta = 163°$ in Figure 29.9 have a wavelength of 0.1867 nm. Find (a) the wavelength of an incident photon, (b) the energy of an incident photon, (c) the energy of a scattered photon, and (d) the kinetic energy of the recoil electron. (For accuracy, use $h = 6.626 \times 10^{-34}$ J · s and $c = 2.998 \times 10^8$ m/s.)

***21.** **ssm www** What is the maximum amount by which the wavelength of an incident photon could change when it undergoes Compton scattering from a nitrogen molecule (N_2)?

****22.** Review Conceptual Example 3 before attempting this problem. A beam of visible light has a wavelength of 395 nm and shines perpendicularly on a surface. As a result, there are 3.0×10^{18} photons per second striking the surface. By using the impulse–momentum theorem (Section 7.1), obtain the average force that this beam applies to the surface when (a) the surface is a mirror, so the momentum of each photon is reversed after reflection, and (b) the surface is black, so each photon is absorbed and the momentum of the photon is reduced to zero in the process.

15) 5.1×10^{-33} kg m/s
16) 1.9
17) 2.124×10^{-24} kg m/s
2.096×10^{-24} kg m/s
18) 4.86×10^{-12} m, 3.26×10^{-13} m
19) 4.692×10^{-24} kg m/s
20) 0.1819 nm, 1.092×10^{-15} J
1.064×10^{-15} J, 2.8×10^{-17} J
21) 9.5×10^{-17} m
22) 1.0×10^{-8} N, 5.0×10^{-9} N

Section 29.5 The de Broglie Wavelength and the Wave Nature of Matter

23. A particle has a speed of 1.2×10^6 m/s. Its de Broglie wavelength is 8.4×10^{-14} m. What is the mass of the particle?

24. The interatomic spacing in a crystal of table salt is 0.282 nm. This crystal is being studied in a neutron diffraction experiment, similar to the one that produced the photograph in Figure 29.12*a*. How fast must a neutron (mass = 1.67×10^{-27} kg) be moving to have a de Broglie wavelength of 0.282 nm?

25. **ssm** The de Broglie wavelength of a proton in a particle accelerator is 1.30×10^{-14} m. Determine the kinetic energy (in joules) of the proton.

26. What is (a) the wavelength of a 5.0-eV photon and (b) the de Broglie wavelength of a 5.0-eV electron?

27. Recall from Section 14.3 that the average kinetic energy of an atom in a monatomic ideal gas is given by $\overline{KE} = \frac{3}{2}kT$, where $k = 1.38 \times 10^{-23}$ J/K and T is the Kelvin temperature of the gas. Determine the de Broglie wavelength of a helium atom (mass = 6.65×10^{-27} kg) that has the average kinetic energy at room temperature (293 K).

28. Neutrons ($m = 1.67 \times 10^{-27}$ kg) are being emitted by two objects. The neutrons are moving with a speed of 2.80×10^3 m/s and pass through a circular aperture whose diameter is 0.100 mm. This situation is analogous to that shown in Figure 27.31*a* for light. According to the Rayleigh criterion, what is the minimum angle θ_{min} (in radians) between the two objects, such that they are just resolved using neutrons?

* 29. **ssm** The width of the central bright fringe in a diffraction pattern on a screen is identical when either electrons or red light (vacuum wavelength = 661 nm) pass through a single slit. The distance between the screen and the slit is the same in each case and is large compared to the slit width. How fast are the electrons moving?

*30. In a Young's double-slit experiment performed with electrons, the two slits are separated by a distance of 2.0×10^{-6} m. The first-order bright fringes are located on the observation screen at an angle given by $\theta = 1.6 \times 10^{-4}$ degrees in Equation 27.1. Find (a) the wavelength, (b) the momentum, and (c) the kinetic energy of the electrons.

*31. In a television picture tube the electrons are accelerated from rest through a potential difference V. Just before an electron strikes the screen, its de Broglie wavelength is 1.0×10^{-11} m. What is the potential difference?

**32. The kinetic energy of a particle is equal to the energy of a photon. The particle moves at 5.0% of the speed of light. Find the ratio of the photon wavelength to the de Broglie wavelength of the particle.

23) 6.6×10^{-27} kg
24) 1.41×10^3 m/s
25) 7.77×10^{-13} J
26) 2.5×10^{-7} m
5.6×10^{-10} m
27) 7.38×10^{-11} m
28) 1.73×10^{-6} rad
29) 1.10×10^3 m/s
30) 5.6×10^{-12} m,
1.2×10^{-22} kg m/s
7.7×10^{-15} J

Section 29.6 The Heisenberg Uncertainty Principle

33. **ssm www** In the lungs there are tiny sacs of air, which are called alveoli. The average diameter of one of these sacs is 0.25 mm. Consider an oxygen molecule (mass = 5.3×10^{-26} kg) trapped within a sac. What is the minimum uncertainty in the velocity of this oxygen molecule?

34. The speed of a golf ball (mass = 0.045 kg) and of an electron is 95 m/s. If the uncertainty in the speed is 2.0%, estimate the minimum uncertainty in the position of each object.

35. A proton is confined to a nucleus whose diameter is 5.5×10^{-15} m. If this distance is considered to be the uncertainty in the position of the proton, what is the minimum uncertainty in its momentum?

36. Review Conceptual Example 6 as background for this problem. When electrons pass through a single slit, as in Figure 29.14, they form a diffraction pattern. As Section 29.6 discusses, the central bright fringe extends to either side of the midpoint, according to an angle θ given by $\sin\theta = \lambda/W$, where λ is the de Broglie wavelength of the electron and W is the width of the slit. When λ is the same size as W, $\theta = 90°$, and the central fringe fills the entire observation screen. In this case, an electron passing through the slit has roughly the same probability of hitting the screen either straight ahead or anywhere off to one side or the other. Now, imagine yourself in a world where Planck's constant is large enough so you exhibit similar effects when you walk through a 0.90-m-wide doorway. If your mass is 82 kg and you walk at a speed of 0.50 m/s, how large would Planck's constant have to be in this hypothetical world?

37. **ssm www** Particles pass through a single slit of width 0.200 mm (see Figure 29.14). The de Broglie wavelength of each particle is 633 nm. After the particles pass through the slit, they spread out over a range of angles. Use the Heisenberg uncertainty principle to determine the minimum range of angles.

*38. Suppose the minimum uncertainty in the position of a particle is equal to its de Broglie wavelength. If the particle has an average speed of 4.5×10^5 m/s, what is the minimum uncertainty in its speed?

31) 1.5×10^{-4} V
32) 4.0×10^1
33) 8.0×10^{-6} m/s
34) 1.2×10^{-5} m, 1.2×10^{-33} m
35) 1.9×10^{-20} kg m/s
36) 37 J·s
37) $-0.0289° \le \theta \le +0.0289°$
38) 7.2×10^4 m/s

Atomic Physics

AP Physics B

Mr. DiBucci

The Nature of the Atom

PREVIEW

In this chapter you will study the nature of the atom. You will learn about line spectra, the Bohr model of the hydrogen atom, energy level diagrams, the quantum mechanical picture of the hydrogen atom, the Pauli exclusion principle, X-rays, and lasers.

QUICK REFERENCE

Important Terms

Line spectrum
A series of discrete electromagnetic wavelengths emitted by the atoms of a low-pressure gas that is subjected to a sufficiently high potential difference. Certain groups of discrete wavelengths are referred to as a series.

Bohr model
The electron in a single electron atom exists in circular orbits called stationary orbits. A photon is emitted when an electron changes from a higher energy orbit to a lower energy orbit.

Ionization energy
The energy needed to remove an electron completely from an atom.

Principal quantum number
An integer number, n, which determines the total energy of the hydrogen atom.

Orbital quantum number
An integer number, l, which determines the angular momentum of the electron due to its orbital motion.

Magnetic quantum number
An integer number, m_l, which determines the component of the angular momentum along a specific direction.

Spin quantum number
A number, m_s, which is used to describe the intrinsic spin angular momentum of an electron.

Pauli exclusion principle
No two electrons in an atom can have the same set values for the four quantum numbers n, l, m_l, m_s. This determines the way in which the electrons in multiple-electron atoms are distributed into shells and subshells, thus explaining the periodic table.

X-rays
Electromagnetic waves emitted when high-energy electrons strike a metal target contained within an evacuated glass tube.

Laser
A device that generates electromagnetic waves via a process known as "stimulated emission". The process produces waves that are coherent and may be confined to a very narrow beam.

Equations

The wavelengths of the **spectral line series of hydrogen** are given by:

Lyman series: $$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \qquad n = 2, 3, 4, \ldots \qquad (30.1)$$

Balmer series: $$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \qquad n = 3, 4, 5, \ldots \qquad (30.2)$$

Paschen series: $$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right) \qquad n = 4, 5, 6, \ldots \qquad (30.3)$$

The **energy of the photon** emitted when an electron changes from a higher to a lower energy level is:

$$E_i - E_f = hf \qquad (30.4)$$

The **total energy of the atom** (Bohr theory) is given by:

$$E = KE + EPE = \frac{1}{2}mv^2 - \frac{kZe^2}{r} \qquad (30.5)$$

Equating the centripetal and Coulomb forces yields:

$$mv^2 = \frac{kZe^2}{r} \qquad (30.6)$$

The total energy can therefore be written:

$$E = \frac{1}{2}\left(\frac{kZe^2}{r}\right) - \frac{kZe^2}{r} = -\frac{kZe^2}{2r} \qquad (30.7)$$

According to Bohr, the **angular momentum L_n** is given by:

$$L_n = mv_n r_n = n\frac{h}{2\pi} \qquad n = 1, 2, 3, \ldots \qquad (30.8)$$

The **radius of the nth Bohr orbit** is:

$$r_n = \left(\frac{h^2}{4\pi^2 mke^2}\right)\frac{n^2}{Z} \qquad n = 1, 2, 3, \ldots \qquad (30.9)$$

Substituting the constants into equation (37.9) yields:

$$r_n = (5.29 \times 10^{-11}\ \text{m})\frac{n^2}{Z} \qquad n = 1, 2, 3, \ldots \qquad (30.10)$$

The **energy of the nth Bohr orbit** is:

$$E_n = -\left(\frac{2\pi^2 m k^2 e^4}{h^2}\right)\frac{Z^2}{n^2} \qquad n = 1, 2, 3, \ldots \qquad (30.11)$$

The **Bohr energy levels** can then be written as:

$$E_n = -(2.18 \times 10^{-18}\,\mathrm{J})\frac{Z^2}{n^2} \qquad n = 1, 2, 3, \ldots \qquad (30.12)$$

The **Bohr energy levels** expressed **in electron volts**:

$$E_n = -(13.6\,\mathrm{eV})\frac{Z^2}{n^2} \qquad n = 1, 2, 3, \ldots \qquad (30.13)$$

The **wavelengths radiated by the hydrogen** are given by:

$$\frac{1}{\lambda} = \frac{2\pi^2 m k^2 e^4}{h^3 c}(Z^2)\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \qquad (30.14)$$

$$n_i, n_f = 1, 2, 3, \ldots \text{ and } n_i > n_f$$

The **magnitude of the angular momentum of an electron** is:

$$L = \sqrt{l\,(l+1)}\,\frac{h}{2\pi} \qquad (30.15)$$

The **angular momentum in the z direction** is:

$$L_z = m_l \frac{h}{2\pi} \qquad (30.16)$$

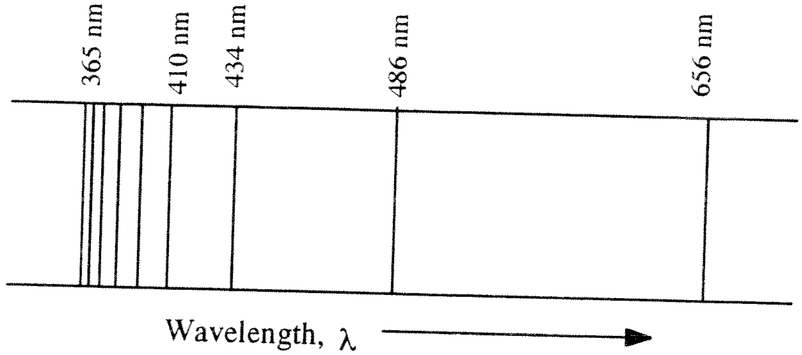
The **minimum wavelength of the photon emitted by an X-ray tube** is:

$$\lambda_0 = \frac{hc}{eV} \qquad (30.17)$$

DISCUSSION OF SELECTED SECTIONS

30.2 Line Spectra

When a low-pressure gas contained within an evacuated tube is subjected to a sufficiently large potential difference, the gas will emit electromagnetic waves. With a grating spectrometer the individual wavelengths emitted by the gas can be separated and identified as a series of bright fringes or lines. The series of lines is called a **line spectrum**. The simplest line spectrum is that of the hydrogen atom. In schematic form, the following figure illustrates one series of lines for atomic hydrogen, the **Balmer series**.



Balmer found an empirical equation that gave the values for the observed wavelengths. This equation is given below, along with similar equations that apply to the **Lyman** and **Paschen** series of hydrogen lines.

Lyman series: $$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \qquad n = 2, 3, 4, \ldots$$

Balmer series: $$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \qquad n = 3, 4, 5, \ldots$$

Paschen series: $$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right) \qquad n = 4, 5, 6, \ldots$$

In these equations, the constant term R, known as the **Rydberg constant**, has the value $R = 1.097 \times 10^7\ m^{-1}$.

Example 1
Find the wavelength of the n = 4 Balmer line. Using equation (30.2) we obtain,

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) = (1.097 \times 10^7\ m^{-1})\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = 2.057 \times 10^6\ m^{-1} \Rightarrow \lambda = 486\ nm$$

30.3 The Bohr Model of the Hydrogen Atom

Bohr's theory of the hydrogen atom has a positively charged nucleus and a negatively charged electron moving around it in a circular orbit. Bohr hypothesized that in a hydrogen atom there can only be certain values of the total energy (electron kinetic energy plus potential energy). These allowed energy levels correspond to different orbits for the electron as it moves around the nucleus, the larger orbits being associated with larger total energies. An electron in one of these orbits does not radiate electromagnetic waves. These orbits are sometimes referred to as **stationary orbits** or **stationary states**.

Bohr theorized that a photon is emitted only when the electron changes orbits from a larger one with higher energy to a smaller one with a lower energy. If E_i is the energy of the initial orbit (higher energy orbit), E_f the energy of the final orbit (lower energy), and the energy of the photon is hf, where f is the frequency and h is Planck's constant, then

$$E_i - E_f = hf$$

For an electron of mass m and speed v in an orbit of radius r, the total energy is the kinetic energy of the electron plus the electric potential energy. Therefore, we can write

$$E = KE + PE = \frac{1}{2}mv^2 - \frac{kZe^2}{r}$$

Equation (30.5) can be modified to yield the radii of the various orbits r_n and the energies of these orbits E_n. As detailed in the textbook, by using the Coulomb electrostatic force as the centripetal force (equation 30.6), and by assuming that the allowed energy levels have angular momenta that are integral multiples of Planck's constant (equation 30.8), the radii for the **Bohr orbits** are shown to be

$$r_n = (5.29 \times 10^{-11}\ m)\frac{n^2}{Z} \qquad n = 1, 2, 3, \ldots$$

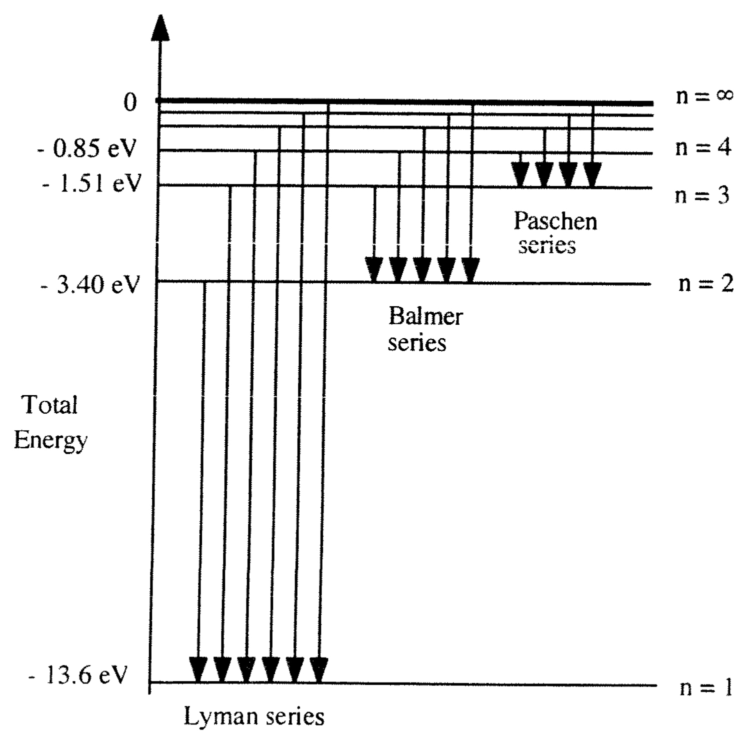
For the hydrogen atom (Z = 1) and the smallest Bohr orbit (n = 1), $r_1 = 5.29 \times 10^{-11}$ m. This particular value is often referred to as the **Bohr radius**. Equation (30.10) can now be used in equation (30.7) to show that the energy of the nth Bohr orbit is given by

$$E_n = -(2.18 \times 10^{-18}\ J)\frac{Z^2}{n^2} \qquad n = 1, 2, 3, \ldots$$

This energy is often expressed in electron volts, where 1 eV = 1.60×10^{-19} J. Therefore, we can write

$$E_n = -(13.6\ eV)\frac{Z^2}{n^2} \qquad n = 1, 2, 3, \ldots$$

It is useful to represent the energy values given by equation (30.13) on an **energy level diagram**. For the hydrogen atom (Z = 1), the highest energy level corresponds to n = ∞ in equation (30.13) and has an energy of 0 eV. This is the energy of the atom when the electron is completely removed (r = ∞) from the nucleus and is at rest. The lowest energy level corresponds to n = 1 and has a value of - 13.6 eV. The lowest energy level is called the **ground state** while higher energy levels are called **excited levels**. To determine the wavelengths radiated by the atom, equation (30.13) for the energies can be used in equation (30.4) with f = c/λ to yield equation (30.14). The Lyman, Balmer, and Paschen series of lines in the hydrogen atom spectrum correspond to transitions that the electron makes between higher and lower energy levels, as shown in the diagram below.



The wavelengths of the line spectra are given by:

$$\frac{1}{\lambda} = \frac{2\pi^2 m k^2 e^4}{h^3 c}(Z^2)\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$n_i, n_f = 1, 2, 3, \ldots \text{ and } n_i > n_f$$

Example 2
Lines in the Paschen series are produced when electrons, excited to higher levels, make transitions to the n = 3 level. Determine (a) the longest wavelength in the series, (b) the wavelength that corresponds to the transition from $n_i = 7$ to $n_f = 3$. (c) What spectral region are these lines found in?

(a) The longest wavelength corresponds to the transition which has the smallest energy change. This occurs between levels $n_i = 4$ to $n_f = 3$. Using equation (30.14) with Z = 1, we find that

$$\frac{1}{\lambda} = (1.097 \times 10^7\,\text{m}^{-1})\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = 5.33 \times 10^5\,\text{m}^{-1} \Rightarrow \lambda = 1880\text{ nm}$$

(b) For the transition from $n_i = 7$ to $n_f = 3$:

$$\frac{1}{\lambda} = (1.097 \times 10^7\,\text{m}^{-1})\left(\frac{1}{3^2} - \frac{1}{7^2}\right) = 9.95 \times 10^5\,\text{m}^{-1} \Rightarrow \lambda = 1010\text{ nm}$$

(c) According to Figure 24.9, these lines lie in the **infrared** region of the electromagnetic spectrum.

30.5 The Quantum Mechanical Picture of the Hydrogen Atom

The picture of the hydrogen atom that quantum mechanics provides differs in a number of ways from the Bohr model. The Bohr model uses a single integer number n to identify the various electron orbits and the associated energies. Because this number can have only discrete values, rather than a continuous range of values, n is called a **quantum number**. Quantum mechanics, on the other hand, reveals that four different quantum numbers are needed to describe each state of the hydrogen atom. They are described below.

1. **The principal quantum number n**. This is the same number used by Bohr to describe the total energy of the atom and can have only integer values: n = 1, 2, 3, . . . $E_n = -(13.6\text{ eV})\,Z^2/n^2$.

2. **The orbital quantum number l**. This number determines the angular momentum of the electron due to its orbital motion. Only the following integer values of l are allowed:

$$l = 0, 1, 2, \ldots, (n - 1)$$

For instance, if n = 1, the orbital quantum number can only have the value $l = 0$. For n = 5, possible values for l are $l = 0, 1, 2, 3$, and 4. According to the Schrodinger equation, the magnitude L of the angular m omentum of the electron is

$$L = \sqrt{l\,(l + 1)}\,\frac{h}{2\pi}$$

3. **The magnetic quantum number m_l.** This number determines the component of the angular momentum along a specific direction, which is called the z direction by convention. The values that m_l can have depend on the value of l, with only the following positive and negative integers being permitted:

$$m_l = 0, \pm 1, \pm 2, \ldots, \pm l$$

For example, if the angular momentum quantum number is $l = 2$, then the magnetic quantum number can have values $m_l = -2, -1, 0, +1, +2$. The Schrodinger equations shows that the component L_z of the angular momentum in the z direction is

$$L_z = m_l \frac{h}{2\pi}$$

4. **The spin quantum number m_s.** This number is needed because the electron has an intrinsic property called spin angular momentum. There are only two possible values for the spin quantum number. They are:

$$m_s = +\frac{1}{2} \quad \text{(spin up)} \qquad \text{or} \qquad m_s = -\frac{1}{2} \quad \text{(spin down)}$$

Example 3
Write down the eighteen possible sets of four quantum numbers that exist when the principal quantum number is 3.

Using the rules listed above, for n = 3 we have:

n	l	m_l	m_s
3	0	0	1/2
3	0	0	-1/2
3	1	1	1/2
3	1	1	-1/2
3	1	0	1/2
3	1	0	-1/2
3	1	-1	1/2
3	1	-1	-1/2
3	2	2	1/2
3	2	2	-1/2
3	2	1	1/2
3	2	1	-1/2
3	2	0	1/2
3	2	0	-1/2
3	2	-1	1/2
3	2	-1	-1/2
3	2	-2	1/2
3	2	-2	-1/2

30.6 The Pauli Exclusion Principle and the Periodic Table of the Elements

In multiple-electron atoms, all electrons with the same value of n are said to be in the same **shell**. For example, electrons with n = 1 are in a single shell (sometimes called the K shell), those with n = 2 are in another shell (the L shell), those with n = 3 are in another (the M shell), and so on. Those electrons with the same values for both n and l are said to be in the same **subshell**. So the n = 1 shell has one subshell, $l = 0$. The n = 2 shell has two subshells, $l = 0$ and $l = 1$, and so forth.

In most atoms, the electron spends most of its time in the lowest energy level n = 1 called the **ground state**. However, when a multiple-electron atom is in the ground state, not every electron is crowded into the n = 1 shell. The reason for this is that electrons obey the **Pauli exclusion principle**. This principle states that no two electrons in an atom can have the same set of values for the four quantum numbers n, l, m_l, and m_s.

Because of the Pauli exclusion principle, there is a maximum number of electrons that can fit into an energy level or subshell. For example, the n = 1, l = 0 subshell can hold at most two electrons. The n = 2, l = 1 subshell, however, can hold six electrons, because with l = 1 there are three possibilities for m_l (-1, 0, 1), each of which have two values of m_s (+1/2, -1/2). The maximum number of electrons the l the subshell can hold is $2(2l + 1)$. So, for instance, the n = 5, l = 4 subshell can hold 2[2(4) + 1] = 18 electrons.

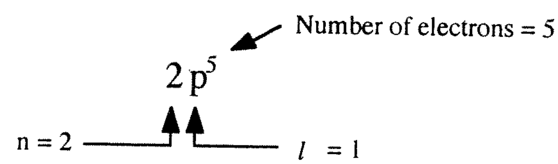
Example 4

How many electrons can be put into (a) the n = 4 shell and (b) the n = 5 shell?

(a) Since there are $2(2l + 1)$ possible electrons in each subshell, and since l can have values up to (n - 1), for n = 4, we have l = 0, 1, 2, and 3. So for l = 0 there are two electrons, for l = 1 there are six electrons, for l = 2 there are ten electrons and for l = 3 there are fourteen electrons. For the n = 4 shell, therefore, there are a maximum of N = 2 + 6 + 10 + 14 = **32 electrons.**

(b) For the n = 5 shell, the l = 4 subshell can have 18 possible electrons. Therefore, in the entire n = 5 shell there can be as many as N = 2 + 6 + 10 + 14 + 18 = **50 electrons.**

A shorthand notation is often used for the electronic configuration of the atom. For instance, the l = 0 subshell is called the s subshell. The l = 1 and 2 subshells are known as the p and d subshells, respectively. Higher values of l are referred to as f, g, h, etc. This convention of letters is used in a shorthand notation that indicates the principal quantum number n, the orbital quantum number l, and the number of electrons in the n, l, subshell. We have



The ground state configurations for a few elements in the periodic table are listed below.

Element	*Number of Electrons*	*Electronic Configuration*
Hydrogen	1	$1s^1$
Helium	2	$1s^2$
Lithium	3	$1s^22s^1$
Boron	5	$1s^22s^22p^1$
Neon	10	$1s^22s^22p^6$
Sodium	11	$1s^22s^22p^63s^1$
Aluminum	13	$1s^22s^22p^63s^23p^1$
Argon	18	$1s^22s^22p^63s^23p^6$
Potassium	19	$1s^22s^22p^63s^23p^64s^1$

PRACTICE PROBLEMS

1. Determine the radius and energy of the hydrogen atom in the n = 3 state of the Bohr model.

2. What are the wavelengths of the longest and shortest wavelength lines in the Brackett series which has $n_f = 4$.

3. Compute the ionization energy of doubly ionized lithium Li^{2+} (Z = 3).

4. Light of 1090-nm wavelength is emitted from a hydrogen discharge tube. If the transition that produces this emission occurs to the $n_f = 3$ level, from which level n_i did it originate?

5. Assume that 1200 hydrogen atoms are initially in the n = 4 state. The electrons then make transitions to lower energy levels. (a) How many distinct spectral lines will be emitted? (b) Assume, for simplicity, that all possible transitions to lower levels are equally probable from a given excited state. Determine the total number of photons emitted.

6. Write down the fourteen sets of the four quantum numbers that correspond to the electrons in a completely filled 4f shell.

7. Write down the ground state electronic configuration for iron (Z = 26).

8. Suppose tungsten (Z = 74) has all but one of its electrons removed. (a) How much energy (in eV) is required to remove the last electron completely if the ionized atom is initially in the ground state? (b) What would be the wavelength of the photon emitted if the remaining electron were to make the transition n = 2 to n = 1? (c) In what region of the spectrum is such a photon found?

9. Suppose the X-ray machine in a dentist's office uses a potential difference of 25 kV to operate the X-ray tube. What is the shortest X-ray wavelength emitted by the machine?

10. Calculate the approximate minimum energy necessary to observe the n = 2 to n = 1 characteristic-X ray of gold.

CHAPTER QUIZ

1. Which of the following is **not** a feature of the Bohr model of the atom?
   a. a small, positively charged nucleus.
   b. accelerating electrons that do not radiate energy.
   c. electrons in planetary-like orbits.
   d. quantized energy levels.
   e. an electron probability cloud.

2. A hydrogen atom is in the ground state when its electron
   a. is in the middle of the atom
   b. is in the innermost orbit
   c. has been completely ionized
   d. has absorbed a photon

3. Light emitted from excited atoms has frequencies that are related to
   a. the mass of the atom
   b. the color of the atom
   c. the difference in energy levels of the atom
   d. the size of the orbits in the atom

4. According to the modern view (quantum mechanical picture) of the atom,
   a. electrons revolve around the nucleus in well-defined circular orbits.
   b. electrons must be treated as particles, not waves.
   c. emitted photons can have any range of continuous energies.
   d. electrons do not have a localized orbit, but are spatially distributed, like a cloud.

5. According to the Pauli exclusion principle,
   a. no two electrons can the same set of values for the four quantum numbers.
   b. electrons are excluded from entering the nucleus.
   c. electrons must exist in discrete energy states.
   d. electrons cannot be removed from an atom.

6. In the n = 1 state, the energy of the hydrogen atom is - 13.6 eV. What is its energy in the n = 2 state?
   a. - 6.80 eV
   b. - 10.2 eV
   c. - 3.40 eV
   d. - 1.51 eV

7. If r_1 is the radius of the first Bohr orbit, the radius of the fourth Bohr orbit is
   a. $4r_1$
   b. $r_1/4$
   c. $16r_1$
   d. $r_1/16$

8. A neutral atom has a ground state electronic configuration $1s^2 2s^2 2p^6 3s^2 3p^5$. What is its atomic number?
   a. 5
   b. 11
   c. 17
   d. 39

9. A neutral atom has an electronic configuration of $1s^2 2s^2 2p^6 3s^2 3p^6$. What is the electronic configuration for a neutral atom holding one additional electron in its orbit?
   a. $1s^2 2s^2 2p^6 3s^2 3p^6 3d^1$
   c. $1s^2 2s^2 2p^6 3s^2 3p^6 4d^1$
   c. $1s^2 2s^2 2p^6 3s^2 3p^5$
   d. $1s^2 2s^2 2p^6 3s^2 3p^6 4s^1$

10. In the ground state, the possible quantum numbers (n, l, m_l, m_s) for hydrogen are.
    a. 1, 1, 1, 1 respectively
    b. 1, 0, 0, ±1/2 respectively
    c. 1, 0, 0, 0 respectively
    d. 1, 1, 0, ±1/2 respectively

11. The number of electrons that can be accommodated in an l = 6 subshell is
    a. 6
    b. 12
    c. 13
    d. 26

SOLUTIONS AND ANSWERS

Practice Problems

---

1. Using equation (30.10) we have,

$$r_3 = (5.29 \times 10^{-11}\ m)(3)^2/1 = \mathbf{4.76 \times 10^{-10}\ m}.$$

Equation (30.13) gives,

$$E_3 = -(13.6\ eV)(1)^2/(3)^2 = \mathbf{-1.51\ eV}.$$

---

2. The longest wavelength occurs from $n_i = 5$ to $n_f = 4$. Equation (30.14) gives,

$$(1/\lambda) = (1.097 \times 10^7\ m^{-1})(1/4^2 - 1/5^2) = 2.47 \times 10^5\ m^{-1},$$

so that λ = **4050 nm**.
The shortest wavelength occurs when $n_i = \infty$. Therefore,

$$(1/\lambda) = (1.097 \times 10^7\ m^{-1})(1/4^2 - 1/\infty^2) = 6.86 \times 10^5\ m^{-1},$$

so that λ = **1460 nm**.

---

3. The ionization energy for doubly ionized lithium (Z = 3) can be found using equation (30.13).

$$E_n = -(13.6\ eV)\ Z^2/n^2 = -(13.6\ eV)(3)^2/(1)^2 = \mathbf{122\ eV}.$$

---

4. Using equation (30.14),

$$(1/\lambda) = (1.097 \times 10^7\ m^{-1})(1/n_f^2 - 1/n_i^2).$$

With λ = 1090 nm = 1.09×10^{-6} m and $n_f = 3$ we obtain n_i = **6**.

---

5. (a) Transitions beginning on n = 4 and ending on n = 1 can take the following paths:

n = 4 → n = 3 → n = 2 → n = 1 (3 spectral lines),

or

n = 4 → n = 2 → n = 1 (one new line),

or

n = 4 → n = 1 (one new line),

and also

n = 4 → n = 3 → n = 1 (one new line).

The total number of distinct lines = **6**.

(b) If all transitions are equally probable, begin with 1200 in the n = 4 state. One-third goes to n = 3, one third goes to n = 2 and one third goes to n = 1 (for a total of 400 + 400 + 400 = 1200 photons).
In the n = 3 state, half goes to n = 2 and half to n = 1 (200 + 200 = 400 photons). In the n = 2 state, where there are now 600 electrons, all 600 produce photons in going to the n = 1 state. The total number of photons produced is therefore,

1200 + 400 + 600 = **2200 photons**.

6.

n	l	m_l	m_s
4	3	3	1/2
4	3	3	-1/2
4	3	2	1/2
4	3	2	-1/2
4	3	1	1/2
4	3	1	-1/2
4	3	0	1/2
4	3	0	-1/2
4	3	-1	1/2
4	3	-1	-1/2
4	3	-2	1/2
4	3	-2	-1/2
4	3	-3	1/2
4	3	-3	-1/2

---

7. The ground state configuration for iron (Z = 26) is

$$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^6$$

---

8. (a) To find the ionization energy use equation (30.13),

$$E_I = (13.6\text{ eV})(74)^2 = \mathbf{74\ keV}.$$

(b) Use equation (30.14),

$$(1/\lambda) = (1.097 \times 10^7\ m^{-1})(74)^2(1/1^2 - 1/2^2) = 4.5 \times 10^{10}\ m^{-1}$$

which gives $\lambda = \mathbf{2.2 \times 10^{-11}\ m}$.

(c) This wavelength corresponds to a photon in the **X-ray** region of the spectrum.

---

9. Use equation (30.17) to obtain

$$\lambda_0 = hc/eV = (6.63 \times 10^{-34}\ J\ s)(3.00 \times 10^8\ m/s)/[(1.60 \times 10^{-19}\ C)(25\ 000\ V)]$$

$$\lambda_0 = \mathbf{5.0 \times 10^{-11}\ m}.$$

---

10. To account for the shielding effect Z = 78 rather than 79 should be used in equation (30.13). We have,

$$E_n = -(13.6\text{ eV})\ Z^2/n^2 = -(13.6\text{ eV})\ (78)^2[1/(1)^2 - 1/(2)^2] = \mathbf{-\ 6.2 \times 10^4\ eV}.$$

Quiz answers

1. e
2. b
3. c
4. d
5. a
6. c
7. c
8. c
9. d
10. b
11. d

Section 30.1 Rutherford Scattering and the Nuclear Atom

1. **ssm** The nucleus of the hydrogen atom has a radius of about 1×10^{-15} m. The electron is normally at a distance of about 5.3×10^{-11} m from the nucleus. Assuming the hydrogen atom is a sphere with a radius of 5.3×10^{-11} m, find (a) the volume of the atom, (b) the volume of the nucleus, and (c) the percentage of the volume of the atom that is occupied by the nucleus.

2. Review Conceptual Example 1 as background for this problem. An electron orbits a certain nucleus at a radius of 2.0×10^{-10} m. The radius of the nucleus is 4.0×10^{-15} m. (a) How many nuclei would have to be lined up side by side to reach the distance at which the electron is located? (b) A marble has a radius of 0.8 cm. If the nucleus were the size of the marble, what would be the length (in kilometers) of the line of nuclei in part (a)?

3. The mass of an α particle is 6.64×10^{-27} kg. An α particle used in a scattering experiment has a kinetic energy of 7.00×10^{-13} J. What is the de Broglie wavelength of the particle?

4. The nucleus of a hydrogen atom is a single proton, which has a radius of about 1.0×10^{-15} m. The single electron in a hydrogen atom normally orbits the nucleus at a distance of 5.3×10^{-11} m. What is the ratio of the density of the hydrogen nucleus to the density of the complete hydrogen atom?

*5. **ssm** The nucleus of a copper atom contains 29 protons and has a radius of 4.8×10^{-15} m. How much work (in electron volts) is done by the electric force as a proton is brought from infinity, where it is at rest, to the "surface" of a copper nucleus?

*6. There are Z protons in the nucleus of an atom, where Z is the atomic number of the element. An α particle carries a charge of $+2e$. In a scattering experiment, an α particle, heading directly toward a nucleus in a metal foil, will come to a halt when all the particle's kinetic energy is converted to electric potential energy. In such a situation, how close will an α particle with a kinetic energy of 5.0×10^{-13} J come to a gold nucleus ($Z = 79$)?

1) 6.2×10^{-31} m^3
4×10^{-45} m^3
7×10^{-13} %

2) 25,000 Nuclei
0.40 km

3) 6.88×10^{-15} m

4) 1.5×10^{-14}

5) -8.7×10^{6} eV

6) 7.3×10^{-14} m

Section 30.2 Line Spectra, Section 30.3 The Bohr Model of the Hydrogen Atom

7. If the line with the longest wavelength in the Balmer series for atomic hydrogen is counted as the first line, what is the wavelength of the third line?

8. The electron in a hydrogen atom is in the first excited state, when the electron acquires an additional 2.86 eV of energy. What is the quantum number n of the state into which the electron moves?

9. **ssm www** What is the radius for the $n = 5$ Bohr orbit in a doubly ionized lithium atom Li^{2+} ($Z = 3$)?

10. Using the Bohr model, compare the nth orbit of a triply ionized beryllium atom Be^{3+} ($Z = 4$) to the nth orbit of a hydrogen atom (H) by calculating the ratio (Be^{3+}/H) of the following quantities: (a) the energies and (b) the radii.

11. In the line spectrum of atomic hydrogen there is also a group of lines known as the Pfund series. These lines are produced when electrons, excited to high energy levels, make transitions to the $n = 5$ level. Determine (a) the longest wavelength and (b) the shortest wavelength in this series. (c) Refer to Figure 24.9 and identify the region of the electromagnetic spectrum in which these lines are found.

12. Determine the ionization energy (in electron volts) that is needed to remove the remaining electron from a singly ionized helium atom He^+ ($Z = 2$).

13. **ssm** Find the energy (in joules) of the photon that is emitted when the electron in a hydrogen atom undergoes a transition from the $n = 7$ energy level to produce a line in the Paschen series.

14. Consider the Bohr energy expression (Equation 30.13) as it applies to singly ionized helium He^+ ($Z = 2$) and doubly ionized lithium Li^{2+} ($Z = 3$). This expression predicts equal electron energies for these two species for certain values of the quantum number n (the quantum number is different for each species). For quantum numbers less than or equal to 9, what are the lowest three energies (in electron volts) for which the helium energy level is equal to the lithium energy level?

15. In the hydrogen atom, what is the total energy (in electron volts) of an electron that is in an orbit with a radius of 4.761×10^{-10} m?

*16. In the Bohr model, Equation 30.13 gives the total energy (kinetic plus potential). In a certain Bohr orbit, the total energy is −4.90 eV. For this orbit, determine the kinetic energy and the electric potential energy of the electron.

*17. **ssm** For atomic hydrogen, the Paschen series of lines occurs when $n_f = 3$, while the Brackett series occurs when $n_f = 4$ in Equation 30.14. Using this equation, show that the ranges of wavelengths in these two series overlap.

*18. The energy of the $n = 2$ Bohr orbit is −30.6 eV for an unidentified ionized atom in which only one electron moves about the nucleus. What is the radius of the $n = 5$ orbit for this species?

*19. **ssm** The Bohr model can be applied to singly ionized helium He^+ ($Z = 2$). Using this model, consider the series of lines that is produced when the electron makes a transition from higher energy levels into the $n_f = 4$ level. Some of the lines in this series lie in the visible region of the spectrum (380–750 nm). What are the values of n_i for the energy levels from which the electron makes the transitions corresponding to these lines?

**20. A diffraction grating is used in the first order to separate the wavelengths in the Balmer series of atomic hydrogen. (Section 27.7 discusses diffraction gratings.) The grating and an observation screen are separated by a distance of 81.0 cm, as Figure 27.34 illustrates. You may assume that θ is small, so $\sin\theta \approx \theta$ when radian measure is used for θ. How many lines per centimeter should the grating have, so the longest and the next-to-the-longest wavelengths in the series are separated by 3.00 cm on the screen?

**21. (a) Derive an expression for the speed of the electron in the nth Bohr orbit, in terms of Z, n, and the constants k, e, and h. For the hydrogen atom, determine the speed in (b) the $n = 1$ orbit and (c) the $n = 2$ orbit. (d) Generally, when speeds are less than one-tenth the speed of light, the effects of special relativity can be ignored. Are the speeds found in (b) and (c) consistent with ignoring relativistic effects in the Bohr model?

7) $\lambda = 434.1$ nm

8) 5

9) 4.41×10^{-10} m

10) 16, 1/4

11) 7458 nm, 2279 nm
infrared

12) 54.4 eV

13) 1.98×10^{-19} J

14) See solution manual

15) −1.51 eV

16) −9.8 eV, +4.9 eV

17) 8.204×10^{-7} m,
1.875×10^{-6} m,
1.459×10^{-6} m
4.051×10^{-6} m

18) 4.41×10^{-10} m

19) $6 \le n_i \le 19$

20) 2180 lines/cm

21) $v_n = \frac{2\pi k e^2 Z}{nh}$
2.19×10^6 m/s
1.09×10^6 m/s
well below the speed of light
and are consistent with ignoring

28)

Nuclear Physics

AP Physics B

Mr. DiBucci

Nuclear Physics and Radioactivity

PREVIEW

One of the great successes of physics during the twentieth century was the development of the model of the atom and the subsequent description of its nucleus. In this chapter you will learn about the nucleon inhabitants of the nucleus: the neutron and proton. You will learn how the size of the nucleus depends on the number of nucleons.

Also, you will learn about the force which binds the nucleons together and the energy associated with this binding. If the nucleus is "split", some of this energy can be released. This fact provides the basis for nuclear power.

One-half of radioactive nuclei spontaneously disintegrate after a certain amount of time. You will be able to calculate this half-life in some cases and to determine how much radioactive material has disintegrated after a given time. This idea is used for dating archeological artifacts.

QUICK REFERENCE

Important Terms

Nucleons
Protons and neutrons.

Atomic number (Z)
The total number of protons in a nucleus.

Atomic mass number (A)
The total number of nucleons in a nucleus.

Isotopes
Nuclei which have the same number of protons but different number of neutrons.

Strong nuclear force
One of the four known fundamental forces in nature. The strong force is the force exerted between nucleons which binds them together in a nucleus.

Radioactivity
The spontaneous disintegration or rearrangement of the internal structure of nuclei.

Binding energy
The energy required to break apart a nucleus into separated nucleons.

Mass defect
The difference in the mass of an intact nucleus and total mass of the separated nucleons. This is the mass equivalent of the binding energy of the nucleus.

α particle
A helium nucleus consisting of two protons and two neutrons.

α decay
The radioactive emission of an α particle from a nucleus.

Transmutation
The changing of one element to another by a loss or gain of one or more protons. Often transmutation is caused by α decay.

β decay
The radioactive decay of a nucleus by the emission of an electron or positron.

γ decay
The radioactive decay of a nucleus by emission of a high frequency "gamma" photon.

Neutrino
A particle created during the β decay of a nucleus.

Half-life
The time necessary for one-half of a particular radioactive substance to undergo spontaneous decay.

Radioactive decay series
The sequential decay of one nuclear species after another.

Geiger counter
A common gas filled device used for detecting α, β, and γ emissions of radioactive nuclei.

Scintillation counter
A device consisting of a material which emits light when struck by ionizing radiation. The material is mounted on a photomultiplier tube.

Semiconductor detector
A device which detects ionizing radiation by the electrons and holes produced in a semiconductor material.

Cloud chamber
A device which detects ionizing radiation by its condensation track left in a supercooled gas.

Bubble chamber
A device which detects ionizing radiation by its vapor track left in a superheated liquid.

Equations

The **relationship** between the **atomic number**, Z, number of neutrons, N, and the **atomic mass number**, A, of a nucleus.

$$A = Z + N \tag{31.1}$$

The approximate **radius of a nucleus**

$$r \cong (1.2 \text{ X } 10^{-15}\text{ m})A^{1/3} \tag{31.2}$$

The **binding energy** of a nucleus

$$\text{Binding energy} = (\Delta m)c^2 \tag{31.3}$$

The **activity** of a radioactive sample is the magnitude of

$$\frac{\Delta N}{\Delta t} = -\lambda N \tag{31.4}$$

The **number of radioactive nuclei** present at time, t, in a sample

$$N = N_0 e^{-\lambda t} \tag{31.5}$$

The **half-life** of a radioactive material

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \tag{31.6}$$

DISCUSSION OF SELECTED SECTIONS

31.1 Nuclear Structure

Example 1
Two isotopes of chlorine occur in nature. The ${}^{35}_{17}Cl$ isotope has an atomic mass of 34.968 85 u and a natural abundance of 75.77%. The ${}^{37}_{17}Cl$ has an atomic mass of 36.965 90 u and a natural abundance of 24.23%. By a calculation of your own, verify that the value of 35.45 u listed in the periodic table is a weighted average of these individual atomic masses.

The weighted average of the masses is

$$(0.7577)(34.968\ 85\ u) + (0.2423)(36.965\ 90\ u) = 35.45\ u$$

Example 2
What is the approximate radius of lead ${}^{207}_{82}Pb$?

The atomic mass number of the lead is A = 207. Equation (31.2) gives the approximate radius to be

$$r \approx (1.2 \text{ X } 10^{-15}\ m)\ A^{1/3} = (1.2 \text{ X } 10^{-15}\ m)(207)^{1/3} = 7.1 \text{ X } 10^{-15}\ m$$

31.3 The Mass Defect of the Nucleus and Nuclear Binding Energy

Relativity tells us that mass and energy are equivalent and related by $E = mc^2$. A nucleus has less energy than the total energy of its separated constituents since some energy is released in the formation of the nucleus (binding energy). Hence, the mass of the nucleus is less than the total mass of its separated nucleons. The difference in mass is called the mass defect and is the mass equivalent of the binding energy of the nucleus. The specific relationship between the binding energy and mass defect is given by equation (31.3).

Note: The total mass of an atom, given in the table in Appendix F, includes the mass of its complement of electrons. The mass of the electrons must be taken into account when calculating mass defects.

Example 3
What is the mass defect and the binding energy for ${}^{239}_{93}Np$?

The total mass of the atom including electrons is found in Appendix F to be 239.052 933 u. The atom has 93 protons, 93 electrons and 146 neutrons. The total mass of these separated particles is

$$93(1.007\ 276\ u) + 93(5.485\ 799 \text{ X } 10^{-4}\ u) + 146(1.008\ 665\ u) = 240.992\ 776\ u.$$

The mass defect of the nucleus is then

$$\Delta m = 240.992\ 776\ u - 239.052\ 933\ u = 1.939\ 843\ u$$

The corresponding binding energy is

$$(\Delta m)c^2 = (1.939\ 843\ u)((931.5\ MeV/1\ u) = 1807\ MeV$$

31.4 Radioactivity

During radioactive decay a nucleus emits an α particle, a β particle or a γ ray. During any of these decay processes, the mass/energy, charge, linear momentum, angular momentum and nucleon number are conserved.

Example 4
What daughter nucleus would be formed if $^{238}_{92}U$ could undergo (a) α decay. (b) β decay. (c) γ decay?

(a) Write the reaction as

$$^{238}_{92}U \rightarrow {}^{A}_{Z}X + {}^{4}_{2}He$$

The conservation of nucleon number requires

$$238 = A + 4 \rightarrow A = 234$$

The conservation of charge requires

$$92 = Z + 2 \rightarrow Z = 90$$

Using Appendix F in the text we find that the daughter nucleus is $^{234}_{90}Th$.

(b) The reaction is

$$^{239}_{92}U \rightarrow {}^{A}_{Z}X + {}^{0}_{-1}e$$

The conservation of nucleon number gives

$$238 = A + 0 \rightarrow A = 238$$

The conservation of charge requires

$$92 = Z + (-1) \rightarrow Z = 93$$

The periodic chart in Appendix F shows that the daughter nucleus is $^{238}_{93}Np$.

(c) The reaction in this case is

$$^{238}_{92}U \rightarrow {}^{A}_{Z}X + \gamma$$

The γ ray does not have a nucleon number or a charge, hence A = 238 and Z = 92. The daughter nucleus is $^{238}_{92}U$.

31.6 Radioactive Decay and Activity

Example 5
The radioactive isotopes of strontium are particularly dangerous to animals since strontium behaves chemically like calcium and is taken up by the body and deposited in bone. Much of the radioactive strontium in the atmosphere is ${}^{90}_{38}Sr$ which was released by atmospheric nuclear bomb testing during the 1950's and early 1960's. Assume that no strontium has been introduced into the atmosphere for 28.0 years. What percentage of the original radioactive nuclei is still present in the atmosphere? Strontium has a half-life of 28.5 years.

In any given volume of air there are still

$$N = N_0 e^{-\lambda t}$$

radioactive nuclei left. The percentage left is then

$$\frac{N}{N_0} X\ 100\ \% = 100\ \%\ X\ e^{-\lambda t}$$

The decay constant, λ, of the strontium isotope is

$$\lambda = (0.693)/T_{1/2} = (0.693)/(28.5\text{ yr}) = 0.02432\text{ yr}^{-1}$$

Note that we have carried an extra figure for additional accuracy. The percent of strontium left in the atmosphere is

$$e^{-(0.02432\text{ yr}^{-1})(28.0\text{ yr})} \times 100\% = 50.6\%$$

31.7 Radioactive Dating

Example 6
The shroud of Turin was originally believed to be about two thousand years old, but was determined by dating techniques to be about 700 years old. Assuming that the shroud had a carbon-14 activity of 0.23 Bq per gram when it was made, what is its activity now?

The number of nuclei left in a given amount of the shroud is

$$N = N_0 e^{-\lambda t}$$

The activity, $A = \lambda N$ so

$$A = A_0 e^{-\lambda t}$$

The decay constant for carbon-14 is

$$\lambda = (0.693)/T_{1/2} = (0.693)/(5730\text{ yr}) = 1.21\ X\ 10^{-4}\text{ yr}^{-1}$$

Now the per gram activity is

$$A = (0.23\text{ Bq})e^{-(1.21\times10^{-4}\text{ yr}^{-1})(700\text{ yr})} = 0.21\text{ Bq}$$

PRACTICE PROBLEMS

1. What is the radius of a nucleus of ${}^{28}_{14}Si$?

2. What is the binding energy per nucleon of ${}^{28}_{14}Si$?

3. Polonium ${}^{214}_{84}Po$ undergoes both α and γ decay. What daughter nucleus is produced in each case?

4. An intermediate nucleus in the decay series of ${}^{238}_{92}U$ is ${}^{230}_{90}Th$. After several successive α decays, this intermediate nucleus leads to the isotope ${}^{214}_{82}Pb$. How many α decays have occurred?

5. The isotope $^{89}_{36}Kr$ has a lifetime of 3.16 min. What percentage of Krypton nuclei are left after 15.0 min?

6. The activity of a sample of a radioactive material is 1.50 Ci and the half-life is 1.6 X 10^3 yr. How many of the radioactive nuclei are left after 250 yr?

7. Plutonium $^{239}_{94}Pu$ (atomic mass = 239.052 157 u) undergoes α decay. Assuming all the released energy is in the form of kinetic energy of the α particle and ignoring the recoil of the daughter nucleus, find the speed of the α particle.

8. One gram of charcoal from an ancient campfire is analyzed for $^{14}_{6}C$ and found to have an activity of 0.11 Bq. How old is the charcoal?

HELPFUL SUGGESTIONS

1. When you are calculating mass defects, you should carry as many significant figures in the atomic masses as possible. This is because the mass defects are often quite small.

2. When calculating the decay constant for use in an exponential, it is desirable to carry an extra figure.

3. Only one-half of a radioactive substance remains after one half-life. After n half-lives only $1/2^n$ of the original amount remains.

EVERYDAY PHYSICS

1. Radioactivity is a concern to many people, particularly those living close to nuclear power plants. Accidental releases of radioactive materials pose a potential hazard to the health of people exposed to the radiation. Additionally, there is the possibility of long term contamination of the area surrounding the site of a nuclear accident. Try to assess and compare the possible long term effects of an accidental release of radioactive gases, such as radon, to the release of heavier radioactive nuclei. Can any clear conclusions be drawn?

2. The potential danger of radioactive radon gas buildup in basements of houses is a cause of concern. (See examples 6 and 7 in the text.) It has been estimated that as many as 50 % of the houses in the United States may have dangerous levels of radon. Inexpensive home test kits are now available for monitoring the radon levels in homes. It might be a good idea to obtain one and determine the radon level in your house.

3. Radon gas itself, apparently, is not as dangerous as its radioactive daughters which attach themselves to particles in the air and can become lodged in the lungs when inhaled. According to Table 31.2 in your text, radon decays by α and γ emissions. What daughters would you expect radon to produce? Do some library research to find out what radioactive daughters radon does produce and their lifetimes.

CHAPTER QUIZ

1. How many neutrons are in the nucleus of $^{260}_{103}Lr$?
   a. 260 b. 103 c. 363 d. 157

2. A certain nucleus has an atomic mass eight times larger than another. The first nucleus has a radius _______ the other.
   a. the same size as b. twice as large as c. four times as large as d. eight times as large as

3. Which of the following is a helium nucleus?
   a. α ray b. β ray c. γ ray d. positron

4. The binding energy of a nucleus is the energy needed to _______.
   a. form it b. tear it apart c. keep it from decaying d. accelerate it to c

5. A nucleus has a mass defect of 1.235 u. What is its binding energy in MeV?
   a. 754.2 b. 1.111×10^{17} c. 1.150×10^{3} d. 931.5

6. A 10.0 g sample of radioactive substance is left to decay for five half-lives? How much of the radioactive substance remains?
   a. Not enough information is given to find an answer.
   b. 0.313 g
   c. 0.625 g
   d. 1.25 g

7. A radioactive nucleus produces a 0.0483 MeV γ ray. How much has the mass of the nucleus changed due to the γ ray emission?
   a. 8.59×10^{-32} kg b. 7.73×10^{-21} kg c. None. The nucleus lost energy not mass.

8. Many of the radiation detectors discussed in the text rely on the fact that the detected particles or rays______.
   a. have charge
   b. have mass
   c. produce ions in the detector
   d. are actually electrons

9. A radioactive series ultimately results in_______.
   a. hydrogen b. lead c. a stable nucleus d. a β decay

10. In most β^- decay processes a(n) ________ is emitted along with the e^-.
    a. α ray b. γ ray c. neutrino d. antineutrino

SOLUTIONS AND ANSWERS

Practice Problems

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1. Equation (31.2) gives the approximate radius to be

$$r \approx (1.2 \times 10^{-15}\ \text{m})(28)^{1/3} = \mathbf{3.6 \times 10^{-15}\ m.}$$

---

2. The mass defect for the nucleus is

$$\Delta m = 14(1.007\,825\ \text{u}) + 14(1.008\,665\ \text{u}) - 27.976\,927\ \text{u} = 0.253\,933\ \text{u}$$

The energy equivalent of this mass defect is

$$\text{Binding energy} = (931.5\ \text{MeV/1 u})(0.253\,933\ \text{u}) = 236.5\ \text{MeV}.$$

There are 28 nucleons, so the energy corresponds to an energy/nucleon of **8.448 MeV/nucleon**.

---

3. For α decay the atomic mass number is conserved so the daughter nucleus has A = 214 - 4 = 210. The charge is also conserved so the daughter has an atomic number of Z = 84 - 2 = 82. The table in Appendix F shows the daughter to be ${}^{210}_{82}\text{Pb}$.
In a gamma decay the nucleus loses only energy, hence the daughter is a de-excited ${}^{214}_{84}\text{Po}$.

---

4. The reaction is

$${}^{230}_{90}\text{Th} \rightarrow {}^{214}_{82}\text{Pb} + N\,{}^{4}_{2}\text{He}$$

where N is the number of a decays which have occurred.

The conservation of charge gives

$$90 = 82 + 2N \Rightarrow N = 4.$$

The conservation of atomic mass number gives

$$230 = 214 + 4N \Rightarrow N = 4.$$

The number of α decays needed is **4.**

---

5. Equation (31.5) gives the percentage of nuclei left.

$$(N/N_0) \times 100\ \% = e^{-\lambda t} \times 100\ \%$$

where $\lambda = (0.693)/T_{1/2} = 0.2193\ \text{min}^{-1}$.

The percentage of nuclei left is **3.73 %**.

---

6. Equation (31.4) gives for the initial case Activity = $-\lambda N_0$, where $\lambda = (0.693)/T_{1/2}$. The number of nuclei present initially is

$$N_0 = (1.5 \text{ Ci})(3.70 \text{ X } 10^{10} \text{ Bq/ } 1 \text{ Ci})/[(0.693)/(1.6 \text{ X } 10^3 \text{ yr})] = 1.3 \text{ X } 10^{14}.$$

The number of nuclei remaining after 250 yr is

$$N = (1.3 \text{ X } 10^{14})\, e^{-(0.000433)(250)} = \mathbf{1.2 \text{ X } 10^{14}}.$$

---

7. The conservation of atomic mass number requires that the daughter have a mass number of 235 and the conservation of charge requires that it have an atomic number of 92. The daughter is uranium. The kinetic energy of the α particle is the equivalent of the total mass defect of the process.

$$M_{lost} = 239.052\,157 \text{ u} - 235.043\,924 \text{ u} - 4.002\,603 \text{ u} = 0.005\,630 \text{ u}.$$

The kinetic energy of the α is KE = $M_{lost}\, c^2$

The speed of the a particle is then

$$v = \sqrt{\frac{2M_{lost}}{M_\alpha}}\, c = \sqrt{\frac{2(0.005\,630 \text{ u})}{4.002\,603 \text{ u}}}\, (3.00 \text{ X } 10^8 \text{ m/s}) = \mathbf{1.59 \text{ X } 10^7 \text{ m/s}}$$

---

8. Assume that the original activity of the radioactive carbon in the wood was 0.23 Bq. The activity at any time is

$$A = A_0\, e^{-\lambda t} \text{ where } \lambda = (0.693)/(5730 \text{ yr}) = 1.21 \text{ X } 10^{-4} \text{ yr}^{-1}$$

Taking natural logarithm of the above equation and solving for the time yields

$$t = -(1/\lambda) \ln (A/A_0) = -(1/1.21 \text{ X } 10^{-4} \text{ yr}^{-1}) \ln (0.11 \text{ Bq}/0.23 \text{ Bq}) = \mathbf{6100 \text{ yr}}.$$

Quiz answers

1. d	3. a	5. c	7. a	9. c
2. b	4. b	6. b	8. c	10. d

Section 31.1 Nuclear Structure, Section 31.2 The Strong Nuclear Force and the Stability of the Nucleus

1. **ssm** In each of the following cases, what element does the symbol X represent and how many neutrons are in the nucleus: (a) $^{195}_{78}X$, (b) $^{32}_{16}X$, (c) $^{63}_{29}X$, (d) $^{11}_{5}X$, and (e) $^{239}_{94}X$? Use the periodic table on the inside of the back cover as needed.

2. What is the radius of a nucleus of titanium $^{48}_{22}Ti$?

3. By what factor does the nucleon number of a nucleus have to increase in order for the nuclear radius to double?

4. For $^{208}_{82}Pb$ find (a) the net electrical charge of the nucleus, (b) the number of neutrons, (c) the number of nucleons, (d) the approximate radius of the nucleus, and (e) the nuclear density.

5. **ssm** **www** The largest stable nucleus has a nucleon number of 209, while the smallest has a nucleon number of 1. If each nucleus is assumed to be a sphere, what is the ratio (largest/smallest) of the surface areas of these spheres?

6. In the nucleus of gold $^{197}_{79}Au$, what is the magnitude of the electrostatic force of repulsion that one proton exerts on another, assuming that the centers of the protons are located at opposite ends of a diameter of the gold nucleus?

***7.** The nucleus of an atom has a volume of 9.4×10^{-43} m³. This nucleus contains 78 neutrons. Identify the nucleus in the form $^{A}_{Z}X$. Use the periodic table on the inside of the back cover as needed.

***8.** One isotope (X) contains an equal number of protons and neutrons. Another isotope (Y) of the same element has twice the number of neutrons as the first isotope does. Determine the ratio r_Y/r_X of the nuclear radii of the isotopes.

****9.** **ssm** **www** Refer to Conceptual Example 1 for a discussion on nuclear densities. A neutron star is composed of neutrons and has a density that is approximately the same as that of a nucleus. What is the radius of a neutron star whose mass is 0.40 times the mass of the sun?

****10.** Conceptual Example 1 provides some useful background for this problem. (a) Determine an approximate value for the density (in kg/m³) of the nucleus. (b) If a BB (radius = 2.3 mm) from an air rifle had a density equal to the nuclear density, what mass would the BB have? (c) Assuming the mass of a supertanker is about 1.5×10^8 kg, how many "supertankers" of mass would this hypothetical BB have?

1) 117 Neutrons, platinum
16 " , sulfur
34 " , Copper
6 " , boron
145 " , Plutonium

2) 4.4×10^{-15} m

3) 8

4) 1.31×10^{-17} C, 126, 208, 7.1×10^{-15} m, 2.3×10^{17} kg/m3

5) 35.2

6) 1.2 N

7) $^{A}_{Z}X = ^{130}_{52}Te$ (tellurium)

8) 1.14

9) 9.4×10^{3} m

10) 2.3×10^{17} kg/m3
1.2×10^{10} kg
80

Section 31.3 The Mass Defect of the Nucleus and Nuclear Binding Energy

11. Determine the mass defect of the nucleus for cobalt $^{59}_{27}Co$, which has an atomic mass of 58.933 198 u. Express your answer in (a) atomic mass units and (b) kilograms.

12. The $^{202}_{80}Hg$ isotope of mercury has an atomic mass of 201.970 617 u. Obtain the binding energy *per nucleon* (in MeV/nucleon).

13. **ssm** Use the binding energy per nucleon curve in Figure 31.6 to determine the total binding energy of $^{16}_{8}O$.

14. The earth revolves around the sun, and the two represent a bound system that has a binding energy of 2.6×10^{33} J. Suppose the earth and sun were completely separated, so that they were infinitely far apart and at rest. What would be the difference between the mass of the separated system and that of the bound system?

15. For radium $^{226}_{88}Ra$ (atomic mass = 226.025 402 u) obtain (a) the mass defect in atomic mass units, (b) the binding energy in MeV, and (c) the binding energy per nucleon.

***16.** (a) Energy is required to separate a nucleus into its constituent nucleons, as Figure 31.3 indicates; this energy is the *total* binding energy of the nucleus. In a similar way one can speak of the energy that binds a single nucleon to the remainder of the nucleus. For example, separating nitrogen $^{14}_{7}N$ into nitrogen $^{13}_{7}N$ and a neutron takes energy equal to the binding energy of the neutron, as shown below:

$$^{14}_{7}N + \text{Energy} \longrightarrow {}^{13}_{7}N + {}^{1}_{0}n$$

Find the energy (in MeV) that binds the neutron to the $^{14}_{7}N$ nucleus by considering the mass of $^{13}_{7}N$ and the mass of $^{1}_{0}n$, as compared to the mass of $^{14}_{7}N$ (see Appendix F for masses). (b) Similarly, one can speak of the energy that binds a single proton to the $^{14}_{7}N$ nucleus:

$$^{14}_{7}N + \text{Energy} \longrightarrow {}^{13}_{6}C + {}^{1}_{1}H$$

Following the procedure outlined in part (a), determine the energy (in MeV) that binds the proton to the $^{14}_{7}N$ nucleus. (c) Which nucleon is more tightly bound, the neutron or the proton?

***17.** **ssm** Two isotopes of a certain element have binding energies that differ by 5.03 MeV. The isotope with the larger binding energy contains one more neutron than the other isotope. Find the difference in atomic mass between the two isotopes.

11) 0,5553357u
9,2217x10^{-28} kg

12) 7.9 MeV per Nucleon

13) 128 MeV

14) 2.9x10^{16} Kg

15) 1,858968u, 1732 MeV
7.66 MeV/nucleon

16) 10.55 MeV, 7.55 Mev, Neutron

17) 1,00326 u

Section 31.4 Radioactivity

18. α decay occurs for each of the nuclei given below. Write the decay process for each, including the chemical symbols and values for Z and A for the daughter nuclei: (a) ${}^{212}_{84}Po$ and (b) ${}^{232}_{92}U$.

19. Complete the following decay processes by stating what the symbol "X" represents (X = α, β^-, β^+, or γ): (a) ${}^{211}_{82}Pb \rightarrow {}^{211}_{83}Bi + X$, (b) ${}^{11}_{6}C \rightarrow {}^{11}_{5}B + X$, (c) ${}^{231}_{90}Th^* \rightarrow {}^{231}_{90}Th + X$, and (d) ${}^{210}_{84}Po \rightarrow {}^{206}_{82}Pb + X$.

20. For the following nuclei, each undergoing β^- decay, write the decay process, identifying each daughter nucleus with its chemical symbol and values for Z and A: (a) ${}^{14}_{6}C$ and (b) ${}^{212}_{82}Pb$.

21. **ssm** Write the β^- decay process for ${}^{35}_{16}S$, including the chemical symbol and values for Z and A.

22. Carbon ${}^{14}_{6}C$ (atomic mass = 14.003 241 u) is converted into nitrogen ${}^{14}_{7}N$ (atomic mass = 14.003 074 u) via β^- decay. (a) Write this process in symbolic form, giving Z and A for the parent and daughter nuclei and the β^- particle. (b) Determine the energy (in MeV) released.

23. Tritium ${}^{3}_{1}H$ (atomic mass = 3.016 050 u) undergoes β^- decay and produces ${}^{3}_{2}He$ (atomic mass = 3.016 030 u). Determine the energy (in MeV) released.

24. A ${}^{32}_{15}P$ nucleus undergoes β^- decay, emitting an electron and an antineutrino. (Section 31.5 discusses antineutrinos.) Suppose the kinetic energy of the electron is 0.90 MeV. How much energy (in MeV) is carried by the antineutrino? Use Appendix F to find the masses of ${}^{32}_{15}P$ and its daughter nucleus.

25. **ssm www** Write the β^+ decay process for each of the following nuclei, being careful to include Z and A and the proper chemical symbol for each daughter nucleus: (a) ${}^{18}_{9}F$ and (b) ${}^{15}_{8}O$.

***26.** Thorium ${}^{228}_{90}Th$ produces a daughter nucleus that is radioactive. The daughter, in turn, produces its own radioactive daughter. This process continues until bismuth ${}^{212}_{83}Bi$ is reached. What is the total number of (a) α particles and (b) β^- particles generated in this series of radioactive decays?

***27.** Plutonium ${}^{239}_{94}Pu$ (atomic mass = 239.052 16 u) undergoes α decay. Assuming that all the released energy is in the form of kinetic energy of the α particle and ignoring the recoil of the daughter nucleus, find the speed of the α particle. Ignore relativistic effects.

***28.** Review Conceptual Example 5 as background for this problem. The α decay of uranium ${}^{238}_{92}U$ produces thorium ${}^{234}_{90}Th$ (atomic mass = 234.0436 u). In Example 4, the energy released in this decay is determined to be 4.3 MeV. Determine how much of this energy is carried away by the recoiling ${}^{234}_{90}Th$ daughter nucleus and how much by the α particle. Assume that the energy of each particle is kinetic energy, and ignore the small amount of energy carried away by the γ ray that is also emitted. In addition, ignore relativistic effects.

****29.** **ssm** Find the energy (in MeV) released when β^+ decay converts sodium ${}^{22}_{11}Na$ (atomic mass = 21.994 434 u) into neon ${}^{22}_{10}Ne$ (atomic mass = 21.991 383 u). Notice that the atomic mass for ${}^{24}_{11}Na$ includes the mass of 11 electrons, whereas the atomic mass for ${}^{22}_{10}Ne$ includes the mass of only 10 electrons.

****30.** Sodium ${}^{24}_{11}Na$ emits a γ ray that has an energy of 0.423 MeV. Assuming that the ${}^{24}_{11}Na$ nucleus is initially at rest, use the conservation of linear momentum to find the speed with which the nucleus recoils. Ignore relativistic effects.

18) ${}^{212}_{84}Po \rightarrow {}^{208}_{82}Pb + {}^{4}_{2}He$

${}^{232}_{92}U \rightarrow {}^{228}_{90}Th + {}^{4}_{2}He$

19) β^-, β^+, γ, α

20) ${}^{14}_{6}C \rightarrow {}^{14}_{7}N + {}^{0}_{-1}e$

${}^{212}_{82}Pb \rightarrow {}^{212}_{83}Bi + {}^{0}_{-1}e$

21) ${}^{35}_{16}S \rightarrow {}^{35}_{17}Cl + {}^{0}_{-1}e$

22) ${}^{14}_{6}C \rightarrow {}^{14}_{7}N + {}^{0}_{-1}e$, 0.156 MeV

23) 0.019 MeV

24) 0.81 MeV

25) ${}^{18}_{9}F \rightarrow {}^{18}_{8}O + {}^{0}_{+1}e$

${}^{15}_{8}O \rightarrow {}^{15}_{7}N + {}^{0}_{+1}e$

26) $N_\alpha = 4$, $N_\beta = 1$

27) 1.59×10^7 m/s

28) 0.072 MeV, 4.2 MeV

29) 1.82 MeV

30) 5.67×10^3 m/s

31.8 Radioactive Decay Series

When an unstable parent nucleus decays, the resulting daughter nucleus is sometimes also unstable. If so, the daughter then decays and produces its own daughter, and so on, until a completely stable nucleus is produced. This sequential decay of one nucleus after another is called a ***radioactive decay series.*** Examples 4–6 discuss the first two steps of a series that begins with uranium $^{238}_{92}U$:

$$\underset{\text{Uranium}}{^{238}_{92}U} \longrightarrow \underset{\text{Thorium}}{^{234}_{90}Th} + {}^{4}_{2}He$$

$$^{234}_{90}Th \longrightarrow \underset{\text{Protactinium}}{^{234}_{91}Pa} + {}^{0}_{-1}e$$

Furthermore, Example 8 deals with radon $^{222}_{86}Rn$, which is formed down the line in the $^{238}_{92}U$ radioactive decay series. Figure 31.15 shows the entire series. At several points in the series, branches occur, because more than one kind of decay is possible for an intermediate species. Ultimately, however, the series ends with lead $^{206}_{82}Pb$, which is stable.

The $^{238}_{92}U$ series and other such series are the only sources of some of the radioactive elements found in nature. Radium $^{226}_{88}Ra$, for instance, has a half-life of 1600 yr, which is short enough that all the $^{226}_{88}Ra$ created when the earth was formed billions of years ago has now disappeared. The $^{238}_{92}U$ series provides a continuing supply of $^{226}_{88}Ra$, however.

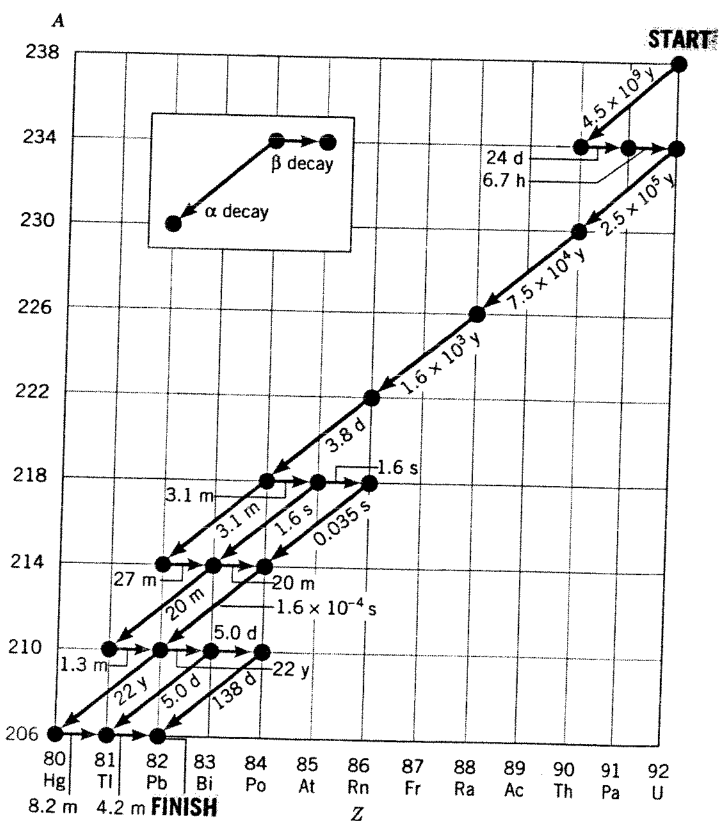

Figure 31.15 The radioactive decay series that begins with uranium $^{238}_{92}U$ and ends with lead $^{206}_{82}Pb$. Half-lives are given in seconds (s), minutes (m), hours (h), days (d), or years (y). The insert in the upper left identifies the type of decay that each nucleus undergoes.

Periodic Table of the Elements

Group I	Group II	Transition elements										Group III	Group IV	Group V	Group VI	Group VII	Group 0
H 1 1.00794 $1s^1$																	**He** 2 4.00260 $1s^2$
Li 3 6.941 $2s^1$	**Be** 4 9.01218 $2s^2$											**B** 5 10.81 $2p^1$	**C** 6 12.011 $2p^2$	**N** 7 14.0067 $2p^3$	**O** 8 15.9994 $2p^4$	**F** 9 18.9984 $2p^5$	**Ne** 10 20.179 $2p^6$
Na 11 22.9898 $3s^1$	**Mg** 12 24.305 $3s^2$											**Al** 13 26.9815 $3p^1$	**Si** 14 28.0855 $3p^2$	**P** 15 30.9738 $3p^3$	**S** 16 32.06 $3p^4$	**Cl** 17 35.453 $3p^5$	**Ar** 18 39.948 $3p^6$
K 19 39.0983 $4s^1$	**Ca** 20 40.08 $4s^2$	**Sc** 21 44.9559 $3d^14s^2$	**Ti** 22 47.88 $3d^24s^2$	**V** 23 50.9415 $3d^34s^2$	**Cr** 24 51.996 $3d^54s^1$	**Mn** 25 54.9380 $3d^54s^2$	**Fe** 26 55.847 $3d^64s^2$	**Co** 27 58.9332 $3d^74s^2$	**Ni** 28 58.69 $3d^84s^2$	**Cu** 29 63.546 $3d^{10}4s^1$	**Zn** 30 65.39 $3d^{10}4s^2$	**Ga** 31 69.72 $4p^1$	**Ge** 32 72.59 $4p^2$	**As** 33 74.9216 $4p^3$	**Se** 34 78.96 $4p^4$	**Br** 35 79.904 $4p^5$	**Kr** 36 83.80 $4p^6$
Rb 37 85.4678 $5s^1$	**Sr** 38 87.62 $5s^2$	**Y** 39 88.9059 $4d^15s^2$	**Zr** 40 91.224 $4d^25s^2$	**Nb** 41 92.9064 $4d^45s^1$	**Mo** 42 95.94 $4d^55s^1$	**Tc** 43 (98) $4d^55s^2$	**Ru** 44 101.07 $4d^75s^1$	**Rh** 45 102.906 $4d^85s^1$	**Pd** 46 106.42 $4d^{10}5s^0$	**Ag** 47 107.868 $4d^{10}5s^1$	**Cd** 48 112.41 $4d^{10}5s^2$	**In** 49 114.82 $5p^1$	**Sn** 50 118.71 $5p^2$	**Sb** 51 121.75 $5p^3$	**Te** 52 127.60 $5p^4$	**I** 53 126.905 $5p^5$	**Xe** 54 131.29 $5p^6$
Cs 55 132.905 $6s^1$	**Ba** 56 137.33 $6s^2$	57–71	**Hf** 72 178.49 $5d^26s^2$	**Ta** 73 180.948 $5d^36s^2$	**W** 74 183.85 $5d^46s^2$	**Re** 75 186.207 $5d^56s^2$	**Os** 76 190.2 $5d^66s^2$	**Ir** 77 192.22 $5d^76s^2$	**Pt** 78 195.08 $5d^96s^2$	**Au** 79 196.967 $5d^{10}6s^1$	**Hg** 80 200.59 $5d^{10}6s^2$	**Tl** 81 204.383 $6p^1$	**Pb** 82 207.2 $6p^2$	**Bi** 83 208.980 $6p^3$	**Po** 84 (209) $6p^4$	**At** 85 (210) $6p^5$	**Rn** 86 (222) $6p^6$
Fr 87 (223) $7s^1$	**Ra** 88 226.025 $7s^2$	89–103	**Rf** 104 (261) $6d^27s^2$	**Ha** 105 (262) $6d^37s^2$	106 (263)	107 (262)	108 (265)	109 (266)									

Symbol — **Cl** 17 — Atomic number
Atomic mass* — 35.453
$3p^5$ — Electron configuration

Lanthanide series

La 57 138.906 $5d^16s^2$	**Ce** 58 140.12 $4f^26s^2$	**Pr** 59 140.908 $4f^36s^2$	**Nd** 60 144.24 $4f^46s^2$	**Pm** 61 (145) $4f^56s^2$	**Sm** 62 150.36 $4f^66s^2$	**Eu** 63 151.96 $4f^76s^2$	**Gd** 64 157.25 $5d^14f^76s^2$	**Tb** 65 158.925 $4f^96s^2$	**Dy** 66 162.50 $4f^{10}6s^2$	**Ho** 67 164.930 $4f^{11}6s^2$	**Er** 68 167.26 $4f^{12}6s^2$	**Tm** 69 168.934 $4f^{13}6s^2$	**Yb** 70 173.04 $4f^{14}6s^2$	**Lu** 71 174.967 $5d^14f^{14}6s^2$

Actinide series

Ac 89 227.028 $6d^17s^2$	**Th** 90 232.038 $6d^27s^2$	**Pa** 91 231.036 $5f^26d^17s^2$	**U** 92 238.029 $5f^36d^17s^2$	**Np** 93 237.048 $5f^46d^17s^2$	**Pu** 94 (244) $5f^66d^07s^2$	**Am** 95 (243) $5f^76d^07s^2$	**Cm** 96 (247) $5f^76d^17s^2$	**Bk** 97 (247) $5f^96d^07s^2$	**Cf** 98 (251) $5f^{10}6d^07s^2$	**Es** 99 (252) $5f^{11}6d^07s^2$	**Fm** 100 (257) $5f^{12}6d^07s^2$	**Md** 101 (258) $5f^{13}6d^07s^2$	**No** 102 (259) $6d^07s^2$	**Lr** 103 (260) $6d^17s^2$

* Atomic mass values are averaged over isotopes according to the percentages that occur on the earth's surface. For many unstable elements, the mass number of the most stable known isotope is given in parentheses. *Source: Handbook of Chemistry and Physics*, 68th ed., CRC Press, Boca Raton, FL. Reprinted by permission.

4. Right circular cylinder of radius r and height h (see Figure E7):

$$\text{Surface area} = 2\pi r^2 + 2\pi rh$$
$$\text{Volume} = \pi r^2 h$$

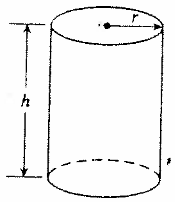

Figure E7

E2 Trigonometry

Basic Trigonometric Functions

1. For a right triangle, the sine, cosine, and tangent of an angle θ are defined as follows (see Figure E8):

$$\sin\theta = \frac{\text{Side opposite } \theta}{\text{Hypotenuse}} = \frac{h_o}{h}$$

$$\cos\theta = \frac{\text{Side adjacent to } \theta}{\text{Hypotenuse}} = \frac{h_a}{h}$$

$$\tan\theta = \frac{\text{Side opposite } \theta}{\text{Side adjacent to } \theta} = \frac{h_o}{h_a}$$

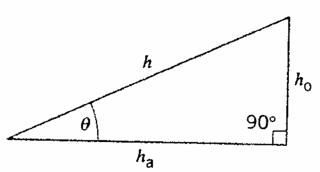

Figure E8

2. The secant (sec θ), cosecant (csc θ), and cotangent (cot θ) of an angle θ are defined as follows:

$$\sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Triangles and Trigonometry

1. The ***Pythagorean theorem*** states that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides (see Figure E8):

$$h^2 = h_o^2 + h_a^2$$

2. The ***law of cosines*** and the ***law of sines*** apply to any triangle, not just a right triangle, and they relate the angles and the lengths of the sides (see Figure E9):

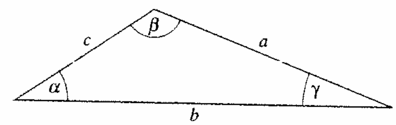

Figure E9

Law of cosines $\quad c^2 = a^2 + b^2 - 2ab\cos\gamma$

Law of sines $\quad \dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

Other Trigonometric Identities

1. $\sin(-\theta) = -\sin\theta$
2. $\cos(-\theta) = \cos\theta$
3. $\tan(-\theta) = -\tan\theta$
4. $(\sin\theta)/(\cos\theta) = \tan\theta$
5. $\sin^2\theta + \cos^2\theta = 1$
6. $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$$\text{If } \alpha = 90°,\ \sin(90° \pm \beta) = \cos\beta$$
$$\text{If } \alpha = \beta,\ \sin 2\beta = 2\sin\beta\cos\beta$$

7. $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$$\text{If } \alpha = 90°,\ \cos(90° \pm \beta) = \mp\sin\beta$$
$$\text{If } \alpha = \beta,\ \cos 2\beta = \cos^2\beta - \sin^2\beta = 1 - 2\sin^2\beta$$

Appendix F Selected Isotopes[a]

Atomic No. Z	Element	Symbol	Atomic Mass No. A	Atomic Mass u	% Abundance, or Decay Mode If Radioactive	Half-life (If Radioactive)
0	(Neutron)	n	1	1.008 665	β^-	10.37 min
1	Hydrogen	H	1	1.007 825	99.985	
	Deuterium	D	2	2.014 102	0.015	
	Tritium	T	3	3.016 050	β^-	12.33 yr

[a] Data for atomic masses are taken from *Handbook of Chemistry and Physics,* 66th ed., CRC Press, Boca Raton, FL. The masses are those for the neutral atom, including the Z electrons. Data for percent abundance, decay mode, and half-life are taken from E. Browne and R. Firestone, *Table of Radioactive Isotopes,* V. Shirley, Ed., Wiley, New York, 1986. α = alpha particle emission, β^- = negative beta emission, β^+ = positron emission, γ = γ-ray emission, EC = electron capture.

APPENDIX F Selected Isotopes *(continued)*

Atomic No. Z	Element	Symbol	Atomic Mass No. A	Atomic Mass u	% Abundance, or Decay Mode If Radioactive	Half-life (If Radioactive)
2	Helium	He	3	3.016 030	0.000 138	
			4	4.002 603	≈100	
3	Lithium	Li	6	6.015 121	7.5	
			7	7.016 003	92.5	
4	Beryllium	Be	7	7.016 928	EC, γ	53.29 days
			9	9.012 182	100	
5	Boron	B	10	10.012 937	19.9	
			11	11.009 305	80.1	
6	Carbon	C	11	11.011 432	β^+, EC	20.39 min
			12	12.000 000	98.90	
			13	13.003 355	1.10	
			14	14.003 241	β^-	5730 yr
7	Nitrogen	N	13	13.005 738	β^+, EC	9.965 min
			14	14.003 074	99.634	
			15	15.000 108	0.366	
8	Oxygen	O	15	15.003 065	β^+, EC	122.2 s
			16	15.994 915	99.762	
			18	17.999 160	0.200	
9	Fluorine	F	18	18.000 937	EC, β^+	1.8295 h
			19	18.998 403	100	
10	Neon	Ne	20	19.992 435	90.51	
			22	21.991 383	9.22	
11	Sodium	Na	22	21.994 434	β^+, EC, γ	2.602 yr
			23	22.989 767	100	
			24	23.990 961	β^-, γ	14.659 h
12	Magnesium	Mg	24	23.985 042	78.99	
13	Aluminum	Al	27	26.981 539	100	
14	Silicon	Si	28	27.976 927	92.23	
			31	30.975 362	β^-, γ	2.622 h
15	Phosphorus	P	31	30.973 762	100	
			32	31.973 907	β^-	14.282 days
16	Sulfur	S	32	31.972 070	95.02	
			35	34.969 031	β^-	87.51 days
17	Chlorine	Cl	35	34.968 852	75.77	
			37	36.965 903	24.23	
18	Argon	Ar	40	39.962 384	99.600	
19	Potassium	K	39	38.963 707	93.2581	
			40	39.963 999	β^-, EC, γ	1.277×10^9 yr
20	Calcium	Ca	40	39.962 591	96.941	
21	Scandium	Sc	45	44.955 910	100	
22	Titanium	Ti	48	47.947 947	73.8	
23	Vanadium	V	51	50.943 962	99.750	
24	Chromium	Cr	52	51.940 509	83.789	
25	Manganese	Mn	55	54.938 047	100	

APPENDIX F Selected Isotopes *(continued)*

Atomic No. Z	Element	Symbol	Atomic Mass No. A	Atomic Mass u	% Abundance, or Decay Mode If Radioactive	Half-life (If Radioactive)
26	Iron	Fe	56	55.934 939	91.72	
27	Cobalt	Co	59	58.933 198	100	
			60	59.933 819	β^-, γ	5.271 yr
28	Nickel	Ni	58	57.935 346	68.27	
			60	59.930 788	26.10	
29	Copper	Cu	63	62.939 598	69.17	
			65	64.927 793	30.83	
30	Zinc	Zn	64	63.929 145	48.6	
			66	65.926 034	27.9	
31	Gallium	Ga	69	68.925 580	60.1	
32	Germanium	Ge	72	71.922 079	27.4	
			74	73.921 177	36.5	
33	Arsenic	As	75	74.921 594	100	
34	Selenium	Se	80	79.916 520	49.7	
35	Bromine	Br	79	78.918 336	50.69	
36	Krypton	Kr	84	83.911 507	57.0	
			89	88.917 640	β^-, γ	3.16 min
			92	91.926 270	β^-, γ	1.840 s
37	Rubidium	Rb	85	84.911 794	72.165	
38	Strontium	Sr	86	85.909 267	9.86	
			88	87.905 619	82.58	
			90	89.907 738	β^-	28.5 yr
			94	93.915 367	β^-, γ	1.235 s
39	Yttrium	Y	89	88.905 849	100	
40	Zirconium	Zr	90	89.904 703	51.45	
41	Niobium	Nb	93	92.906 377	100	
42	Molybdenum	Mo	98	97.905 406	24.13	
43	Technecium	Tc	98	97.907 215	β^-, γ	4.2×10^6 yr
44	Ruthenium	Ru	102	101.904 348	31.6	
45	Rhodium	Rh	103	102.905 500	100	
46	Palladium	Pd	106	105.903 478	27.33	
47	Silver	Ag	107	106.905 092	51.839	
			109	108.904 757	48.161	
48	Cadmium	Cd	114	113.903 357	28.73	
49	Indium	In	115	114.903 880	95.7; β^-	4.41×10^{14} yr
50	Tin	Sn	120	119.902 200	32.59	
51	Antimony	Sb	121	120.903 821	57.3	
52	Tellurium	Te	130	129.906 229	38.8; β^-	2.5×10^{21} yr
53	Iodine	I	127	126.904 473	100	
			131	130.906 114	β^-, γ	8.040 days
54	Xenon	Xe	132	131.904 144	26.9	
			136	135.907 214	8.9	
			140	139.921 620	β^-, γ	13.6 s

APPENDIX F Selected Isotopes *(continued)*

Atomic No. Z	Element	Symbol	Atomic Mass No. A	Atomic Mass u	% Abundance, or Decay Mode If Radioactive	Half-life (If Radioactive)
55	Cesium	Cs	133	132.905 429	100	
			134	133.906 696	β^-, EC, γ	2.062 yr
56	Barium	Ba	137	136.905 812	11.23	
			138	137.905 232	71.70	
			141	140.914 363	β^-, γ	18.27 min
57	Lanthanum	La	139	138.906 346	99.91	
58	Cerium	Ce	140	139.905 433	88.48	
59	Praseodymium	Pr	141	140.907 647	100	
60	Neodymium	Nd	142	141.907 719	27.13	
61	Promethium	Pm	145	144.912 743	EC, α, γ	17.7 yr
62	Samarium	Sm	152	151.919 729	26.7	
63	Europium	Eu	153	152.921 225	52.2	
64	Gadolinium	Gd	158	157.924 099	24.84	
65	Terbium	Tb	159	158.925 342	100	
66	Dysprosium	Dy	164	163.929 171	28.2	
67	Holmium	Ho	165	164.930 319	100	
68	Erbium	Er	166	165.930 290	33.6	
69	Thulium	Tm	169	168.934 212	100	
70	Ytterbium	Yb	174	173.938 859	31.8	
71	Lutetium	Lu	175	174.940 770	97.41	
72	Hafnium	Hf	180	179.946 545	35.100	
73	Tantalum	Ta	181	180.947 992	99.988	
74	Tungsten (wolfram)	W	184	183.950 928	30.67	
75	Rhenium	Re	187	186.955 744	62.60; β^-	4.6×10^{10} yr
76	Osmium	Os	191	190.960 920	β^-, γ	15.4 days
			192	191.961 467	41.0	
77	Iridium	Ir	191	190.960 584	37.3	
			193	192.962 917	62.7	
78	Platinum	Pt	195	194.964 766	33.8	
79	Gold	Au	197	196.966 543	100	
			198	197.968 217	β^-, γ	2.6935 days
80	Mercury	Hg	202	201.970 617	29.80	
81	Thallium	Tl	205	204.974 401	70.476	
			208	207.981 988	β^-, γ	3.053 min
82	Lead	Pb	206	205.974 440	24.1	
			207	206.975 872	22.1	
			208	207.976 627	52.4	
			210	209.984 163	α, β^-, γ	22.3 yr
			211	210.988 735	β^-, γ	36.1 min
			212	211.991 871	β^-, γ	10.64 h
			214	213.999 798	β^-, γ	26.8 min

APPENDIX F Selected Isotopes *(continued)*

Atomic No. Z	Element	Symbol	Atomic Mass No. A	Atomic Mass u	% Abundance, or Decay Mode If Radioactive	Half-life (If Radioactive)
83	Bismuth	Bi	209	208.980 374	100	
			211	210.987 255	α, β^-, γ	2.14 min
			212	211.991 255	β^-, α, γ	1.0092 h
84	Polonium	Po	210	209.982 848	α, γ	138.376 days
			212	211.988 842	α, γ	45.1 s
			214	213.995 176	α, γ	163.69 μs
			216	216.001 889	α, γ	150 ms
85	Astatine	At	218	218.008 684	α, β^-	1.6 s
86	Radon	Rn	220	220.011 368	α, γ	55.6 s
			222	222.017 570	α, γ	3.825 days
87	Francium	Fr	223	223.019 733	α, β^-, γ	21.8 min
88	Radium	Ra	224	224.020 186	α, γ	3.66 days
			226	226.025 402	α, γ	1.6×10^3 yr
			228	228.031 064	β^-, γ	5.75 yr
89	Actinium	Ac	227	227.027 750	α, β^-, γ	21.77 yr
			228	228.031 015	β^-, γ	6.13 h
90	Thorium	Th	228	228.028 715	α, γ	1.913 yr
			231	231.036 298	β^-, γ	1.0633 days
			232	232.038 054	100; α, γ	1.405×10^{10} yr
			234	234.043 593	β^-, γ	24.10 days
91	Protactinium	Pa	231	231.035 880	α, γ	3.276×10^4 yr
			234	234.043 303	β^-, γ	6.70 h
			237	237.051 140	β^-, γ	8.7 min
92	Uranium	U	232	232.037 130	α, γ	68.9 yr
			233	233.039 628	α, γ	1.592×10^5 yr
			235	235.043 924	0.7200; α, γ	7.037×10^8 yr
			236	236.045 562	α, γ	2.342×10^7 yr
			238	238.050 784	99.2745; α, γ	4.468×10^9 yr
			239	239.054 289	β^-, γ	23.47 min
93	Neptunium	Np	239	239.052 933	β^-, γ	2.355 days
94	Plutonium	Pu	239	239.052 157	α, γ	2.411×10^4 yr
			242	242.058 737	α, γ	3.763×10^5 yr
95	Americium	Am	243	243.061 375	α, γ	7.380×10^3 yr
96	Curium	Cm	245	245.065 483	α, γ	8.5×10^3 yr
97	Berkelium	Bk	247	247.070 300	α, γ	1.38×10^3 yr
98	Californium	Cf	249	249.074 844	α, γ	350.6 yr
99	Einsteinium	Es	254	254.088 019	α, γ, β^-	275.7 days
100	Fermium	Fm	253	253.085 173	EC, α, γ	3.00 days
101	Mendelevium	Md	255	255.091 081	EC, α	27 min
102	Nobelium	No	255	255.093 260	EC, α	3.1 min
103	Lawrencium	Lr	257	257.099 480	α, EC	646 ms
104	Rutherfordium	Rf	261	261.108 690	α	1.08 min
105	Hahnium	Ha	262	262.113 760	α	34 s