Chapter 6

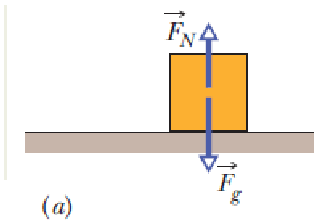
Force and Motion-II

Frictional forces are very common in our everyday lives.

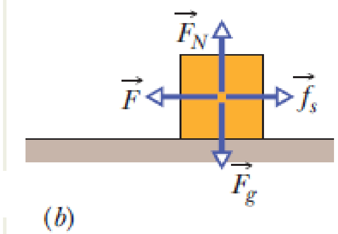
Examples:

1. If you send a book sliding down a horizontal surface, the book will finally slow down and stop.
2. If you push a heavy crate and the crate does not move, then the applied force must be counteracted by frictional forces.

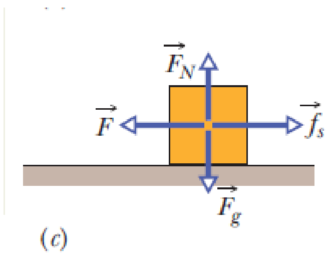
6.2 Frictional Force: Motion of a crate with applied forces



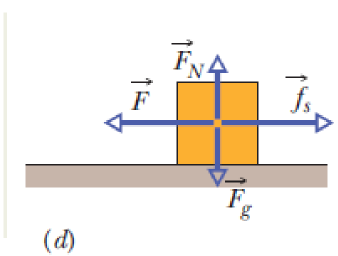
There is no attempt at sliding. Thus, no friction and no motion.
NO FRICTION



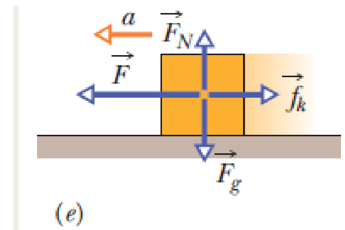
Force *F attempts* sliding but is balanced by the frictional force.
No motion.
STATIC FRICTION



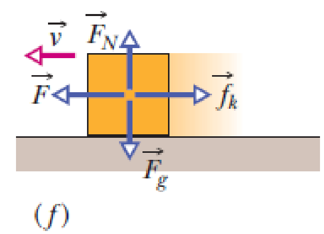
Force *F is now* stronger but is still balanced by the frictional force.
No motion.
LARGER STATIC FRICTION



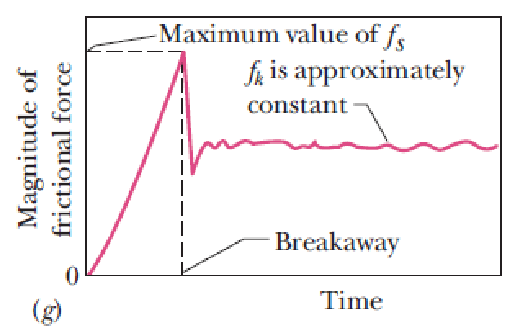
Force *F is now even* stronger but is still balanced by the frictional force.
No motion.
EVEN LARGER STATIC FRICTION



Finally, the applied force has overwhelmed the static frictional force. Block slides and accelerates.
WEAK KINETIC FRICTION



To maintain the speed, weaken force *F to match* the weak frictional force.
SAME WEAK KINETIC FRICTION



Static frictional force can only match growing applied force.

Kinetic frictional force has only one value (no matching).

f_s is the **static frictional force**

f_k is the **kinetic frictional force**

- Static frictional force acts when there is no relative motion between the body and the contact surface

- The magnitude of the static frictional force increases as the applied force to the body is increased

- Finally when the there is relative motion between the body and the contact surface, kinetic friction starts to act.

- Usually, the magnitude of the kinetic frictional force, which acts when there is motion, is less than the maximum magnitude of the static frictional force, which acts when there is no motion.

Often, the sliding motion of one surface over another is “jerky” because the two surfaces alternately stick together and then slip.

Examples:

- Tires skid on dry pavement
- Fingernails scratch on a chalkboard
- A rusty hinge is forced to open
- A bow is drawn on a violin string

6.3 Properties of Friction

Property 1. If the body does not move, then the static frictional force and the component of **F** that is parallel to the surface balance each other. They are equal in magnitude, and is f_s directed opposite that component of **F**.

Property 2. The magnitude of has a maximum value $f_{s,max}$ *that is given by*

$$f_{s,\max} = \mu_s F_N.$$

where μ_s *is the* ***coefficient of static friction*** and F_N ***is the magnitude of the normal force*** on the body from the surface. If the magnitude of the component of ***F*** that is parallel to the surface exceeds $f_{s,max}$, then the body begins to slide along the surface.

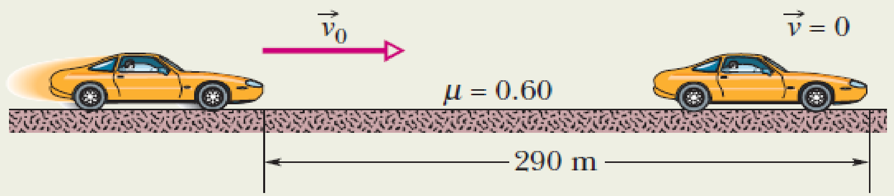
Property 3. If the body begins to slide along the surface, the magnitude of the frictional force rapidly decreases to a value f_k *given by*

$$f_k = \mu_k F_N.$$

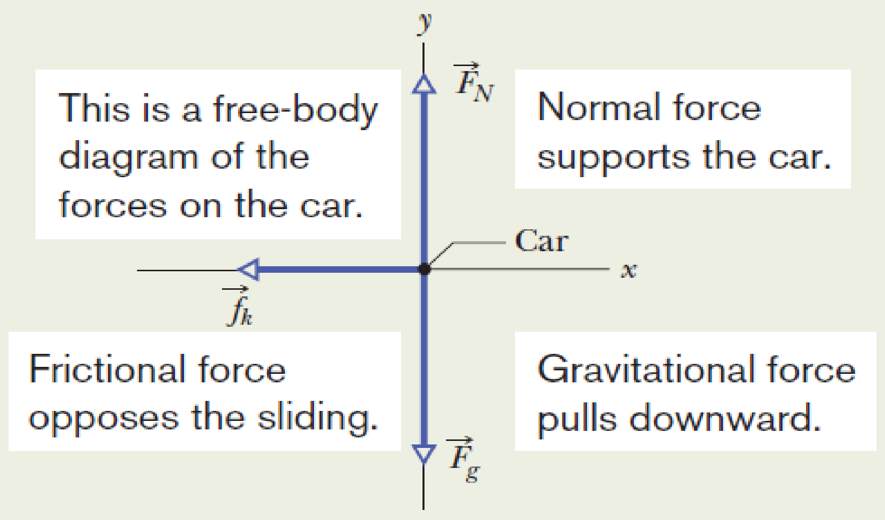
where μ_k *is the* ***coefficient of kinetic friction.*** Thereafter, during the sliding, ***a kinetic frictional force*** $\mathbf{f}_k$ opposes the motion.

Sample Problem

If a car's wheels are "locked" (kept from rolling) during emergency braking, the car slides along the road. Ripped-off bits of tire and small melted sections of road form the "skid marks" that reveal that cold-welding occurred during the slide. The record for the longest skid marks on a public road was reportedly set in 1960 by a Jaguar on the M1 highway in England (Fig. 6-3*a*)—the marks were 290 m long! Assuming that $\mu_k = 0.60$ and the car's acceleration was constant during the braking, how fast was the car going when the wheels became locked?



(*a*)



(*b*)

Assume that the constant acceleration *a was due only to a kinetic frictional* force on the car from the road, directed opposite the direction of the car's motion. This results in:

$$-f_k = ma,$$

where *m is the car's mass. The minus sign indicates the direction* of the kinetic frictional force.

Calculations: The frictional force has the magnitude $f_k = \mu_k F_N$,
where F_N is the magnitude of the normal force on the car from the road. Because the car is not accelerating vertically,

$$F_N = mg.$$

Thus, $$f_k = \mu_k F_N = \mu_k mg$$

$$a = -f_k/m = -\mu_k mg/m = -\mu_k g,$$

where the minus sign indicates that the acceleration is in the negative direction. Use

$$v^2 = v_o^2 + 2a(x - x_o)$$

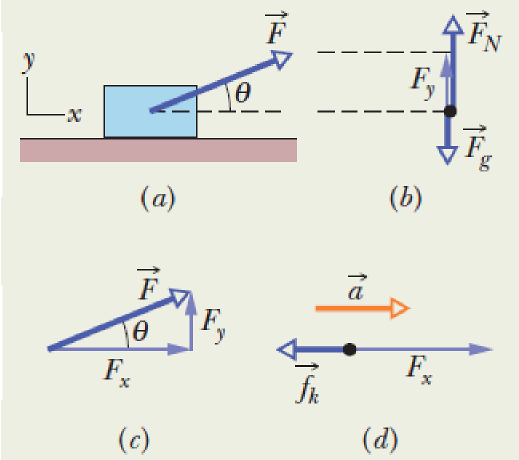
where $(x-x_o)$ = 290 m, and the final speed is 0.
Solving for v_o,

$$v_o = \sqrt{2\mu_k g(x - x_o)} = 58\,m/s$$

We assumed that *v = 0 at the far end of the skid marks.* Actually, the marks ended only because the Jaguar left the road after 290 m. So *v_0 was at least 210 km/h.*

Sample Problem: Friction applied at an angle

In Fig. 6-4*a*, a block of mass $m = 3.0$ kg slides along a floor while a force $\vec{F}$ of magnitude 12.0 N is applied to it at an upward angle θ. The coefficient of kinetic friction between the block and the floor is $\mu_k = 0.40$. We can vary θ from 0 to 90° (the block remains on the floor). What θ gives the maximum value of the block's acceleration magnitude a?



Calculating F_N: Because we need the magnitude f_k of the frictional force, we first must calculate the magnitude F_N of the normal force. Figure 6-4*b* is a free-body diagram showing the forces along the vertical y axis. The normal force is upward, the gravitational force $\vec{F}_g$ with magnitude mg is downward, and (note) the vertical component F_y of the applied force is upward. That component is shown in Fig. 6-4*c*, where we can see that $F_y = F \sin\theta$. We can write Newton's second law ($\vec{F}_{net} = m\vec{a}$) for those forces along the y axis as

$$F_N + F\sin\theta - mg = m(0),$$

$$F_N = mg - F\sin\theta.$$

Calculating acceleration a: Figure 6-4*d* is a free-body diagram for motion along the x axis. The horizontal component F_x of the applied force is rightward; from Fig. 6-4*c*, we see that $F_x = F\cos\theta$. The frictional force has magnitude f_k ($= \mu_k F_N$) and is leftward. Writing Newton's second law for motion along the x axis gives us

$$F\cos\theta - \mu_k F_N = ma.$$

$$a = \frac{F}{m}\cos\theta - \mu_k\left(g - \frac{F}{m}\sin\theta\right).$$

Finding a maximum: To find the value of θ that maximizes a, we take the derivative of a with respect to θ and set the result equal to zero:

$$\frac{da}{d\theta} = -\frac{F}{m}\sin\theta + \mu_k\frac{F}{m}\cos\theta = 0.$$

$$\tan\theta = \mu_k.$$

$$\theta = \tan^{-1}\mu_k$$

$$= 21.8^\circ \approx 22^\circ.$$

6.4 The Drag Force and Terminal Speed

When there is a relative velocity between a fluid and a body (either because the body moves through the fluid or because the fluid moves past the body), the body experiences a **drag force, D, that opposes the relative motion and points in the direction in** which the fluid flows relative to the body.

Fig. 6-5 This skier crouches in an "egg position" so as to minimize her effective cross-sectional area and thus minimize the air drag acting on her. *(Karl-Josef Hildenbrand/dpa/Landov LLC)*

6.4 The Drag Force and Terminal Speed

For cases in which air is the fluid, and the body is blunt (like a baseball) rather than slender (like a javelin), and the relative motion is fast enough so that the air becomes turbulent (breaks up into swirls) behind the body,

$$D = \tfrac{1}{2}C\rho A v^2,$$

where ρ is the air density (mass per volume), *A is the **effective cross-sectional*** **area of the body (the area of a cross section taken perpendicular to the velocity**), and C is the drag coefficient .

When a blunt body falls from rest through air, the drag force is directed upward; its magnitude gradually increases from zero as the speed of the body increases. From Newton's second law along *y axis*

$$D - F_g = ma,$$

where m is the mass of the body. Eventually, a = 0, and the body then falls at a constant speed, called the **terminal speed v_t .**

$$\tfrac{1}{2}C\rho A v_t^2 - F_g = 0,$$

$$v_t = \sqrt{\frac{2F_g}{C\rho A}}.$$

Some typical values of terminal speed

TABLE 6-1

Some Terminal Speeds in Air

Object	Terminal Speed (m/s)	95% Distance[a] (m)
Shot (from shot put)	145	2500
Sky diver (typical)	60	430
Baseball	42	210
Tennis ball	31	115
Basketball	20	47
Ping-Pong ball	9	10
Raindrop (radius = 1.5 mm)	7	6
Parachutist (typical)	5	3

[a]This is the distance through which the body must fall from rest to reach 95% of its terminal speed.

Source: Adapted from Peter J. Brancazio, *Sport Science,* 1984, Simon & Schuster, New York.

Sample Problem: Terminal Speed

A raindrop with radius $R = 1.5$ mm falls from a cloud that is at height $h = 1200$ m above the ground. The drag coefficient C for the drop is 0.60. Assume that the drop is spherical throughout its fall. The density of water ρ_w is 1000 kg/m^3, and the density of air ρ_a is 1.2 kg/m^3.

(a) As Table 6-1 indicates, the raindrop reaches terminal speed after falling just a few meters. What is the terminal speed?

KEY IDEA

The drop reaches a terminal speed v_t when the gravitational force on it is balanced by the air drag force on it, so its acceleration is zero. We could then apply Newton's second law and the drag force equation to find v_t, but Eq. 6-16 does all that for us.

Calculations: To use Eq. 6-16, we need the drop's effective cross-sectional area A and the magnitude F_g of the gravitational force. Because the drop is spherical, A is the area of a circle (πR^2) that has the same radius as the sphere. To find F_g, we use three facts: (1) $F_g = mg$, where m is the drop's mass; (2) the (spherical) drop's volume is $V = \frac{4}{3}\pi R^3$; and (3) the density of the water in the drop is the mass per volume, or $\rho_w = m/V$. Thus, we find

$$F_g = V\rho_w g = \tfrac{4}{3}\pi R^3 \rho_w g.$$

We next substitute this, the expression for A, and the given data into Eq. 6-16. Being careful to distinguish between the air density ρ_a and the water density ρ_w, we obtain

$$v_t = \sqrt{\frac{2F_g}{C\rho_a A}} = \sqrt{\frac{8\pi R^3 \rho_w g}{3C\rho_a \pi R^2}} = \sqrt{\frac{8R\rho_w g}{3C\rho_a}}$$

$$= \sqrt{\frac{(8)(1.5 \times 10^{-3}\text{ m})(1000\text{ kg/m}^3)(9.8\text{ m/s}^2)}{(3)(0.60)(1.2\text{ kg/m}^3)}}$$

$$= 7.4\text{ m/s} \approx 27\text{ km/h}. \qquad \text{(Answer)}$$

Note that the height of the cloud does not enter into the calculation.

(b) What would be the drop's speed just before impact if there were no drag force?

KEY IDEA

With no drag force to reduce the drop's speed during the fall, the drop would fall with the constant free-fall acceleration g, so the constant-acceleration equations of Table 2-1 apply.

Calculation: Because we know the acceleration is g, the initial velocity v_0 is 0, and the displacement $x - x_0$ is $-h$, we use Eq. 2-16 to find v:

$$v = \sqrt{2gh} = \sqrt{(2)(9.8\text{ m/s}^2)(1200\text{ m})}$$

$$= 153\text{ m/s} \approx 550\text{ km/h}. \qquad \text{(Answer)}$$

Had he known this, Shakespeare would scarcely have written, "it droppeth as the gentle rain from heaven, upon the place beneath." In fact, the speed is close to that of a bullet from a large-caliber handgun!

6.5 Uniform Circular Motion

Uniform circular motion:

A body moving with speed v in uniform circular motion feels a centripetal acceleration directed towards the center of the circle of radius R.

$$a = \frac{v^2}{R}$$

Examples:

1. When a car moves in the circular arc, it has an acceleration that is directed toward the center of the circle. The frictional force on the tires from the road provide the centripetal force responsible for that.

2. In a space shuttle around the earth, both the rider and the shuttle are in uniform circular motion and have accelerations directed toward the center of the circle. Centripetal forces, causing these accelerations, are gravitational pulls exerted by Earth and directed radially inward, toward the center of Earth.

Example of a hockey puck:

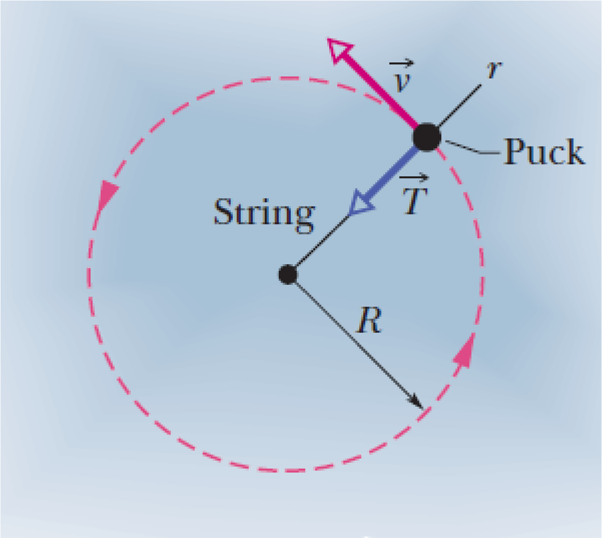


Fig. 6-8 An overhead view of a hockey puck moving with constant speed v in a circular path of radius R on a horizontal frictionless surface. The centripetal force on the puck is T, the pull from the string, directed inward along the radial axis r extending through the puck.

A centripetal force accelerates a body by changing the direction of the body's velocity without changing the body's speed.

From Newton's 2nd Law:

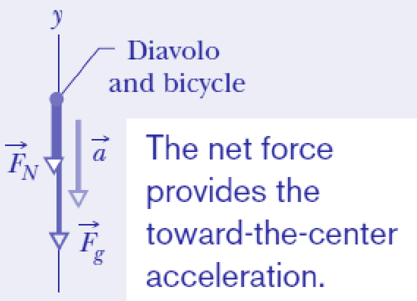
$$F = m\frac{v^2}{R} \qquad \text{(magnitude of centripetal force).}$$

Since the speed v here is constant, the magnitudes of the acceleration and the force are also constant.

Sample Problem: Vertical circular loop

In a 1901 circus performance, Allo "Dare Devil" Diavolo introduced the stunt of riding a bicycle in a loop-the-loop (Fig. 6-9*a*). Assuming that the loop is a circle with radius $R = 2.7$ m, what is the least speed v that Diavolo and his bicycle could have at the top of the loop to remain in contact with it there?

(*a*)



(*b*)

Fig. 6-9

KEY IDEA

We can assume that Diavolo and his bicycle travel through the top of the loop as a single particle in uniform circular motion. Thus, at the top, the acceleration $\vec{a}$ of this particle must have the magnitude $a = v^2/R$ given by Eq. 6-17 and be directed downward, toward the center of the circular loop.

Calculations: The forces on the particle when it is at the top of the loop are shown in the free-body diagram of Fig 6-9*b*. The gravitational force $\vec{F}_g$ is downward along a y axis; so is the normal force $\vec{F}_N$ on the particle from the loop; so also is the centripetal acceleration of the particle. Thus, Newton's second law for y components ($F_{net,y} = ma_y$) gives us

$$-F_N - F_g = m(-a)$$

and

$$-F_N - mg = m\left(-\frac{v^2}{R}\right).$$

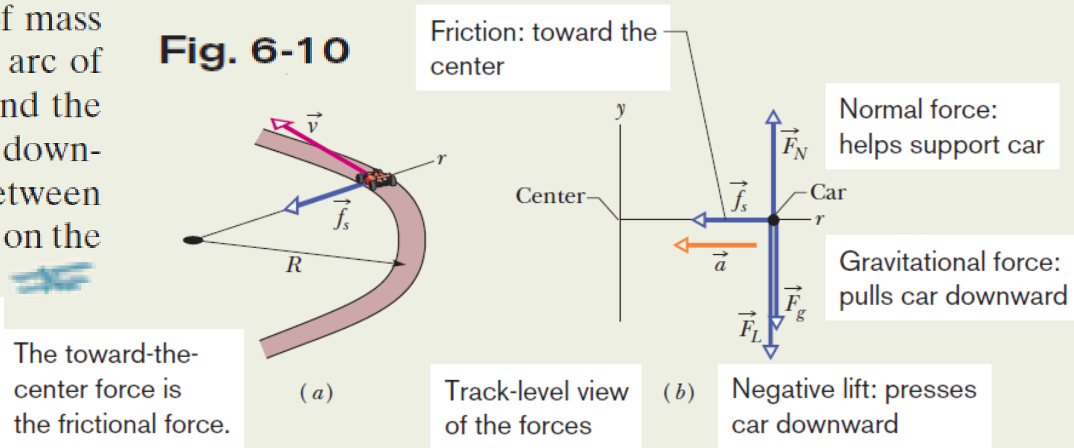
If the particle has the *least speed* v needed to remain in contact, then it is on the *verge of losing contact* with the loop (falling away from the loop), which means that $F_N = 0$ at the top of the loop (the particle and loop touch but without any normal force). Substituting 0 for F_N in Eq. 6-19, solving for v, and then substituting known values give us

$$v = \sqrt{gR} = \sqrt{(9.8\ \text{m/s}^2)(2.7\ \text{m})}$$
$$= 5.1\ \text{m/s}. \qquad \text{(Answer)}$$

Sample Problem: Car in flat circular turn

Figure 6-10*a* represents a Grand Prix race car of mass $m = 600$ kg as it travels on a flat track in a circular arc of radius $R = 100$ m. Because of the shape of the car and the wings on it, the passing air exerts a negative lift $\vec{F}_L$ downward on the car. The coefficient of static friction between the tires and the track is 0.75. (Assume that the forces on the four tires are identical.)

Fig. 6-10



Radial calculations: The frictional force $\vec{f}_s$ is shown in the free-body diagram of Fig. 6-10*b*. It is in the negative direction of a radial axis r that always extends from the center of curvature through the car as the car moves. The force produces a centripetal acceleration of magnitude v^2/R. We can relate the force and acceleration by writing Newton's second law for components along the r axis ($F_{\text{net},r} = ma_r$) as

$$-f_s = m\left(-\frac{v^2}{R}\right).$$

Substituting $f_{s,\max} = \mu_s F_N$ for f_s leads us to

$$\mu_s F_N = m\left(\frac{v^2}{R}\right).$$

Vertical calculations: Next, let's consider the vertical forces on the car. The normal force $\vec{F}_N$ is directed up, in the positive direction of the y axis in Fig. 6-10*b*. The gravitational force $\vec{F}_g = m\vec{g}$ and the negative lift $\vec{F}_L$ are directed down. The acceleration of the car along the y axis is zero. Thus we can write Newton's second law for components along the y axis ($F_{\text{net},y} = ma_y$) as

$$F_N - mg - F_L = 0,$$

or

$$F_N = mg + F_L.$$

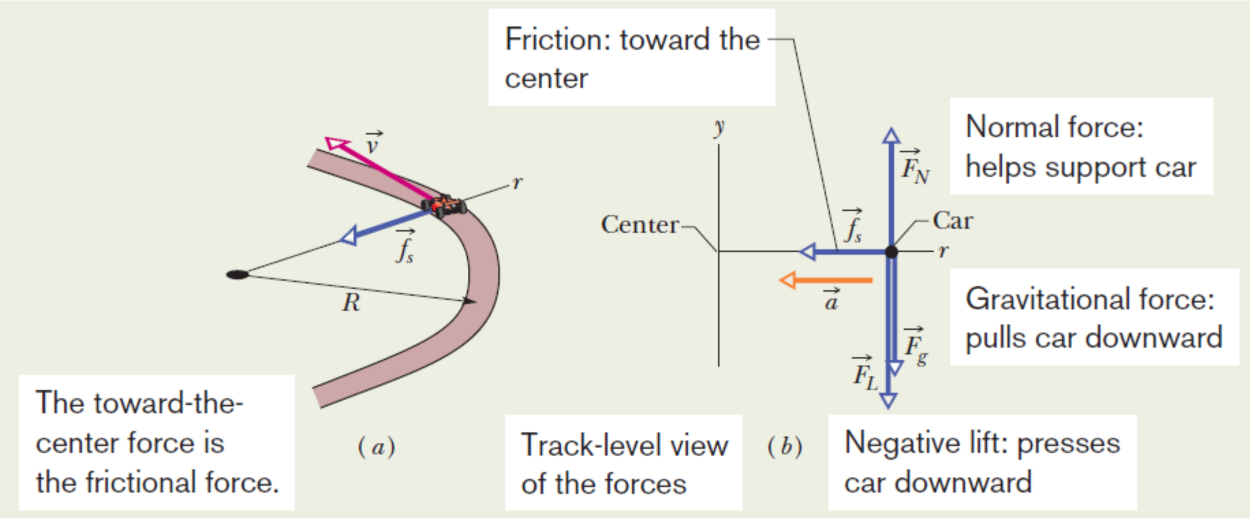
Combining results:

$$F_L = m\left(\frac{v^2}{\mu_s R} - g\right)$$

$$= (600\text{ kg})\left(\frac{(28.6\text{ m/s})^2}{(0.75)(100\text{ m})} - 9.8\text{ m/s}^2\right)$$

$$= 663.7\text{ N} \approx 660\text{ N}. \qquad \text{(Answer)}$$

Sample Problem: Car in flat circular turn, cont.



(b) The magnitude F_L *of the negative lift on a car depends* on the square of the car's speed v^2, *just as the drag force* does .Thus, the negative lift on the car here is greater when the car travels faster, as it does on a straight section of track. What is the magnitude of the negative lift for a speed of 90 m/s?

Calculations: Thus we can write a ratio of the negative lift $F_{L,90}$ at v =90 m/s to our result for the negative lift F_L at v =28.6 m/s as

$$\frac{F_{L,90}}{F_L} = \frac{(90\text{ m/s})^2}{(28.6\text{ m/s})^2}.$$

Using F_L =*663.7 N* ,

$$F_{L,90} = 6572\text{ N} \approx 6600\text{ N}.$$

Upside-down racing: The gravitational force is, of course, the force to beat if there is a chance of racing upside down:

$$F_g = mg = (600\text{ kg})(9.8\text{ m/s}^2) = 5880\text{ N}.$$