# Fundamental Particles, Interactions and Special Relativity

Advanced Placement Physics B
Mr. DiBucci

## 30.4 ELEMENTARY PARTICLES

The word "atom" is from the Greek word *atomos*, meaning "indivisible." At one time, atoms were thought to be the indivisible constituents of matter; that is, they were regarded as elementary particles. Discoveries in the early part of the 20th century revealed that the atom is not elementary, but has protons, neutrons, and electrons as its constituents. Until 1932, physicists viewed these three constituent particles as elementary because, with the exception of the free neutron, they are highly stable. The theory soon fell apart, however, and beginning in 1937, many new particles were discovered in experiments involving high-energy collisions between known particles. These new particles are characteristically unstable and have very short half-lives, ranging between $10^{-23}$ s and $10^{-6}$ s. So far more than 300 of them have been cataloged.

Until the 1960s, physicists were bewildered by the large number and variety of subatomic particles being discovered. They wondered whether the particles were like animals in a zoo or whether a pattern could emerge that would provide a better understanding of the elaborate structure in the subnuclear world. In the last 30 years, physicists have made tremendous advances in our knowledge of the structure of matter by recognizing that all particles (with the exception of electrons, photons, and a few others) are made of smaller particles called *quarks*. Protons and neutrons, for example, are not truly elementary but are systems of tightly bound quarks. The quark model has reduced the bewildering array of particles to a manageable number and has predicted new quark combinations that were subsequently found in many experiments.

## 30.5 THE FUNDAMENTAL FORCES OF NATURE

The key to understanding the properties of elementary particles is to be able to describe the forces between them. All particles in nature are subject to four fundamental forces: strong, electromagnetic, weak, and gravitational.

The **strong force** is responsible for the tight binding of quarks to form neutrons and protons and for the nuclear force, a sort of residual strong force, binding neutrons and protons into nuclei. This force represents the "glue" that holds the nucleons together and is the strongest of all the fundamental forces. It is a very short-range force and is negligible for separations greater than about $10^{-15}$ m (the approximate size of the nucleus). The **electromagnetic force**, which is about $10^{-2}$ times the strength of the strong force, is responsible for the binding of atoms and molecules. It is a long-range force that decreases in strength as the inverse square of the separation between interacting particles. The **weak force** is a short-range nuclear force that tends to produce instability in certain nuclei. It is responsible for beta decay, and its strength is only about $10^{-6}$ times that of the strong force. (As we discuss later, scientists now believe that the weak and electromagnetic forces are two manifestations of a single force called the *electroweak* force). Finally, the **gravitational force** is a long-range force with a strength only about $10^{-43}$ times that of the strong force. Although this familiar interaction is the force that holds the planets, stars, and galaxies together, its effect on elementary

TABLE 30.1

**Particle Interactions**

| Interaction (Force) | Relative Strength[a] | Range of Force | Mediating Field Particle |
|---|---|---|---|
| Strong | 1 | Short ($\approx 1$ fm) | Gluon |
| Electromagnetic | $10^{-2}$ | Long ($\propto 1/r^2$) | Photon |
| Weak | $10^{-6}$ | Short ($\approx 10^{-3}$ fm) | $W^{\pm}$ and Z bosons |
| Gravitational | $10^{-43}$ | Long ($\propto 1/r^2$) | Graviton |

[a] For two quarks separated by $3 \times 10^{-17}$ m.

particles is negligible. The gravitational force is by far the weakest of all the fundamental forces.

Modern physics often describes the forces between particles in terms of the actions of field particles or quanta. In the case of the familiar electromagnetic interaction, the field particles are photons. In the language of modern physics, the electromagnetic force is *mediated* (carried) by photons, which are the quanta of the electromagnetic field. Likewise, the strong force is mediated by field particles called *gluons*, the weak force is mediated by particles called the W and Z *bosons*, and the gravitational force is thought to be mediated by quanta of the gravitational field called *gravitons*. All of these field quanta have been detected except for the graviton, which may never be found directly because of the weakness of the gravitational field. These interactions, their ranges, and their relative strengths are summarized in Table 30.1.

## 30.6 POSITRONS AND OTHER ANTIPARTICLES

In the 1920s, the theoretical physicist Paul Adrien Maurice Dirac (1902–1984) developed a version of quantum mechanics that incorporated special relativity. Dirac's theory successfully explained the origin of the electron's spin and its magnetic moment. But it had one major problem: its relativistic wave equation required solutions corresponding to negative energy states even for free electrons, and if negative energy states existed, we would expect a normal free electron in a state of positive energy to make a rapid transition to one of these lower states, emitting a photon in the process. Normal electrons would not exist and we would be left with a universe of photons and electrons locked up in negative energy states.

Dirac circumvented this difficulty by postulating that all negative energy states are normally filled. The electrons that occupy the negative energy states are said to be in the "Dirac sea" and are not directly observable when all negative energy states are filled. However, if one of these negative energy states is vacant, leaving a hole in the sea of filled states, the hole can react to external forces and therefore can be observed as the electron's positive antiparticle. The general and profound implication of Dirac's theory is that **for every particle, there is an antiparticle with the same mass as the particle, but the opposite charge**. For example, the electron's antiparticle, the *positron*, has a mass of 0.511 MeV/$c^2$ and a positive charge of $1.6 \times 10^{-19}$ C. As noted in Chapter 29, we usually designate an antiparticle with a bar over the symbol for the particle. For example, $\overline{p}$ denotes the antiproton and $\overline{\nu}$ the antineutrino. In this book, the notation $e^+$ is preferred for the positron.

Courtesy AIP Emilio Segrè Visual Archives

**PAUL ADRIEN MAURICE DIRAC**
(1902–1984)

Dirac was instrumental in the understanding of antimatter and in the unification of quantum mechanics and relativity. He made numerous contributions to the development of quantum physics and cosmology, and won the Nobel Prize for physics in 1933.

The positron was discovered by Carl Anderson in 1932, and in 1936 he was awarded the Nobel prize for his achievement. Anderson discovered the positron while examining tracks created by electron-like particles of positive charge in a cloud chamber. (These early experiments used cosmic rays—mostly energetic protons passing through interstellar space—to initiate high-energy reactions on the order of several GeV.) In order to discriminate between positive and negative charges, the cloud chamber was placed in a magnetic field, causing moving charges to follow curved paths. Anderson noted that some of the

**TIP 30.1 Antiparticles**

An antiparticle is not identified solely on the basis of opposite charge: even neutral particles have antiparticles.

electronlike tracks deflected in a direction corresponding to a positively charged particle.

Since Anderson's initial discovery, the positron has been observed in a number of experiments. Perhaps the most common process for producing positrons is pair production, introduced in Chapter 26. In this process, a gamma ray with sufficiently high energy collides with a nucleus, creating an electron–positron pair. Because the total rest energy of the pair is $2m_ec^2 = 1.02$ MeV, the gamma ray must have at least this much energy to create such a pair.

Practically every known elementary particle has a distinct antiparticle. Among the exceptions are the photon and the neutral pion ($\pi^0$), which are their own antiparticles. Following the construction of high-energy accelerators in the 1950s, many of these antiparticles were discovered. They included the antiproton $\overline{\text{p}}$, discovered by Emilio Segrè and Owen Chamberlain in 1955, and the antineutron $\overline{\text{n}}$, discovered shortly thereafter.

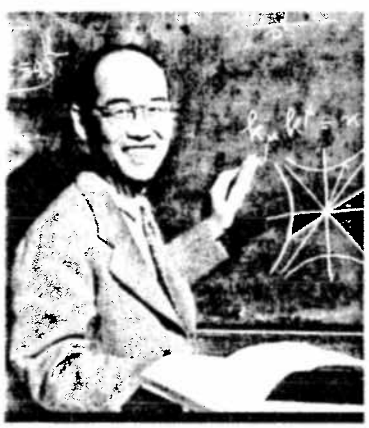
The process of electron–positron annihilation is used in the medical diagnostic technique of positron emission tomography (PET). The patient is injected with a glucose solution containing a radioactive substance that decays by positron emission. Examples of such substances are oxygen-15, nitrogen-13, carbon-11, and fluorine-18. The radioactive material is carried to the brain. When a decay occurs, the emitted positron annihilates with an electron in the brain tissue, resulting in two gamma ray photons. With the assistance of a computer, an image can be created of the sites in the brain at which the glucose accumulates.

**APPLICATION**

Positron Emission Tomography

The images from a PET scan can point to a wide variety of disorders in the brain, including Alzheimer's disease. In addition, because glucose metabolizes more rapidly in active areas of the brain, the PET scan can indicate which areas of the brain are involved in various processes such as language, music, and vision.

## 30.7 MESONS AND THE BEGINNING OF PARTICLE PHYSICS


UPI/Corbis-Bettmann

**HIDEKI YUKAWA,** Japanese Physicist (1907–1981)

Yukawa was awarded the Nobel Prize in 1949 for predicting the existence of mesons. This photograph of Yukawa at work was taken in 1950 in his office at Columbia University.

Physicists in the mid-1930s had a fairly simple view of the structure of matter. The building blocks were the proton, the electron, and the neutron. Three other particles were known or postulated at the time: the photon, the neutrino, and the positron. These six particles were considered the fundamental constituents of matter. Although the accepted picture of the world was marvelously simple, no one was able to provide an answer to the following important question: Because the many protons in proximity in any nucleus should strongly repel each other due to their like charges, what is the nature of the force that holds the nucleus together? Scientists recognized that this mysterious nuclear force must be much stronger than anything encountered up to that time.

The first theory to explain the nature of the nuclear force was proposed in 1935 by the Japanese physicist Hideki Yukawa (1907–1981), an effort that later earned him the Nobel prize. In order to understand Yukawa's theory, it is useful to first note that **two atoms can form a covalent chemical bond by the exchange of electrons**. Similarly, in the modern view of electromagnetic interactions, **charged particles interact by exchanging a photon**. Yukawa used this same idea to explain the nuclear force by proposing a new particle that is exchanged by nucleons in the nucleus to produce the strong force. Further, he demonstrated that the range of the force is inversely proportional to the mass of this particle, and predicted that the mass would be about 200 times the mass of the electron. Because the new particle would have a mass between that of the electron and the proton, it was called a **meson** (from the Greek *meso*, meaning "middle").

**TIP 30.2 The Nuclear Force and the Strong Force**

The nuclear force discussed in Chapter 29 was originally called the *strong* force. Once the quark theory was established, however, the phrase *strong force* was reserved for the force between quarks. We will follow this convention: the strong force is between quarks and the nuclear force is between nucleons.

In an effort to substantiate Yukawa's predictions, physicists began looking for the meson by studying cosmic rays that enter the Earth's atmosphere. In 1937, Carl Anderson and his collaborators discovered a particle with mass 106 MeV/$c^2$, about 207 times the mass of the electron. However, subsequent experiments showed that the particle interacted very weakly with matter and hence could not be the carrier of the nuclear force. This puzzling situation inspired several theoreticians to propose that there are two mesons with slightly different masses, an idea that was confirmed in 1947 with the discovery of the pi meson ($\pi$), or simply *pion*, by Cecil Frank Powell (1903–1969) and Guiseppe P. S. Occhialini (1907–1993). The lighter meson discovered earlier by Anderson, now called a *muon* ($\mu$), has only weak and electromagnetic interactions and plays no role in the strong interaction.

The pion comes in three varieties, corresponding to three charge states: $\pi^+$, $\pi^-$, and $\pi^0$. The $\pi^+$ and $\pi^-$ particles have masses of 139.6 MeV/$c^2$, and the $\pi^0$ has a mass of 135.0 MeV/$c^2$. Pions and muons are highly unstable particles. For example, the $\pi^-$, which has a lifetime of about $2.6 \times 10^{-8}$ s, decays into a muon and an antineutrino. The muon, with a lifetime of 2.2 $\mu$s, then decays into an electron, a neutrino, and an antineutrino. The sequence of decays is

$$\pi^- \rightarrow \mu^- + \bar{\nu} \qquad [30.6]$$

$$\mu^- \rightarrow e^- + \nu + \bar{\nu}$$

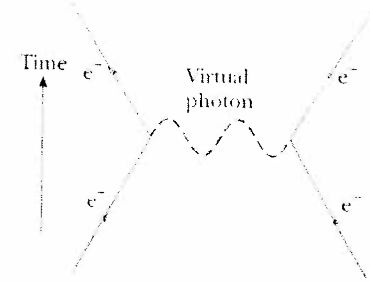



**Figure 30.6** Feynman diagram representing a photon mediating the electromagnetic force between two electrons.

The interaction between two particles can be understood in general with a simple illustration called a *Feynman diagram*, developed by Richard P. Feynman (1918–1988). Figure 30.6 is a Feynman diagram for the electromagnetic interaction between two electrons. In this simple case, a photon is the field particle that mediates the electromagnetic force between the electrons. The photon transfers energy and momentum from one electron to the other in the interaction. Such a photon, called a *virtual photon*, can never be detected directly because it is absorbed by the second electron very shortly after being emitted by the first electron. The existence of a virtual photon might be expected to violate the law of conservation of energy, but it does not because of the time–energy uncertainty principle. Recall that the uncertainty principle says that the energy is uncertain or not conserved by an amount $\Delta E$ for a time $\Delta t$ such that $\Delta E \, \Delta t \approx \hbar$.

© Shelly Gazin/CORBIS

**RICHARD FEYNMAN,** American Physicist (1918–1988)

Feynman, together with Julian S. Schwinger and Shinichiro Tomonaga, won the 1965 Nobel Prize for physics for fundamental work in the principles of quantum electrodynamics. His many important contributions to physics include work on the first atomic bomb in the Manhattan project, the invention of simple diagrams to represent particle interactions graphically, the theory of the weak interaction of subatomic particles, a reformulation of quantum mechanics, and the theory of superfluid helium. Later he served on the commission investigating the *Challenger* tragedy, demonstrating the problem with the O-rings by dipping a scale-model O-ring in his glass of ice water and then shattering it with a hammer. He also contributed to physics education through the magnificent three-volume text *The Feynman Lectures on Physics.*

Now consider the pion exchange between a proton and a neutron via the nuclear force (Fig. 30.7). The energy needed to create a pion of mass $m_\pi$ is given by $\Delta E = m_\pi c^2$. Again, the existence of the pion is allowed in spite of conservation of energy if this energy is surrendered in a short enough time $\Delta t$, the time it takes the pion to transfer from one nucleon to the other. From the uncertainty principle, $\Delta E \, \Delta t \approx \hbar$, we get

$$\Delta t \approx \frac{\hbar}{\Delta E} = \frac{\hbar}{m_\pi c^2} \qquad [30.7]$$

Because the pion can't travel faster than the speed of light, the maximum distance $d$ it can travel in a time $\Delta t$ is $c\,\Delta t$. Using Equation 30.7 and $d = c\,\Delta t$, we find this maximum distance to be

$$d \approx \frac{\hbar}{m_\pi c} \qquad [30.8]$$

The measured range of the nuclear force is about $1.5 \times 10^{-15}$ m. Using this value for $d$ in Equation 30.8, the rest energy of the pion is calculated to be

$$m_\pi c^2 = \frac{\hbar c}{d} = \frac{(1.05 \times 10^{-34}\,\text{J}\cdot\text{s})(3.00 \times 10^8\,\text{m/s})}{1.5 \times 10^{-15}\,\text{m}}$$

$$= 2.1 \times 10^{-11}\,\text{J} \cong 130\,\text{MeV}$$

This corresponds to a mass of 130 MeV/$c^2$ (about 250 times the mass of the electron), which is in good agreement with the observed mass of the pion.

The concept we have just described is quite revolutionary. In effect, it says that a proton can change into a proton plus a pion, as long as it returns to its original state in a very short time. High-energy physicists often say that a nucleon undergoes "fluctuations" as it emits and absorbs pions. As we have seen, these fluctuations are a consequence of a combination of quantum mechanics (through the uncertainty principle) and special relativity (through Einstein's energy–mass relation $E = mc^2$).

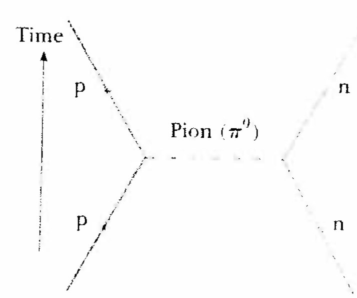



**Figure 30.7** Feynman diagram representing a proton interacting with a neutron via the strong force. In this case, the pion mediates the nuclear force.

This section has dealt with the early theory of Yukawa of particles that mediate the nuclear force, pions, and the mediators of the electromagnetic force, photons. Although his model led to the modern view, it has been superseded by the more basic quark–gluon theory, as explained in Sections 30.12 and 30.13.

## 30.8 CLASSIFICATION OF PARTICLES

### Hadrons

All particles other than photons can be classified into two broad categories, hadrons and leptons, according to their interactions. Particles that interact through the strong force are called *hadrons*. There are two classes of hadrons, known as *mesons* and *baryons*, distinguished by their masses and spins. All mesons are known to decay finally into electrons, positrons, neutrinos, and photons. The pion is the lightest of known mesons, with a mass of about 140 MeV/$c^2$ and a spin of 0. Another is the K meson, with a mass of about 500 MeV/$c^2$ and spin 0 also.

Baryons have masses equal to or greater than the proton mass (the name *baryon* means "heavy" in Greek), and their spin is always a non-integer value (1/2 or 3/2). Protons and neutrons are baryons, as are many other particles. With the exception of the proton, all baryons decay in such a way that the end products include a proton. For example, the baryon called the $\Xi$ hyperon first decays to a $\Lambda^0$ in about $10^{-10}$ s. The $\Lambda^0$ then decays to a proton and a $\pi^-$ in about $3 \times 10^{-10}$ s.

Today it is believed that hadrons are composed of quarks. (Later, we will have more to say about the quark model.) Some of the important properties of hadrons are listed in Table 30.2.

TABLE 30.2

**Some Particles and Their Properties**

| Category | Particle Name | Symbol | Antiparticle | Mass (MeV/$c^2$) | $B$ | $L_e$ | $L_\mu$ | $L_\tau$ | $S$ | Lifetime(s) | Principal Decay Modes[a] |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Leptons** | Electron | $e^-$ | $e^+$ | 0.511 | 0 | +1 | 0 | 0 | 0 | Stable | |
| | Electron–neutrino | $\nu_e$ | $\overline{\nu}_e$ | $< 7\text{eV}/c^2$ | 0 | +1 | 0 | 0 | 0 | Stable | |
| | Muon | $\mu^-$ | $\mu^+$ | 105.7 | 0 | 0 | +1 | 0 | 0 | $2.20 \times 10^{-6}$ | $e^-\overline{\nu}_e\nu_\mu$ |
| | Muon–neutrino | $\nu_\mu$ | $\overline{\nu}_\mu$ | < 0.3 | 0 | 0 | +1 | 0 | 0 | Stable | |
| | Tau | $\tau^-$ | $\tau^+$ | 1 784 | 0 | 0 | 0 | +1 | 0 | $< 4 \times 10^{-13}$ | $\mu^-\overline{\nu}_\mu\nu_\tau$, $e^-\overline{\nu}_e\nu_\tau$ |
| | Tau–neutrino | $\nu_\tau$ | $\overline{\nu}_\tau$ | < 30 | 0 | 0 | 0 | +1 | 0 | Stable | |
| **Hadrons** | | | | | | | | | | | |
| **Mesons** | Pion | $\pi^+$ | $\pi^-$ | 139.6 | 0 | 0 | 0 | 0 | 0 | $2.60 \times 10^{-8}$ | $\mu^+\nu_\mu$ |
| | | $\pi^0$ | Self | 135.0 | 0 | 0 | 0 | 0 | 0 | $0.83 \times 10^{-16}$ | $2\gamma$ |
| | Kaon | $K^+$ | $K^-$ | 493.7 | 0 | 0 | 0 | 0 | +1 | $1.24 \times 10^{-8}$ | $\mu^+\nu_\mu$, $\pi^+\pi^0$ |
| | | $K_S^0$ | $\overline{K}_S^0$ | 497.7 | 0 | 0 | 0 | 0 | +1 | $0.89 \times 10^{-10}$ | $\pi^+\pi^-$, $2\pi^0$ |
| | | $K_L^0$ | $\overline{K}_L^0$ | 497.7 | 0 | 0 | 0 | 0 | +1 | $5.2 \times 10^{-8}$ | $\pi^\pm e^\mp\overline{\nu}_e$, $3\pi^0$<br>$\pi^\pm\mu^\mp\overline{\nu}_\mu$ |
| | Eta | $\eta$ | Self | 548.8 | 0 | 0 | 0 | 0 | 0 | $< 10^{-18}$ | $2\gamma$, $3\pi$ |
| | | $\eta'$ | Self | 958 | 0 | 0 | 0 | 0 | 0 | $2.2 \times 10^{-21}$ | $\eta\pi^+\pi^-$ |
| **Baryons** | Proton | p | $\overline{p}$ | 938.3 | +1 | 0 | 0 | 0 | 0 | Stable | |
| | Neutron | n | $\overline{n}$ | 939.6 | +1 | 0 | 0 | 0 | 0 | 920 | $pe^-\overline{\nu}_e$ |
| | Lambda | $\Lambda^0$ | $\overline{\Lambda}^0$ | 1 115.6 | +1 | 0 | 0 | 0 | −1 | $2.6 \times 10^{-10}$ | $p\pi^-$, $n\pi^0$ |
| | Sigma | $\Sigma^+$ | $\overline{\Sigma}^-$ | 1 189.4 | +1 | 0 | 0 | 0 | −1 | $0.80 \times 10^{-10}$ | $p\pi^0$, $n\pi^+$ |
| | | $\Sigma^0$ | $\overline{\Sigma}^0$ | 1 192.5 | +1 | 0 | 0 | 0 | −1 | $6 \times 10^{-20}$ | $\Lambda^0\gamma$ |
| | | $\Sigma^-$ | $\overline{\Sigma}^+$ | 1 197.3 | +1 | 0 | 0 | 0 | −1 | $1.5 \times 10^{-10}$ | $n\pi^-$ |
| | Xi | $\Xi^0$ | $\overline{\Xi}^0$ | 1 315 | +1 | 0 | 0 | 0 | −2 | $2.9 \times 10^{-10}$ | $\Lambda^0\pi^0$ |
| | | $\Xi^-$ | $\overline{\Xi}^+$ | 1 321 | +1 | 0 | 0 | 0 | −2 | $1.64 \times 10^{-10}$ | $\Lambda^0\pi^-$ |
| | Omega | $\Omega^-$ | $\Omega^-$ | 1 672 | +1 | 0 | 0 | 0 | −3 | $0.82 \times 10^{-10}$ | $\Xi^0\pi^0$, $\Lambda^0K^-$ |

[a] Notations in this column, such as $p\pi^-$, $n\pi^0$ mean two possible decay modes. In this case, the two possible decays are $\Lambda^0 \rightarrow p + \pi^-$ and $\Lambda^0 \rightarrow n + \pi^0$.

### Leptons

Leptons (from the Greek *leptos*, meaning "small" or "light") are a group of particles that participate in the weak interaction. All leptons have a spin of 1/2. Included in this group are electrons, muons, and neutrinos, which are less massive than the lightest hadron. Although hadrons have size and structure, leptons appear to be truly elementary, with no structure down to the limit of resolution of experiment (about $10^{-19}$ m).

Unlike hadrons, the number of known leptons is small. Currently, scientists believe there are only six leptons (each having an antiparticle): the electron, the muon, the tau, and a neutrino associated with each:

$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \quad \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \quad \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$$

The tau lepton, discovered in 1975, has a mass about twice that of the proton.

Although neutrinos have masses of about zero, there is strong indirect evidence that the electron neutrino has a nonzero mass of about 3 eV/$c^2$, or 1/180 000 of the electron mass. A firm knowledge of the neutrino's mass could have great significance in cosmological models and in our understanding of the future of the Universe.

## 30.9 CONSERVATION LAWS

A number of conservation laws are important in the study of elementary particles. Although the two described here have no theoretical foundation, they are supported by abundant empirical evidence.

### Baryon Number

The law of conservation of baryon number tells us that whenever a baryon is created in a reaction or decay, an antibaryon is also created. This information can be quantified by assigning a baryon number: $B = +1$ for all baryons, $B = -1$ for all antibaryons, and $B = 0$ for all other particles. Thus, the **law of conservation of baryon number** states that whenever a nuclear reaction or decay occurs, the sum of the baryon numbers before the process equals the sum of the baryon numbers after the process.

◀ Conservation of baryon number

Note that if the baryon number is absolutely conserved, the proton must be absolutely stable: if it were not for the law of conservation of baryon number, the proton could decay into a positron and a neutral pion. However, such a decay has never been observed. At present, we can only say that the proton has a half-life of at least $10^{31}$ years. (The estimated age of the Universe is about $10^{10}$ years.) In one recent version of a so-called grand unified theory (GUT), physicists have predicted that the proton is actually unstable. According to this theory, the baryon number (sometimes called the *baryonic charge*) is not absolutely conserved, whereas electric charge is always conserved.

### EXAMPLE 30.4 Checking Baryon Numbers

**Goal** Use conservation of baryon number to determine whether a given reaction can occur.

**Problem** Determine whether the following reaction can occur based on the law of conservation of baryon number.

$$p + n \rightarrow p + p + n + \bar{p}$$

Count the barvons on the right: There are three baryons and one antibaryon, so $1 + 1 + 1 + (-1) = 2$.

**Remark** Baryon number is conserved in this reaction, so it can occur, provided the incoming proton has sufficient energy.

***Exercise 30.4***

Can the following reaction occur, based on the law of conservation of baryon number?

$$p + n \rightarrow p + p + \bar{p}$$

**Answer** No. (Show this by computing the baryon number on both sides and finding that they're not equal.)

## Lepton Number

Conservation of lepton number ▶

There are three conservation laws involving lepton numbers, one for each variety of lepton. The **law of conservation of electron-lepton number** states that the sum of the electron-lepton numbers before a reaction or decay must equal the sum of the electron-lepton numbers after the reaction or decay. The electron and the electron neutrino are assigned a positive electron-lepton number $L_e = +1$, the antileptons $e^+$ and $\bar{\nu}_e$ are assigned the electron-lepton number $L_e = -1$, and all other particles have $L_e = 0$. For example, consider neutron decay:

Neutron decay ▶

$$n \rightarrow p^+ + e^- + \bar{\nu}_e$$

Before the decay, the electron-lepton number is $L_e = 0$; after the decay, it is $0 + 1 + (-1) = 0$, so the electron-lepton number is conserved. It's important to recognize that the baryon number must also be conserved. This can easily be seen by noting that before the decay $B = +1$, whereas after the decay $B = +1 + 0 + 0 = +1$.

Similarly, when a decay involves muons, the muon-lepton number $L_\mu$ is conserved. The $\mu^-$ and the $\nu_\mu$ are assigned $L_\mu = +1$, the antimuons $\mu^+$ and $\bar{\nu}_\mu$ are assigned $L_\mu = -1$, and all other particles have $L_\mu = 0$. Finally, the tau-lepton number $L_\tau$ is conserved, and similar assignments can be made for the $\tau$ lepton and its neutrino.

## EXAMPLE 30.5 Checking Lepton Numbers

**Goal** Use conservation of lepton number to determine whether a given process is possible.

**Problem** Determine which of the following decay schemes can occur on the basis of conservation of lepton number.

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \qquad (1)$$

$$\pi^+ \rightarrow \mu^+ + \nu_\mu + \nu_e \qquad (2)$$

***Exercise 30.5***

Determine whether the decay $\mu^- \rightarrow e^- + \bar{\nu}_e$ can occur.

**Quick Quiz 30.2**

Which of the following reactions cannot occur?
(a) $p + p \rightarrow p + p + \overline{p}$ (b) $n \rightarrow p + e^- + \overline{\nu}_e$
(c) $\mu^- \rightarrow e^- + \overline{\nu}_e + \nu_\mu$ (d) $\pi^- \rightarrow \mu^- + \overline{\nu}_\mu$

**Quick Quiz 30.3**

Which of the following reactions cannot occur?
(a) $p + \overline{p} \rightarrow 2\gamma$ (b) $\gamma + p \rightarrow n + \pi^0$
(c) $\pi^0 + n \rightarrow K^+ + \Sigma^-$ (d) $\pi^+ + p \rightarrow K^+ + \Sigma^+$

**Quick Quiz 30.4**

Suppose a claim is made that the decay of a neutron is given by $n \rightarrow p^+ + e^-$. Which of the following conservation laws are necessarily violated by this proposed decay scheme? (a) energy (b) linear momentum (c) electric charge (d) lepton number (e) baryon number

## 30.10 STRANGE PARTICLES AND STRANGENESS

Many particles discovered in the 1950s were produced by the nuclear interaction of pions with protons and neutrons in the atmosphere. A group of these particles, namely the K, Λ, and Σ particles, was found to exhibit unusual properties in their production and decay and hence were called *strange particles.*

One unusual property of strange particles is that they are always produced in pairs. For example, when a pion collides with a proton, two neutral strange particles are produced with high probability (Fig. 30.8) following the reaction

$$\pi^- + p^+ \rightarrow K^0 + \Lambda^0$$

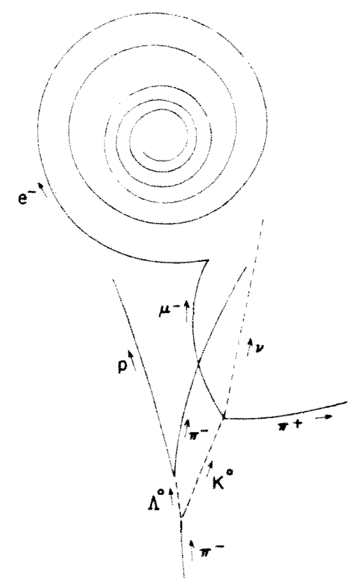
On the other hand, the reaction $\pi^- + p^+ \rightarrow K^0 + n$ has never occurred, even though it violates no known conservation laws and the energy of the pion is sufficient to initiate the reaction.

The second peculiar feature of strange particles is that although they are produced by the strong interaction at a high rate, they do not decay into particles that interact via the strong force at a very high rate. Instead, they decay very slowly, which is characteristic of the weak interaction. Their half-lives are in the range from $10^{-10}$ s to $10^{-8}$ s; most other particles that interact via the strong force have lifetimes on the order of $10^{-23}$ s.

To explain these unusual properties of strange particles, a law called *conservation of strangeness* was introduced, together with a new quantum number $S$ called **strangeness.** The strangeness numbers for some particles are given in Table 30.2. The production of strange particles in pairs is explained by assigning $S = +1$ to one of the particles and $S = -1$ to the other. All nonstrange particles are assigned strangeness $S = 0$. The **law of conservation of strangeness** states that whenever a nuclear reaction or decay occurs, the sum of the strangeness numbers before the process must equal the sum of the strangeness numbers after the process.

◀ Conservation of strangeness number

The slow decay of strange particles can be explained by assuming that the strong and electromagnetic interactions obey the law of conservation of strangeness, whereas the weak interaction does not. Because the decay reaction involves the loss of one strange particle, it violates strangeness conservation and hence proceeds slowly via the weak interaction.




Courtesy Lawrence Berkeley Laboratory, University of California

**Figure 30.8** This drawing represents tracks of many events obtained by analyzing a bubble-chamber photograph. The strange particles $\Lambda^0$ and $K^0$ are formed (at the bottom) as the $\pi^-$ interacts with a proton according to the interaction $\pi^- + p \rightarrow \Lambda^0 + K^0$. (Note that the neutral particles leave no tracks, as is indicated by the dashed lines.) The $\Lambda^0$ and $K^0$ then decay according to the interactions $\Lambda^0 \rightarrow \pi + p$ and $K^0 \rightarrow \pi + \mu^- + \nu_\mu$.

## Applying Physics 30.2 Breaking Conservation Laws

A student claims to have observed a decay of an electron into two neutrinos traveling in opposite directions. What conservation laws would be violated by this decay?

**Explanation** Several conservation laws are violated. Conservation of electric charge is violated because the negative charge of the electron has disappeared. Conservation of electron lepton number is also violated, because there is one lepton before the decay and two afterward. If both neutrinos were electron-neutrinos, electron lepton number conservation would be violated in the final state. However, if one of the product neutrinos were other than an electron-neutrino, then another lepton conservation law would be violated, because there were no other leptons in the initial state.

Other conservation laws are obeyed by this decay. Energy can be conserved—the rest energy of the electron appears as the kinetic energy (and possibly some small rest energy) of the neutrinos. The opposite directions of the velocities of the two neutrinos allow for the conservation of momentum. Conservation of baryon number and conservation of other lepton numbers are also upheld in this decay.

## EXAMPLE 30.6 Is Strangeness Conserved?

**Goal** Apply conservation of strangeness to determine whether a process can occur.

**Problem** Determine whether the following reactions can occur on the basis of conservation of strangeness:

$$\pi^0 + \text{n} \rightarrow \text{K}^+ + \Sigma^- \qquad (1)$$

$$\pi^- + \text{p} \rightarrow \pi^- + \Sigma^+ \qquad (2)$$

***Exercise 30.6***

Does the reaction $\text{p}^+ + \pi^- \rightarrow \text{K}^0 + \Lambda^0$ obey the law of conservation of strangeness? Show why or why not.

## 30.11 THE EIGHTFOLD WAY

Quantities such as spin, baryon number, lepton number, and strangeness are labels we associate with particles. Many classification schemes that group particles into families based on such labels have been proposed. First, consider the first eight baryons listed in Table 30.2, all having a spin of 1/2. The family consists of the proton, the neutron, and six other particles. If we plot their strangeness versus their charge using a sloping coordinate system, as in Figure 30.9a, a fascinating pattern emerges: six of the baryons form a hexagon, and the remaining two are at the hexagon's center. (Particles with spin quantum number 1/2 or 3/2 are called fermions.)

Now consider the family of mesons listed in Table 30.2 with spins of zero. (Particles with spin quantum number 0 or 1 are called bosons.) If we count both particles and antiparticles, there are nine such mesons. Figure 30.9b is a plot of strangeness versus charge for *this* family. Again, a fascinating hexagonal pattern emerges. In this case, the particles on the perimeter of the hexagon lie opposite their antiparticles, and the remaining three (which form their own antiparticles)

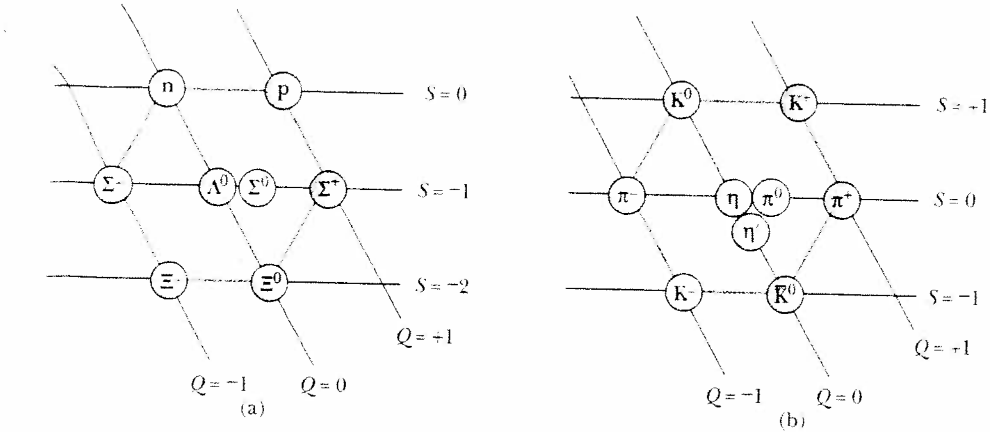



**Figure 30.9** (a) The hexagonal eightfold-way pattern for the eight spin $-\frac{1}{2}$ baryons. This strangeness versus charge plot uses a horizontal axis for the strangeness values $S$, but a sloping axis for the charge number $Q$. (b) The eightfold-way pattern for the nine spin-zero mesons.

are at its center. These and related symmetric patterns, called the **eightfold way**, were proposed independently in 1961 by Murray Gell-Mann and Yuval Ne'eman.

The groups of baryons and mesons can be displayed in many other symmetric patterns within the framework of the eightfold way. For example, the family of spin-3/2 baryons contains ten particles arranged in a pattern like the tenpins in a bowling alley. After the pattern was proposed, one of the particles was missing—it had yet to be discovered. Gell-Mann predicted that the missing particle, which he called the *omega minus* ($\Omega^-$), should have a spin of 3/2, a charge of $-1$, a strangeness of $-3$, and a mass of about 1 680 MeV/$c^2$. Shortly thereafter, in 1964, scientists at the Brookhaven National Laboratory found the missing particle through careful analyses of bubble chamber photographs and confirmed all its predicted properties.

The patterns of the eightfold way in the field of particle physics have much in common with the periodic table. Whenever a vacancy (a missing particle or element) occurs in the organized patterns, experimentalists have a guide for their investigations.

# 30.12 QUARKS

As we have noted, leptons appear to be truly elementary particles because they have no measurable size or internal structure, are limited in number, and do not seem to break down into smaller units. Hadrons, on the other hand, are complex particles with size and structure. Further, we know that hadrons decay into other hadrons and are many in number. Table 30.2 lists only those hadrons that are stable against hadronic decay; hundreds of others have been discovered. These facts strongly suggest that hadrons cannot be truly elementary but have some substructure.

## The Original Quark Model

In 1963 Gell-Mann and George Zweig independently proposed that hadrons have an elementary substructure. According to their model, all hadrons are composite systems of two or three fundamental constitutents called **quarks**, which rhymes with "forks" (though some rhyme it with "sharks"). Gell-Mann borrowed the word *quark* from the passage "Three quarks for Muster Mark" in James Joyce's book *Finnegans Wake*. In the original model there were three types of quarks designated by the symbols u, d, and s. These were given the arbitrary names *up*, *down*, and *sideways* (or, now more commonly, *strange*).

An unusual property of quarks is that they have fractional electronic charges, as shown—along with other properties—in Table 30.3 (page 994). Associated with each quark is an antiquark of opposite charge, baryon number, and strangeness. The compositions of all hadrons known when Gell-Mann and Zweig presented their models could be completely specified by three simple rules:

Photo courtesy of Michael R. Dressler

**MURRAY GELL-MANN,**
American Physicist (1929–)

Gell-Mann was awarded the Nobel Prize in 1969 for his theoretical studies dealing with subatomic particles.

TABLE 30.3

**Properties of Quarks and Antiquarks**

**Quarks**

| Name | Symbol | Spin | Charge | Baryon Number | Strangeness | Charm | Bottomness | Topness |
|---|---|---|---|---|---|---|---|---|
| Up | u | $\frac{1}{2}$ | $+\frac{2}{3}e$ | $\frac{1}{3}$ | 0 | 0 | 0 | 0 |
| Down | d | $\frac{1}{2}$ | $-\frac{1}{3}e$ | $\frac{1}{3}$ | 0 | 0 | 0 | 0 |
| Strange | s | $\frac{1}{2}$ | $-\frac{1}{3}e$ | $\frac{1}{3}$ | −1 | 0 | 0 | 0 |
| Charmed | c | $\frac{1}{2}$ | $+\frac{2}{3}e$ | $\frac{1}{3}$ | 0 | +1 | 0 | 0 |
| Bottom | b | $\frac{1}{2}$ | $-\frac{1}{3}e$ | $\frac{1}{3}$ | 0 | 0 | +1 | 0 |
| Top | t | $\frac{1}{2}$ | $+\frac{2}{3}e$ | $\frac{1}{3}$ | 0 | 0 | 0 | +1 |

**Antiquarks**

| Name | Symbol | Spin | Charge | Baryon Number | Strangeness | Charm | Bottomness | Topness |
|---|---|---|---|---|---|---|---|---|
| Anti-up | $\bar{u}$ | $\frac{1}{2}$ | $-\frac{2}{3}e$ | $-\frac{1}{3}$ | 0 | 0 | 0 | 0 |
| Anti-down | $\bar{d}$ | $\frac{1}{2}$ | $+\frac{1}{3}e$ | $-\frac{1}{3}$ | 0 | 0 | 0 | 0 |
| Anti-strange | $\bar{s}$ | $\frac{1}{2}$ | $+\frac{1}{3}e$ | $-\frac{1}{3}$ | +1 | 0 | 0 | 0 |
| Anti-charmed | $\bar{c}$ | $\frac{1}{2}$ | $-\frac{2}{3}e$ | $-\frac{1}{3}$ | 0 | −1 | 0 | 0 |
| Anti-bottom | $\bar{b}$ | $\frac{1}{2}$ | $+\frac{1}{3}e$ | $-\frac{1}{3}$ | 0 | 0 | −1 | 0 |
| Anti-top | $\bar{t}$ | $\frac{1}{2}$ | $-\frac{2}{3}e$ | $-\frac{1}{3}$ | 0 | 0 | 0 | −1 |

TABLE 30.4

**Quark Composition of Several Hadrons**

| Particle | Quark Composition |
|---|---|
| **Mesons** | |
| $\pi^+$ | $\bar{d}u$ |
| $\pi^-$ | $\bar{u}d$ |
| $K^+$ | $\bar{s}u$ |
| $K^-$ | $\bar{u}s$ |
| $K^0$ | $\bar{s}d$ |
| **Baryons** | |
| p | uud |
| n | udd |
| $\Lambda^0$ | uds |
| $\Sigma^+$ | uus |
| $\Sigma^0$ | uds |
| $\Sigma^-$ | dds |
| $\Xi^0$ | uss |
| $\Xi^-$ | dss |
| $\Omega^-$ | sss |

1. Mesons consist of one quark and one antiquark, giving them a baryon number of 0, as required.
2. Baryons consist of three quarks.
3. Antibaryons consist of three antiquarks.

Table 30.4 lists the quark compositions of several mesons and baryons. Note that just two of the quarks, u and d, are contained in all hadrons encountered in ordinary matter (protons and neutrons). The third quark, s, is needed only to construct strange particles with a strangeness of either +1 or −1. Active Figure 30.10 is a pictorial representation of the quark compositions of several particles.

## Applying Physics 30.3 Conservation of Meson Number?

We have seen a law of conservation of lepton number and a law of conservation of baryon number. Why isn't there a law of conservation of meson number?

**Explanation** We can argue this from the point of view of creating particle–antiparticle pairs from available energy. If energy is converted to the rest energy of a lepton–antilepton pair, then there is no net change in lepton number, because the lepton has a lepton number of +1 and the antilepton −1. Energy could also be transformed into the rest energy of a baryon–antibaryon pair. The baryon has baryon number +1, the antibaryon −1, and there is no net change in baryon number.

But now suppose energy is transformed into the rest energy of a quark–antiquark pair. By definition in quark theory, a quark–antiquark pair is a meson. There was no meson before, and now there's a meson, so already there is violation of conservation of meson number. With more energy, we can create more mesons, with no restriction from a conservation law other than that of energy.

### Charm and Other Recent Developments

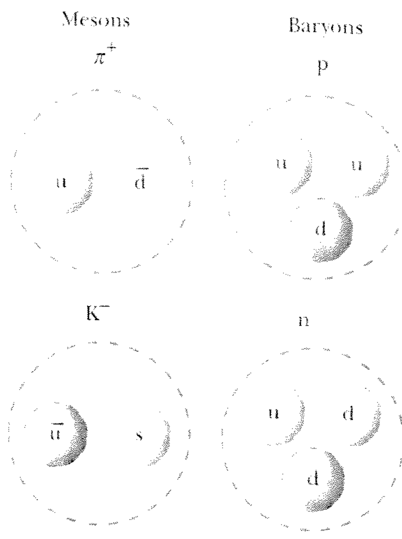
Although the original quark model was highly successful in classifying particles into families, there were some discrepancies between predictions of the model and certain experimental decay rates. Consequently, a fourth quark was proposed by several physicists in 1967. The fourth quark, designated by c, was given a property called **charm**. A charmed quark would have the charge $+2e/3$, but its charm would distinguish it from the other three quarks. The new quark would have a charm $C = +1$, its antiquark would have a charm $C = -1$, and all other quarks would have $C = 0$, as indicated in Table 30.3. Charm, like strangeness, would be conserved in strong and electromagnetic interactions but not in weak interactions.

In 1974 a new heavy meson called the $J/\psi$ particle (or simply, $\psi$) was discovered independently by a group led by Burton Richter at the Stanford Linear Accelerator (SLAC) and another group led by Samuel Ting at the Brookhaven National Laboratory. Richter and Ting were awarded the Nobel Prize in 1976 for this work. The $J/\psi$ particle didn't fit into the three-quark model, but had the properties of a combination of a charmed quark and its antiquark ($c\bar{c}$). It was much heavier than the other known mesons ($\sim 3\ 100\ \text{MeV}/c^2$) and its lifetime was much longer than those of other particles that decay via the strong force. In 1975, researchers at Stanford University reported strong evidence for the existence of the tau ($\tau$) lepton, with a mass of $1\ 784\ \text{MeV}/c^2$. Such discoveries led to more elaborate quark models and the proposal of two new quarks, named *top* (t) and *bottom* (b). To distinguish these quarks from the old ones, quantum numbers called *topness* and *bottomness* were assigned to these new particles and are included in Table 30.3. In 1977 researchers at the Fermi National Laboratory, under the direction of Leon Lederman, reported the discovery of a very massive new meson $\Upsilon$ with composition $b\bar{b}$. In March of 1995, researchers at Fermilab announced the discovery of the top quark (supposedly the last of the quarks to be found) having mass $173\ \text{GeV}/c^2$.

You are probably wondering whether such discoveries will ever end. How many "building blocks" of matter really exist? The numbers of different quarks and leptons have implications for the primordial abundance of certain elements, so at present it appears there may be no further fundamental particles. Some properties of quarks and leptons are given in Table 30.5.

Despite extensive experimental efforts, no isolated quark has ever been observed. Physicists now believe that quarks are permanently confined inside ordinary particles because of an exceptionally strong force that prevents them from escaping. This force, called the color force (which will be discussed in Section 30.13), increases with separation distance (similar to the force of a spring). The great strength of the force between quarks has been described by one author as follows:[2]

> Quarks are slaves of their own color charge, . . . bound like prisoners of a chain gang. . . . Any locksmith can break the chain between two prisoners, but no locksmith is expert enough to break the gluon chains between quarks. Quarks remain slaves forever.




ACTIVE FIGURE 30.10
Quark compositions of two mesons and two baryons. Note that the mesons on the left contain two quarks, and the baryons on the right contain three quarks.

Physics Now™
Log into PhysicsNow at **www.cp7e.com** and go to Active Figure 30.10 to observe the quark compositions for the mesons and baryons.

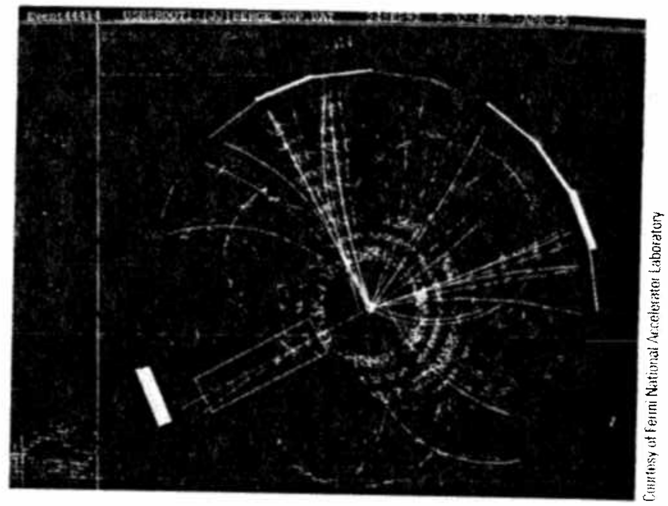
## TABLE 30.5

**The Fundamental Particles and Some of Their Properties**

| Particle | Rest Energy | Charge |
|---|---|---|
| **Quarks** | | |
| u | 360 MeV | $+\frac{2}{3}e$ |
| d | 360 MeV | $-\frac{1}{3}e$ |
| c | 1500 MeV | $+\frac{2}{3}e$ |
| s | 540 MeV | $-\frac{1}{3}e$ |
| t | 173 GeV | $+\frac{2}{3}e$ |
| b | 5 GeV | $-\frac{1}{3}e$ |
| **Leptons** | | |
| $e^-$ | 511 keV | $-e$ |
| $\mu^-$ | 107 MeV | $-e$ |
| $\tau^-$ | 1784 MeV | $-e$ |
| $\nu_e$ | <30 eV | 0 |
| $\nu_\mu$ | <0.5 MeV | 0 |
| $\nu_\tau$ | <250 MeV | 0 |

[2]Harald Fritzsch, *Quarks: The Stuff of Matter* (London: Allen Lane, 1983).

Computers at Fermilab create a pictorial representation such as this of the paths of particles after a collision.




## 30.13 COLORED QUARKS

Shortly after the theory of quarks was proposed, scientists recognized that certain particles had quark compositions that were in violation of the Pauli exclusion principle. Because all quarks have spins of 1/2, they are expected to follow the exclusion principle. One example of a particle that violates the exclusion principle is the $\Omega^-$ (sss) baryon, which contains three *s* quarks having parallel spins, giving it a total spin of 3/2. Other examples of baryons that have identical quarks with parallel spins are the $\Delta^{++}$ (uuu) and the $\Delta^-$ (ddd). To resolve this problem, Moo-Young Han and Yoichiro Nambu suggested in 1965 that quarks possess a new property called **color** or **color charge**. This "charge" property is similar in many respects to electric charge, except that it occurs in three varieties, labeled *red*, *green*, and *blue*! (The antiquarks are labeled *anti-red*, *anti-green*, and *anti-blue.*) To satisfy the exclusion principle, all three quarks in a baryon must have different colors. Just as a combination of actual colors of light can produce the neutral color white, a combination of three quarks with different colors is also "white," or colorless. A meson consists of a quark of one color and an antiquark of the corresponding anticolor. The result is that baryons and mesons are always colorless (or white).

**TIP 30.3 Color is Not Really Color**

When we use the word *color* to describe a quark, it has nothing to do with visual sensation from light. It is simply a convenient name for a property analgous to electric charge.

Although the concept of color in the quark model was originally conceived to satisfy the exclusion principle, it also provided a better theory for explaining certain experimental results. For example, the modified theory correctly predicts the lifetime of the $\pi^0$ meson. The theory of how quarks interact with each other by means of color charge is called **quantum chromodynamics**, or QCD, to parallel quantum electrodynamics (the theory of interactions among electric charges). In QCD, the quark is said to carry a **color charge**, in analogy to electric charge. The strong force between quarks is often called the **color force**. The force is carried by massless particles called **gluons** (which are analogous to photons for the electromagnetic force). According to QCD, there are eight gluons, all with color charge. When a quark emits or absorbs a gluon, its color changes. For example, a blue quark that emits a gluon may become a red quark, and a red quark that absorbs this gluon becomes a blue quark. The color force between quarks is analogous to the electric force between charges: Like colors repel and opposite colors attract. Therefore, two red quarks repel each other, but a red quark will be attracted to an anti-red quark. The attraction between quarks of opposite color to form a meson ($q\bar{q}$) is indicated in Figure 30.11a.

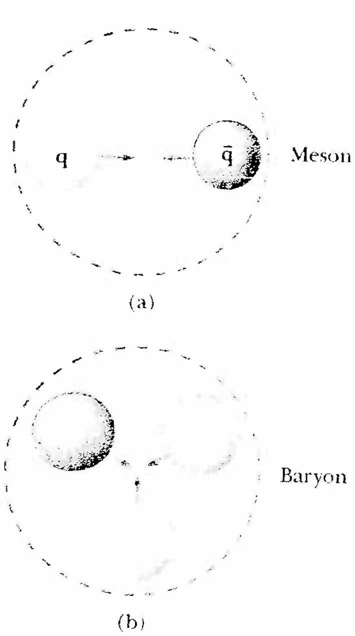



**Figure 30.11** (a) A green quark is attracted to an anti-green quark to form a meson with quark structure ($q\bar{q}$). (b) Three different-colored quarks attract each other to form a baryon.

Different-colored quarks also attract each other, but with less intensity than opposite colors of quark and antiquark. For example, a cluster of red, blue, and green quarks all attract each other to form baryons, as indicated in Figure 30.11b. Every baryon contains three quarks of three different colors.

Although the color force between two color-neutral hadrons (such as a proton and a neutron) is negligible at large separations, the strong color force between their constituent quarks does not exactly cancel at small separations of about 1 fm. **This residual strong force is in fact the nuclear force that binds protons and**

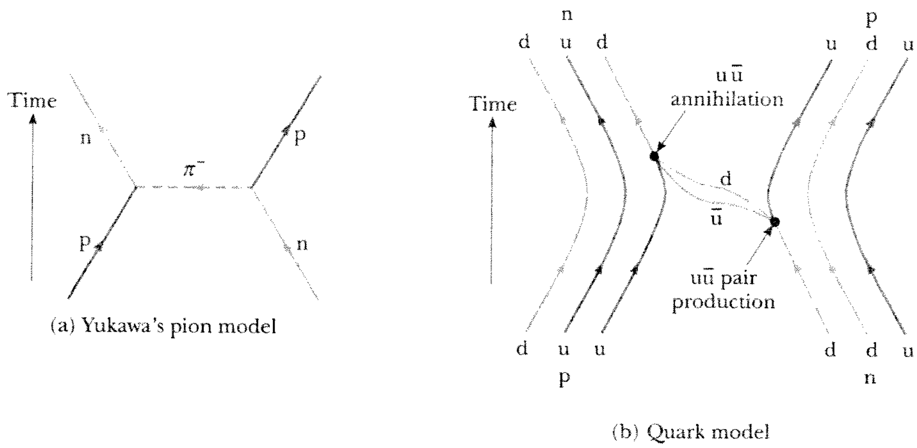



**Figure 30.12** (a) A nuclear interaction between a proton and a neutron explained in terms of Yukawa's pion exchange model. Because the pion carries charge, the proton and neutron switch identities. (b) The same interaction explained in terms of quarks and gluons. Note that the exchanged $\bar{u}d$ quark pair makes up a $\pi^-$ meson.

**neutrons to form nuclei**. It is similar to the residual electromagnetic force that binds neutral atoms into molecules. According to QCD, a more basic explanation of nuclear force can be given in terms of quarks and gluons, as shown in Figure 30.12, which shows contrasting Feynman diagrams of the same process. Each quark within the neutron and proton is continually emitting and absorbing virtual gluons and creating and annihilating virtual ($q\bar{q}$) pairs. When the neutron and proton approach within 1 fm of each other, these virtual gluons and quarks can be exchanged between the two nucleons, and such exchanges produce the nuclear force. Figure 30.12b depicts one likely possibility or contribution to the process shown in Figure 30.12a: a down quark emits a virtual gluon (represented by a wavy line in Fig. 30.12b), which creates a $u\bar{u}$ pair. Both the recoiling d quark and the $\bar{u}$ are transmitted to the proton where the $\bar{u}$ annihilates a proton u quark (with the creation of a gluon) and the d is captured.

## 30.14 ELECTROWEAK THEORY AND THE STANDARD MODEL

Recall that the weak interaction is an extremely short range force having an interaction distance of approximately $10^{-18}$ m (Table 30.1). Such a short-range interaction implies that the quantized particles which carry the weak field (the spin one $W^+$, $W^-$, and $Z^0$ bosons) are extremely massive, as is indeed the case. These amazing bosons can be thought of as structureless, pointlike particles as massive as krypton atoms! The weak interaction is responsible for the decay of the c, s, b, and t quarks into lighter, more stable u and d quarks, as well as the decay of the massive $\mu$ and $\tau$ leptons into (lighter) electrons. **The weak interaction is very important because it governs the stability of the basic particles of matter.**

A mysterious feature of the weak interaction is its lack of symmetry, especially when compared to the high degree of symmetry shown by the strong, electromagnetic, and gravitational interactions. For example, the weak interaction, unlike the strong interaction, is not symmetric under mirror reflection or charge exchange. (*Mirror reflection* means that all the quantities in a given particle reaction are exchanged as in a mirror reflection—left for right, an inward motion toward the mirror for an outward motion, etc. *Charge exchange* means that all the electric charges in a particle reaction are converted to their opposites—all positives to negatives and vice versa.) When we say that the weak interaction is not symmetric, we mean that the reaction with all quantities changed occurs less frequently than the direct reaction. For example, the decay of the $K^0$, which is governed by the weak interaction, is not symmetric under charge exchange because the reaction $K^0 \rightarrow \pi^- + e^+ + \nu_e$ occurs much more frequently than the reaction $K^0 \rightarrow \pi^+ + e^- + \bar{\nu}_e$.

In 1979, Sheldon Glashow, Abdus Salam, and Steven Weinberg won a Nobel prize for developing a theory called the **electroweak theory** that unified the electromagnetic and weak interactions. This theory postulates that the weak and electromagnetic interactions have the same strength at very high particle energies,

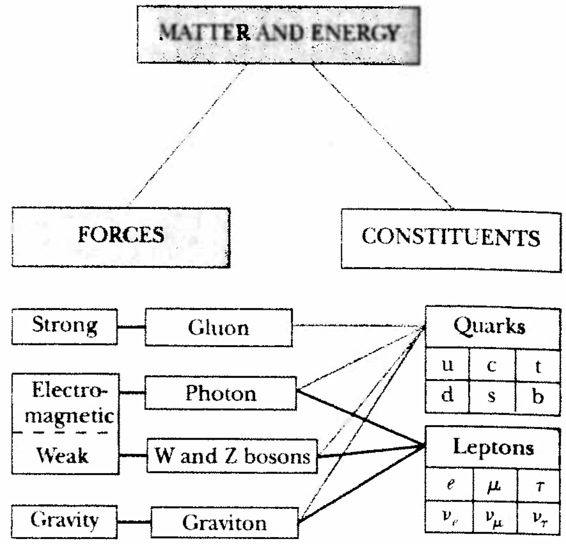



**Figure 30.13** The Standard Model of particle physics.

and are different manifestations of a single unifying electroweak interaction. The photon and the three massive bosons ($W^{\pm}$ and $Z^0$) play a key role in the electroweak theory. The theory makes many concrete predictions, but perhaps the most spectacular is the prediction of the masses of the W and Z particles at about 82 GeV/$c^2$ and 93 GeV/$c^2$, respectively. A 1984 Nobel Prize was awarded to Carlo Rubbia and Simon van der Meer for their work leading to the discovery of these particles at just those energies at the CERN Laboratory in Geneva, Switzerland.

The combination of the electroweak theory and QCD for the strong interaction form what is referred to in high energy physics as the **Standard Model**. Although the details of the Standard Model are complex, its essential ingredients can be summarized with the help of Figure 30.13. The strong force, mediated by gluons, holds quarks together to form composite particles such as protons, neutrons, and mesons. Leptons participate only in the electromagnetic and weak interactions. The electromagnetic force is mediated by photons, and the weak force is mediated by W and Z bosons. Note that all fundamental forces are mediated by bosons (particles with spin 1) whose properties are given, to a large extent, by symmetries involved in the theories.

However, the Standard Model does not answer all questions. A major question is why the photon has no mass while the W and Z bosons do. Because of this mass difference, the electromagnetic and weak forces are quite distinct at low energies, but become similar in nature at very high energies, where the rest energies of the W and Z bosons are insignificant fractions of their total energies. This behavior during the transition from high to low energies, called **symmetry breaking**, doesn't answer the question of the origin of particle masses. To resolve that problem, a hypothetical particle called the **Higgs boson** has been proposed which provides a mechanism for breaking the electroweak symmetry and bestowing different particle masses on different particles. The Standard Model, including the Higgs mechanism, provides a logically consistent explanation of the massive nature of the W and Z bosons. Unfortunately, the Higgs boson has not yet been found, but physicists know that its mass should be less than 1 TeV/$c^2$ ($10^{12}$ eV).

In order to determine whether the Higgs boson exists, two quarks of at least 1 TeV of energy must collide, but calculations show that this requires injecting 40 TeV of energy within the volume of a proton. Scientists are convinced that because of the limited energy available in conventional accelerators using fixed targets, it is necessary to build colliding-beam accelerators called **colliders**. The concept of a collider is straightforward. In such a device, particles with equal masses and kinetic energies, traveling in opposite directions in an accelerator ring, collide head-on to produce the required reaction and the formation of new particles. Because the total momentum of the interacting particles is zero, all of their kinetic energy is available for the reaction. The Large Electron–Positron (LEP) collider at CERN, near Geneva, Switzerland, and the Stanford Linear Collider in California collide both electrons and positrons. The Super Proton Synchrotron at CERN accelerates

Courtesy of CERN

A view from inside the Large Electron–Positron (LEP) collider tunnel, which is 27 km in circumference.

protons and antiprotons to energies of 270 GeV, and the world's highest-energy proton acclerator, the Tevatron, at the Fermi National Laboratory in Illinois, produces protons at almost 1 000 GeV (or 1 TeV). CERN has started construction of the Large Hadron Collider (LHC), a proton–proton collider that will provide a center-of-mass energy of 14 TeV and allow an exploration of Higgs-boson physics. The accelerator is being constructed in the same 27-km circumference tunnel as CERN's LEP collider, and construction is expected to be completed in 2005.

Following the success of the electroweak theory, scientists attempted to combine it with QCD in a **grand unification theory** (GUT). In this model, the electroweak force was merged with the strong color force to form a grand unified force. One version of the theory considers leptons and quarks as members of the same family that are able to change into each other by exchanging an appropriate particle. Many GUT theories predict that protons are unstable and will decay with a lifetime of about $10^{31}$ years, a period far greater than the age of the Universe. As yet, proton decays have not been observed.

## Applying Physics 30.4 Head-on Collisions

Consider a car making a head-on collision with an identical car moving in the opposite direction at the same speed. Compare that collision to one in which one of the cars collides with a second car that is at rest. In which collision is there a larger transformation of kinetic energy to other forms? How does this idea relate to producing exotic particles in collisions?

**Explanation** In the head-on collision with both cars moving, conservation of momentum causes most, if not all, of the kinetic energy to be transformed to other forms. In the collision between a moving car and a stationary car, the cars are still moving after the collision in the direction of the moving car, but with reduced speed. Thus, only part of the kinetic energy is transformed to other forms. This suggests the advantage of using colliding beams to produce exotic particles, as opposed to firing a beam into a stationary target. When particles moving in opposite directions collide, all of the kinetic energy is available for transformation into other forms—in this case, the creation of new particles. When a beam is fired into a stationary target, only part of the energy is available for transformation, so particles of higher mass cannot be created.

## 30.15 THE COSMIC CONNECTION

In this section we describe one of the most fascinating theories in all of science—the Big Bang theory of the creation of the Universe—and the experimental evidence that supports it. This theory of cosmology states that the Universe had a beginning and that this beginning was so cataclysmic that it is impossible to look back beyond it. According to the theory, the Universe erupted from an infinitely dense singularity about 15 to 20 billion years ago. The first few minutes after the Big Bang saw such extremes of energy that it is believed that all four interactions of physics were unified and all matter was contained in an undifferentiated "quark soup."

The evolution of the four fundamental forces from the Big Bang to the present is shown in Figure 30.14. During the first $10^{-43}$ s (the ultrahot epoch, with

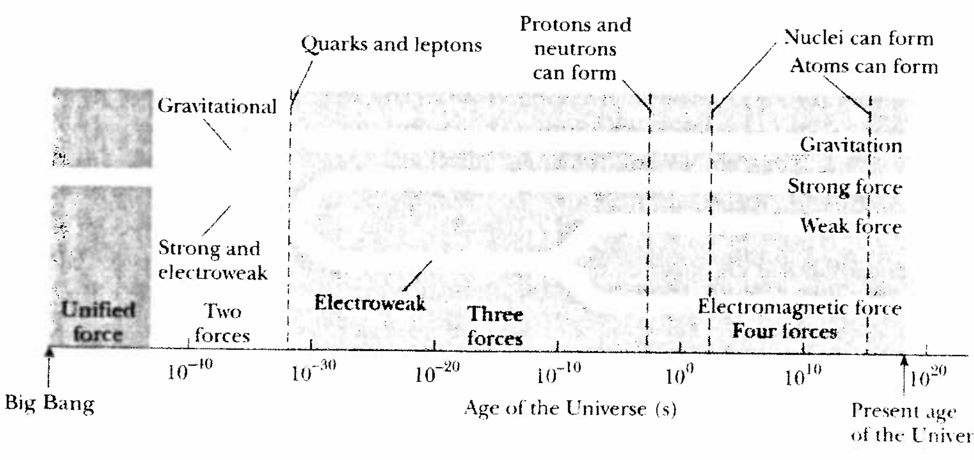



**Figure 30.14** A brief history of the Universe from the Big Bang to the present. The four forces became distinguishable during the first microsecond. Following this, all the quarks combined to form particles that interact via the strong force. The leptons remained separate, however, and exist as individually observable particles to this day.

Courtesy of AIP Emilio Segre Visual Archives

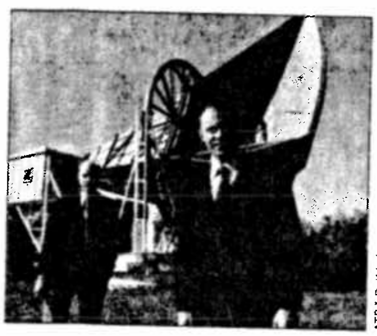
**GEORGE GAMOW (1904–1968)**

Gamow and two of his students, Ralph Alpher and Robert Herman, were the first to take the first half hour of the Universe seriously. In a mostly overlooked paper published in 1948, they made truly remarkable cosmological predictions. They correctly calculated the abundances of hydrogen and helium after the first half hour (75% H and 25% He) and predicted that radiation from the Big Bang should still be present and have an apparent temperature of about 5 K.

$T \approx 10^{32}$ K), it is presumed that the strong, electroweak, and gravitational forces were joined to form a completely unified force. In the first $10^{-35}$ s following the Big Bang (the hot epoch, with $T \approx 10^{29}$ K), gravity broke free of this unification and the strong and electroweak forces remained as one, described by a grand unification theory. This was a period when particle energies were so great ($> 10^{16}$ GeV) that very massive particles as well as quarks, leptons, and their antiparticles, existed. Then, after $10^{-35}$ s, the Universe rapidly expanded and cooled (the warm epoch, with $T \approx 10^{29}$ to $10^{15}$ K), the strong and electroweak forces parted company, and the grand unification scheme was broken. As the Universe continued to cool, the electroweak force split into the weak force and the electromagnetic force about $10^{-10}$ s after the Big Bang.

After a few minutes, protons condensed out of the hot soup. For half an hour the Universe underwent thermonuclear detonation, exploding like a hydrogen bomb and producing most of the helium nuclei now present. The Universe continued to expand, and its temperature dropped. Until about 700 000 years after the Big Bang, the Universe was dominated by radiation. Energetic radiation prevented matter from forming single hydrogen atoms because collisions would instantly ionize any atoms that might form. Photons underwent continuous Compton scattering from the vast number of free electrons, resulting in a Universe that was opaque to radiation. By the time the Universe was about 700 000 years old, it had expanded and cooled to about 3 000 K, and protons could bind to electrons to form neutral hydrogen atoms. Because the energies of the atoms were quantized, far more wavelengths of radiation were not absorbed by atoms than were, and the Universe suddenly became transparent to photons. Radiation no longer dominated the Universe, and clumps of neutral matter grew steadily—first atoms, followed by molecules, gas clouds, stars, and finally galaxies.

## Observation of Radiation from the Primordial Fireball



AT&T Bell Laboratories

**Figure 30.15** Robert W. Wilson (*left*) and Arno A. Penzias (*right*), with Bell Telephone Laboratories' horn-reflector antenna.

In 1965 Arno A. Penzias (b. 1933) and Robert W. Wilson (b. 1936) of Bell Laboratories made an amazing discovery while testing a sensitive microwave receiver. A pesky signal producing a faint background hiss was interfering with their satellite communications experiments. In spite of their valiant efforts, the signal remained. Ultimately it became clear that they were observing microwave background radiation (at a wavelength of 7.35 cm) representing the leftover "glow" from the Big Bang.

The microwave horn that served as their receiving antenna is shown in Figure 30.15. The intensity of the detected signal remained unchanged as the antenna was pointed in different directions. The fact that the radiation had equal strengths in all directions suggested that the entire Universe was the source of this radiation. Evicting a flock of pigeons from the 20-foot horn and cooling the microwave detector both failed to remove the signal. Through a casual conversation, Penzias and Wilson discovered that a group at Princeton had predicted the residual radiation from the Big Bang and were planning an experiment to confirm the theory. The excitement in the scientific community was high when Penzias and Wilson announced that they had already observed an excess microwave background compatible with a 3-K blackbody source.

Because Penzias and Wilson made their measurements at a single wavelength, they did not completely confirm the radiation as 3-K blackbody radiation. Subsequent experiments by other groups added intensity data at different wavelengths, as shown in Figure 30.16. The results confirm that the radiation is that of a blackbody at 2.9 K. This figure is perhaps the most clear-cut evidence for the Big Bang theory. The 1978 Nobel Prize in physics was awarded to Penzias and Wilson for their important discovery.

The discovery of the cosmic background radiation produced a problem, however: the radiation was too uniform. Scientists believed there had to be slight fluctuations in this background in order for such objects as galaxies to form. In 1989, NASA launched a satellite called the Cosmic Background Explorer (COBE, pronounced KOH-bee) to study this radiation in greater detail. In 1992, George Smoot

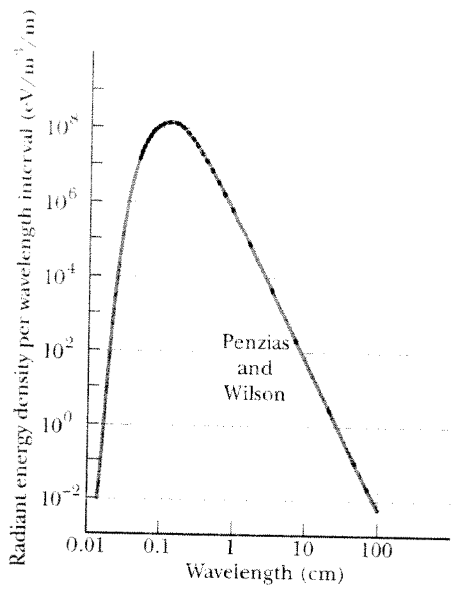



**Figure 30.16** Theoretical blackbody (brown curve) and measured radiation spectra (blue points) of the Big Bang. Most of the data were collected from the Cosmic Background Explorer (COBE) satellite. The datum of Wilson and Penzias is indicated.

(b. 1945) at the Lawrence Berkeley Laboratory found that the background was not perfectly uniform, but instead contained irregularities corresponding to temperature variations of 0.000 3 K. It is these small variations that provided nucleation sites for the formation of the galaxies and other objects we now see in the sky.

## 30.16 PROBLEMS AND PERSPECTIVES

While particle physicists have been exploring the realm of the very small, cosmologists have been exploring cosmic history back to the first microsecond of the Big Bang. Observation of the events that occur when two particles collide in an accelerator is essential in reconstructing the early moments in cosmic history. Perhaps the key to understanding the early Universe is first to understand the world of elementary particles. Cosmologists and particle physicists find that they have many common goals and are joining efforts to study the physical world at its most fundamental level.

Our understanding of physics at short and long distances is far from complete. Particle physics is faced with many questions: why is there so little antimatter in the Universe? Do neutrinos have a small mass, and if so, how much do they contribute to the "dark matter" holding the universe together gravitationally? How can we understand the latest astronomical measurements, which show that the expansion of the universe is accelerating and that there may be a kind of "antigravity force" acting between widely separated galaxies? Is it possible to unify the strong and electroweak theories in a logical and consistent manner? Why do quarks and leptons form three similar but distinct families? Are muons the same as electrons (apart from their different masses), or do they have subtle differences that have not been detected? Why are some particles charged and others neutral? Why do quarks carry a fractional charge? What determines the masses of the fundamental particles? The questions go on and on. Because of the rapid advances and new discoveries in the related fields of particle physics and cosmology, by the time you read this book some of these questions may have been resolved and others may have emerged.

An important question that remains is whether leptons and quarks have a substructure. If they do, one could envision an infinite number of deeper structure levels. However, if leptons and quarks are indeed the ultimate constituents of matter, as physicists today tend to believe, we should be able to construct a final theory of the structure of matter, as Einstein dreamed of doing. In the view of many physicists, the end of the road is in sight, but how long it will take to reach that goal is anyone's guess.

# SUMMARY

**Physics Now™** Take a practice test by logging into PhysicsNow at **www.cp7e.com** and clicking on the Pre-Test link for this chapter.

## 30.1 Nuclear Fission &

## 30.2 Nuclear Reactors

In **nuclear fission**, the total mass of the products is always less than the original mass of the reactants. Nuclear fission occurs when a heavy nucleus splits, or fissions, into two smaller nuclei. The lost mass is transformed into energy, electromagnetic radiation, and the kinetic energy of daughter particles.

A **nuclear reactor** is a system designed to maintain a self-sustaining chain reaction. Nuclear reactors using controlled fission events are currently being used to generate electric power. A useful parameter for describing the level of reactor operation is the reproduction constant $K$, which is the average number of neutrons from each fission event that will cause another event. A self-sustaining reaction is achieved when $K = 1$.

## 30.3 Nuclear Fusion

In nuclear fusion, two light nuclei combine to form a heavier nucleus. This type of nuclear reaction occurs in the Sun, assisted by a quantum tunneling process that helps particles get through the Coulomb barrier.

Controlled fusion events offer the hope of plentiful supplies of energy in the future. The nuclear fusion reactor is considered by many scientists to be the ultimate energy source because its fuel is water. **Lawson's criterion** states that a fusion reactor will provide a net output power if the product of the plasma ion density $n$ and the plasma confinement time $\tau$ satisfies the following relationships:

$n\tau \geq 10^{14}\ \mathrm{s/cm^3}$ Deuterium–tritium interaction **[30.5]**

$n\tau \geq 10^{16}\ \mathrm{s/cm^3}$ Deuterium–deuterium interaction

## 30.5 The Fundamental Forces of Nature

There are four fundamental forces of nature: the **strong** (hadronic), **electromagnetic**, **weak**, and **gravitational** forces. The strong force is the force between nucleons that keeps the nucleus together. The weak force is responsible for beta decay. The electromagnetic and weak forces are now considered to be manifestations of a single force called the **electroweak** force.

Every fundamental interaction is said to be mediated by the exchange of field particles. The electromagnetic interaction is mediated by the photon, the weak interaction by the $W^{\pm}$ and $Z^0$ bosons, the gravitational interaction by gravitons, and the strong interaction by gluons.

## 30.6 Positrons and Other Antiparticles

An antiparticle and a particle have the same mass, but opposite charge, and may also have other properties with opposite values, such as lepton number and baryon number. It is possible to produce particle–antiparticle pairs in nuclear reactions if the available energy is greater than $2mc^2$, where $m$ is the mass of the particle (or antiparticle).

## 30.8 Classification of Particles

Particles other than photons are classified as hadrons or leptons. **Hadrons** interact primarily through the strong force. They have size and structure and hence are not elementary particles. There are two types of hadrons: *baryons* and *mesons*. Mesons have a baryon number of zero and have either zero or integer spin. Baryons, which generally are the most massive particles, have nonzero baryon numbers and spins of 1/2 or 3/2. The neutron and proton are examples of baryons.

**Leptons** have no known structure, down to the limits of current resolution (about $10^{-19}$ m). Leptons interact only through the weak and electromagnetic forces. There are six leptons: the electron, $e^-$; the muon, $\mu^-$; the tau, $\tau^-$; and their associated neutrinos, $\nu_e$, $\nu_\mu$, and $\nu_\tau$.

## 30.9 Conservation Laws &

## 30.10 Strange Particles and Strangeness

In all reactions and decays, quantities such as energy, linear momentum, angular momentum, electric charge, baryon number, and lepton number are strictly conserved. Certain particles have properties called **strangeness** and **charm**. These unusual properties are conserved only in those reactions and decays that occur via the strong force.

## 30.12 Quarks &
## 30.13 Colored Quarks

Recent theories postulate that all hadrons are composed of smaller units known as **quarks** which have fractional electric charges and baryon numbers of 1/3 and come in six "flavors": up, down, strange, charmed, top, and bottom. Each baryon contains three quarks, and each meson contains one quark and one antiquark.

According to the theory of **quantum chromodynamics**, quarks have a property called **color**, and the strong force between quarks is referred to as the **color force**. The color force increases as the distance between particles increases, so quarks are confined and are never observed in isolation. When two bound quarks are widely separated, a new quark–antiquark pair forms between them, and the single particle breaks into two new particles, each composed of a quark–antiquark pair.

## 30.15 The Cosmic Connection

Observation of background microwave radiation by Penzias and Wilson strongly confirmed that the Universe started with a Big Bang about 15 billion years ago and has been expanding ever since. The background radiation is equivalent to that of a blackbody at a temperature of about 3 K.

The cosmic microwave background has very small irregularities, corresponding to temperature variations of 0.000 3 K. Without these irregularities acting as nucleation sites, particles would never have clumped together to form galaxies and stars.

# CONCEPTUAL QUESTIONS

1. If high-energy electrons with de Broglie wavelengths smaller than the size of the nucleus are scattered from nuclei, the behavior of the electrons is consistent with scattering from very massive structures much smaller in size than the nucleus, namely, quarks. How is this similar to a classic experiment that detected small structures in an atom?
2. What factors make a fusion reaction difficult to achieve?
3. Doubly charged baryons are known to exist. Why are there no doubly charged mesons?
4. Why would a fusion reactor produce less radioactive waste than a fission reactor?
5. Atoms didn't exist until hundreds of thousands of years after the Big Bang. Why?
6. Particles known as resonances have very short half-lives, on the order of $10^{-23}$ s. Would you guess they are hadrons or leptons?
7. Describe the quark model of hadrons, including the properties of quarks.
8. In the theory of quantum chromodynamics, quarks come in three colors. How would you justify the statement "All baryons and mesons are colorless?"
9. Describe the properties of baryons and mesons and the important differences between them.
10. Identify the particle decays in Table 30.2 that occur by the electromagnetic interaction. Justify your answer.
11. Kaons all decay into final states that contain no protons or neutrons. What is the baryon number of kaons?
12. When an electron and a positron meet at low speeds in free space, why are *two* 0.511-MeV gamma rays produced, rather than *one* gamma ray with an energy of 1.02 MeV?
13. Two protons in a nucleus interact via the strong interaction. Are they also subject to a weak interaction?
14. Why is a neutron stable inside the nucleus? (In free space, the neutron decays in 900 s.)
15. An antibaryon interacts with a meson. Can a baryon be produced in such an interaction? Explain.
16. Why is water a better shield against neutrons than lead or steel is?
17. How many quarks are there in (a) a baryon, (b) an antibaryon, (c) a meson, and (d) an antimeson? How do you account for the fact that baryons have half-integral spins and mesons have spins of 0 or 1? [*Hint*: quarks have spin $\frac{1}{2}$.]

**18.** A typical chemical reaction is one in which a water molecule is formed by combining hydrogen and oxygen. In such a reaction, about 2.5 eV of energy is released. Compare this reaction to a nuclear event such as $^{1}_{0}n + ^{235}_{92}U \rightarrow ^{136}_{53}I + ^{98}_{39}Y + 2^{1}_{0}n$. Would you expect the energy released in this nuclear event to be much greater, much less, or about the same as that released in the chemical reaction? Explain.

**19.** The neutral $\rho$ meson decays by the strong interaction into two pions according to $\rho^0 \rightarrow \pi^+ + \pi^-$, with a half-life of about $10^{-23}$ s. The neutral K meson also decays into two pions according to $K^0 \rightarrow \pi^+ + \pi^-$, but with a much longer half-life of about $10^{-10}$ s. How do you explain these observations?

## PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging ☐ = full solution available in *Student Solutions Manual/Study Guide*
**Physics Now™** = coached problem with hints available at **www.cp7e.com** = biomedical application

### Section 30.1 Nuclear Fission
### Section 30.2 Nuclear Reactors

**1.** If the average energy released in a fission event is 208 MeV, find the total number of fission events required to operate a 100-W lightbulb for 1.0 h.

**2.** Find the energy released in the fission reaction

$$n + ^{235}_{92}U \rightarrow ^{98}_{40}Zr + ^{135}_{52}Te + 3n$$

The atomic masses of the fission products are 97.912 0 u for $^{98}_{40}Zr$ and 134.908 7 u for $^{135}_{52}Te$.

**3.** Find the energy released in the following fission reaction:

$$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{88}_{38}Sr + ^{136}_{54}Xe + 12^{1}_{0}n$$

**4.** Strontium-90 is a particularly dangerous fission product of $^{235}U$ because it is radioactive and it substitutes for calcium in bones. What other direct fission products would accompany it in the neutron-induced fission of $^{235}U$? [*Note*: This reaction may release two, three, or four free neutrons.]

**5.** Assume that ordinary soil contains natural uranium in amounts of 1 part per million by mass. (a) How much uranium is in the top 1.00 meter of soil on a 1-acre (43 560-ft$^2$) plot of ground, assuming the specific gravity of soil is 4.00? (b) How much of the isotope $^{235}U$, appropriate for nuclear reactor fuel, is in this soil? [*Hint*: See Appendix B for the percent abundance of $^{235}_{92}U$.]

**6.** A typical nuclear fission power plant produces about 1.00 GW of electrical power. Assume that the plant has an overall efficiency of 40.0% and that each fission produces 200 MeV of thermal energy. Calculate the mass of $^{235}U$ consumed each day.

**7.** **Physics Now™** Suppose that the water exerts an average frictional drag of $1.0 \times 10^5$ N on a nuclear-powered ship. How far can the ship travel per kilogram of fuel if the fuel consists of enriched uranium containing 1.7% of the fissionable isotope $^{235}U$ and the ship's engine has an efficiency of 20%? (Assume 208 MeV is released per fission event.)

**8.** It has been estimated that the Earth contains $1.0 \times 10^9$ tons of natural uranium that can be mined economically. If all the world's energy needs ($7.0 \times 10^{12}$ J/s) were supplied by $^{235}U$ fission, how long would this supply last? [*Hint*: See Appendix B for the percent abundance of $^{235}_{92}U$.]

**9.** An all-electric home uses approximately 2 000 kWh of electric energy per month. How much uranium-235 would be required to provide this house with its energy needs for 1 year? (Assume 100% conversion efficiency and 208 MeV released per fission.)

### Section 30.3 Nuclear Fusion

**10.** Find the energy released in the fusion reaction

$$^{1}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{2}He + \gamma$$

**11.** When a star has exhausted its hydrogen fuel, it may fuse other nuclear fuels. At temperatures above $1.0 \times 10^8$ K, helium fusion can occur. Write the equations for the following processes: (a) Two alpha particles fuse to produce a nucleus $A$ and a gamma ray. What is nucleus $A$? (b) Nucleus $A$ absorbs an alpha particle to produce a nucleus $B$ and a gamma ray. What is nucleus $B$? (c) Find the total energy released in the reactions given in (a) and (b). [*Note*: The mass of $^{8}_{4}\text{Be}$ = 8.005 305 u.]

**12.** Another series of nuclear reactions that can produce energy in the interior of stars is the cycle described below. This cycle is most efficient when the central temperature in a star is above $1.6 \times 10^7$ K. Because the temperature at the center of the Sun is only $1.5 \times 10^7$ K, the following cycle produces less than 10% of the Sun's energy. (a) A high-energy proton is absorbed by $^{12}\text{C}$. Another nucleus, $A$, is produced in the reaction, along with a gamma ray. Identify nucleus $A$. (b) Nucleus $A$ decays through positron emission to form nucleus $B$. Identify nucleus $B$. (c) Nucleus B absorbs a proton to produce nucleus $C$ and a gamma ray. Identify nucleus $C$. (d) Nucleus $C$ absorbs a proton to produce nucleus $D$ and a gamma ray. Identify nucleus $D$. (e) Nucleus $D$ decays through positron emission to produce nucleus $E$. Identify nucleus $E$. (f) Nucleus $E$ absorbs a proton to produce nucleus $F$ plus an alpha particle. What is nucleus $F$? [*Note*: If nucleus $F$ is not $^{12}\text{C}$—that is, the nucleus you started with—you have made an error and should review the sequence of events.]

**13.** If an all-electric home uses approximately 2 000 kWh of electric energy per month, how many fusion events described by the reaction $^{2}_{1}\text{H} + ^{3}_{1}\text{H} \rightarrow ^{4}_{2}\text{He} + ^{1}_{0}\text{n}$ would be required to keep this home running for one year?

14. To understand why plasma containment is necessary, consider the rate at which an unconfined plasma would be lost. (a) Estimate the rms speed of deuterons in a plasma at $4.00 \times 10^8$ K. (b) Estimate the order of magnitude of the time such a plasma would remain in a 10-cm cube if no steps were taken to contain it.

15. The oceans have a volume of 317 million cubic miles and contain $1.32 \times 10^{21}$ kg of water. Of all the hydrogen nuclei in this water, 0.030 0% of the mass is deuterium. (a) If all of these deuterium nuclei were fused to helium via the first reaction in Equation 30.4, determine the total amount of energy that could be released. (b) The present world electric power consumption is about $7.00 \times 10^{12}$ W. If consumption were 100 times greater, how many years would the energy supply calculated in part (a) last?

### Section 30.6 Positrons and Other Antiparticles

**16.** Two photons are produced when a proton and an antiproton annihilate each other. What is the minimum frequency and corresponding wavelength of each photon?

**17.** **PhysicsNow™** A photon produces a proton–antiproton pair according to the reaction $\gamma \rightarrow \text{p} + \bar{\text{p}}$. What is the minimum possible frequency of the photon? What is its wavelength?

18. A photon with an energy of 2.09 GeV creates a proton–antiproton pair in which the proton has a kinetic energy of 95.0 MeV. What is the kinetic energy of the antiproton?

### Section 30.7 Mesons and the Beginning of Particle Physics

**19.** When a high-energy proton or pion traveling near the speed of light collides with a nucleus, it travels an average distance of $3.0 \times 10^{-15}$ m before interacting with another particle. From this information, estimate the time for the strong interaction to occur.

**20.** Calculate the order of magnitude of the range of the force that might be produced by the virtual exchange of a proton.

**21.** One of the mediators of the weak interaction is the $Z^0$ boson, which has a mass of 96 GeV/$c^2$. Use this information to find an approximate value for the range of the weak interaction.

**22.** If a $\pi^0$ at rest decays into two $\gamma$'s, what is the energy of each of the $\gamma$'s?

### Section 30.9 Conservation Laws
### Section 30.10 Strange Particles and Strangeness

**23.** Each of the following reactions is forbidden. Determine a conservation law that is violated for each reaction.
(a) $\text{p} + \bar{\text{p}} \rightarrow \mu^+ + e^-$
(b) $\pi^- + \text{p} \rightarrow \text{p} + \pi^+$
(c) $\text{p} + \text{p} \rightarrow \text{p} + \pi^+$
(d) $\text{p} + \text{p} \rightarrow \text{p} + \text{p} + \text{n}$
(e) $\gamma + \text{p} \rightarrow \text{n} + \pi^0$

24. For the following two reactions, the first may occur but the second cannot. Explain.

$$K^0 \rightarrow \pi^+ + \pi^- \quad \text{(can occur)}$$
$$\Lambda^0 \rightarrow \pi^+ + \pi^- \quad \text{(cannot occur)}$$

25. **Physics Now™** Identify the unknown particle on the left side of the reaction

$$? + p \rightarrow n + \mu^+$$

26. Determine the type of neutrino or antineutrino involved in each of the following processes:
(a) $\pi^+ \rightarrow \pi^0 + e^+ + ?$
(b) $? + p \rightarrow \mu^- + p + \pi^+$
(c) $\Lambda^0 \rightarrow p + \mu^- + ?$
(d) $\tau^+ \rightarrow \mu^+ + ? + ?$

27. The following reactions or decays involve one or more neutrinos. Supply the missing neutrinos.
(a) $\pi^- \rightarrow \mu^- + ?$
(b) $K^+ \rightarrow \mu^+ + ?$
(c) $? + p \rightarrow n + e^+$
(d) $? + n \rightarrow p + e^-$
(e) $? + n \rightarrow p + \mu^-$
(f) $\mu^- \rightarrow e^- + ? + ?$

28. Determine which of the reactions below can occur. For those that cannot occur, determine the conservation law (or laws) that each violates:
(a) $p \rightarrow \pi^+ + \pi^0$
(b) $p + p \rightarrow p + p + \pi^0$
(c) $p + p \rightarrow p + \pi^+$
(d) $\pi^+ \rightarrow \mu^+ + \nu_\mu$
(e) $n \rightarrow p + e^- + \bar{\nu}_e$
(f) $\pi^+ \rightarrow \pi^+ + n$

29. Which of the following processes are allowed by the strong interaction, the electromagnetic interaction, the weak interaction, or no interaction at all?
(a) $\pi^- + p \rightarrow 2\eta^0$
(b) $K^- + n \rightarrow \Lambda^0 + \pi^-$
(c) $K^- \rightarrow \pi^- + \pi^0$
(d) $\Omega^- \rightarrow \Xi^- + \pi^0$
(e) $\eta^0 \rightarrow 2\gamma$

30. A $K^0$ particle at rest decays into a $\pi^+$ and a $\pi^-$. What will be the speed of each of the pions? The mass of the $K^0$ is 497.7 MeV/$c^2$ and the mass of each pion is 139.6 MeV/$c^2$.

31. Determine whether or not strangeness is conserved in the following decays and reactions:
(a) $\Lambda^0 \rightarrow p + \pi^-$
(b) $\pi^- + p \rightarrow \Lambda^0 + K^0$
(c) $\bar{p} + p \rightarrow \bar{\Lambda}^0 + \Lambda^0$
(d) $\pi^- + p \rightarrow \pi^- + \Sigma^+$
(e) $\Xi^- \rightarrow \Lambda^0 + \pi^-$
(f) $\Xi^0 \rightarrow p + \pi^-$

32. Fill in the missing particle. Assume that (a) occurs via the strong interaction while (b) and (c) involve the weak interaction.
(a) $K^+ + p \rightarrow$ ____ $+ p$
(b) $\Omega^- \rightarrow$ ____ $+ \pi^-$
(c) $K^+ \rightarrow$ ____ $+ \mu^+ + \nu_\mu$

33. Identify the conserved quantities in the following processes:
(a) $\Xi^- \rightarrow \Lambda^0 + \mu^- + \nu_\mu$
(b) $K^0 \rightarrow 2\pi^0$
(c) $K^- + p \rightarrow \Sigma^0 + n$
(d) $\Sigma^0 \rightarrow \Lambda^0 + \gamma$
(e) $e^+ + e^- \rightarrow \mu^+ + \mu^-$
(f) $\bar{p} + n \rightarrow \bar{\Lambda}^0 + \Sigma^-$

## Section 30.12 Quarks
## Section 30.13 Colored Quarks

34. The quark composition of the proton is uud, while that of the neutron in udd. Show that the charge, baryon number, and strangeness of these particles equal the sums of these numbers for their quark constituents.

35. Find the number of electrons, and of each species of quark, in 1 L of water.

36. The quark compositions of the $K^0$ and $\Lambda^0$ particles are $d\bar{s}$ and uds, respectively. Show that the charge, baryon number, and strangeness of these particles equal the sums of these numbers for the quark constituents.

37. Identify the particles corresponding to the following quark states: (a) suu; (b) $\bar{u}d$; (c) $\bar{s}d$; (d) ssd.

38. What is the electrical charge of the baryons with the quark compositions (a) $\bar{u}\bar{u}\bar{d}$ and (b) $\bar{u}\bar{d}\bar{d}$? What are these baryons called?

39. Analyze the first three of the following reactions at the quark level, and show that each conserves the net number of each type of quark; then, in the last reaction, identify the mystery particle:
(a) $\pi^- + p \rightarrow K^0 + \Lambda^0$
(b) $\pi^+ + p \rightarrow K^+ + \Sigma^+$
(c) $K^- + p \rightarrow K^+ + K^0 + \Omega^-$
(d) $p + p \rightarrow K^0 + p + \pi^+ + ?$

40. **Physics Now™** Assume binding energies can be neglected. Find the masses of the u and d quarks from the masses of the proton and neutron.

## ADDITIONAL PROBLEMS

41. A $\Sigma^0$ particle traveling through matter strikes a proton and a $\Sigma^+$, and a gamma ray, as well as a third particle, emerges. Use the quark model of each to determine the identity of the third particle.

42. It was stated in the text that the reaction $\pi^- + p^+ \rightarrow K^0 + \Lambda^0$ occurs with high probability, whereas the reaction $\pi^- + p^+ \rightarrow K^0 + n$ never occurs. Analyze these reactions at the quark level and show that the first conserves the net number of each type of quark while the second does not.

43. Two protons approach each other with equal and opposite velocities. Find the minimum kinetic energy of each of the protons if they are to produce a $\pi^+$ meson at rest in the reaction

$$p + p \rightarrow p + n + \pi^+$$

44. Name at least one conservation law that prevents each of the following reactions from occurring:
(a) $\pi^- + p \rightarrow \Sigma^+ + \pi^0$
(b) $\mu^- \rightarrow \pi^- + \nu_e$
(c) $p \rightarrow \pi^+ + \pi^+ + \pi^-$

45. Find the energy released in the fusion reaction

$$H + {}^3_2He \rightarrow {}^4_2He + e^+ + \nu$$

46. Occasionally, high-energy muons collide with electrons and produce two neutrinos according to the reaction $\mu^+ + e^- \rightarrow 2\nu$. What kind of neutrinos are these?

47. Each of the following decays is forbidden. For each process, determine a conservation law that is violated:
(a) $\mu^- \rightarrow e^- + \gamma$
(b) $n \rightarrow p + e^- + \nu_e$
(c) $\Lambda^0 \rightarrow p + \pi^0$
(d) $p \rightarrow e^+ + \pi^0$
(e) $\Xi^0 \rightarrow n + \pi^0$

48. Two protons approach each other with 70.4 MeV of kinetic energy and engage in a reaction in which a proton and a positive pion emerge at rest. What third particle, obviously uncharged and therefore difficult to detect, must have been created?

49. The atomic bomb dropped on Hiroshima on August 6, 1945, released $5 \times 10^{13}$ J of energy (equivalent to that from 12 000 tons of TNT). Estimate (a) the number of ${}^{235}_{92}U$ nuclei fissioned and (b) the mass of this ${}^{235}_{92}U$.

50. A $\Sigma^0$ particle at rest decays according to $\Sigma^0 \rightarrow \Lambda^0 + \gamma$. Find the gamma-ray energy. [*Hint*: remember to conserve momentum.]

51. If baryon number is not conserved, then one possible mechanism by which a proton can decay is

$$p \rightarrow e^+ + \gamma$$

Show that this reaction violates the conservation of baryon number. (b) Assuming that the reaction occurs and that the proton is initially at rest, determine the energy and momentum of the photon after the reaction. [*Hint*: recall that energy and momentum must be conserved in the reaction.] (c) Determine the speed of the positron after the reaction.

52. Classical general relativity views the space–time manifold as a deterministic structure completely well defined down to arbitrarily small distances. On the other hand, quantum general relativity forbids distances smaller than the Planck length $L = (\hbar G/c^3)^{1/2}$. (a) Calculate the value of $L$. The answer suggests that, after the Big Bang (when all the known Universe was reduced to a singularity), nothing could be observed until that singularity grew larger than the Planck length, $L$. Since the size of the singularity grew at the speed of light, we can infer that during the time it took for light to travel the Planck length, no observations were possible. (b) Determine this time (known as the Planck time $T$), and compare it to the ultra-hot epoch discussed in the text. (c) Does your answer to part (b) suggest that we may never know what happened between the time $t = 0$ and the time $t = T$?

53. (a) Show that about $1.0 \times 10^{10}$ J would be released by the fusion of the deuterons in 1.0 gal of water. Note that 1 out of every 6 500 hydrogen atoms is a deuteron. (b) The average energy consumption rate of a person living in the United States is about $1.0 \times 10^4$ J/s (an average power of 10 kW). At this rate, how long would the energy needs of one person be supplied by the fusion of the deuterons in 1.0 gal of water? Assume that the energy released per deuteron is 1.64 MeV.

54. Calculate the mass of $^{235}U$ required to provide the total energy requirements of a nuclear submarine

Standard Model of

# FUNDAMENTAL PARTICLES AND INTERACTIONS

The Standard Model summarizes the current knowledge in Particle Physics. It is the quantum theory that includes the theory of strong interactions (quantum chromodynamics or QCD) and the unified theory of weak and electromagnetic interactions (electroweak). Gravity is included on this chart because it is one of the fundamental interactions even though not part of the "Standard Model."

## FERMIONS

matter constituents
spin = 1/2, 3/2, 5/2, ...

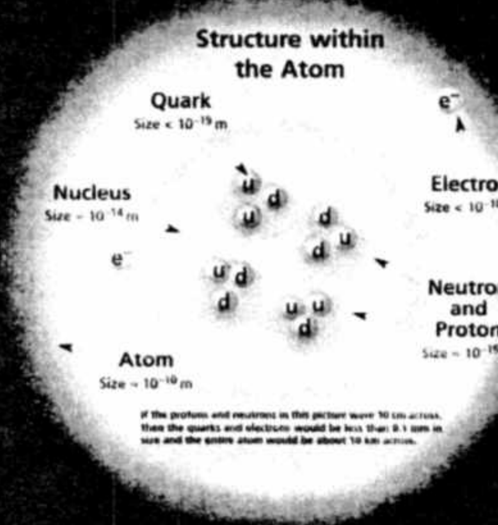
| Leptons spin = 1/2 | | |
|---|---|---|
| Flavor | Mass GeV/c² | Electric charge |
| $\nu_e$ electron neutrino | $<1\times10^{-8}$ | 0 |
| e electron | 0.000511 | −1 |
| $\nu_\mu$ muon neutrino | <0.0002 | 0 |
| $\mu$ muon | 0.106 | −1 |
| $\nu_\tau$ tau neutrino | <0.02 | 0 |
| $\tau$ tau | 1.7771 | −1 |

| Quarks spin = 1/2 | | |
|---|---|---|
| Flavor | Approx. Mass GeV/c² | Electric charge |
| u up | 0.003 | 2/3 |
| d down | 0.006 | −1/3 |
| c charm | 1.3 | 2/3 |
| s strange | 0.1 | −1/3 |
| t top | 175 | 2/3 |
| b bottom | 4.3 | −1/3 |

**Spin** is the intrinsic angular momentum of particles. Spin is given in units of ħ, which is the quantum unit of angular momentum, where $\hbar = h/2\pi = 6.58\times10^{-25}$ GeV s $= 1.05\times10^{-34}$ J s.

**Electric charges** are given in units of the proton's charge. In SI units the electric charge of the proton is $1.60\times10^{-19}$ coulombs.

The **energy** unit of particle physics is the electronvolt (eV), the energy gained by one electron in crossing a potential difference of one volt. **Masses** are given in GeV/c² (remember $E = mc^2$), where 1 GeV = $10^9$ eV = $1.60\times10^{-10}$ joule. The mass of the proton is 0.938 GeV/c² = $1.67\times10^{-27}$ kg.




If the protons and neutrons in this picture were 10 cm across, then the quarks and electrons would be less than 0.1 mm in size and the entire atom would be about 10 km across.

## BOSONS

force carriers
spin = 0, 1, 2, ...

| Unified Electroweak spin = 1 | | |
|---|---|---|
| Name | Mass GeV/c² | Electric charge |
| γ photon | 0 | 0 |
| $W^-$ | 80.4 | −1 |
| $W^+$ | 80.4 | +1 |
| $Z^0$ | 91.187 | 0 |

| Strong (color) spin = 1 | | |
|---|---|---|
| Name | Mass GeV/c² | Electric charge |
| g gluon | 0 | 0 |

### Color Charge

Each quark carries one of three types of "strong charge," also called "color charge." These charges have nothing to do with the colors of visible light. There are eight possible types of color charge for gluons. Just as electrically-charged particles interact by exchanging photons, in strong interactions color-charged particles interact by exchanging gluons. Leptons, photons, and **W** and **Z** bosons have no strong interactions and hence no color charge.

### Quarks Confined in Mesons and Baryons

One cannot isolate quarks and gluons; they are confined in color-neutral particles called **hadrons**. This confinement (binding) results from multiple exchanges of gluons among the color-charged constituents. As color-charged particles (quarks and gluons) move apart, the energy in the color-force field between them increases. This energy eventually is converted into additional quark-antiquark pairs (see figure below). The quarks and antiquarks then combine into hadrons; these are the particles seen to emerge. Two types of hadrons have been observed in nature: **mesons** $q\bar{q}$ and **baryons** qqq.

### Residual Strong Interaction

The strong binding of color-neutral protons and neutrons to form nuclei is due to residual strong interactions between their color-charged constituents. It is similar to the residual electrical interaction that binds electrically neutral atoms to form molecules. It can also be viewed as the exchange of mesons between the hadrons.

## PROPERTIES OF THE INTERACTIONS

| Baryons qqq and Antibaryons $\bar{q}\bar{q}\bar{q}$ — Baryons are fermionic hadrons. There are about 120 types of baryons. | | | | | |
|---|---|---|---|---|---|
| Symbol | Name | Quark content | Electric charge | Mass GeV/c² | Spin |
| p | proton | uud | 1 | 0.938 | 1/2 |
| $\bar{p}$ | anti-proton | $\bar{u}\bar{u}\bar{d}$ | −1 | 0.938 | 1/2 |
| n | neutron | udd | 0 | 0.940 | 1/2 |
| Λ | lambda | uds | 0 | 1.116 | 1/2 |
| $\Omega^-$ | omega | sss | −1 | 1.672 | 3/2 |

| Property \ Interaction | Gravitational | | | Strong: Fundamental | Strong: Residual |
|---|---|---|---|---|---|
| Acts on: | Mass – Energy | Flavor | Electric Charge | Color Charge | See Residual Strong Interaction Note |
| Particles experiencing: | All | Quarks, Leptons | Electrically charged | Quarks, Gluons | Hadrons |
| Particles mediating: | Graviton (not yet observed) | $W^+$ $W^-$ $Z^0$ | γ | Gluons | Mesons |
| Strength relative to electromag for two u quarks at: $10^{-18}$ m | $10^{-41}$ | 0.8 | 1 | 25 | Not applicable to quarks |
| $3\times10^{-17}$ m | $10^{-41}$ | $10^{-4}$ | 1 | 60 | |
| for two protons in nucleus | $10^{-36}$ | $10^{-7}$ | 1 | Not applicable to hadrons | 20 |

| Mesons $q\bar{q}$ — Mesons are bosonic hadrons. There are about 140 types of mesons. | | | | | |
|---|---|---|---|---|---|
| Symbol | Name | Quark content | Electric charge | Mass GeV/c² | Spin |
| $\pi^+$ | pion | $u\bar{d}$ | +1 | 0.140 | 0 |
| $K^-$ | kaon | $s\bar{u}$ | −1 | 0.494 | 0 |
| $\rho^+$ | rho | $u\bar{d}$ | +1 | 0.770 | 1 |
| $B^0$ | B-zero | $d\bar{b}$ | 0 | 5.279 | 0 |
| $\eta_c$ | eta-c | $c\bar{c}$ | 0 | 2.980 | 0 |

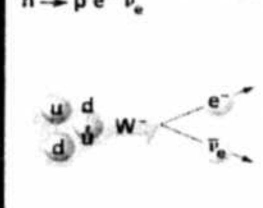
### Matter and Antimatter

For every particle type there is a corresponding antiparticle type, denoted by a bar over the particle symbol (unless + or − charge is shown). Particle and antiparticle have identical mass and spin but opposite charges. Some electrically neutral bosons (e.g., $Z^0$, γ, and $\eta_c = c\bar{c}$, but not $K^0 = d\bar{s}$) are their own antiparticles.

### Figures

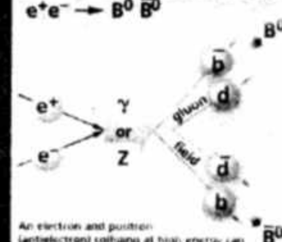
These diagrams are an artist's conception of physical processes. They are *not* exact and have *no* meaningful scale. Green shaded areas represent the cloud of gluons or the gluon field, and red lines the quark paths.

$n \rightarrow p\, e^-\, \bar{\nu}_e$



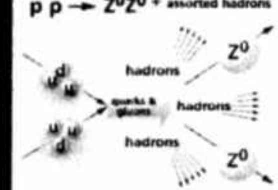

A neutron decays to a proton, an electron, and an antineutrino via a virtual (mediating) W boson. This is neutron β decay.

$e^+e^- \rightarrow B^0\,\bar{B}^0$




An electron and positron (antielectron) colliding at high energy can annihilate to produce $B^0$ and $\bar{B}^0$ mesons via a virtual Z boson or a virtual photon.

$p\,p \rightarrow Z^0Z^0$ + assorted hadrons




Two protons colliding at high energy can produce various hadrons plus very high mass particles such as Z bosons. Events such as this one are rare but can yield vital clues to the structure of matter.

### The Particle Adventure

Visit the award-winning web feature *The Particle Adventure* at
**http://ParticleAdventure.org**


**This chart has been made possible by the generous support of:**
U.S. Department of Energy
U.S. National Science Foundation
Lawrence Berkeley National Laboratory
Stanford Linear Accelerator Center
American Physical Society, Division of Particles and Fields
BURLE INDUSTRIES, INC.




**http://CPEPweb.org**

# SPECIAL RELATIVITY


*Ask anyone to name the most famous scientist of the twentieth century and you are likely to get Albert Einstein (1879–1955) for an answer. Few individuals have become as well known as he, in recognition for the remarkable number, diversity, and fundamental significance of his scientific accomplishments. He is best known for the theory of special relativity, published in 1905 when he was 26 years old. Even today, this theory is a rich source of amazement, for it alters in fundamental ways many of our basic ideas about the physical universe. We will see that time no longer has a unique meaning in the aftermath of Einstein's theory. And neither does length. For example, due to relativistic effects, space travelers would make a high-speed journey to a distant planet in a much shorter time than that registered on the clocks left behind on earth. In addition, we will find that mass (inertia) and energy are not independent ideas as classical physics assumes. Instead, they are equivalent, with the result that the energy $E_0$ of a stationary object is related to its mass m by Einstein's famous equation, $E_0 = mc^2$, where c is the speed of light in a vacuum. Another surprising result of special relativity is that an object with mass cannot be accelerated to speeds at or beyond the speed of light, no matter how much force is applied. Thus, the speed of light represents the ultimate speed for any moving object that has mass. We will find that relativistic effects become significant when the speed of an object is an appreciable fraction of the speed of light. Special relativity also modifies our notions about how relative velocities are measured by different observers, ideas that were first presented in Section 3.4. All of these features of special relativity will be discussed in this chapter. But first, we need to review the idea of an inertial reference frame, since this idea plays such a fundamental role in the theory.*

Courtesy of the Archives, California Institute of Technology

# Relativity

Most of our everyday experiences and observations have to do with objects that move at speeds much less than the speed of light. Newtonian mechanics was formulated to describe the motion of such objects, and its formalism is quite successful in describing a wide range of phenomena that occur at low speeds. It fails, however, when applied to particles having speeds approaching that of light.

This chapter introduces Einstein's theory of special relativity and includes a section on general relativity. The concepts of special relativity often violate our common sense. Moving clocks run slow, and the length of a moving meter stick is contracted. Nonetheless, the theory has been rigorously tested, correctly predicting the results of experiments involving speeds near the speed of light. The theory is verified daily in particle accelerators around the world.

## 26.1 INTRODUCTION

Experimentally, the predictions of Newtonian theory can be tested at high speeds by accelerating electrons or other charged particles through a large electric potential difference. For example, it's possible to accelerate an electron to a speed of $0.99c$ (where $c$ is the speed of light) by using a potential difference of several million volts. According to Newtonian mechanics, if the potential difference is increased by a factor of 4, the electron's kinetic energy is four times greater and its speed should double to $1.98c$. However, experiments show that the speed of the electron—as well as the speed of any other particle that has mass—always remains *less* than the speed of light, regardless of the size of the accelerating voltage.

The existence of a universal speed limit has far-reaching consequences. It means that the usual concepts of force, momentum, and energy no longer apply for rapidly moving objects. Less obvious consequences include the fact that observers moving at different speeds will measure different time intervals and displacements between the same two events. Newtonian mechanics was contradicted by experimental observations, so it was necessary to replace it with another theory.

In 1905, at the age of 26, Einstein published his special theory of relativity. Regarding the theory, Einstein wrote:

> The relativity theory arose from necessity, from serious and deep contradictions in the old theory from which there seemed no escape. The strength of the new theory lies in the consistency and simplicity with which it solves all these difficulties, using only a few very convincing assumptions.[1]

Although Einstein made many other important contributions to science, his theory of relativity alone represents one of the greatest intellectual achievements of all time. With this theory, experimental observations can be correctly predicted over the range of speeds from $v = 0$ to speeds approaching the speed of light. Newtonian mechanics, which was accepted for more than 200 years, remains valid, but only for speeds much smaller than the speed of light.

At the foundation of special relativity is reconciling the measurements of two observers moving relative to each other. Normally, two such observers will measure different outcomes for the same event. If the measurement is the speed of a car, for example, an observer standing on the road will measure a different speed for the car than an observer in a truck traveling at speed $v$ relative the stationary observer. Special relativity is all about relating two such measurements—and this rather innocuous relating of measurements leads to some of the most bizarre consequences in physics!

## 26.2 THE PRINCIPLE OF GALILEAN RELATIVITY

In order to describe a physical event, it's necessary to choose a *frame of reference.* For example, when you perform an experiment in a laboratory, you select a coordinate system, or frame of reference, that is at rest with respect to the laboratory. However, suppose an observer in a passing car moving at a constant velocity with respect to the lab were to observe your experiment. Would the observations made by the moving observer differ dramatically from yours? That is, if you found Newton's first law to be valid in your frame of reference, would the moving observer agree with you?

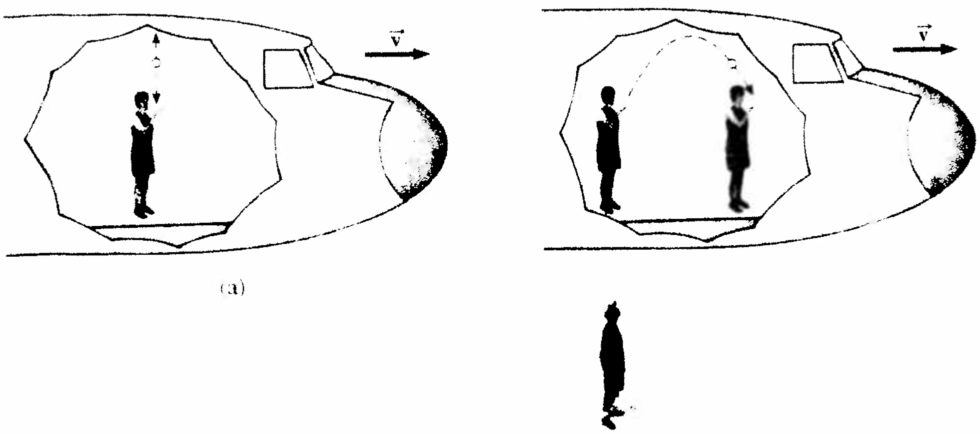
According to the principle of Galilean relativity, **the laws of mechanics must be the same in all inertial frames of reference**. Inertial frames of reference are those reference frames in which Newton's laws are valid. Practically, such frames are those in which objects subjected to no forces move in straight lines at constant speed—thus the name "inertial frame" because objects observed from these frames obey Newton's first law, the law of inertia. For the situation described in the previous paragraph, the laboratory coordinate system and the coordinate system of the moving car are both inertial frames of reference. Consequently, if the laws of mechanics are found to be true in the laboratory, then the person in the car must also observe the same laws.[2]

Consider an airplane in flight, moving with a constant velocity, as in Figure 26.1a. If a passenger in the airplane throws a ball straight up in the air, the passenger observes that the ball moves in a vertical path. The motion of the ball is precisely the same as it would be if the ball were thrown while at rest on Earth. The law of gravity and the equations of motion under constant acceleration are obeyed whether the airplane is at rest or in uniform motion.

Now consider the same experiment when viewed by another observer at rest on Earth. This stationary observer views the path of the ball in the plane to be a parabola, as in Figure 26.1b. Further, according to this observer, the ball has a velocity to the right equal to the velocity of the plane. Although the two observers disagree on the shape of the ball's path, both agree that the motion of the ball obeys the law of gravity and Newton's laws of motion, and even agree on how long

[1]A. Einstein and L. Infeld, *The Evolution of Physics* (New York: Simon and Schuster, 1961).

[2]What is an example of a *non*inertial frame? A frame undergoing translational acceleration or a frame rotating with respect to the two inertial frames just mentioned.



(a)

(b)

**Figure 26.1** (a) The observer on the airplane sees the ball move in a vertical path when thrown upward. (b) The observer on Earth views the path of the ball to be a parabola.

the ball is in the air. We draw the following important conclusion: **There is no preferred frame of reference for describing the laws of mechanics.**

## 26.3 THE SPEED OF LIGHT

It's natural to ask whether the concept of Galilean relativity in mechanics also applies to experiments in electricity, magnetism, optics, and other areas. Experiments indicate the answer is no. For example, if we assume that the laws of electricity and magnetism are the same in all inertial frames, a paradox concerning the speed of light immediately arises. This can be understood by recalling that, according to electromagnetic theory, the speed of light always has the fixed value of $2.997\,924\,58 \times 10^8$ m/s in free space. But this is in direct contradiction to common sense. For example, suppose a light pulse is sent out by an observer in a boxcar moving with a velocity $\vec{v}$ (Fig. 26.2). The light pulse has a velocity $\vec{c}$ relative to observer S' in the boxcar. According to Galilean relativity, the speed of the pulse relative to the stationary observer S outside the boxcar should be $c + v$. This obviously contradicts Einstein's theory, which postulates that the velocity of the light pulse is the same for all observers.

In order to resolve this paradox, we must conclude that either (1) the addition law for velocities is incorrect or (2) the laws of electricity and magnetism are not the same in all inertial frames. Assume that the second conclusion is true; then a preferred reference frame must exist in which the speed of light has the value $c$, but in any other reference frame the speed of light must have a value that is greater or less than $c$. It's useful to draw an analogy with sound waves, which propagate through a medium such as air. The speed of sound in air is about 330 m/s when measured in a reference frame in which the air is stationary. However, the speed of sound is greater or less than this value when measured from a reference frame that is moving with respect to the air.

In the case of light signals (electromagnetic waves), recall that electromagnetic theory predicted that such waves must propagate through free space with a speed

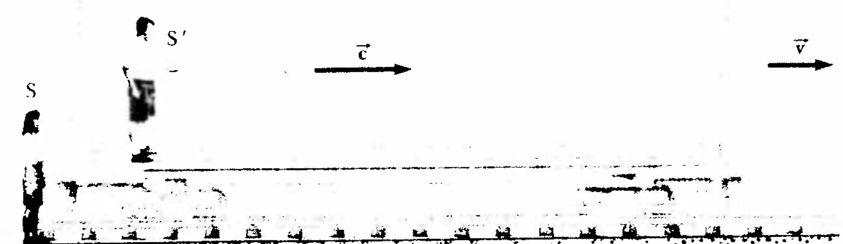



**Figure 26.2** A pulse of light is sent out by a person in a moving boxcar. According to Newtonian relativity, the speed of the pulse should be $\vec{c} + \vec{v}$ relative to a stationary observer.

equal to the speed of light. However, the theory doesn't require the presence of a medium for wave propagation. This is in contrast to other types of waves, such as water and sound waves, that do require a medium to support the disturbances. In the 19th century, physicists thought that electromagnetic waves also required a medium in order to propagate. They proposed that such a medium existed and gave it the name **luminiferous ether**. The ether was assumed to be present everywhere, even in empty space, and light waves were viewed as ether oscillations. Further, the ether would have to be a massless but rigid medium with no effect on the motion of planets or other objects. These are strange concepts indeed. In addition, it was found that the troublesome laws of electricity and magnetism would take on their simplest forms in a special frame of reference at *rest* with respect to the ether. This frame was called the *absolute frame*. The laws of electricity and magnetism would be valid in this absolute frame, but they would have to be modified in any reference frame moving with respect to the absolute frame.

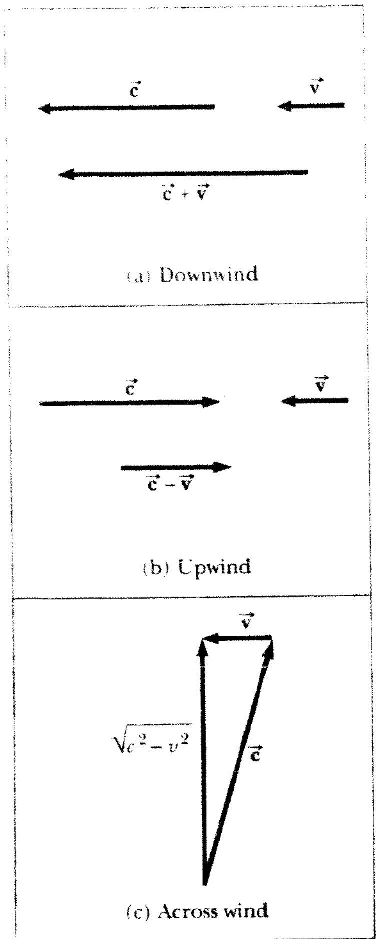



**Figure 26.3** If the speed of the ether wind relative to Earth is $v$, and $c$ is the speed of light relative to the ether, the speed of light relative to Earth is (a) $c + v$ in the downwind direction, (b) $c - v$ in the upwind direction, and (c) $\sqrt{c^2 - v^2}$ in the direction perpendicular to the wind.

As a result of the importance attached to the ether and the absolute frame, it became of considerable interest in physics to prove by experiment that they existed. Since it was considered likely that Earth was in motion through the ether, from the view of an experimenter on Earth, there was an "ether wind" blowing through his laboratory. A direct method for detecting the ether wind would use an apparatus fixed to Earth to measure the wind's influence on the speed of light. If $v$ is the speed of the ether relative to Earth, then the speed of light should have its maximum value, $c + v$, when propagating downwind, as shown in Figure 26.3a. Likewise, the speed of light should have its minimum value, $c - v$, when propagating upwind, as in Figure 26.3b, and an intermediate value, $(c^2 - v^2)^{1/2}$, in the direction perpendicular to the ether wind, as in Figure 26.3c. If the Sun were assumed to be at rest in the ether, then the velocity of the ether wind would be equal to the orbital velocity of Earth around the Sun, which has a magnitude of approximately $3 \times 10^4$ m/s. Because $c = 3 \times 10^8$ m/s, it should be possible to detect a change in speed of about 1 part in $10^4$ for measurements in the upwind or downwind directions. However, as we will see in the next section, all attempts to detect such changes and establish the existence of the ether (and hence the absolute frame) proved futile.

In conclusion, we see that the second hypothesis in our introduction to this section is false—and we now believe that **the laws of electricity and magnetism are the same in all inertial frames**. It is the simple classical addition laws for velocities that are incorrect and must be modified, as shown in Section 26.8.

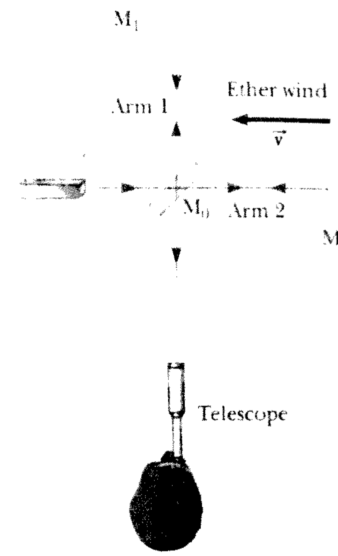
## 26.4 THE MICHELSON–MORLEY EXPERIMENT

The most famous experiment designed to detect small changes in the speed of light was first performed in 1881 by Albert A. Michelson (1852–1931) and later repeated under various conditions by Michelson and Edward W. Morley (1838–1923). We state at the outset that the outcome of the experiment contradicted the ether hypothesis.

The experiment was designed to determine the velocity of Earth relative to the hypothetical ether. The experimental tool used was the Michelson interferometer, which was discussed in Section 25.7 and is shown again in Active Figure 26.4. Arm 2 is aligned along the direction of Earth's motion through space. Earth's moving through the ether at speed $v$ is equivalent to the ether's flowing past Earth in the opposite direction with speed $v$. This ether wind blowing in the direction opposite the direction of Earth's motion should cause the speed of light measured in Earth frame to be $c - v$ as the light approaches mirror $M_2$ and $c + v$ after reflection, where $c$ is the speed of light in the ether frame.

The two beams reflected from $M_1$ and $M_2$ recombine, and an interference pattern consisting of alternating dark and bright fringes is formed. The interference pattern was observed while the interferometer was rotated through an angle of 90°. This rotation supposedly would change the speed of the ether wind along the direction of arm 1. The effect of such rotation should have been to cause the fringe pattern to shift slightly but measurably; however, measurements failed to show any change in the interference pattern! The Michelson–Morley experiment was repeated at different times of the year when the ether wind was expected to change direction, but the results were always the same: **no fringe shift of the magnitude required was ever observed.**

The negative results of the Michelson–Morley experiment not only contradicted the ether hypothesis, but also showed that it was impossible to measure the absolute velocity of Earth with respect to the ether frame. However, as we will see in the next section, Einstein suggested a postulate in the special theory of relativity that places quite a different interpretation on these negative results. In later years, when more was known about the nature of light, the idea of an ether that permeates all of space was relegated to the theoretical graveyard. **Light is now understood to be an electromagnetic wave, which requires no medium for its propagation.** As a result, the idea of an ether in which these waves could travel became unnecessary.




**ACTIVE FIGURE 26.4**
According to the ether wind theory, the speed of light should be $c - v$ as the beam approaches mirror $M_2$ and $c + v$ after reflection.

**Physics Now™**
Log into PhysicsNow at **www.cp7e.com** and go to Active Figure 26.4, where you can adjust the speed of a fictitious ether wind and observe the effect on beams of light.

## Details of the Michelson–Morley Experiment

As we mentioned earlier, the Michelson–Morley experiment was designed to detect the motion of Earth with respect to the ether. Before we examine the details of this historical experiment, it is instructive to consider a race between two airplanes, as shown in Figure 26.5a. One airplane flies from point $O$ to point $A$ perpendicular to the direction of the wind, and the second airplane flies from point $O$ to point $B$ parallel to the wind. We will assume that they start at $O$ at the same time, travel the same distance $L$ with the same cruising speed $c$ with respect to the wind, and return to $O$. Which airplane will win the race? In order to answer this question, we calculate the time of flight for both airplanes.

First, consider the airplane that moves along path I parallel to the wind. As it moves to the right, its speed is enhanced by the wind, and its speed with respect to Earth is $c + v$. As it moves to the left on its return journey, it must fly opposite the wind; hence, its speed with respect to Earth is $c - v$. The times of flight to the right and to the left are, respectively,

$$t_R = \frac{L}{c+v} \quad \text{and} \quad t_L = \frac{L}{c-v}$$

and the total time of flight for the airplane moving along path I is

$$t_1 = t_R + t_L = \frac{L}{c+v} + \frac{L}{c-v} = \frac{2Lc}{c^2 - v^2}$$

$$= \frac{2L}{c\left(1 - \frac{v^2}{c^2}\right)} \qquad [26.1]$$

Now consider the airplane flying along path II. If the pilot aims the airplane directly toward point A, it will be blown off course by the wind and won't reach its

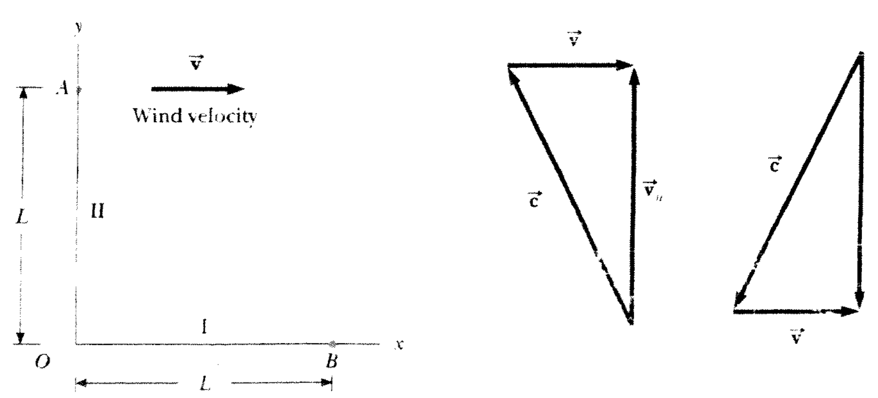



**Figure 26.5** (a) If an airplane travels from $O$ to $A$ with a wind blowing to the right, it must head into the wind at some angle. (b) Vector diagram for determining the airplane's direction for the trip from $O$ to $A$. (c) Vector diagram for determining its direction for the trip from $A$ to $O$.

destination. To compensate for the wind, the pilot must point the airplane into the wind at some angle, as shown in Figure 26.5a. This angle must be selected so that the vector sum of $\vec{c}$ and $\vec{v}$ leads to a velocity vector pointed directly toward $A$. The resultant vector diagram is shown in Figure 26.5b, where $\vec{v}_u$ is the velocity of the airplane with respect to the ground as it moves from $O$ to $A$. From the Pythagorean theorem, the magnitude of the vector $\vec{v}_u$ is

$$v_u = \sqrt{c^2 - v^2} = c\sqrt{1 - \frac{v^2}{c^2}}$$

Likewise, on the return trip from $A$ to $O$, the pilot must again head into the wind so that the airplane's velocity $\vec{v}_d$ with respect to Earth will be directed toward $O$, as shown in Figure 26.5c. From this figure, we see that

$$v_d = \sqrt{c^2 - v^2} = c\sqrt{1 - \frac{v^2}{c^2}}$$

The total time of flight for the trip along path II is therefore

$$t_2 = \frac{L}{v_u} + \frac{L}{v_d} = \frac{L}{c\sqrt{1 - \frac{v^2}{c^2}}} + \frac{L}{c\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{2L}{c\sqrt{1 - \frac{v^2}{c^2}}} \qquad \textbf{[26.2]}$$

Comparing Equations 26.1 and 26.2, we see that the airplane flying along path II wins the race. The difference in flight times is given by

$$\Delta t = t_1 - t_2 = \frac{2L}{c}\left[\frac{1}{\left(1 - \frac{v^2}{c^2}\right)} - \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\right]$$

This expression can be simplified by noting that the ratio of wind speed to plane speed, $v/c$, is usually much smaller than 1, and by using the following binomial expansions in $v/c$ after dropping all terms higher than second order:

$$\left(1 - \frac{v^2}{c^2}\right)^{-1} \approx 1 + \frac{v^2}{c^2}$$

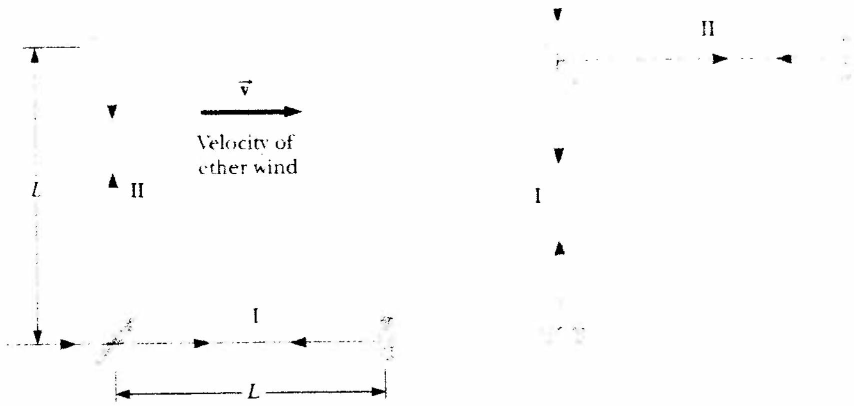
and

$$\left(1 - \frac{v^2}{c^2}\right)^{-1/2} \approx 1 + \frac{1}{2}\frac{v^2}{c^2}$$

The difference in flight times is therefore

$$\Delta t \approx \frac{Lv^2}{c^3} \qquad \text{for} \qquad v/c \ll 1 \qquad \textbf{[26.3]}$$

The analogy between this airplane race and the Michelson–Morley experiment is shown in Figure 26.6a. Two beams of light travel along two arms of an interferometer. In this case, the "wind" is the ether blowing across Earth from left to right as Earth moves through the ether from right to left. Because the speed of Earth in its orbital path is approximately $3 \times 10^4$ m/s, it is reasonable to use that value for the speed of the ether wind. Notice in this case that $v/c \approx 1 \times 10^{-4} \ll 1$. The two light beams start out in phase and return to form an interference pattern. We assume that the interferometer is adjusted for parallel fringes and that a telescope is focused on one of these fringes. The time difference between the two light beams gives rise to a phase difference between the beams, producing an interference pattern when they combine at the position of the telescope. The difference in the pattern is detected by rotating the interferometer through 90° in a horizontal plane, so that the two beams exchange roles (Fig. 26.6b). This results in a net




(a) (b)

**Figure 26.6** (a) Top view of the Michelson–Morley interferometer, where $\vec{v}$ is the velocity of the ether and $L$ is the length of each arm. (b) When the interferometer is rotated by 90°, the role of each arm is reversed.

time shift of twice the time difference given by Equation 26.3. The net time difference is therefore

$$\Delta t_{\text{net}} = 2\,\Delta t = \frac{2Lv^2}{c^3} \qquad [26.4]$$

The corresponding path difference is

$$\Delta d = c\,\Delta t_{\text{net}} = \frac{2Lv^2}{c^2} \qquad [26.5]$$

In the first experiments by Michelson and Morley, each light beam was reflected by the mirrors many times to give an increased effective path length $L$ of about 11 meters. Using this value and taking $v$ to be equal to $3 \times 10^4$ m/s gives a path difference of

$$\Delta d = \frac{2(11\text{ m})(3.0 \times 10^4\text{ m/s})^2}{(3.0 \times 10^8\text{ m/s})^2} = 2.2 \times 10^{-7}\text{ m}$$

This extra travel distance should produce a noticeable shift in the fringe pattern. Specifically, calculations show that if the pattern is viewed while the interferometer is rotated through 90°, a shift of about 0.4 fringe should be observed. The instrument used by Michelson and Morley was capable of detecting a shift in the fringe pattern as small as 0.01 fringe. However, *it detected no shift whatsoever in the fringe pattern.* Since then, the experiment has been repeated many times by different scientists under a wide variety of conditions and no fringe shift has ever been detected. The inescapable conclusion is that motion of Earth with respect to the ether can't be detected.

Many efforts were made to explain the null results of the Michelson–Morley experiment and to save the ether frame concept and the Galilean addition law for the velocity of light. All proposals resulting from these efforts have been shown to be wrong. No experiment in the history of physics has received such valiant efforts to explain the absence of an expected result as was the Michelson–Morley experiment. The stage was set for Einstein, who, at the age of only 26, solved the problem in 1905 with his special theory of relativity.

## 26.5 EINSTEIN'S PRINCIPLE OF RELATIVITY

In the previous section we noted the serious contradiction between the Galilean addition law for velocities and the fact that the speed of light is the same for all observers. In 1905 Albert Einstein proposed a theory that resolved this contradiction but at the same time completely altered our notions of space and time. He based his special theory of relativity on two postulates:

AIP Niels Bohr Library

**ALBERT EINSTEIN,**
**German-American Physicist (1879–1955)**

One of the greatest physicists of all time, Einstein was born in Ulm, Germany. In 1905, at the age of 26, he published four scientific papers that revolutionized physics. Two of these papers were concerned with what is now considered his most important contribution: the special theory of relativity. In 1916, Einstein published his work on the general theory of relativity. The most dramatic prediction of this theory is the degree to which light is deflected by a gravitational field. Measurements made by astronomers on bright stars in the vicinity of the eclipsed Sun in 1919 confirmed Einstein's prediction, and as a result, Einstein became a world celebrity. Einstein was deeply disturbed by the development of quantum mechanics in the 1920s despite his own role as a scientific revolutionary. In particular, he could never accept the probabilistic view of events in nature that is a central feature of quantum theory. The last few decades of his life were devoted to an unsuccessful search for a unified theory that would combine gravitation and electromagnetism.

Postulates of relativity ▶

1. **The principle of relativity:** All the laws of physics are the same in all inertial frames.
2. **The constancy of the speed of light:** The speed of light in a vacuum has the same value, $c = 2.997\,924\,58 \times 10^8$ m/s, in all inertial reference frames, regardless of the velocity of the observer or the velocity of the source emitting the light.

The first postulate asserts that *all* the laws of physics are the same in all reference frames moving with constant velocity relative to each other. This postulate is a sweeping generalization of the principle of Galilean relativity, which refers only to the laws of mechanics. From an experimental point of view, Einstein's principle of relativity means that *any* kind of experiment—mechanical, thermal, optical, or electrical—performed in a laboratory at rest, must give the same result when performed in a laboratory moving at a constant speed past the first one. Hence, no preferred inertial reference frame exists, and it is impossible to detect absolute motion.

Although postulate 2 was a brilliant theoretical insight on Einstein's part in 1905, it has since been confirmed experimentally in many ways. Perhaps the most direct demonstration involves measuring the speed of photons emitted by particles traveling at 99.99% of the speed of light. The measured photon speed in this case agrees to five significant figures with the speed of light in empty space.

The null result of the Michelson–Morley experiment can be readily understood within the framework of Einstein's theory. According to his principle of relativity, the premises of the Michelson–Morley experiment were incorrect. In the process of trying to explain the expected results, we stated that when light traveled against the ether wind its speed was $c - v$. However, if the state of motion of the observer or of the source has no influence on the value found for the speed of light, the measured value must always be $c$. Likewise, the light makes the return trip after reflection from the mirror at a speed of $c$, not at a speed of $c + v$. Thus, the motion of Earth does not influence the fringe pattern observed in the Michelson–Morley experiment, and a null result should be expected.

If we accept Einstein's theory of relativity, we must conclude that uniform relative motion is unimportant when measuring the speed of light. At the same time, we have to adjust our commonsense notions of space and time and be prepared for some rather bizarre consequences.

## 26.6 CONSEQUENCES OF SPECIAL RELATIVITY

Almost everyone who has dabbled even superficially in science is aware of some of the startling predictions that arise because of Einstein's approach to relative motion. As we examine some of the consequences of relativity in this section, we'll find that they conflict with some of our basic notions of space and time. We will restrict our discussion to the concepts of length, time, and simultaneity, which are quite different in relativistic mechanics from what they are in Newtonian mechanics. For example, in relativistic mechanics, the distance between two points and the time interval between two events depend on the frame of reference in which they are measured. **In relativistic mechanics, there is no such thing as absolute length or absolute time.** Further, **events at different locations that are observed to occur simultaneously in one frame are not observed to be simultaneous in another frame moving uniformly past the first.**

Absolute length and absolute time intervals are meaningless in relativity. ▶

### Simultaneity and the Relativity of Time

A basic premise of Newtonian mechanics is that a universal time scale exists that is the same for all observers. In fact, Newton wrote, "Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external." Newton and his followers simply took simultaneity for granted. In his special theory of relativity, Einstein abandoned that assumption.

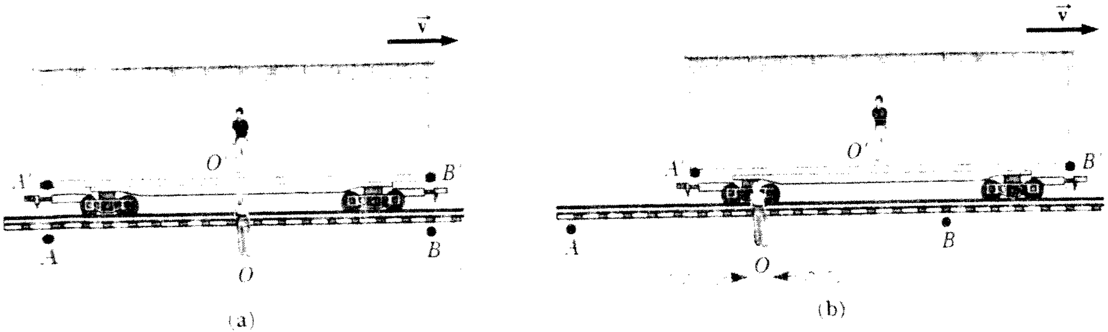



**Figure 26.7** Two lightning bolts strike the ends of a moving boxcar. (a) The events appear to be simultaneous to the stationary observer at $O$, who is midway between $A$ and $B$. (b) The events don't appear to be simultaneous to the observer at $O'$, who claims that the front of the train is struck *before* the rear.

Einstein devised the following thought experiment to illustrate this point: a boxcar moves with uniform velocity, and two lightning bolts strike its ends, as in Figure 26.7a, leaving marks on the boxcar and the ground. The marks on the boxcar are labeled $A'$ and $B'$, and those on the ground are labeled $A$ and $B$. An observer at $O'$ moving with the boxcar is midway between $A'$ and $B'$, and an observer on the ground at $O$ is midway between $A$ and $B$. The events recorded by the observers are the striking of the boxcar by the two lightning bolts.

The light signals recording the instant at which the two bolts struck reach observer $O$ at the same time, as indicated in Figure 26.7b. This observer realizes that the signals have traveled at the same speed over equal distances, and so rightly concludes that the events at $A$ and $B$ occurred simultaneously. Now consider the same events as viewed by observer $O'$. By the time the signals have reached observer $O$, observer $O'$ has moved as indicated in Figure 26.7b. Thus, the signal from $B'$ has already swept past $O'$, but the signal from $A'$ has not yet reached $O'$. In other words, $O'$ sees the signal from $B'$ before seeing the signal from $A'$. According to Einstein, *the two observers must find that light travels at the same speed.* Therefore, observer $O'$ concludes that the lightning struck the front of the boxcar before it struck the back.

**TIP 26.1 Who's Right?**
Which person is correct concerning the simultaneity of the two events? Both are correct, because the principle of relativity states that no inertial frame of reference is preferred. Although the two observers may reach different conclusions, both are correct in their own reference frame. Any uniformly moving frame of reference can be used to describe events and do physics.

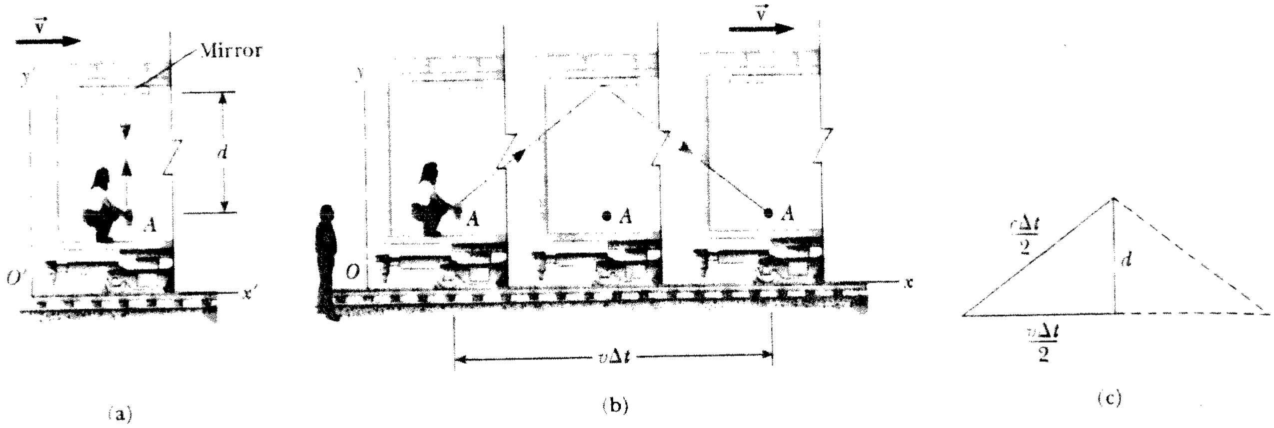
This thought experiment clearly demonstrates that the two events which appear to be simultaneous to observer $O$ do not appear to be simultaneous to observer $O'$. In other words,

Two events that are simultaneous in one reference frame are in general not simultaneous in a second frame moving relative to the first. Simultaneity depends on the state of motion of the observer, and is therefore not an absolute concept.

At this point, you might wonder which observer is right concerning the two events. The answer is that *both* are correct, because the principle of relativity states that **there is no preferred inertial frame of reference**. Although the two observers reach different conclusions, both are correct in their own reference frames because the concept of simultaneity is not absolute. In fact, this is the central point of relativity. Any inertial frame of reference can be used to describe events and do physics.

## Time Dilation

We can illustrate the fact that observers in different inertial frames may measure different time intervals between a pair of events by considering a vehicle moving to the right with a speed $v$ as in Active Figure 26.8a (page 852). A mirror is fixed to the ceiling of the vehicle, and an observer $O'$ at rest in this system holds a laser a distance $d$ below the mirror. At some instant, the laser emits a pulse of light




**ACTIVE FIGURE 26.8**
(a) A mirror is fixed to a moving vehicle, and a light pulse leaves $O'$ at rest in the vehicle. (b) Relative to a stationary observer on Earth, the mirror and $O'$ move with a speed $v$. Note that the distance the pulse travels is greater than $2d$ as measured by the stationary observer. (c) The right triangle for calculating the relationship between $\Delta t$ and $\Delta t_p$.

**Physics Now™**
Log into PhysicsNow at **www.cp7e.com** and go to Active Figure 26.8, where you can observe the bouncing of the light pulse for various speeds of the train.

directed toward the mirror (event 1), and at some later time after reflecting from the mirror, the pulse arrives back at the laser (event 2). Observer $O'$ carries a clock and uses it to measure the time interval $\Delta t_p$ between these two events which she views as occurring at the same place. (The subscript $p$ stands for *proper*, as we'll see in a moment.) Because the light pulse has a speed $c$, the time it takes it to travel from point $A$ to the mirror and back to point $A$ is

$$\Delta t_p = \frac{\text{Distance traveled}}{\text{Speed}} = \frac{2d}{c} \qquad \textbf{[26.6]}$$

The time interval $\Delta t_p$ measured by $O'$ requires only a single clock located at the same place as the laser in this frame.

Now consider the same set of events as viewed by $O$ in a second frame, as shown in Active Figure 26.8b. According to this observer, the mirror and laser are moving to the right with a speed $v$, and as a result, the sequence of events appears different. By the time the light from the laser reaches the mirror, the mirror has moved to the right a distance $v\,\Delta t/2$, where $\Delta t$ is the time it takes the light pulse to travel from point $A$ to the mirror and back to point $A$ as measured by $O$. In other words, $O$ concludes that, because of the motion of the vehicle, if the light is to hit the mirror, it must leave the laser at an angle with respect to the vertical direction. Comparing Active Figures 26.8a and 26.8b, we see that the light must travel farther in (b) than in (a). (Note that neither observer "knows" that he or she is moving. Each is at rest in his or her own inertial frame.)

According to the second postulate of the special theory of relativity, both observers must measure $c$ for the speed of light. Because the light travels farther in the frame of $O$, it follows that the time interval $\Delta t$ measured by $O$ is longer than the time interval $\Delta t_p$ measured by $O'$. To obtain a relationship between these two time intervals, it is convenient to examine the right triangle shown in Active Figure 26.8c. The Pythagorean theorem gives

$$\left(\frac{c\Delta t}{2}\right)^2 = \left(\frac{v\Delta t}{2}\right)^2 + d^2$$

Solving for $\Delta t$ yields

$$\Delta t = \frac{2d}{\sqrt{c^2 - v^2}} = \frac{2d}{c\sqrt{1 - v^2/c^2}}$$

Because $\Delta t_p = 2d/c$, we can express this result as

$$\Delta t = \frac{\Delta t_p}{\sqrt{1 - v^2/c^2}} = \gamma \Delta t_p \quad [26.7]$$

◀ Time dilation

where

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \quad [26.8]$$

Because $\gamma$ is always greater than one, Equation 26.7 says that **the time interval $\Delta t$ between two events measured by an observer moving with respect to a clock**[3] **is longer than the time interval $\Delta t_p$ between the same two events measured by an observer at rest with respect to the clock.** Consequently, $\Delta t > \Delta t_p$, and the proper time interval is expanded or dilated by the factor $\gamma$. Hence, this effect is known as **time dilation**.

For example, suppose the observer at rest with respect to the clock measures the time required for the light flash to leave the laser and return. We assume that the measured time interval in this frame of reference, $\Delta t_p$, is one second. (This would require a very tall vehicle.) Now we find the time interval as measured by observer $O$ moving with respect to the same clock. If observer $O$ is traveling at half the speed of light ($v = 0.500c$), then $\gamma = 1.15$, and according to Equation 26.7, $\Delta t = \gamma \Delta t_p = 1.15(1.00 \text{ s}) = 1.15 \text{ s}$. Therefore, when observer $O'$ claims that 1.00 s has passed, observer $O$ claims that 1.15 s has passed. Observer $O$ considers the clock of $O'$ to be reading too low a value for the elapsed time between the two events and says that the clock of $O'$ is "running slow." From this phenomenon, we may conclude the following:

> A clock moving past an observer at speed $v$ runs more slowly than an identical clock at rest with respect to the observer by a factor of $\gamma^{-1}$.

◀ A clock in motion runs more slowly than an identical stationary clock.

The time interval $\Delta t_p$ in Equations 26.6 and 26.7 is called the **proper time**. In general, **proper time is the time interval between two events as measured by an observer who sees the events occur at the same position.**

Although you may have realized it by now, it's important to spell out that relativity is a scientific democracy: the view of $O'$ that $O$ is really the one moving with speed $v$ to the left and that $O$'s clock is running more slowly is just as valid as the view of $O$. The principle of relativity requires that the views of two observers in uniform relative motion be equally valid and capable of being checked experimentally.

**TIP 26.2 Proper Time Interval**

You must be able to correctly identify the observer who measures the proper time interval. The proper time interval between two events is the time interval measured by an observer for whom the two events take place at the same position.

We have seen that moving clocks run slow by a factor of $\gamma^{-1}$. This is true for ordinary mechanical clocks as well as for the light clock just described. In fact, we can generalize these results by stating that all physical processes, including chemical and biological ones, slow down relative to a clock when those processes occur in a frame moving with respect to the clock. For example, the heartbeat of an astronaut moving through space would keep time with a clock inside the spaceship. Both the astronaut's clock and heartbeat would be slowed down relative to a clock back on Earth (although the astronaut would have no sensation of life slowing down in the spaceship).

Time dilation is a very real phenomenon that has been verified by various experiments involving the ticking of natural clocks. An interesting example of time dilation involves the observation of *muons*—unstable elementary particles that are very similar to electrons, having the same charge, but 207 times the mass. Muons can be produced by the collision of cosmic radiation with atoms high in the atmosphere. These particles have a lifetime of 2.2 $\mu$s when measured in a reference frame at rest with respect to them. If we take 2.2 $\mu$s as the average lifetime of a muon and assume that their speed is close to the speed of light, we find that

[3] Actually, Figure 26.8 shows the clock moving and not the observer, but this is equivalent to observer $O$ moving to the left with velocity $\vec{v}$ with respect to the clock.

these particles can travel only about 600 m before they decay (Fig. 26.9a). Hence, they could never reach Earth from the upper atmosphere where they are produced. However, experiments show that a large number of muons *do* reach Earth, and the phenomenon of time dilation explains how. Relative to an observer on Earth, the muons have a lifetime equal to $\gamma\tau_p$, where $\tau_p = 2.2\ \mu s$ is the lifetime in a frame of reference traveling with the muons. For example, for $v = 0.99c$, $\gamma \approx 7.1$ and $\gamma\tau_p \approx 16\ \mu s$. Hence, the average distance muons travel as measured by an observer on Earth is $\gamma v \tau_p \approx 4\,800$ m, as indicated in Figure 26.9b. Consequently, muons can reach Earth's surface.

In 1976 experiments with muons were conducted at the laboratory of the European Council for Nuclear Research (CERN) in Geneva. Muons were injected into a large storage ring, reaching speeds of about 0.9994$c$. Electrons produced by the decaying muons were detected by counters around the ring, enabling scientists to measure the decay rate, and hence the lifetime of the muons. The lifetime of the moving muons was measured to be about 30 times as long as that of stationary muons to within two parts in a thousand, in agreement with the prediction of relativity.

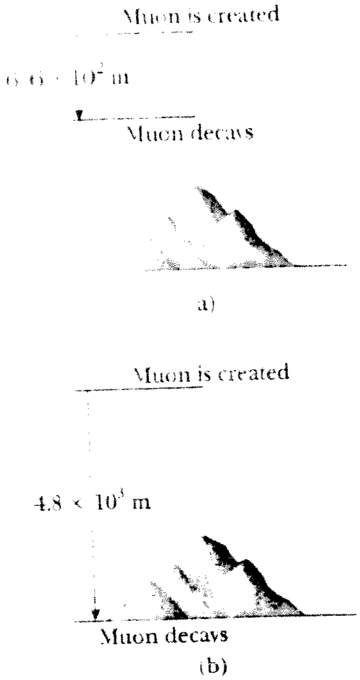



**Figure 26.9** (a) The muons travel only about $6.6 \times 10^2$ m as measured in the muons' reference frame, in which their lifetime is about 2.2 $\mu$s. Because of time dilation, the muons' lifetime is longer as measured by the observer on Earth. (b) Muons traveling with a speed of 0.99$c$ travel a distance of about $4.80 \times 10^3$ m as measured by an observer on Earth.

**Quick Quiz 26.1**

Suppose you're an astronaut being paid according to the time you spend traveling in space. You take a long voyage traveling at a speed near that of light. Upon your return to Earth, you're asked how you'd like to be paid: according to the time elapsed on a clock on Earth or according to your ship's clock. Which should you choose in order to maximize your paycheck? (a) the Earth clock (b) the ship's clock (c) Either clock, it doesn't make a difference.

## EXAMPLE 26.1 Pendulum Periods

**Goal** Apply the concept of time dilation.

**Problem** The period of a pendulum is measured to be 3.00 s in the inertial frame of the pendulum. What is the period as measured by an observer moving at a speed of 0.950$c$ with respect to the pendulum?

***Exercise 26.1***

What pendulum period does a third observer moving at 0.900$c$ measure?

The confusion that arises in problems like Example 26.1 lies in the fact that movement is relative: from the point of view of someone in the pendulum's rest frame, the pendulum is standing still (except, of course, for the swinging motion), whereas to someone in a frame that is moving with respect to the pendulum, it's the pendulum that's doing the moving. To keep this straight, always focus on the observer who is doing the measurement, and ask yourself whether the clock being measured is moving with respect to that observer. If the answer is no, then the observer is in the rest frame of the clock and measures the clock's proper time. If the answer is yes, then the time measured by the observer will be dilated—larger than the clock's proper time.

This confusion of perspectives led to the famous "twin paradox."

## The Twin Paradox

An intriguing consequence of time dilation is the so-called twin paradox (Fig. 26.10). Consider an experiment involving a set of twins named Speedo and Goslo. When they are 20 years old, Speedo, the more adventuresome of the two, sets out on an epic journey to Planet X, located 20 lightyears from Earth. Further, his spaceship is capable of reaching a speed of $0.95c$ relative to the inertial frame of his twin brother back home. After reaching Planet X, Speedo becomes homesick and immediately returns to Earth at the same speed of $0.95c$. Upon his return, Speedo is shocked to discover that Goslo has aged $2D/v = 2(20 \text{ ly})/(0.95 \text{ ly/y}) = 42$ years and is now 62 years old. Speedo, on the other hand, has aged only 13 years.

◀ The space traveler ages more slowly than his twin who remains on Earth.

Some wrongly consider *this* the paradox; that twins could age at different rates and end up after a period of time having very different ages. While contrary to our common sense, this isn't the paradox at all. The paradox lies in the fact that from Speedo's point of view, *he* was at rest while Goslo (on Earth) sped away from *him* at $0.95c$ and returned later. So Goslo's clock was moving relative to Speedo and hence running slow compared with Speedo's clock. The conclusion: Speedo, not Goslo, should be the older of the twins!

To resolve this apparent paradox, consider a third observer moving at a constant speed of $0.5c$ relative to Goslo. To the third observer, Goslo never changes inertial frames: His speed relative to the third observer is always the same. The third observer notes, however, that Speedo accelerates during his journey, *changing reference frames in the process.* From the third observer's perspective, it's clear that there is something very different about the motion of Goslo when compared to Speedo. The roles played by Goslo and Speedo are not symmetric, so it isn't surprising that time flows differently for each. Further, because Speedo accelerates, he is in a noninertial frame of reference—technically outside the bounds of special relativity (though there are methods for dealing with accelerated motion in relativity). Only Goslo, who is in a single inertial frame, can apply the simple time-dilation formula to Speedo's trip. Goslo finds that instead of aging 42 years,

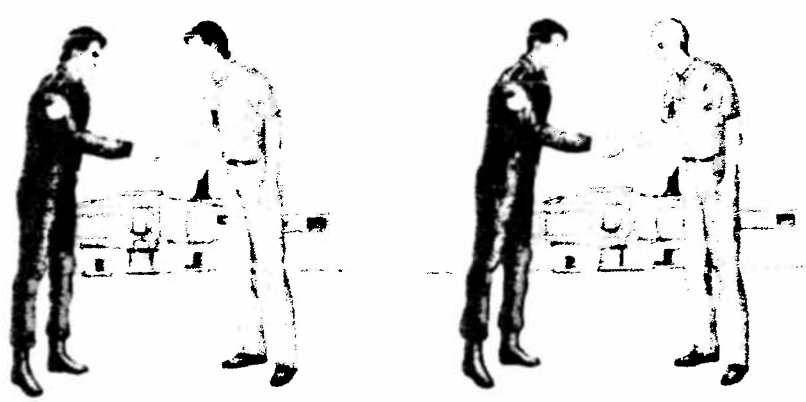

(a) (b)

**Figure 26.10** (a) As the twins depart, they're the same age. (b) When Speedo returns from his journey to Planet X, he's younger than his twin Goslo, who remained on Earth.

Speedo ages only $(1 - v^2/c^2)^{1/2}(42 \text{ years}) = 13$ years. Of these 13 years, Speedo spends 6.5 years traveling to Planet X and 6.5 years returning, for a total travel time of 13 years, in agreement with our earlier statement.

## Length Contraction

**TIP 26.3 The Proper Length**

You must be able to correctly identify the observer who measures the proper length. The proper length between two points in space is the length measured by an observer at rest with respect to the length. Very often, the proper time interval and the proper length are not measured by the same observer.

The measured distance between two points depends on the frame of reference of the observer. The **proper length** $L_p$ of an object is **the length of the object as measured by an observer at rest relative to the object.** The length of an object measured in a reference frame that is moving with respect to the object is always less than the proper length. This effect is known as **length contraction.**

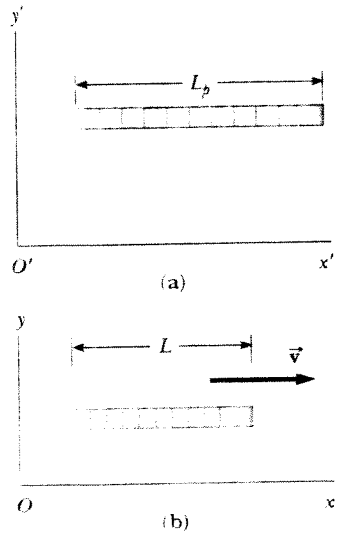
To understand length contraction quantitatively, consider a spaceship traveling with a speed $v$ from one star to another, as seen by two observers, one on Earth and the other in the spaceship. The observer at rest on Earth (and also assumed to be at rest with respect to the two stars) measures the distance between the stars to be $L_p$. According to this observer, the time it takes the spaceship to complete the voyage is $\Delta t = L_p/v$. Because of time dilation, the space traveler, using his spaceship clock, measures a smaller time of travel: $\Delta t_p = \Delta t/\gamma$. The space traveler claims to be at rest and sees the destination star moving toward the spaceship with speed $v$. Because the space traveler reaches the star in time $\Delta t_p$, he concludes that the distance $L$ between the stars is shorter than $L_p$. The distance measured by the space traveler is

$$L = v\,\Delta t_p = v\frac{\Delta t}{\gamma}$$

Because $L_p = v\,\Delta t$, it follows that

Length contraction ▶

$$L = \frac{L_p}{\gamma} = L_p\sqrt{1 - v^2/c^2} \qquad [26.9]$$




**ACTIVE FIGURE 26.11**

A meter stick moves to the right with a speed $v$. (a) The meter stick as viewed by an observer at rest with respect to the meter stick. (b) The meter stick as seen by an observer moving with a speed $v$ with respect to it. The moving meter stick is always measured to be *shorter* than in its own rest frame by a factor of $\sqrt{1 - v^2/c^2}$.

**Physics Now™**

Log into PhysicsNow at **www.cp7e.com** and go to Active Figure 26.11, where you can view the meter stick from the points of view of two observers and compare the measured lengths of the stick.

According to this result, illustrated in Active Figure 26.11, if an observer at rest with respect to an object measures its length to be $L_p$, an observer moving at a speed $v$ relative to the object will find it to be shorter than its proper length by the factor $\sqrt{1 - v^2/c^2}$. Note that **length contraction takes place only along the direction of motion.**

Time-dilation and length contraction effects have interesting applications for future space travel to distant stars. In order for the star to be reached in a fraction of a human lifetime, the trip must be taken at very high speeds. According to an Earth-bound observer, the time for a spacecraft to reach the destination star will be dilated compared with the time interval measured by travelers. As was discussed in the treatment of the twin paradox, the travelers will be younger than their twins when they return to Earth. Therefore, by the time the travelers reach the star, they will have aged by some number of years, while their partners back on Earth will have aged a larger number of years, the exact ratio depending on the speed of the spacecraft. At a spacecraft speed of $0.94c$, this ratio is about 3:1.

**Quick Quiz 26.2**

You are packing for a trip to another star, and on your journey you will be traveling at a speed of $0.99c$. Can you sleep in a smaller cabin than usual, because you will be shorter when you lie down? Explain your answer.

**Quick Quiz 26.3**

You observe a rocket moving away from you. Compared to its length when it was at rest on the ground, you will measure its length to be (a) shorter, (b) longer, or (c) the same. Compared to the passage of time measured by the watch on your wrist, the passage of time on the rocket's clock is (d) faster, (e) slower, or (f) the same. Answer the same questions if the rocket turns around and comes toward you.

## EXAMPLE 26.2 Starship Contraction

**Goal** Apply the concept of length contraction to a moving object.

**Problem** A starship is measured to be 125 m long while it is at rest with respect to an observer. If this starship now flies past the observer at a speed of $0.99c$, what length will the observer measure for the starship?

***Exercise 26.2***
If the ship moves past the observer with a speed of $0.80c$, what length will the observer measure?

## EXAMPLE 26.3 Speedy Plunge

**Goal** Apply the concept of length contraction to a distance.

**Problem** **(a)** An observer on Earth sees a spaceship at an altitude of 4 350 km moving downward toward Earth with a speed of $0.970c$. What is the distance from the spaceship to Earth as measured by the spaceship's captain? **(b)** After firing his engines, the captain measures her ship's altitude as 267 km, while the observer on Earth measures it to be 625 km. What is the speed of the spaceship at this instant?

***Exercise 26.3***
Suppose the observer on the ship measures the distance from Earth as 50.0 km, while the observer on Earth measures the distance as 125 km. At what speed is the spacecraft approaching Earth?

Length contraction occurs only in the direction of the observer's motion. No contraction occurs perpendicular to that direction. For example, a spaceship at rest relative to an observer may have the shape of an equilateral triangle, but if it passes the observer at relativistic speed in a direction parallel to its base, the base will shorten while the height remains the same. Hence, the craft will be observed to be isosceles. An observer traveling with the ship will still observe it to be equilateral.

## 26.7 RELATIVISTIC MOMENTUM

Properly describing the motion of particles within the framework of special relativity requires generalizing Newton's laws of motion and the definitions of momentum and energy. These generalized definitions reduce to the classical (nonrelativistic) definitions when $v$ is much less than $c$.

First, recall that conservation of momentum states that when two objects collide, the total momentum of the system remains constant, assuming that the objects are isolated, reacting only with each other. However, analyzing such collisions from rapidly moving inertial frames, it is found that momentum is not conserved if the classical definition of momentum, $p = mv$, is used. In order to have momentum conservation in all inertial frames—even those moving at an appreciable fraction of $c$—the definition of momentum must be modified to read

Momentum ▶

$$p \equiv \frac{mv}{\sqrt{1 - v^2/c^2}} = \gamma mv \qquad [26.10]$$

where $v$ is the speed of the particle and $m$ is its mass as measured by an observer at rest with respect to the particle. Note that when $v$ is much less than $c$, the denominator of Equation 26.10 approaches one, so that $p$ approaches $mv$. Therefore, the relativistic equation for momentum reduces to the classical expression when $v$ is small compared with $c$.

### EXAMPLE 26.4 The Relativistic Momentum of an Electron

**Goal** Contrast the classical and relativistic definitions of momentum.

**Problem** An electron, which has a mass of $9.11 \times 10^{-31}$ kg, moves with a speed of $0.750c$. Find the classical (nonrelativistic) momentum and compare it to its relativistic counterpart $p_{\rm rel}$.

***Exercise 26.4***

Repeat the calculation for a proton traveling at $0.600c$.

## 26.8 RELATIVISTIC ADDITION OF VELOCITIES

Imagine a motorcycle rider moving with a speed of $0.80c$ past a stationary observer, as shown in Figure 26.12. If the rider tosses a ball in the forward direction with a speed of $0.70c$ relative to himself, what is the speed of the ball as seen by the stationary observer at the side of the road? Common sense and the ideas of Newtonian relativity say that the speed should be the sum of the two speeds, or $1.50c$. This answer must be incorrect because it contradicts the assertion that no material object can travel faster than the speed of light.

Einstein resolved this dilemma by deriving an equation for the relativistic addition of velocities. Here, only one dimension of motion will be considered. Let two frames or reference be labeled $b$ and $d$, and suppose that frame $d$ is moving at velocity $v_{db}$ in the position $x$-direction relative frame $b$. If the velocity of an object $a$ as measured in frame $d$ is called $v_{ad}$, then the velocity of $a$ as measured in frame $b$, $v_{ab}$, is given by

$$v_{ab} = \frac{v_{ad} + v_{db}}{1 + \frac{v_{ad} v_{db}}{c^2}} \qquad [26.11]$$

◀ Relativistic velocity addition

The left side of this equation and the numerator on the right are like the equations of Galilean relativity discussed in Chapter 3, and the evaluation of subscripts is applied in the same way as discussed in Section 3.6. The denominator of Equation 26.11 is a correction to Galilean relativity based on length contraction and time dilation.

We apply Equation 26.11 to Figure 26.13, which shows a motorcyclist, his ball, and a stationary observer. We are given

$v_{bm}$ = the velocity of the ball with respect to the motorcycle = $0.70c$

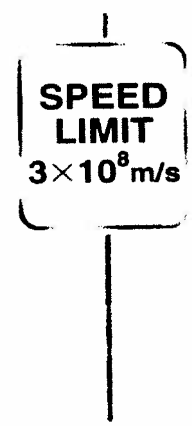
$v_{mo}$ = the velocity of the motorcycle with respect to the stationary observer = $0.80c$,

and we want to find

$v_{bo}$ = the velocity of the ball with respect to the stationary observer.

Thus,

$$v_{bo} = \frac{v_{bm} + v_{mo}}{1 + \frac{v_{bm} v_{mo}}{c^2}} = \frac{0.70c + 0.80c}{1 + \frac{(0.70c)(0.80c)}{c^2}} = 0.96c$$




The speed of light is the speed limit of the Universe.

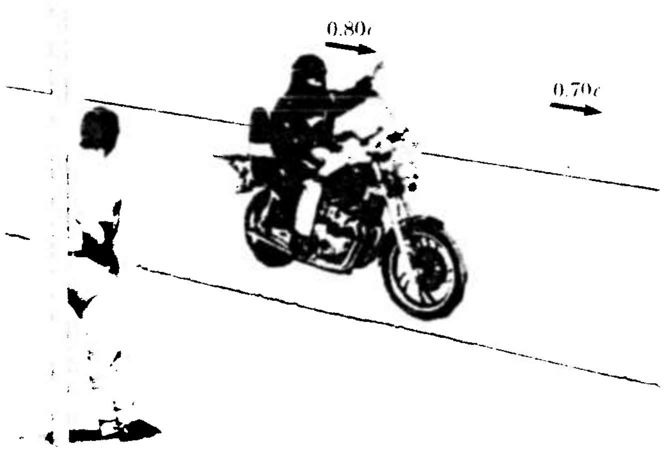



**Figure 26.12** A motorcycle moves past a stationary observer with a speed of $0.80c$; the motorcyclist throws a ball in the direction of motion with a speed of $0.70c$ relative to himself.

## EXAMPLE 26.5 Urgent Course Correction Needed!

**Goal** Apply the concept of the relativistic addition of velocities.

**Problem** Suppose that Bob's spacecraft is traveling at $0.600c$ in the positive $x$-direction, as measured by a nearby observer, while Mike is traveling in his own vehicle directly toward Bob in the negative $x$-direction at $-0.800c$ relative the nearby observer. What's the velocity of Bob relative to Mike?

***Exercise 26.5***
Suppose Bob shines a laser beam in the direction of his ship's motion. What speed would the nearby observer measure for the beam? Don't guess: do the calculation that proves the answer.

## 26.9 RELATIVISTIC ENERGY AND THE EQUIVALENCE OF MASS AND ENERGY

We have seen that the definition of momentum required generalization to make it compatible with the principle of relativity. Likewise, the definition of kinetic energy requires modification in relativistic mechanics. Einstein found that the correct expression for the **kinetic energy** of an object is

Kinetic energy ▶

$$KE = \gamma mc^2 - mc^2 \quad [26.12]$$

The constant term $mc^2$ in Equation 26.12, which is independent of the speed of the object, is called the **rest energy** of the object, $E_R$:

$$E_R = mc^2 \qquad [26.13]$$

◂ Rest energy

The term $\gamma mc^2$ in Equation 26.12 depends on the object's speed and is the sum of the kinetic and rest energies. We define $\gamma mc^2$ to be the **total energy** $E$, so that

total energy = kinetic energy + rest energy

or, using Equation 26.12,

$$E = KE + mc^2 = \gamma mc^2 \qquad [26.14]$$

Because $\gamma = (1 - v^2/c^2)^{-1/2}$, we can also express $E$ as

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \qquad [26.15]$$

◂ Total energy

This is Einstein's famous mass–energy equivalence equation.[4]

The relation $E = \gamma mc^2 = KE + mc^2$ shows the amazing result that **a stationary particle with zero kinetic energy has an energy proportional to its mass**. Further, a small mass corresponds to an enormous amount of energy because the proportionality constant between mass and energy is large: $c^2 = 9 \times 10^{16}\ \mathrm{m^2/s^2}$. The equation $E_R = mc^2$, as Einstein first suggested, indirectly implies that the mass of a particle may be completely convertible to energy and that pure energy—for example, electromagnetic energy—may be converted to particles having mass. This is indeed the case, as has been shown in the laboratory many times. For example, the coming together of a slowly moving electron and its antiparticle, the positron, a particle with the same mass $m_e$ as the electron, but opposite charge, results in the disappearance of both particles and the appearance of a burst of electromagnetic energy in the amount $2m_ec^2$. The reverse process is also fairly easily observed in the laboratory: A high-energy pulse of electromagnetic energy, a gamma ray—disappears near an atom and an electron–positron pair is created with nearly 100% conversion of the gamma ray's energy into mass. Such a pair-production process is shown in the bubble chamber photo of Figure 26.13. We will discuss pair production and annihilation in more detail in Section 26.10.

On a larger scale, nuclear power plants produce energy by the fission of uranium, which involves the conversion of a small amount of the mass of the uranium into energy. The Sun, too, converts mass into energy, and continually loses mass in pouring out a tremendous amount of electromagnetic energy in all directions.

It's extremely interesting that while we have been talking about the interconversion of mass and energy for particles, the expression $E = mc^2$ is universal and applies to all objects, processes, and systems: a hot object has slightly more mass and is slightly more difficult to accelerate than an identical cold object because it has more thermal energy, and a stretched spring has more elastic potential energy and more mass than an identical unstretched spring. A key point, however, is that these changes in mass are often far too small to measure. Our best bet for measuring mass changes is in nuclear transformations, where a measurable fraction of the mass is converted into energy.

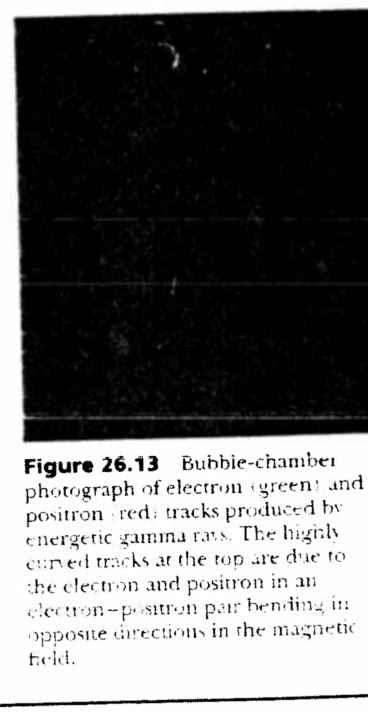




**Figure 26.13** Bubble-chamber photograph of electron (green) and positron (red) tracks produced by energetic gamma rays. The highly curved tracks at the top are due to the electron and positron in an electron–positron pair bending in opposite directions in the magnetic field.

## EXAMPLE 26.6 Pool Heater

**Goal** Combine the concepts of density, rest mass, and heat capacity.

**Problem** Suppose some mechanism allowed the conversion of the rest mass of water completely into energy. **(a)** How much rest energy is contained in 0.500 mm$^3$ of water? **(b)** If all this energy is used to heat an Olympic swim-

[4] Although this doesn't look exactly like the famous equation $E = mc^2$, it used to be common to write $m = \gamma m_0$ (Einstein himself wrote it that way), where $m$ is the effective mass of an object moving at speed $v$ and $m_0$ is the mass of that object as measured by an observer at rest with respect to the object. Then our $E = \gamma m_0c^2$ becomes the familiar $E = mc^2$. It is currently unfashionable to use $m = \gamma m_0$.

ming pool with dimensions 2.00 m deep, 25.0 m wide, and 50.0 m long, what is the change in temperature of the water?

***Exercise 26.6***
(a) What mass, when completely converted into energy, would provide the annual energy needs of the entire world (about $4 \times 10^{20}$ J)? (b) What volume of water contains that much energy?

---

## Energy and Relativistic Momentum

Often the momentum or energy of a particle is measured rather than its speed, so it's useful to have an expression relating the total energy $E$ to the relativistic momentum $p$. This is accomplished by using the expressions $E = \gamma mc^2$ and $p = \gamma mv$. By squaring these equations and subtracting, we can eliminate $v$. The result, after some algebra, is

$$E^2 = p^2c^2 + (mc^2)^2 \qquad [26.16]$$

When the particle is at rest, $p = 0$, so $E = E_R = mc^2$. In this special case, the total energy equals the rest energy. For the case of particles that have zero mass, such as photons (massless, chargeless particles of light), we set $m = 0$ in Equation 26.16 and find that

$$E = pc \qquad [26.17]$$

This equation is an exact expression relating energy and momentum for photons, which always travel at the speed of light.

In dealing with subatomic particles, it's convenient to express their energy in electron volts (eV), because the particles are given energy when accelerated through an electrostatic potential difference. The conversion factor is

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$$

For example, the mass of an electron is $9.11 \times 10^{-31}$ kg. Hence, the rest energy of the electron is

$$m_e c^2 = (9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 = 8.20 \times 10^{-14} \text{ J}$$

Converting to eV, we have

$$m_e c^2 = (8.20 \times 10^{-14} \text{ J})(1 \text{ eV}/1.60 \times 10^{-19} \text{ J}) = 0.511 \text{ MeV}$$

Because we frequently use the expression $E = \gamma mc^2$ in nuclear physics, and because $m$ is usually in atomic mass units, u, it is useful to have the conversion factor $1 \text{ u} = 931.494 \text{ MeV}/c^2$. Using this factor makes it easy, for example, to find the rest energy in MeV of the nucleus of a uranium atom with a mass of 235.043 924 u:

$$E_R = mc^2 = (235.043\,924 \text{ u})(931.494 \text{ MeV/u} \cdot c^2)(c^2) = 2.189\,42 \times 10^5 \text{ MeV}$$

**Quick Quiz 26.4**

A photon is reflected from a mirror. **True or false:** (a) Because a photon has zero mass, it does not exert a force on the mirror. (b) Although the photon has energy, it can't transfer any energy to the surface because it has zero mass. (c) The photon carries momentum, and when it reflects off the mirror, it undergoes a change in momentum and exerts a force on the mirror. (d) Although the photon carries momentum, its change in momentum is zero when it reflects from the mirror, so it can't exert a force on the mirror.

## EXAMPLE 26.7 A Speedy Electron

**Goal** Compute a total energy and a relativistic kinetic energy.

**Problem** An electron moves with a speed $v = 0.850c$. Find its total energy and kinetic energy in mega electron volts (MeV), and compare the latter to the classical kinetic energy ($10^6$ eV = 1 MeV).

**Strategy** Substitute into Equation 26.15 to get the total energy, and subtract the rest mass energy to obtain the kinetic energy.

**Remarks** Notice the large discrepancy between the relativistic kinetic energy and the classical kinetic energy.

***Exercise 26.7***

Calculate the total energy and the kinetic energy in MeV of a proton traveling at $0.600c$. (The rest energy of a proton is approximately 938 MeV.)

---

## EXAMPLE 26.8 The Conversion of Mass to Kinetic Energy in Uranium Fission

**Goal** Understand the production of energy from nuclear sources.

**Problem** The fission, or splitting, of uranium was discovered in 1938 by Lise Meitner, who successfully interpreted some curious experimental results found by Otto Hahn as due to fission. (Hahn received the Nobel prize.) The fission of $^{235}_{92}U$ begins with the absorption of a slow-moving neutron that produces an unstable nucleus of $^{236}U$. The $^{236}U$ nucleus then quickly decays into two heavy fragments moving at high speed, as well as several neutrons. Most of the kinetic energy released in such a fission is carried off by the two large fragments. **(a)** For the typical fission process

$$^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,^{1}_{0}n$$

calculate the kinetic energy in MeV carried off by the fission fragments. **(b)** What percentage of the initial energy is converted into kinetic energy? The atomic masses involved are given below in atomic mass units.

$$^{1}_{0}n = 1.008\,665\text{ u} \qquad {}^{235}_{92}U = 235.043\,924\text{ u} \qquad {}^{141}_{56}Ba = 140.903\,496\text{ u} \qquad {}^{92}_{36}Kr = 91.907\,936\text{ u}$$

**Strategy** This is an application of the conservation of relativistic energy. Write the conservation law as a sum of kinetic energy and rest energy, and solve for the final kinetic energy. Equation 26.15, solved for $v$, then yields the speeds.

***Exercise 26.8***

In a fusion reaction, light elements combine to form a heavier element. Deuterium, which is also called heavy hydrogen, has an extra neutron in its nucleus. Two such particles can fuse into a heavier form of hydrogen, called tritium, plus an ordinary hydrogen atom. The reaction is

$$^{2}_{1}\text{D} + ^{2}_{1}\text{D} \rightarrow ^{3}_{1}\text{T} + ^{1}_{1}\text{H}$$

(a) Calculate the energy released in the form of kinetic energy, assuming for simplicity that the initial kinetic energy is zero. (b) What percentage of the rest mass was converted to energy? The atomic masses involved are as follows:

$$^{2}_{1}\text{D} = 2.014\,102\ \text{u} \qquad ^{3}_{1}\text{T} = 3.016\,049\ \text{u} \qquad ^{1}_{1}\text{H} = 1.007\,825\ \text{u}$$

**Answers** (a) 4.033 37 MeV (b) 0.1075%

## 26.10 PAIR PRODUCTION AND ANNIHILATION

In general, converting mass into energy is a low-yield process. Burning wood or coal, or even the fission or fusion processes presented in Example 26.8, convert only a very small percentage of the available energy. An exception is the reaction of matter with antimatter.

A common process in which a photon creates matter is called **pair production**, illustrated in Figure 26.14. In this process, an electron and a positron are simultaneously produced, while the photon disappears. (Note that the positron is a positively charged particle having the same mass as an electron. The positron is often called the *antiparticle* of the electron.) In order for pair production to occur, energy, momentum, and charge must all be conserved during the process. It's impossible for a photon to produce a single electron because the photon has zero charge and charge would not be conserved in the process.

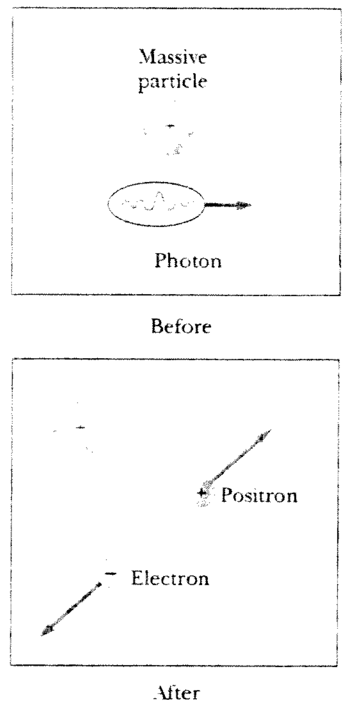



**Figure 26.14** Representation of the process of pair production.

As we explain in more detail in Chapter 27, the energy of a photon having a frequency $f$ is given by $E = hf$, where $h$ is Planck's constant. The *minimum* energy that a photon must have to produce an electron–positron pair can be found using conservation of energy by equating the photon energy $hf_{\text{min}}$ to the total rest energy of the pair. That is,

$$hf_{\text{min}} = 2m_ec^2 \qquad \textbf{[26.18]}$$

Because the energy of an electron is $m_ec^2 = 0.51$ MeV, the minimum energy required for pair production is 1.02 MeV.

Pair production can't occur in a vacuum, but can only take place in the presence of a massive particle such as an atomic nucleus. The massive particle must participate in the interaction in order that energy and momentum be conserved simultaneously.

**Pair annihilation** is a process in which an electron–positron pair produces two photons—the inverse of pair production. Figure 26.15 is one example of pair annihilation in which an electron and positron initially at rest combine with each other, disappear, and create two photons. Because the initial momentum of the pair is zero, it's impossible to produce a single photon. Momentum can be conserved only if two photons moving in opposite directions, both with the same energy and magnitude of momentum, are produced. We will discuss particles and their antiparticles further in Chapter 30.

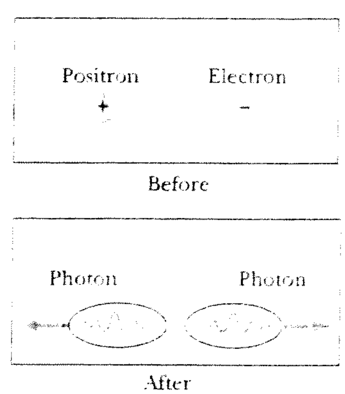



**Figure 26.15** Representation of the process of pair annihilation.

## 26.11 GENERAL RELATIVITY

Special relativity relates observations of inertial observers. Einstein sought a more general theory that would address accelerating systems. His search was motivated in part by the following curious fact: mass determines the inertia of an object and also the strength of the gravitational field. The mass involved in inertia is called inertial mass, $m_i$, whereas the mass responsible for the gravitational field is called the gravitational mass, $m_g$. They appear in Newton's law of gravitation and in the second law of motion:

$$\text{Gravitational property} \qquad F_g = G\frac{m_g m_g'}{r^2}$$

$$\text{Inertial property} \qquad F_i = m_i a$$

The value for the gravitational constant $G$ was chosen to make the magnitudes of $m_g$ and $m_i$ numerically equal. Regardless of how $G$ is chosen, however, the strict proportionality of $m_g$ and $m_i$ has been established experimentally to an extremely high degree: a few parts in $10^{12}$. It appears that gravitational mass and inertial mass may indeed be exactly equal: $m_i = m_g$.

There is no reason a priori, however, why these two very different quantities should be equal. They seem to involve two entirely different concepts: a force of mutual gravitational attraction between two masses and the resistance of a single mass to being accelerated. This question puzzled Newton and many other physicists over the years and was finally incorporated as a fundamental principle of Einstein's remarkable theory of gravitation, known as *general relativity*, in 1916.

In Einstein's view, the remarkable coincidence that $m_g$ and $m_i$ were exactly equal was evidence for an intimate connection between the two concepts. He pointed out that no mechanical experiment (such as releasing a mass) could distinguish between the two situations illustrated in Figures 26.16a and 26.16b. In each case, a mass released by the observer undergoes a downward acceleration of $g$ relative to the floor.

Einstein carried this idea further and proposed that *no* experiment, mechanical or otherwise, could distinguish between the two cases. This extension to include all phenomena (not just mechanical ones) has interesting consequences. For example, suppose that a light pulse is sent horizontally across the box, as in Figure 26.16c. The trajectory of the light pulse bends downward as the box accelerates upward to meet it. Einstein proposed that a beam of light should also be bent downward by a gravitational field (Fig. 26.16d).

The two postulates of Einstein's **general relativity** are as follows:

1. All the laws of nature have the same form for observers in any frame of reference, accelerated or not.
2. In the vicinity of any given point, a gravitational field is equivalent to an accelerated frame of reference without a gravitational field. (This is the *principle of equivalence*.)

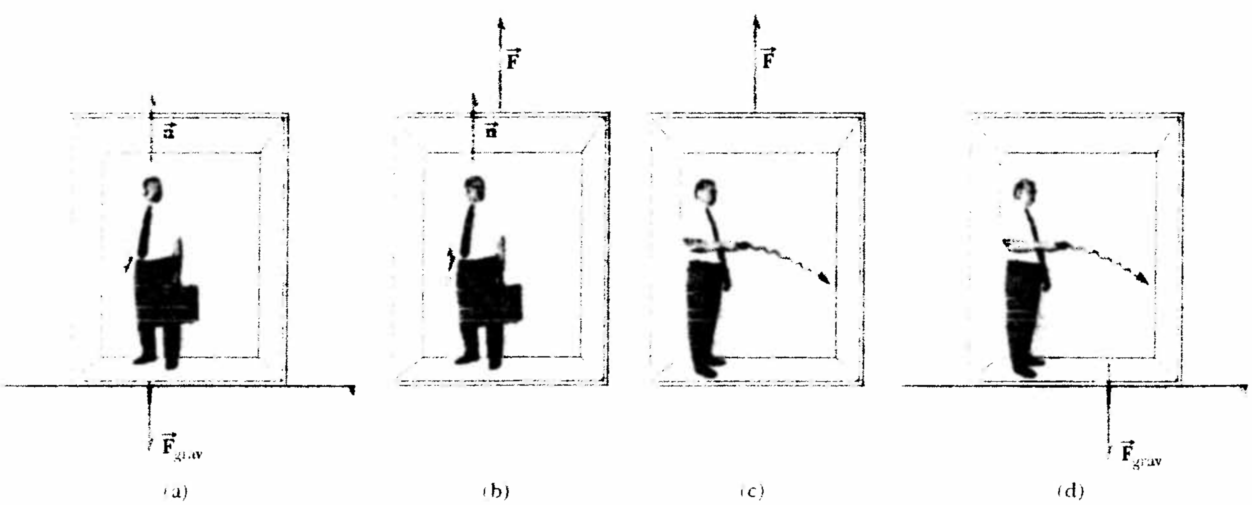



**Figure 26.16** (a) The observer in the cubicle is at rest in a uniform gravitational field $\vec{g}$. He experiences a normal force $\vec{n}$. (b) Now the observer is in a region where gravity is negligible, but an external force $\vec{F}$ acts on the frame of reference, producing an acceleration with magnitude $g$. Again, the man experiences a normal force $\vec{n}$ that accelerates him along with the cubicle. According to Einstein, the frames of reference in parts (a) and (b) are equivalent in every way. No local experiment could distinguish between them. (c) The observer turns on his pocket flashlight. Because of the acceleration of the cubicle, the beam would appear to bend toward the floor, just as a tossed ball would. (d) Given the equivalence of the frames, the same phenomenon should be observed in the presence of a gravity field.

The second postulate implies that gravitational mass and inertial mass are completely equivalent, not just proportional. What were thought to be two different types of mass are actually identical.

One interesting effect predicted by general relativity is that time scales are altered by gravity. A clock in the presence of gravity runs more slowly than one in which gravity is negligible. As a consequence, light emitted from atoms in a strong gravity field, such as the Sun's, is observed to have a lower frequency than the same light emitted by atoms in the laboratory. This gravitational shift has been detected in spectral lines emitted by atoms in massive stars. It has also been verified on Earth by comparing the frequencies of gamma rays emitted from nuclei separated vertically by about 20 m.

**Quick Quiz 26.9**

Two identical clocks are in the same house, one upstairs in a bedroom and the other downstairs in the kitchen. Which statement is correct? (a) The clock in the kitchen runs more slowly than the clock in the bedroom. (b) The clock in the bedroom runs more slowly than the clock in the kitchen. (c) Both clocks keep the same time.

The second postulate suggests that a gravitational field may be "transformed away" at any point if we choose an appropriate accelerated frame of reference—a freely falling one. Einstein developed an ingenious method of describing the acceleration necessary to make the gravitational field "disappear." He specified a certain quantity, the *curvature of spacetime,* that describes the gravitational effect at every point. In fact, the curvature of spacetime completely replaces Newton's gravitational theory. According to Einstein, there is no such thing as a gravitational force. Rather, the presence of a mass causes a curvature of spacetime in the vicinity of the mass. Planets going around the Sun follow the natural contours of the spacetime, much as marbles roll around inside a bowl. In 1979, John Wheeler summarized Einstein's general theory of relativity in a single sentence: "Mass one tells spacetime how to curve; curved spacetime tells mass two how to move." The fundamental equation of general relativity can be roughly stated as a proportion as follows:

$$\text{Average curvature of spacetime} \propto \text{energy density}$$

The equation corresponding to this proportion is solved for a mathematical quantity called the *metric,* which can be used to measure the lengths of vectors and to compute trajectories of bodies through space. The metric looks something like a matrix, with different entries at each point of space and time. (There are a few important differences, beyond the level of this course.)

Einstein pursued a new theory of gravity in large part because of a discrepancy in the orbit of Mercury as calculated from Newton's second law. The closest approach of the Mercury to the Sun, called the perihelion, changes position slowly over time. Newton's theory accounted for all but 43 seconds of arc per century; Einstein's general relativity explained the discrepancy.

The most dramatic test of general relativity came shortly after the end of World War I. The theory predicts that a star would bend a light ray by a certain precise amount. Sir Arthur Eddington mounted an expedition to Africa and, during a solar eclipse, confirmed that starlight bent on passing the Sun in an amount matching the prediction of general relativity (Fig. 26.17). When this discovery was announced, Einstein became an international celebrity.

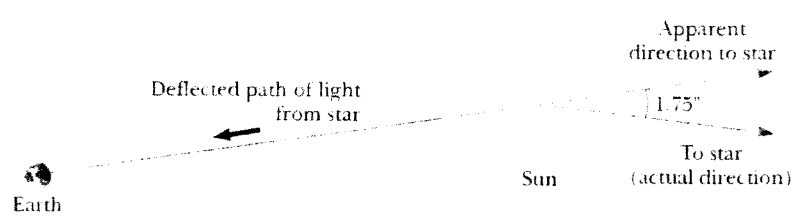



**Figure 26.17** Deflection of starlight passing near the Sun. Because of this effect, the Sun and other remote objects can act as a *gravitational lens.* In his general theory of relativity, Einstein calculated that starlight just grazing the Sun's surface should be deflected by an angle of 1.75".

Other tests were proposed and verified long after Einstein's death, including the time delay of radar bounced off Venus, and the gradual lengthening of the period of binary pulsars due to the emission of gravitational radiation. The latter has been verified with such precision that general relativity can lay claim to being the most accurate theory in physics.

General relativity also predicts extreme states of matter created by gravitational collapse. If the concentration of mass becomes very great, as is believed to occur when a large star exhausts its nuclear fuel and collapses to a very small volume, a **black hole** may form. Here the curvature of spacetime is so extreme that all matter and light within a certain radius becomes trapped. This radius, called the *Schwarzschild radius* or *event horizon*, is about 3 km for a black hole with the mass of our Sun. At the black hole's center may lurk a *singularity*—a point of infinite density and curvature where spacetime comes to an end.

There is strong evidence for the existence of a black hole having a mass of millions of Suns at the center of our galaxy.

## Applying Physics 26.1 Faster Clocks in a "Mile High City"

Atomic clocks are extremely accurate; in fact, an error of 1 second in 3 million years is typical. This error can be described as about one part in $10^{14}$. On the other hand, the atomic clock in Boulder, Colorado, is often 15 ns faster than the one in Washington after only one day. This is an error of about one part in $6 \times 10^{12}$, which is about 17 times larger than the typical error. If atomic clocks are so accurate, why does a clock in Boulder not remain synchronous with one in Washington?

**Explanation** According to the general theory of relativity, the passage of time depends on gravity—clocks run more slowly in strong gravitational fields. Washington is at an elevation very close to sea level, whereas Boulder is about a mile higher in altitude. Hence, the gravitational field at Boulder is weaker than at Washington. As a result, an atomic clock runs more rapidly in Boulder than in Washington. (This effect has been verified by experiment.)

# SUMMARY

**Physics Now™** Take a practice test by logging into PhysicsNow at **www.cp7e.com** and clicking on the Pre-Test link for this chapter.

## 26.5 Einstein's Principle of Relativity

The two basic postulates of the **special theory of relativity** are as follows:

1. The laws of physics are the same in all inertial frames of reference.
2. The speed of light is the same for all inertial observers, independently of their motion or of the motion of the source of light.

## 26.6 Consequences of Special Relativity

Some of the consequences of the special theory of relativity are as follows:

1. Clocks in motion relative to an observer slow down, a phenomenon known as **time dilation**. The relationship between time intervals in the moving and at-rest systems is

$$\Delta t = \gamma \Delta t_p \qquad [26.7]$$

where $\Delta t$ is the time interval measured in the system in relative motion with respect to the clock,

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \qquad [26.8]$$

and $\Delta t_p$ is the proper time interval measured in the system moving with the clock.

2. The length of an object in motion is *contracted* in the direction of motion. The equation for **length contraction** is

$$L = L_p\sqrt{1 - v^2/c^2} \qquad [26.9]$$

where $L$ is the length measured by an observer in motion relative to the object and $L_p$ is the proper length measured by an observer for whom the object is at rest.

3. Events that are simultaneous for one observer are not simultaneous for another observer in motion relative to the first.

## 26.7 Relativistic Momentum

The relativistic expression for the **momentum** of a particle moving with velocity $v$ is

$$p \equiv \frac{mv}{\sqrt{1 - v^2/c^2}} = \gamma mv \qquad [26.10]$$

## 26.8 Relativistic Addition of Velocities

The relativistic expression for the addition of velocities is

$$v_{ab} = \frac{v_{ad} + v_{db}}{1 + \frac{v_{ad}v_{db}}{c^2}} \qquad [26.11]$$

where $v_{ab}$ is the velocity of object $a$ with as measured in frame $b$, $v_{ad}$ is the velocity of object $a$ as measured in frame $d$, and $v_{db}$ is the velocity of frame $d$ as measured in frame $b$.

## 26.9 Relativistic Energy and the Equivalence of Mass and Energy

The relativistic expression for the **kinetic energy** of an object is

$$KE = \gamma mc^2 - mc^2 \qquad [26.12]$$

where $mc^2$ is the **rest energy** of the object, $E_R$.

The **total energy** of a particle is

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \qquad [26.15]$$

This is Einstein's famous mass–energy equivalence equation.

The relativistic momentum is related to the total energy through the equation

$$E^2 = p^2c^2 + (mc^2)^2 \qquad [26.16]$$

## 26.10 Pair Production and Annihilation

**Pair production** is a process in which the energy of a photon is converted into mass. In this process, the photon disappears as an electron–positron pair is created. Likewise, the energy of an electron–positron pair can be converted into electromagnetic radiation by the process of **pair annihilation.**

# OmniTest II



AP Physics

Name: ______________________ Score: ________

1. A spaceship leaves Earth and travels at a speed of 0.60 c for a period of 2.0 years as measured by its shipboard clock. How long was the flight as measured by a clock on Earth? t= ______________

2. A stick whose proper length is 10.0 m moves past you and is observed to have a length of 4.4 m. What is its speed? u= ______________

3. Assume you are cruising the universe in your new sport spaceship when you pass a friend in an identical ship. You know your ship is exactly 16 m long but you determine that his is only 12 m long. What is your speed relative to his? u= ______________

A muon moves downward from the upper atmosphere at a speed of 0.995 c. It goes past a mountain that is 10,000 feet tall as measured by observers on the earth. What is the height of the mountain in the muon's frame of reference?

A) 16,000 feet B) 1000 feet
C) 6300 feet D) 7800 feet
E) 2100 feet

5. Calculate the speed at which a clock would have to move in order to run at half the speed of a similar clock at rest.

A) 0.50c B) 1.1c C) 0.98c D) 0.72c E) 0.87c

6. An astronaut leaves the earth and travels 0.75 c for 1.0 years. How much did her twin sister who remained on earth age while she was gone?

A) 0.90 years B) 1.1 years
C) 1.5 years D) 2.0 years
E) 3.0 years

AP Physics - DiBucci

## Section 28.3 The Relativity of Time: Time Dilation

**1.** **ssm** A law enforcement officer in an intergalactic "police car" turns on a red flashing light and sees it generate a flash every 1.5 s. A person on earth measures that the time between flashes is 2.5 s. How fast is the "police car" moving relative to the earth?

**2.** In 1992, Jenny Thompson set a world's record for the 100-m freestyle. Suppose that this race had been monitored from a spaceship traveling at a speed of $0.8200c$ relative to the earth and that the space travelers measured the time interval of the race to be 95.18 s. What was the time recorded on earth?

**3.** A particle known as a pion lives, on average, for a proper time of $2.6 \times 10^{-8}$ s before breaking apart into other particles. How long does this particle live according to a laboratory observer if the particle moves past the observer at a speed of $0.67c$?

**4.** A Klingon spacecraft has a speed of $0.75c$ with respect to the earth. The Klingons measure 37.0 h for the time interval between two events on the earth. What value for the time interval would they measure if their ship had a speed of $0.94c$ with respect to the earth?

**5.** **ssm** A radar antenna is rotating at an angular speed of 0.25 rad/s, as measured on earth. To an observer moving past the antenna at a speed of $0.80c$, what is its angular speed?

***6.** An astronaut travels at a speed of 7800 m/s relative to the earth, a speed that is very small compared to $c$. According to a clock on the earth, the trip lasts 15 days. Determine the *difference* (in seconds) between the time recorded by the earth clock and the astronaut's clock. [*Hint: When* $v \ll c$, *the following approximation is valid:* $\sqrt{1 - v^2/c^2} \approx 1 - \frac{1}{2}(v^2/c^2)$.]

***7.** A 5.00-kg object oscillates back and forth at the end of a spring whose spring constant is 49.3 N/m. An observer is traveling at a speed of $2.80 \times 10^8$ m/s relative to the fixed end of the spring. What does this observer measure for the period of oscillation?

**8.** As observed on earth, a certain type of bacteria is known to double in number every 24.0 hours. Two cultures of these bacteria are prepared, each consisting initially of one bacterium. One culture is left on earth and the other placed on a rocket that travels at a speed of $0.866c$ relative to the earth. At a time when the earth-bound culture has grown to 256 bacteria, how many bacteria are in the culture on the rocket, according to an earth-based observer?

1) $2.4 \times 10^8$ m/s
2) 54.48 s
3) $3.5 \times 10^{-8}$ s
4) 72 h
5) 0.15 rad/s
6) $4.4 \times 10^{-4}$ s
7) 5.57 s
8) 16

## Section 28.4 The Relativity of Length: Length Contraction

**9.** **ssm** A UFO streaks across the sky at a speed of $0.90c$ relative to the earth. A person on earth determines the length of the UFO to be 230 m along the direction of its motion. What length does the person measure for the UFO when it lands?

**10.** Suppose that the speed of light in a vacuum were 358 m/s. At what speed would a jet have to fly so that the pilot observes the distance between New York and San Francisco to be one-half of its proper value?

**11.** Suppose the straight-line distance between New York and San Francisco is $4.2 \times 10^6$ m (neglecting the curvature of the earth). A UFO is flying between these two cities at a speed of $0.70c$ relative to the earth. What do the voyagers aboard the UFO measure for this distance?

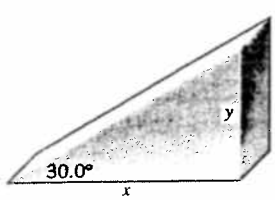
**12.** Our galaxy, the Milky Way, has a diameter (proper length) of about $1.2 \times 10^5$ light-years. As far as an astronaut is concerned, how long (in years) would it take to cross the Milky Way, if the speed of the spacecraft is $0.999\,99c$?

**13.** **ssm www** Two spaceships A and B are exploring a new planet. Relative to this planet, spaceship A has a speed of $0.60c$, while spaceship B has a speed of $0.80c$. What is the ratio $D_A/D_B$ of the values for the planet's diameter that each spaceship measures in a direction that is parallel to its motion?

**14.** Suppose you are traveling in space and pass a rectangular landing pad on a planet. Your spacecraft has a speed of $0.85c$ relative to the planet and moves in a direction parallel to the length of the pad. While moving, you measure the length to be 1800 m and the width to be 1500 m. What are the dimensions of the landing pad according to the engineer who built it?

**15.** A space traveler moving at a speed of $0.60c$ with respect to the earth makes a trip to a distant star that is stationary relative to the earth. He measures the length of this trip to be 8.0 light-years. What would be the length of this same trip (in light-years) as measured by a traveler moving at a speed of $0.80c$ with respect to the earth?

***16.** As the drawing shows, a carpenter on a space station has constructed a 30.0° ramp. A rocket moves past the space station with a relative speed of $0.730c$ in a direction parallel to side $x$. What does a person aboard the rocket measure for the angle of the ramp?




****17.** **ssm www** A rectangle has the dimensions of 3.0 m × 2.0 m when viewed by someone at rest with respect to it. When you move past the rectangle along one of its sides, the rectangle looks like a square. What dimensions do you observe when you move at the same speed along the adjacent side of the rectangle?

****18.** A woman and a man are on separate rockets, which are flying parallel to each other and have a relative speed of $0.940c$. The woman measures the same value for the length of her own rocket and for the length of the man's rocket. What is the ratio of the value that the man measures for the length of his own rocket to the value he measures for the length of the woman's rocket?

9) 530 m
10) 310 m/s
11) $3.0 \times 10^6$ m
12) $5.4 \times 10^2$ yrs.
13) 1.3
14) 3400 m × 1500 m
15) 6.0 ly
16) 40.2°
17) 3.0 m × 1.3 m
18) 8.59

## Section 28.6 The Equivalence of Mass and Energy

**24.** Radium is a radioactive element whose nucleus emits an $\alpha$ particle (a helium nucleus) that has a kinetic energy of about $7.8 \times 10^{-13}$ J (4.9 MeV). To what amount of mass is this energy equivalent?

**25.** **ssm** How much work must be done on an electron to accelerate it from rest to a speed of $0.990c$?

**26.** How fast (relative to $c$) is a proton traveling when its total energy is four times its rest energy?

**27.** The amount of heat required to melt 1 kg of ice at 0 °C is $3.35 \times 10^5$ J. What is the difference between the mass of the water and that of the ice? Which has the greater mass?

**28.** An electron and a positron each have a mass of $9.11 \times 10^{-31}$ kg. They collide and both vanish, with only electromagnetic radiation appearing after the collision. If each particle is moving at a speed of $0.20c$ relative to the laboratory before the collision, determine the energy of the electromagnetic radiation.

**29.** **ssm** Determine the ratio of the relativistic kinetic energy to the nonrelativistic kinetic energy ($\frac{1}{2}mv^2$) when a particle has a speed of (a) $1.00 \times 10^{-3}c$ and (b) $0.970c$.

**30.** Four kilograms of water are heated from 20.0 °C to 60.0 °C. (a) How much heat is required to produce this change in temperature? [The specific heat capacity of water is 4186 J/(kg·C°).] (b) By how much does the mass of the water increase?

***31.** Using Equations 28.3 and 28.4, prove that the total energy of an object is related to its relativistic momentum $p$ according to $E^2 = p^2c^2 + (mc^2)^2$.

***32.** How close would two stationary electrons have to be positioned so that their total mass is twice what it is when the electrons are very far apart?

24) $8.7 \times 10^{-30}$ kg
25) $5.0 \times 10^{-13}$ J
26) $0.968c$
27) $3.72 \times 10^{-12}$ kg
28) $1.7 \times 10^{-13}$ J
29) 6.6
30) $6.7 \times 10^{5}$ J
31) $E^2 = p^2c^2 + (mc^2)^2$
32) $1.40 \times 10^{-15}$ m

## Section 28.5 Relativistic Momentum

**19.** A car, whose mass is 1550 kg, is traveling at 15.0 m/s. If the speed of light were 25.0 m/s, what would be the magnitude of the momentum of the car as measured by a person standing on the ground?

**20.** A spaceship is approaching the earth at a relative speed of 0.85*c*. The mass of the ship is $2.0 \times 10^7$ kg. Find the magnitude of (a) the classical momentum and (b) the relativistic momentum of the ship.

**21.** **ssm** A rocket of mass $1.40 \times 10^5$ kg has a relativistic momentum the magnitude of which is $3.15 \times 10^{13}$ kg · m/s. How fast is the rocket traveling?

**22.** A woman is 1.6 m tall and has a mass of 55 kg. She moves past an observer with the direction of the motion parallel to her height. The observer measures her relativistic momentum to have a magnitude of $2.0 \times 10^{10}$ kg · m/s. What does the observer measure for her height?

***23.** Starting from rest, two skaters "push off" against each other on smooth level ice, where friction is negligible. One is a woman and one is a man. The woman moves away with a velocity of +2.5 m/s relative to the ice. The mass of the woman is 54 kg, and the mass of the man is 88 kg. Assuming that the speed of light is 3.0 m/s, so that the relativistic momentum must be used, find the recoil velocity of the man relative to the ice. *(Hint: This problem is similar to Example 6 in Chapter 7.)*

19) $2.91 \times 10^4$ kg·m/s

20) $5.1 \times 10^{15}$ kg m/s

$9.7 \times 10^{15}$ kg m/s

21) $1.80 \times 10^8$ m/s

22) 1.0 m

23) −2.0 m/s

## Section 28.7 The Relativistic Addition of Velocities

**33.** **ssm** A spacecraft approaching the earth launches an exploration vehicle. After the launch, an observer on earth sees the spacecraft approaching at a speed of 0.50*c* and the exploration vehicle approaching at a speed of 0.70*c*. What is the speed of the exploration vehicle relative to the spaceship?

**34.** The spaceship *Enterprise I* is moving directly away from the earth at a speed that an earth-based observer measures to be 0.65*c*. A sister ship, the *Enterprise II*, is also moving directly away from earth along the same line and is traveling faster than the *Enterprise I*. The relative speed between the ships is 0.31*c*. What is the speed of the *Enterprise II*, as measured by the earth-based observer?

**35.** It has been proposed that spaceships of the future will be powered by ion propulsion engines, in which ions are ejected from the back of the ship to drive it forward. In one such engine the ions are to be ejected with a speed of 0.80*c* relative to the engine. If the ship were traveling away from the earth with a velocity of 0.70*c*, what would be the velocity of the ions relative to the earth? (Be sure to assign the correct plus or minus signs to the velocities, assuming that the direction away from earth is positive.)

***36.** Refer to Conceptual Example 11 as an aid in solving this problem. An intergalactic cruiser has two types of guns: a photon cannon that fires a beam of laser light, and an ion gun that shoots ions at a velocity of 0.950*c* relative to the cruiser. The cruiser closes in on an alien spacecraft at a velocity of 0.800*c* relative to this spacecraft. The captain fires both types of guns. At what velocity do the aliens see (a) the laser light and (b) the ions approach them? At what velocity do the aliens see (c) the laser light and (d) the ions move away from the cruiser?

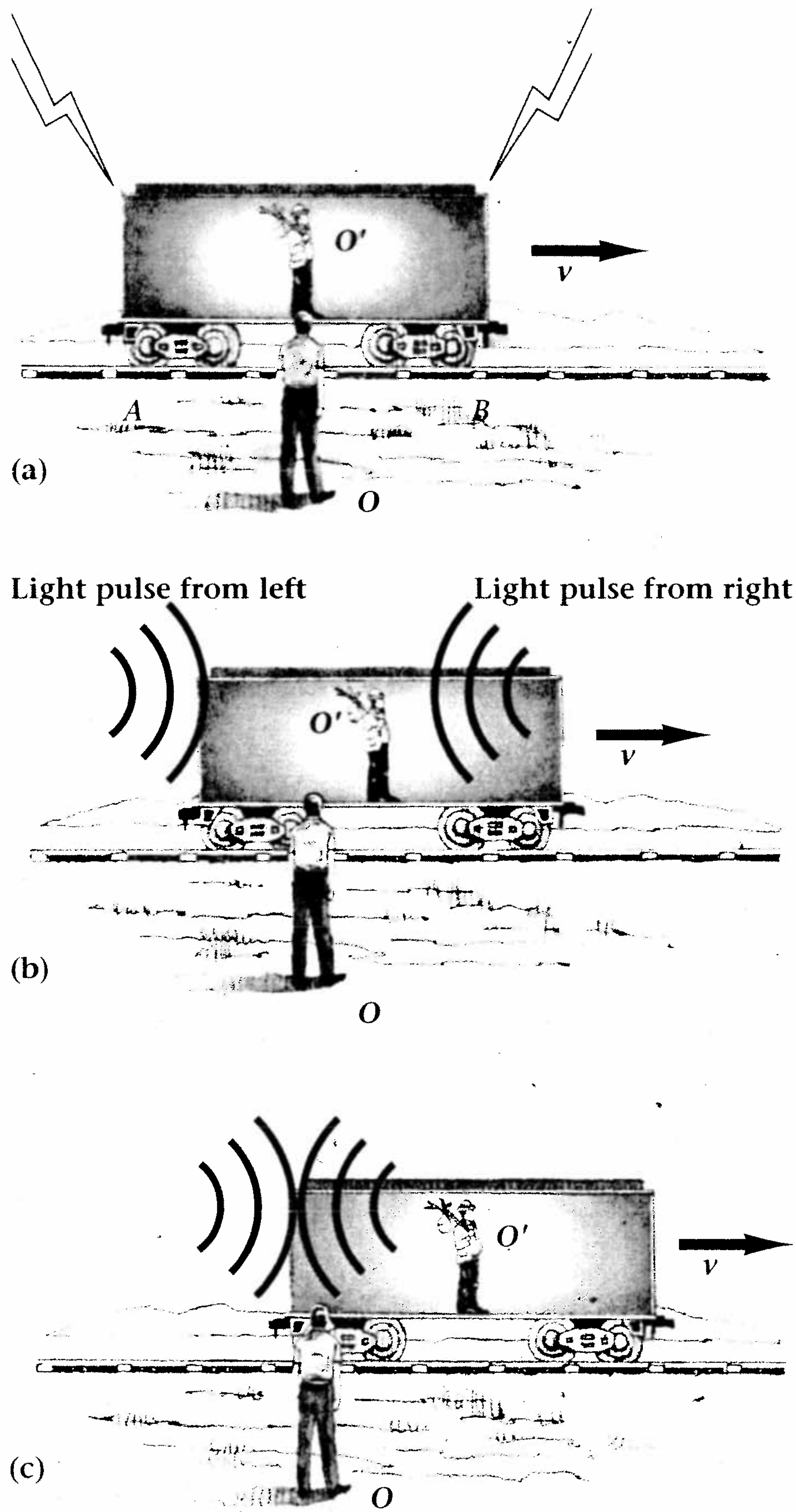
***37.** **ssm www** The crew of a rocket that is moving away from the earth launches an escape pod, which they measure to be 45 m long. The pod is launched toward the earth with a speed of 0.55*c* relative to the rocket. After the launch, the rocket's speed relative to the earth is 0.75*c*. What is the length of the escape pod as determined by an observer on earth?

***38.** Two atomic particles approach each other in a head-on collision. Each particle has a mass of $2.16 \times 10^{-25}$ kg. The speed of each particle is $2.10 \times 10^{8}$ m/s when measured by an observer standing in the laboratory. (a) What is the speed of one particle as seen by the other particle? (b) Determine the relativistic momentum of one particle, as would be observed by the other.

33) 0.31c
34) 0.80c
35) -0.23c
36) c, 0.994c
0.200c
0.194c

37) 42 m
38) 0.940c
$1.8 \times 10^{-16}$ kg m/s

131. Events simultaneous to one observer may not be simultaneous to another. (Fig. 25.7)



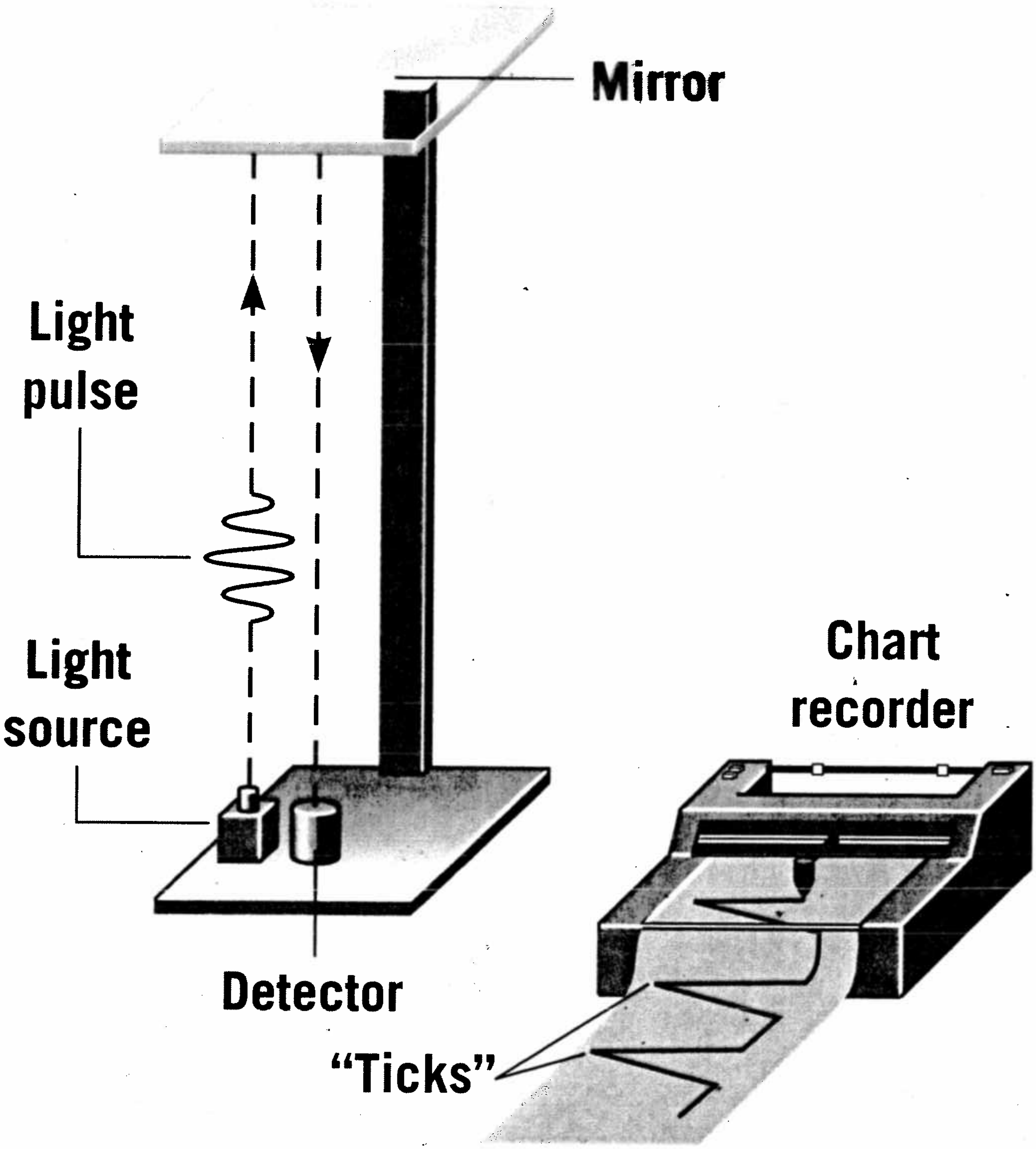
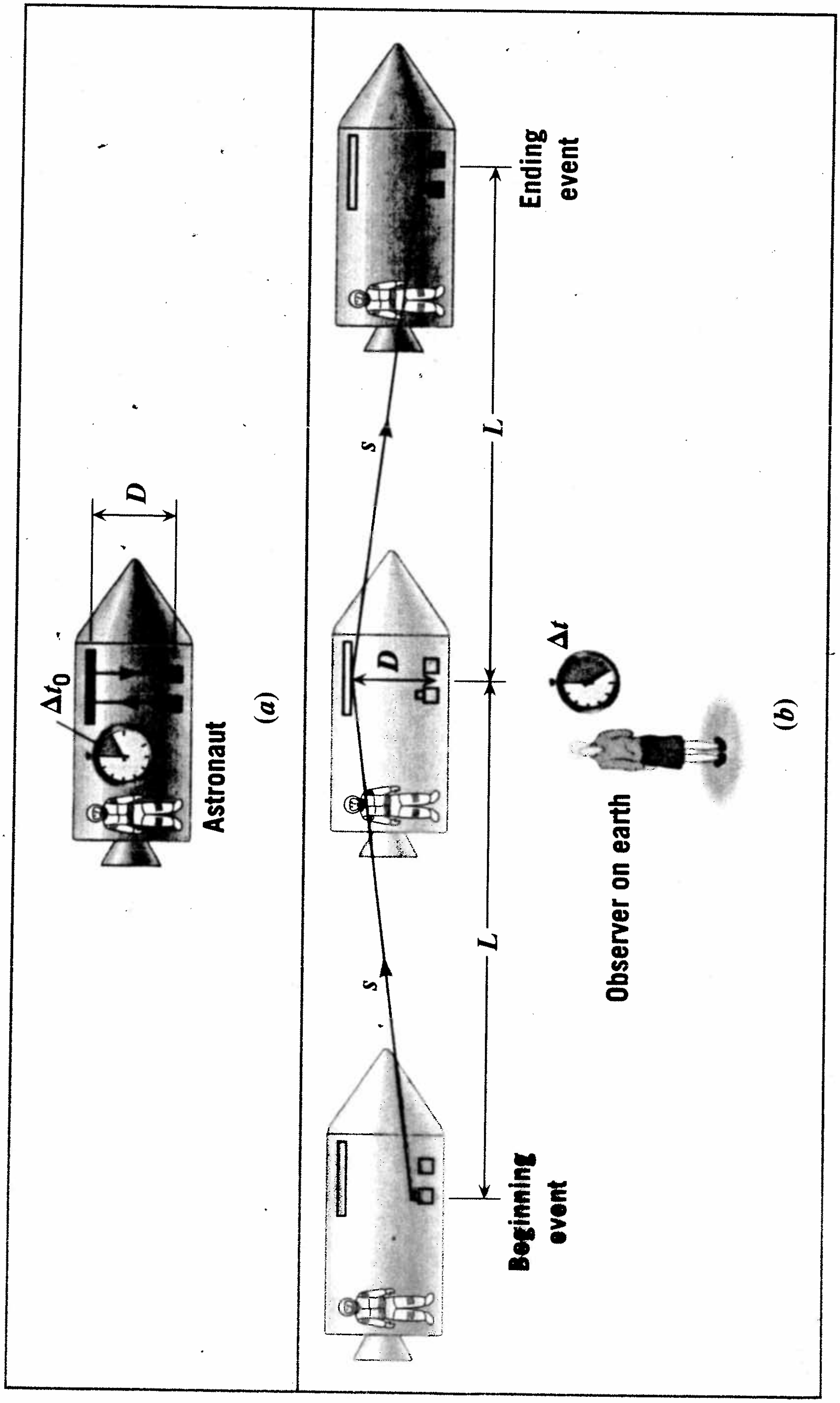

Mirror
Light
pulse
Light
source
Detector
Chart
recorder
"Ticks"



FIGURE 28-4



Δt0
D
Astronaut
(a)
s
s
L
L
D
Δt
Beginning event
Ending event
Observer on earth
(b)

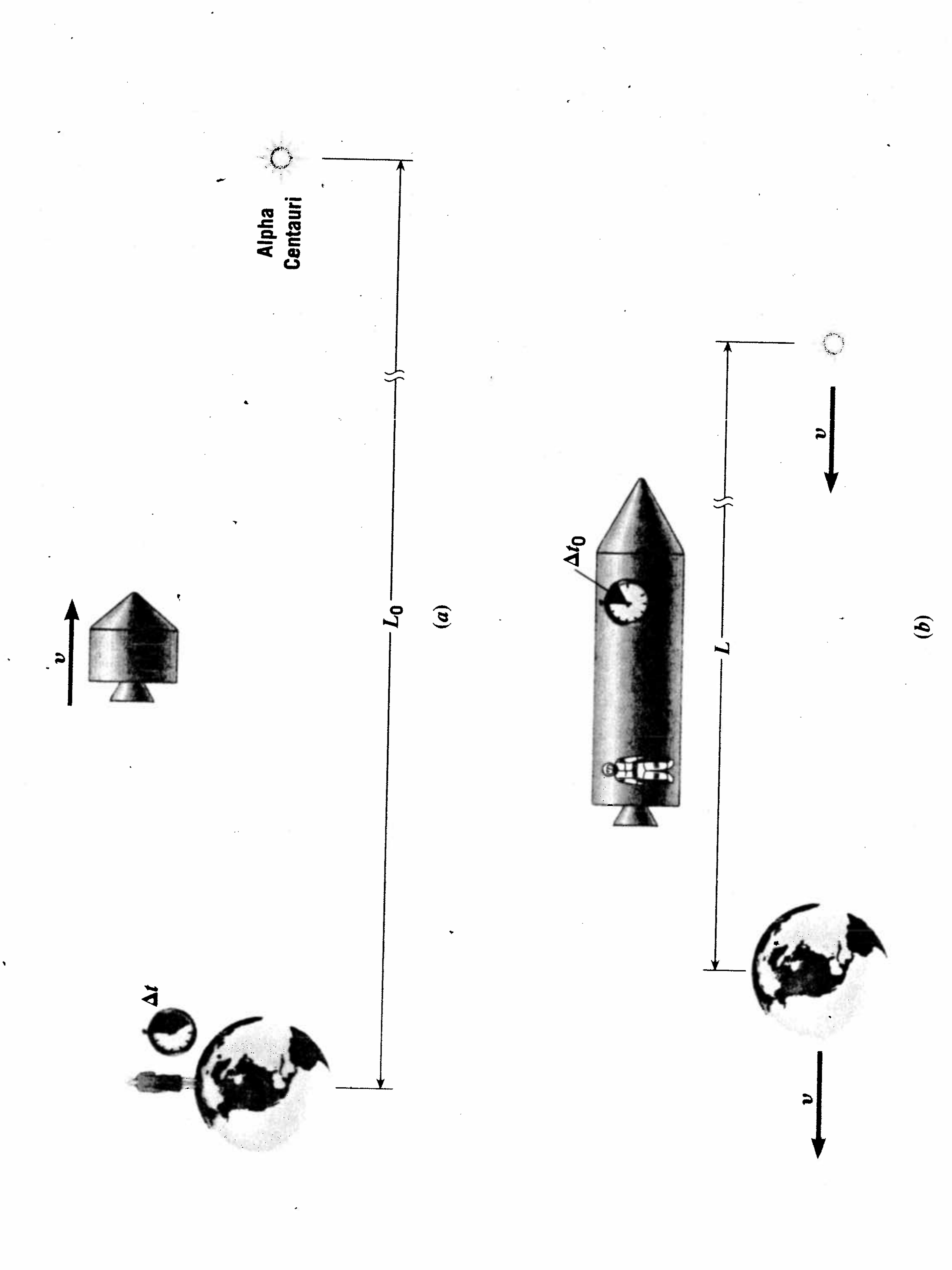





FIGURE 28-6