Chapter 2

Motion along a straight line

We find moving objects all around us.

The study of motion is called *kinematics*.

Examples:

- The Earth orbits around the Sun
- A roadway moves with Earth’s rotation

2.2 Motion

Here we will study motion that takes place in a straight line.

Forces cause motion. We will find out, as a result of application of force, if the objects speed up, slow down, or maintain the same rate.

The moving object here will be considered as a particle. If we deal with a stiff, extended object, we will assume that all particles on the body move in the same fashion. We will study the motion of a particle, which will represent the entire body.

2.3 Position and displacement

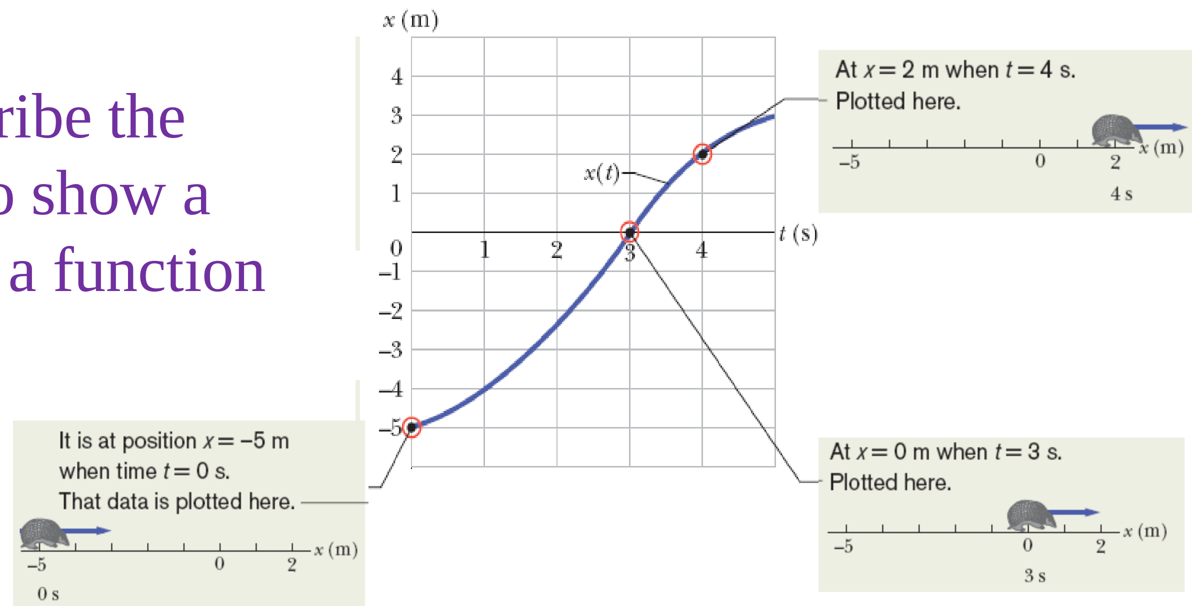
The location of an object is usually given in terms of a standard reference point, called the origin. The positive direction is taken to be the direction where the coordinates are increasing, and the negative direction as that where the coordinates are decreasing.

A change in the coordinates of the position of the body describes the *displacement* of the body.
For example, if the x-coordinate of a body changes from x_1 to x_2, then the displacement, $\Delta x = (x_2 - x_1)$.

Displacement is a vector quantity. That is, a quantity that has both magnitude and direction information.
An object's displacement is x = -4 m means that the object has moved towards decreasing x-axis by 4 m. The direction of motion, here, is toward decreasing x.

2.4 Average Velocity and Average Speed

A common way to describe the motion of an object is to show a graph of the position as a function of time.



Average velocity, or v_{avg}, is defined as the displacement over the time duration.

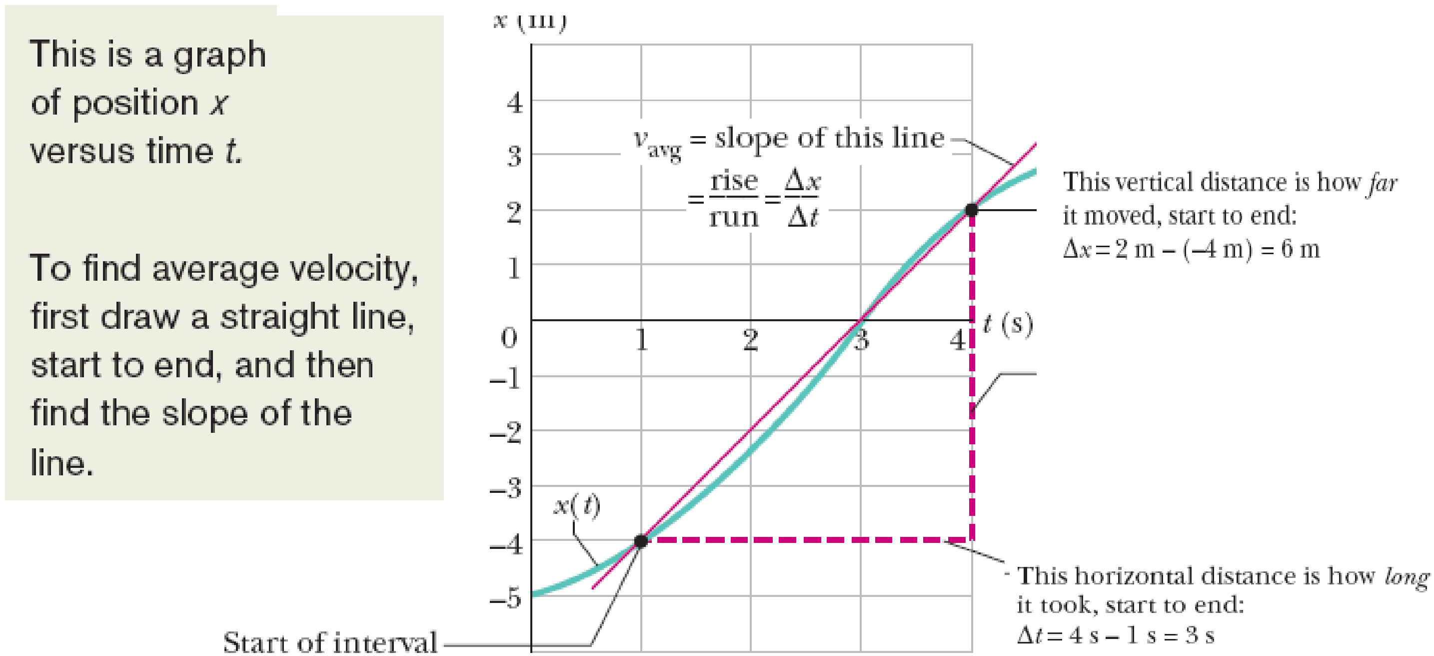
$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}.$$

The average velocity has the same sign as the displacement

The magnitude of the slope of the x-t graph gives the average velocity

Here, the average velocity is:

$$v_{\text{avg}} = \frac{6\text{ m}}{3\text{ s}} = 2\text{ m/s}.$$



2.4 Average Velocity and Average Speed

Average speed is the ratio of the total distance traveled to the total time duration. It is a scalar quantity, and does not carry any sense of direction.

$$s_{avg} = \frac{\text{total distance}}{\Delta t}.$$

Example, motion:

You drive a beat-up pickup truck along a straight road for 8.4 km at 70 km/h, at which point the truck runs out of gasoline and stops. Over the next 30 min, you walk another 2.0 km farther along the road to a gasoline station.

(a) What is your overall displacement from the beginning of your drive to your arrival at the station?

KEY IDEA

Assume, for convenience, that you move in the positive direction of an x axis, from a first position of $x_1 = 0$ to a second position of x_2 at the station. That second position must be at $x_2 = 8.4\ \text{km} + 2.0\ \text{km} = 10.4\ \text{km}$. Then your displacement Δx along the x axis is the second position minus the first position.

$$\Delta x = x_2 - x_1 = 10.4\ \text{km} - 0 = 10.4\ \text{km}. \qquad \text{(Answer)}$$

Thus, your overall displacement is 10.4 km in the positive direction of the x axis.

Example, motion:

(b) What is the time interval Δt from the beginning of your drive to your arrival at the station?

KEY IDEA

We already know the walking time interval Δt_{wlk} (= 0.50 h), but we lack the driving time interval Δt_{dr}. However, we know that for the drive the displacement Δx_{dr} is 8.4 km and the average velocity $v_{avg,dr}$ is 70 km/h. Thus, this average velocity is the ratio of the displacement for the drive to the time interval for the drive.

Calculations: We first write

$$v_{avg,dr} = \frac{\Delta x_{dr}}{\Delta t_{dr}}.$$

Rearranging and substituting data then give us

$$\Delta t_{dr} = \frac{\Delta x_{dr}}{v_{avg,dr}} = \frac{8.4\ \text{km}}{70\ \text{km/h}} = 0.12\ \text{h}.$$

So,
$$\Delta t = \Delta t_{dr} + \Delta t_{wlk}$$
$$= 0.12\ \text{h} + 0.50\ \text{h} = 0.62\ \text{h}. \qquad \text{(Answer)}$$

Example, motion:

(c) What is your average velocity v_{avg} from the beginning of your drive to your arrival at the station? Find it both numerically and graphically.

KEY IDEA

From Eq. 2-2 we know that v_{avg} *for the entire trip* is the ratio of the displacement of 10.4 km *for the entire trip* to the time interval of 0.62 h *for the entire trip.*

Calculation: Here we find

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{10.4 \text{ km}}{0.62 \text{ h}}$$

$$= 16.8 \text{ km/h} \approx 17 \text{ km/h}. \qquad \text{(Answer)}$$

To find v_{avg} graphically, first we graph the function $x(t)$ as shown in Fig. 2-5, where the beginning and arrival points on the graph are the origin and the point labeled as "Station." Your average velocity is the slope of the straight line connecting those points; that is, v_{avg} is the ratio of the *rise* ($\Delta x = 10.4$ km) to the *run* ($\Delta t = 0.62$ h), which gives us $v_{avg} = 16.8$ km/h.

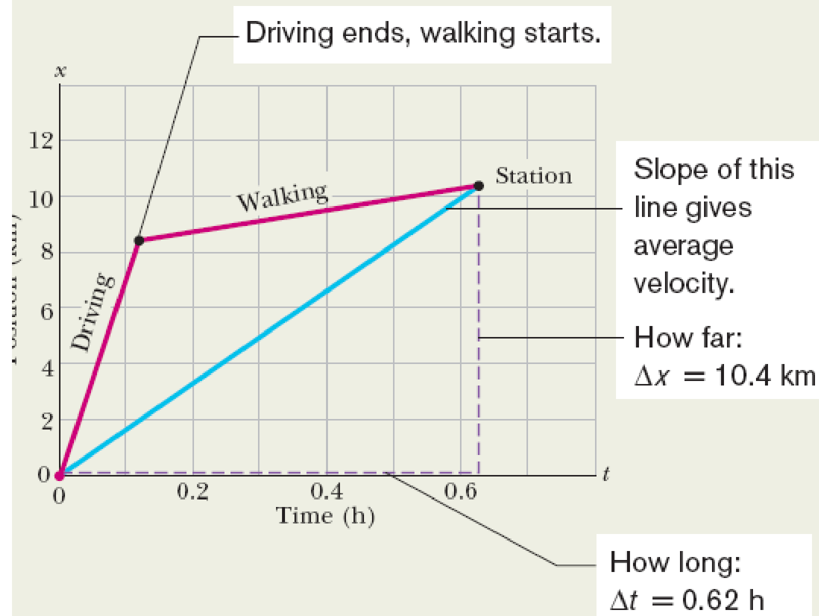
(d) Suppose that to pump the gasoline, pay for it, and walk back to the truck takes you another 45 min. What is your average speed from the beginning of your drive to your return to the truck with the gasoline?

KEY IDEA

Your average speed is the ratio of the total distance you move to the total time interval you take to make that move.

Calculation: The total distance is 8.4 km + 2.0 km + 2.0 km = 12.4 km. The total time interval is 0.12 h + 0.50 h + 0.75 h = 1.37 h. Thus, Eq. 2-3 gives us

$$s_{avg} = \frac{12.4 \text{ km}}{1.37 \text{ h}} = 9.1 \text{ km/h}. \qquad \text{(Answer)}$$



2.5 Instantaneous Velocity and Speed

The instantaneous velocity of a particle at a particular instant is the velocity of the particle at that instant.

Here Δt approaches a limiting value:

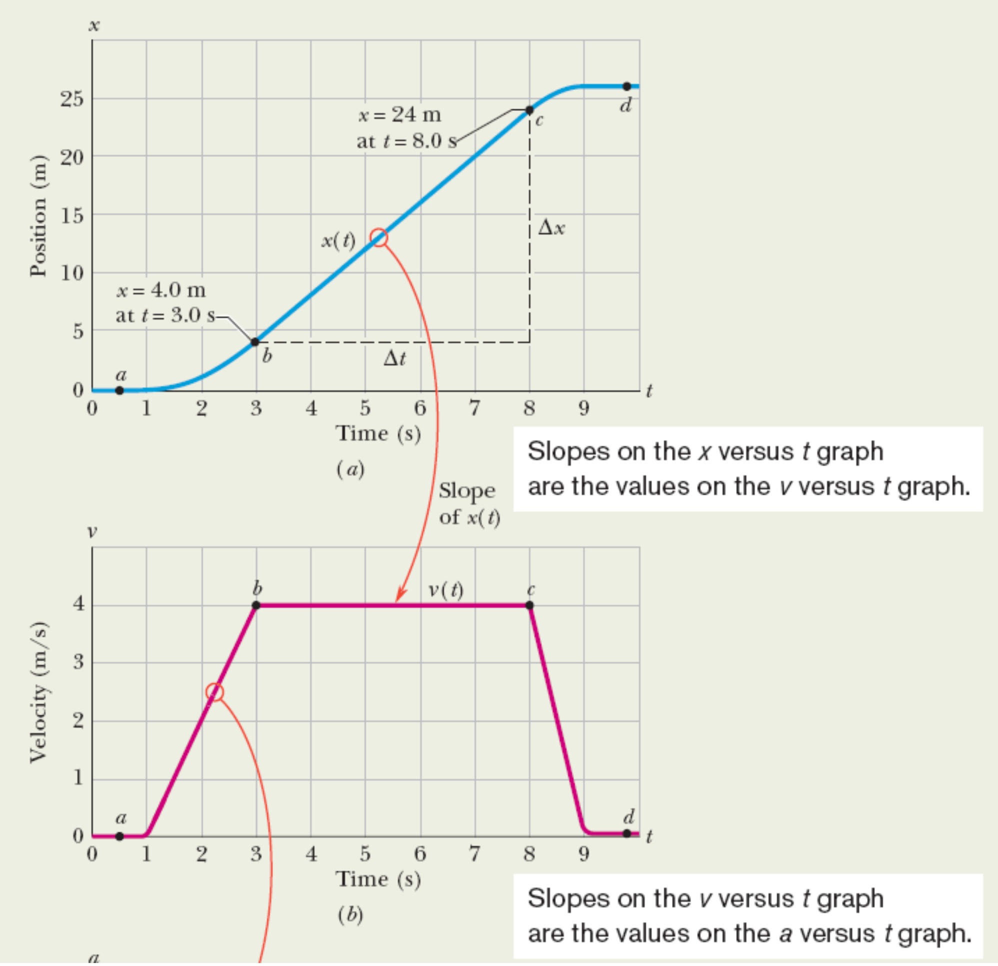
$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}.$$

v, the instantaneous velocity, is the slope of the tangent of the position-time graph at that particular instant of time.

Velocity is a vector quantity and has with it an associated sense of direction.

Example, instantaneous velocity:

Figure 2-6*a* is an $x(t)$ plot for an elevator cab that is initially stationary, then moves upward (which we take to be the positive direction of x), and then stops. Plot $v(t)$.



KEY IDEA

We can find the velocity at any time from the slope of the $x(t)$ curve at that time.

Calculations: The slope of $x(t)$, and so also the velocity, is zero in the intervals from 0 to 1 s and from 9 s on, so then the cab is stationary. During the interval *bc*, the slope is constant and nonzero, so then the cab moves with constant velocity. We calculate the slope of $x(t)$ then as

$$\frac{\Delta x}{\Delta t} = v = \frac{24\text{ m} - 4.0\text{ m}}{8.0\text{ s} - 3.0\text{ s}} = +4.0\text{ m/s}. \qquad (2\text{-}5)$$

The plus sign indicates that the cab is moving in the positive x direction. These intervals (where $v = 0$ and $v = 4$ m/s) are plotted in Fig. 2-6*b*. In addition, as the cab initially begins to move and then later slows to a stop, v varies as indicated in the intervals 1 s to 3 s and 8 s to 9 s. Thus, Fig. 2-6*b* is the required plot. (Figure 2-6*c* is considered in Section 2-6.)

Given a $v(t)$ graph such as Fig. 2-6*b*, we could "work backward" to produce the shape of the associated $x(t)$ graph (Fig. 2-6*a*). However, we would not know the actual values for x at various times, because the $v(t)$ graph indicates only *changes* in x. To find such a change in x during any interval, we must, in the language of calculus, calculate the area "under the curve" on the $v(t)$ graph for that interval. For example, during the interval 3 s to 8 s in which the cab has a velocity of 4.0 m/s, the change in x is

$$\Delta x = (4.0\text{ m/s})(8.0\text{ s} - 3.0\text{ s}) = +20\text{ m}. \qquad (2\text{-}6)$$

(This area is positive because the $v(t)$ curve is above the t axis.) Figure 2-6*a* shows that x does indeed increase by 20 m in that interval. However, Fig. 2-6*b* does not tell us the *values* of x at the beginning and end of the interval. For that, we need additional information, such as the value of x at some instant.

2.6 Acceleration

Average acceleration is the change of velocity over the change of time.
As such,

$$a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

Here the velocity is v_1 at time t_1, and the velocity is v_2 at time t_2.

The instantaneous acceleration is defined as:

$$a = \frac{dv}{dt}.$$

In terms of the position function, the acceleration can be defined as:

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}.$$

The SI units for acceleration are m/s^2.

2.6 Acceleration

If a particle has the same sign for velocity and acceleration, then that particle is speeding up.

Conversely, if a particle has opposite signs for the velocity and acceleration, then the particle is slowing down.

Our bodies often react to accelerations but not to velocities. A fast car often does not bother the rider, but a sudden brake is felt strongly by the rider. This is common in amusement car rides, where the rides change velocities quickly to thrill the riders.

The magnitude of acceleration falling near the Earth's surface is 9.8 m/s^2, and is often referred to as ***g***.

Colonel J. P. Stapp in a rocket sled, which undergoes sudden change in velocities.

Example, acceleration:

A particle's position on the x axis of Fig. 2-1 is given by

$$x = 4 - 27t + t^3,$$

with x in meters and t in seconds.

(a) Because position x depends on time t, the particle must be moving. Find the particle's velocity function $v(t)$ and acceleration function $a(t)$.

KEY IDEAS

(1) To get the velocity function $v(t)$, we differentiate the position function $x(t)$ with respect to time. (2) To get the acceleration function $a(t)$, we differentiate the velocity function $v(t)$ with respect to time.

Calculations: Differentiating the position function, we find

$$v = -27 + 3t^2, \qquad \text{(Answer)}$$

with v in meters per second. Differentiating the velocity function then gives us

$$a = +6t, \qquad \text{(Answer)}$$

with a in meters per second squared.

(b) Is there ever a time when $v = 0$?

Calculation: Setting $v(t) = 0$ yields

$$0 = -27 + 3t^2,$$

which has the solution

$$t = \pm 3 \text{ s}. \qquad \text{(Answer)}$$

Thus, the velocity is zero both 3 s before and 3 s after the clock reads 0.

(c) Describe the particle's motion for $t \geq 0$.

Reasoning: We need to examine the expressions for $x(t)$, $v(t)$, and $a(t)$.

At $t = 0$, the particle is at $x(0) = +4$ m and is moving with a velocity of $v(0) = -27$ m/s—that is, in the negative direction of the x axis. Its acceleration is $a(0) = 0$ because just then the particle's velocity is not changing.

For $0 < t < 3$ s, the particle still has a negative velocity, so it continues to move in the negative direction. However, its acceleration is no longer 0 but is increasing and positive. Because the signs of the velocity and the acceleration are opposite, the particle must be slowing.

Indeed, we already know that it stops momentarily at $t = 3$ s. Just then the particle is as far to the left of the origin in Fig. 2-1 as it will ever get. Substituting $t = 3$ s into the expression for $x(t)$, we find that the particle's position just then is $x = -50$ m. Its acceleration is still positive.

For $t > 3$ s, the particle moves to the right on the axis. Its acceleration remains positive and grows progressively larger in magnitude. The velocity is now positive, and it too grows progressively larger in magnitude.

2.7 Constant Acceleration: A Special Case

When the acceleration is constant, its average and taneous values are the same.

$$a = a_{avg} = \frac{v - v_0}{t - 0}.$$ means that $$v = v_0 + at.$$ ……(1)

Here, velocity at t=0 is v_o.

Similarly, $v_{avg} = \frac{x - x_0}{t - 0}$ which means that $x = x_0 + v_{avg} t,$

finally leading to

$$x - x_0 = v_0 t + \tfrac{1}{2} a t^2.$$ …..(2)

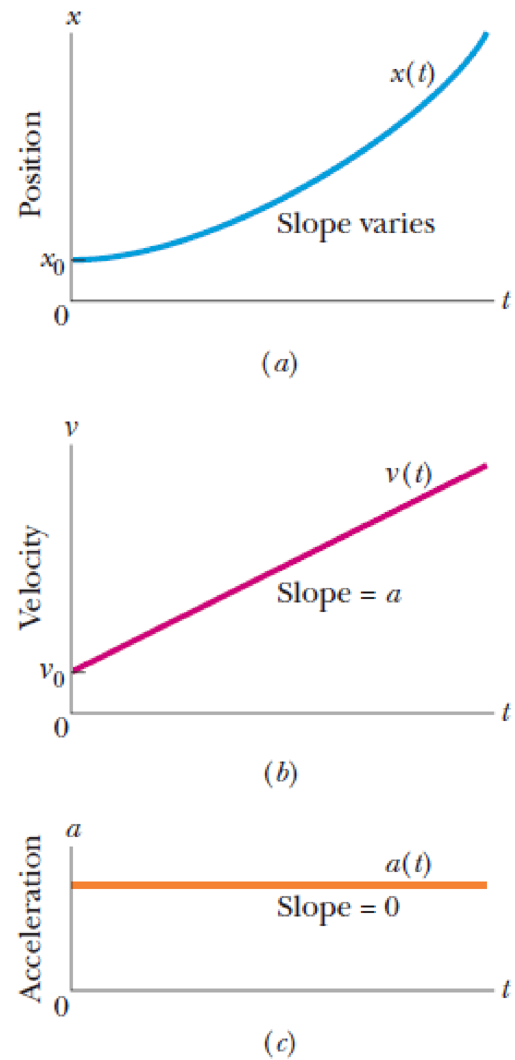
Eliminating t from the Equations (1) and (2):

$$v^2 = v_0^2 + 2a(x - x_0).$$ …..(3)

2.7 Constant Acceleration: A Special Case

Integrating constant acceleration graph for a fixed time duration yields values for velocity graph during that time.

Similarly, integrating velocity graph will yield values for position graph.



Slopes of the position graph are plotted on the velocity graph.

Slope of the velocity graph is plotted on the acceleration graph.

Example, constant acceleration:

Figure 2-9 gives a particle's velocity v versus its position as it moves along an x axis with constant acceleration. What is its velocity at position $x = 0$?

KEY IDEA

We can use the constant-acceleration equations; in particular, we can use Eq. 2-16 ($v^2 = v_0^2 + 2a(x - x_0)$), which relates velocity and position.

First try: Normally we want to use an equation that includes the requested variable. In Eq. 2-16, we can identify x_0 as 0 and v_0 as being the requested variable. Then we can identify a second pair of values as being v and x. From the graph, we have two such pairs: (1) $v = 8$ m/s and $x = 20$ m, and (2) $v = 0$ and $x = 70$ m. For example, we can write Eq. 2-16 as

$$(8 \text{ m/s})^2 = v_0^2 + 2a(20 \text{ m} - 0). \qquad (2\text{-}19)$$

However, we know neither v_0 nor a.

Second try: Instead of directly involving the requested variable, let's use Eq. 2-16 with the two pairs of known data, identifying $v_0 = 8$ m/s and $x_0 = 20$ m as the first pair and $v = 0$ m/s and $x = 70$ m as the second pair. Then we can write

$$(0 \text{ m/s})^2 = (8 \text{ m/s})^2 + 2a(70 \text{ m} - 20 \text{ m}),$$

which gives us $a = -0.64$ m/s^2. Substituting this value into Eq. 2-19 and solving for v_0 (the velocity associated with the position of $x = 0$), we find

$$v_0 = 9.5 \text{ m/s}. \qquad \text{(Answer)}$$

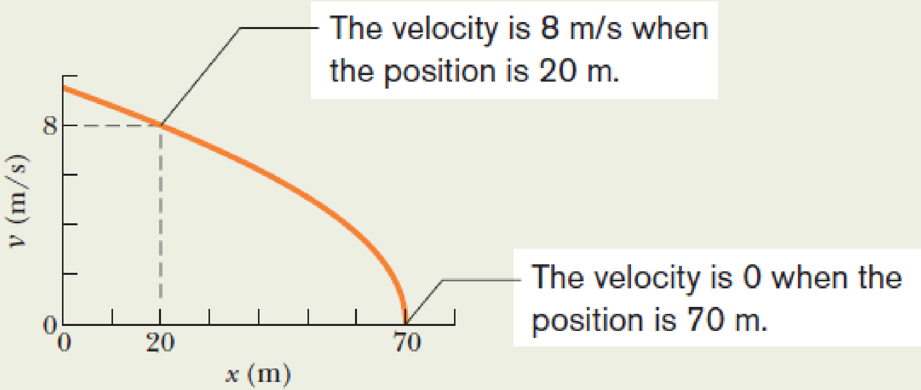


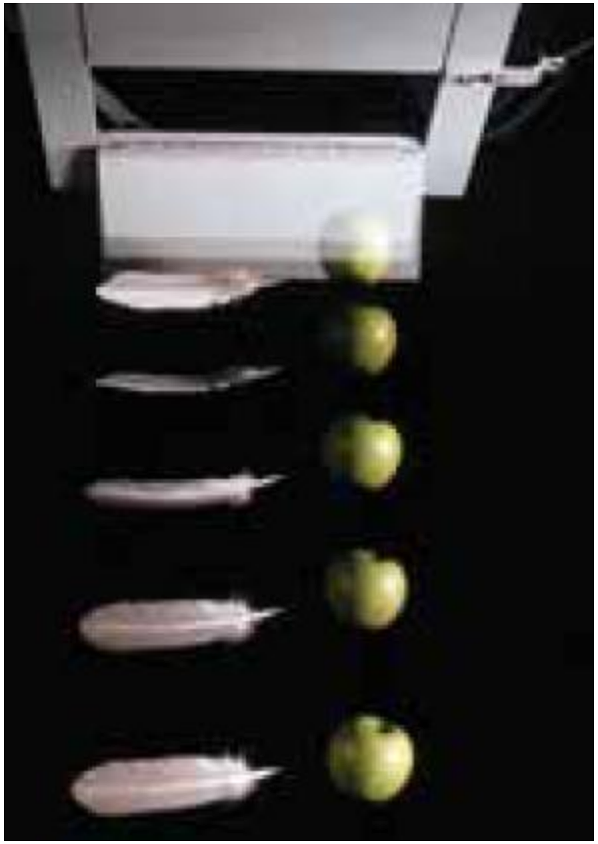
Fig. 2-9 Velocity versus position.

2.9 Free-Fall Acceleration

In this case objects close to the Earth's surface fall towards the Earth's surface with no external forces acting on them except for their weight.

Use the constant acceleration model with "a" replaced by "-g", where $g = 9.8\ m/s^2$ for motion close to the Earth's surface.

In vacuum, a feather and an apple will fall at the same rate.



Sample Problem

In Fig. 2-11, a pitcher tosses a baseball up along a y axis, with an initial speed of 12 m/s.

(a) How long does the ball take to reach its maximum height?

KEY IDEAS

(1) Once the ball leaves the pitcher and before it returns to his hand, its acceleration is the free-fall acceleration $a = -g$. Because this is constant, Table 2-1 applies to the motion. (2) The velocity v at the maximum height must be 0.

Calculation: Knowing v, a, and the initial velocity $v_0 = 12$ m/s, and seeking t, we solve Eq. 2-11, which contains those four variables. This yields

$$t = \frac{v - v_0}{a} = \frac{0 - 12\text{ m/s}}{-9.8\text{ m/s}^2} = 1.2\text{ s.} \qquad \text{(Answer)}$$

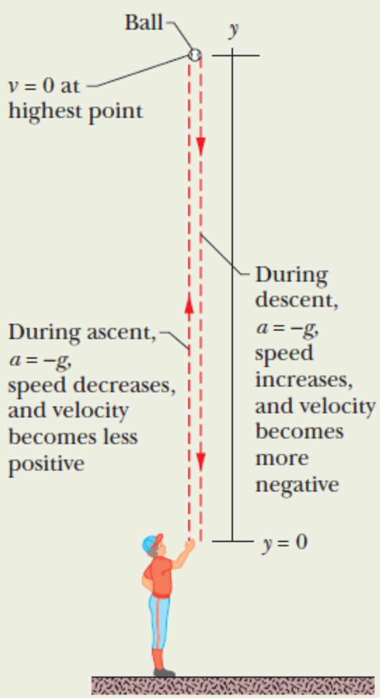


Fig. 2-11 A pitcher tosses a baseball straight up into the air. The equations of free fall apply for rising as well as for falling objects, provided any effects from the air can be neglected.

(b) What is the ball's maximum height above its release point?

Calculation: We can take the ball's release point to be $y_0 = 0$. We can then write Eq. 2-16 in y notation, set $y - y_0 = y$ and $v = 0$ (at the maximum height), and solve for y. We get

$$y = \frac{v^2 - v_0^2}{2a} = \frac{0 - (12\text{ m/s})^2}{2(-9.8\text{ m/s}^2)} = 7.3\text{ m.} \qquad \text{(Answer)}$$

(c) How long does the ball take to reach a point 5.0 m above its release point?

Calculations: We know v_0, $a = -g$, and displacement $y - y_0 = 5.0$ m, and we want t, so we choose Eq. 2-15. Rewriting it for y and setting $y_0 = 0$ give us

$$y = v_0 t - \tfrac{1}{2} g t^2,$$

or

$$5.0\text{ m} = (12\text{ m/s})t - (\tfrac{1}{2})(9.8\text{ m/s}^2)t^2.$$

If we temporarily omit the units (having noted that they are consistent), we can rewrite this as

$$4.9t^2 - 12t + 5.0 = 0.$$

Solving this quadratic equation for t yields

$$t = 0.53\text{ s} \quad \text{and} \quad t = 1.9\text{ s.} \qquad \text{(Answer)}$$

There are two such times! This is not really surprising because the ball passes twice through $y = 5.0$ m, once on the way up and once on the way down.

2-10 Graphical Integration in Motion Analysis

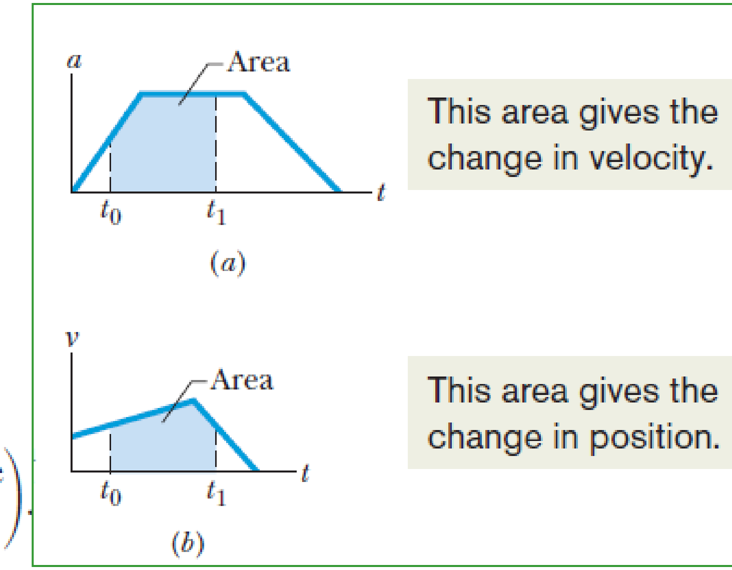
Starting from

$$a = dv/dt$$

we obtain

$$v_1 - v_0 = \int_{t_0}^{t_1} a\, dt.$$

(v_o= velocity at time t=0, and v_1= velocity at time t = t_1).



Note that

$$\int_{t_0}^{t_1} a\, dt = \begin{pmatrix}\text{area between acceleration curve} \\ \text{and time axis, from } t_0 \text{ to } t_1\end{pmatrix}.$$

Similarly, we obtain

$$x_1 - x_0 = \int_{t_0}^{t_1} v\, dt,$$

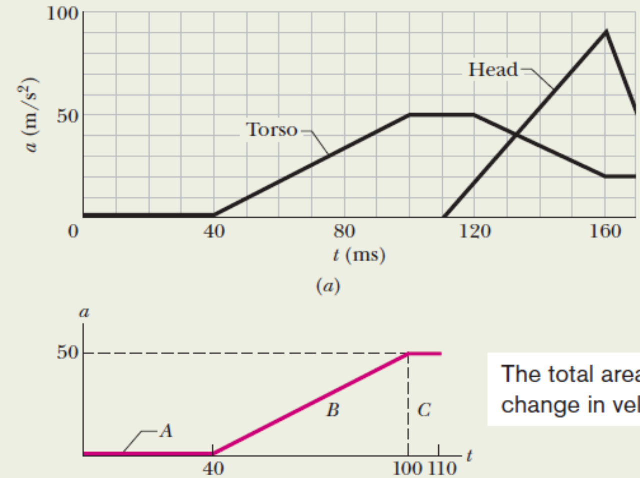
(x_o= position at time t = 0, and x_1 = position at time t=t_1), and

$$\int_{t_0}^{t_1} v\, dt = \begin{pmatrix}\text{area between velocity curve} \\ \text{and time axis, from } t_0 \text{ to } t_1\end{pmatrix}.$$

Example, graphical solution:

"Whiplash injury" commonly occurs in a rear-end collision where a front car is hit from behind by a second car. In the 1970s, researchers concluded that the injury was due to the occupant's head being whipped back over the top of the seat as the car was slammed forward. As a result of this finding, head restraints were built into cars, yet neck injuries in rear-end collisions continued to occur.

In a recent test to study neck injury in rear-end collisions, a volunteer was strapped to a seat that was then moved abruptly to simulate a collision by a rear car moving at 10.5 km/h. Figure 2-13*a* gives the accelerations of the volunteer's torso and head during the collision, which began at time $t = 0$. The torso acceleration was delayed by 40 ms because during that time interval the seat back had to compress against the volunteer. The head acceleration was delayed by an additional 70 ms. What was the torso speed when the head began to accelerate?



(*b*)

The total area gives the change in velocity.

KEY IDEA

We can calculate the torso speed at any time by finding an area on the torso $a(t)$ graph.

Calculations: We know that the initial torso speed is $v_0 = 0$ at time $t_0 = 0$, at the start of the "collision." We want the torso speed v_1 at time $t_1 = 110$ ms, which is when the head begins to accelerate.

$$v_1 - v_0 = \left(\begin{array}{c}\text{area between acceleration curve} \\ \text{and time axis, from } t_0 \text{ to } t_1\end{array}\right).$$

For convenience, let us separate the area into three regions (Fig. 2-13*b*). From 0 to 40 ms, region *A* has no area:

$$\text{area}_A = 0.$$

From 40 ms to 100 ms, region *B* has the shape of a triangle, with area

$$\text{area}_B = \tfrac{1}{2}(0.060\text{ s})(50\text{ m/s}^2) = 1.5\text{ m/s}.$$

From 100 ms to 110 ms, region *C* has the shape of a rectangle, with area

$$\text{area}_C = (0.010\text{ s})(50\text{ m/s}^2) = 0.50\text{ m/s}.$$

Substituting these values and $v_0 = 0$ into Eq. 2-26 gives us

$$v_1 - 0 = 0 + 1.5\text{ m/s} + 0.50\text{ m/s},$$

or

$$v_1 = 2.0\text{ m/s} = 7.2\text{ km/h}. \qquad \text{(Answer)}$$