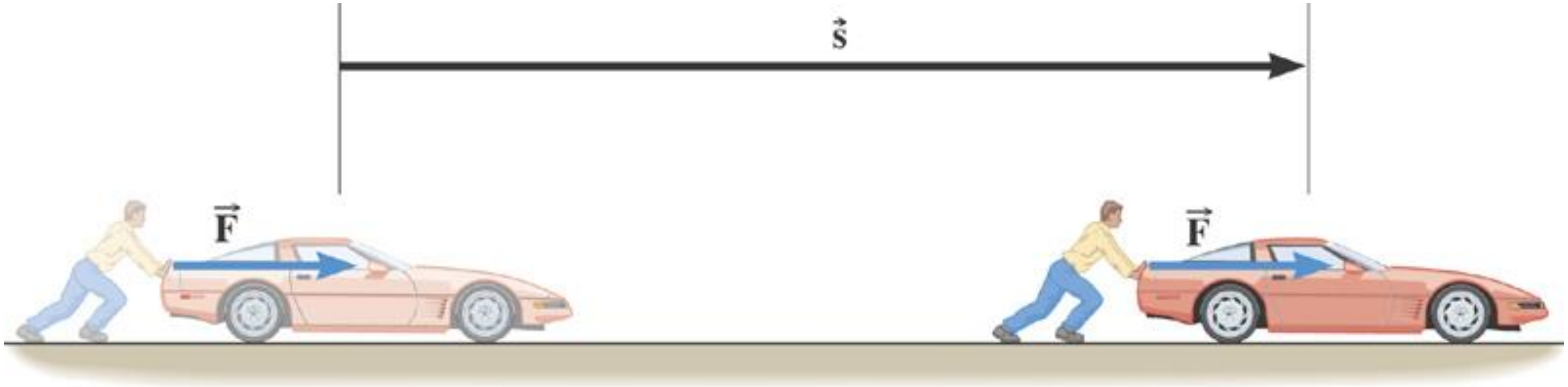
Chapter 6

Work and Energy



$$W = Fs$$

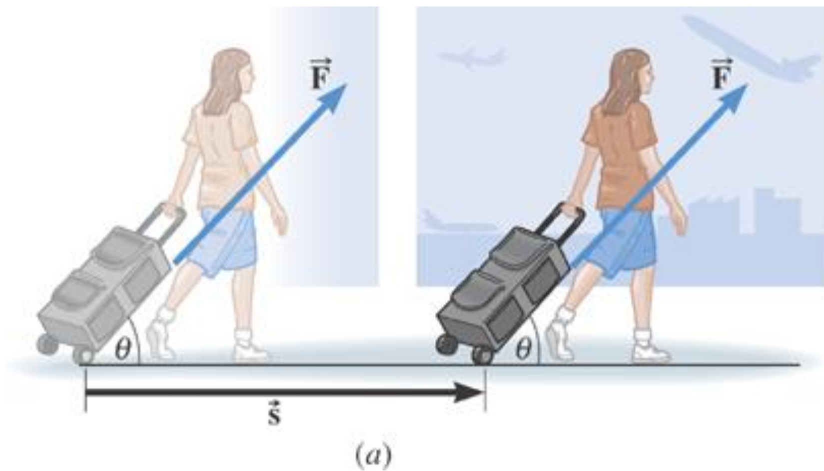
$$1\,\mathrm{N}\cdot\mathrm{m} = 1\text{ joule}\,(\mathrm{J})$$

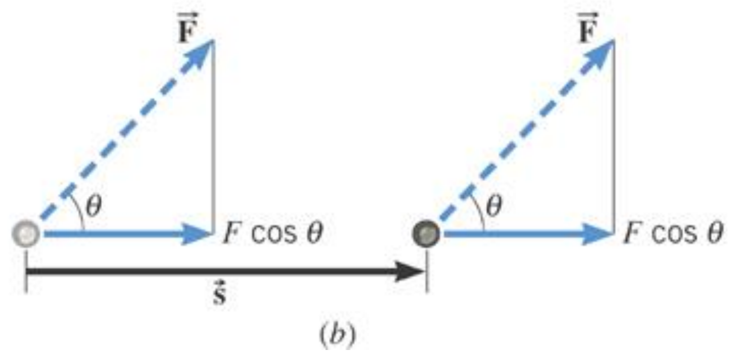
6.1 Work Done by a Constant Force

Table 6.1 Units of Measurement for Work

System	Force	×	Distance	=	Work
SI	newton (N)		meter (m)		joule (J)
CGS	dyne (dyn)		centimeter (cm)		erg
BE	pound (lb)		foot (ft)		foot · pound (ft · lb)

6.1 Work Done by a Constant Force



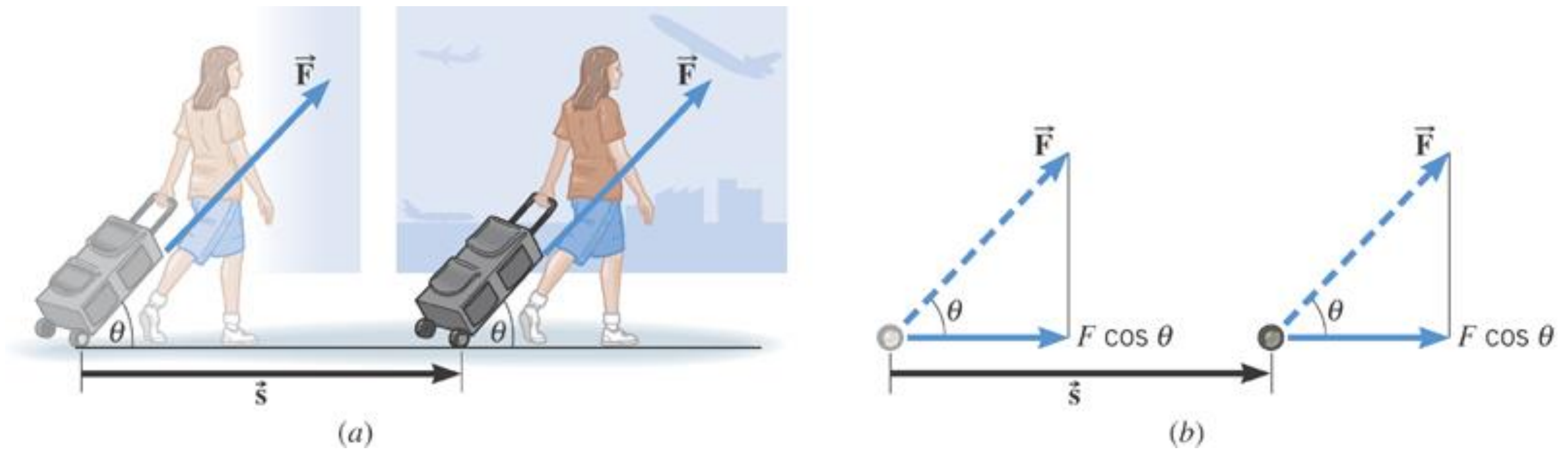


$$W = (F\cos\theta)s$$

$$\cos 0^\circ = 1$$

$$\cos 90^\circ = 0$$

$$\cos 180^\circ = -1$$



Example 1 Pulling a Suitcase-on-Wheels

Find the work done if the force is 45.0-N, the angle is 50.0 degrees, and the displacement is 75.0 m.

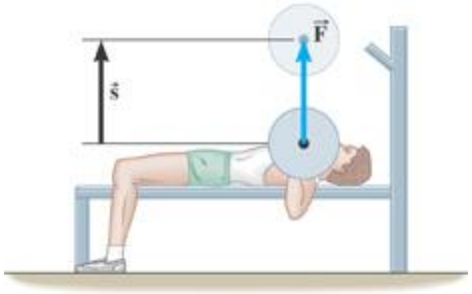
$$W = (F\cos\theta)s = [(45.0\ \text{N})\cos 50.0^{\circ}](75.0\ \text{m})$$

$$= 2170\ \text{J}$$

6.1 Work Done by a Constant Force

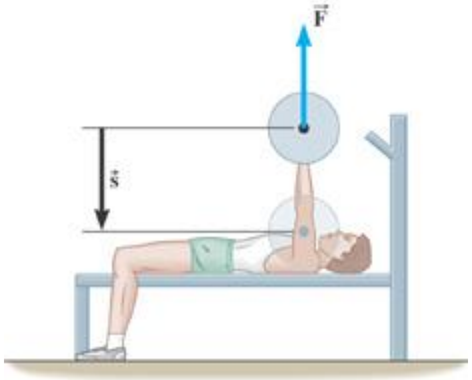
(a)

$$W = (F\cos 0)s = Fs$$



(b)

$$W = (F\cos 180)s = -Fs$$

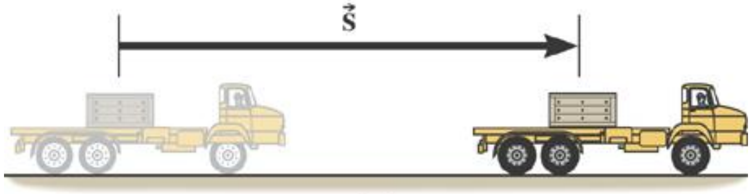


(c)

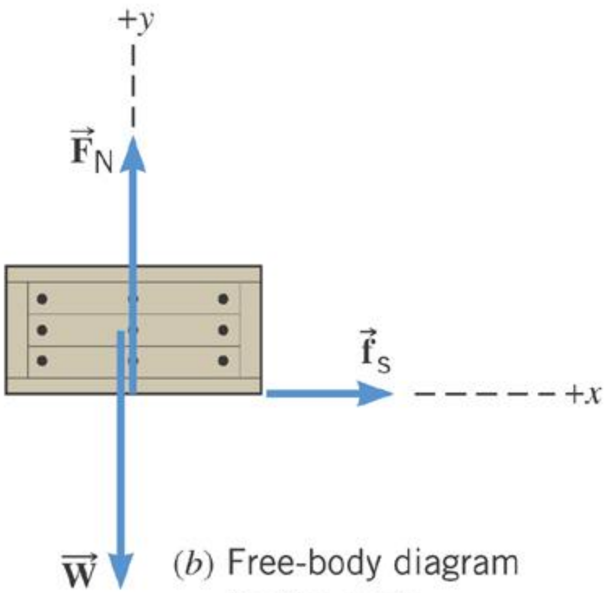
Example 3 Accelerating a Crate

The truck is accelerating at a rate of +1.50 m/s^2. The mass of the crate is 120-kg and it does not slip. The magnitude of the displacement is 65 m.

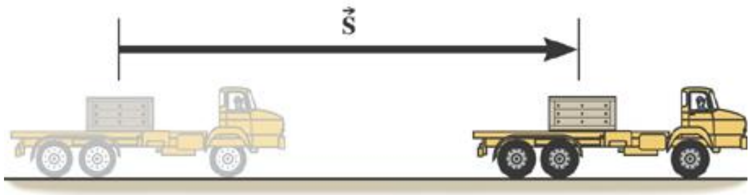
What is the total work done on the crate by all of the forces acting on it?



(*a*)



(*b*) Free-body diagram for the crate

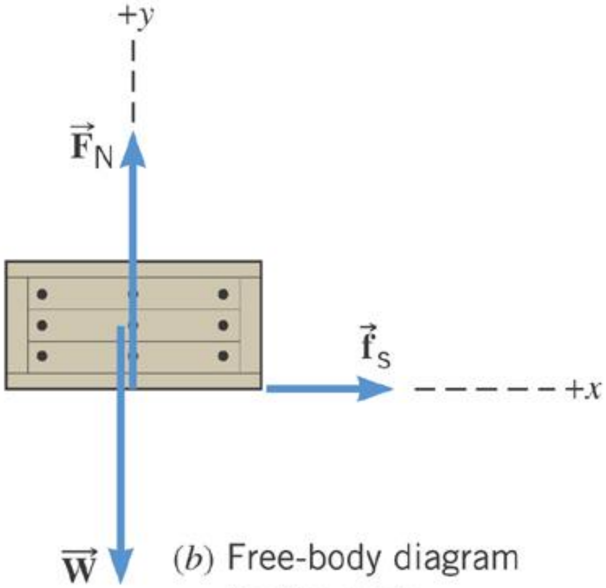


(a)

The angle between the displacement and the normal force is 90 degrees.

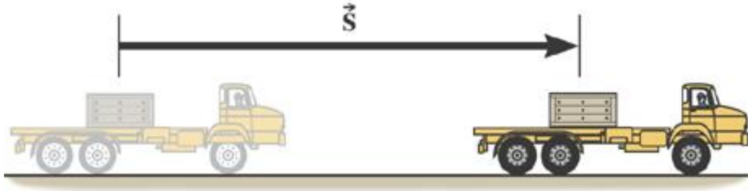
The angle between the displacement and the weight is also 90 degrees.

$$W = (F\cos 90)s = 0$$



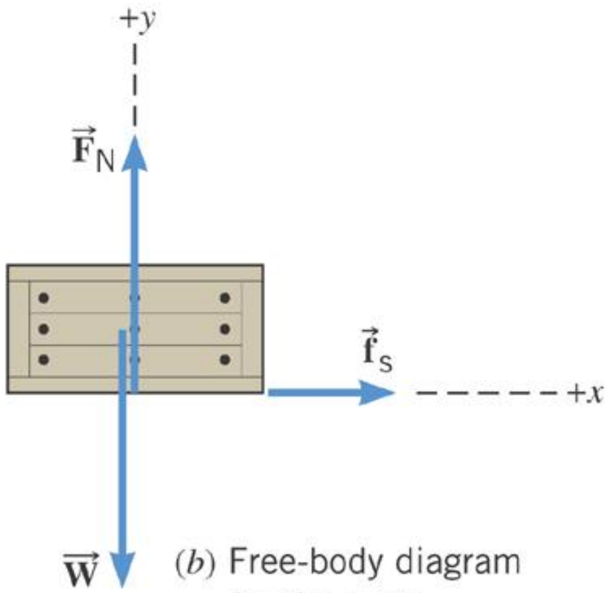
(b) Free-body diagram for the crate

6.1 Work Done by a Constant Force



(a)

The angle between the displacement and the friction force is 0 degrees.



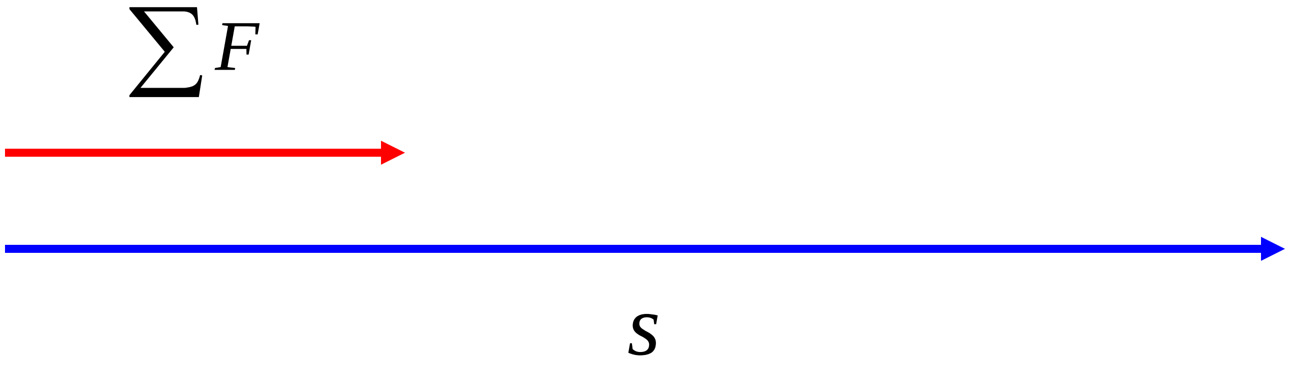
(b) Free-body diagram for the crate

$$f_s = ma = (120\,\text{kg})(1.5\,\text{m/s}^2) = 180\,\text{N}$$

$$W = [(180\,\text{N})\cos 0](65\,\text{m}) = 1.2\times10^4\,\text{J}$$

Consider a constant net external force acting on an object.

The object is displaced a distance *s*, in the same direction as the net force.



The work is simply $W = \left(\sum F\right)s = (ma)s$

6.2 The Work-Energy Theorem and Kinetic Energy

$$W = m(as) = m\tfrac{1}{2}\left(v_f^2 - v_o^2\right) = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_o^2$$

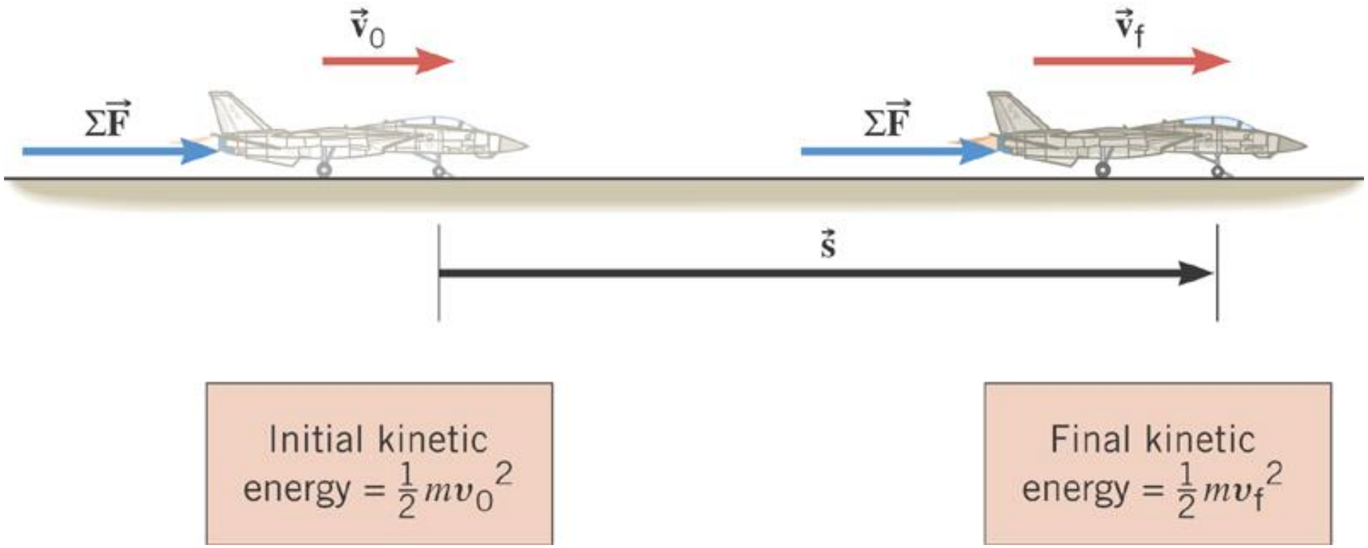
$$v_f^2 = v_o^2 + 2(ax)$$

$$(ax) = \tfrac{1}{2}\left(v_f^2 - v_o^2\right)$$

DEFINITION OF KINETIC ENERGY

The kinetic energy KE of an object with mass *m* and speed *v* is given by

$$\mathrm{KE} = \tfrac{1}{2}mv^2$$



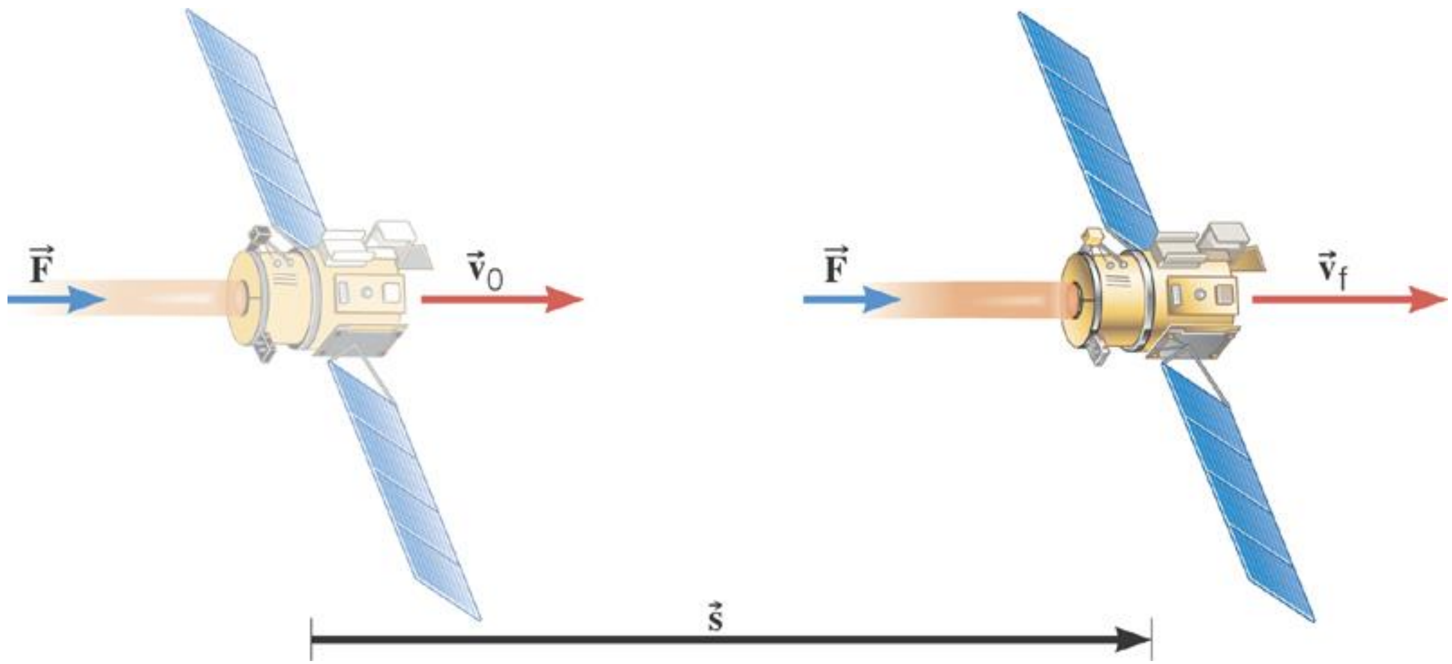
THE WORK-ENERGY THEOREM

When a net external force does work on an object, the kinetic energy of the object changes by the amount of work done on it:

$$W = \mathrm{KE}_f - \mathrm{KE}_o = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_o^2$$

Example 4 Deep Space 1

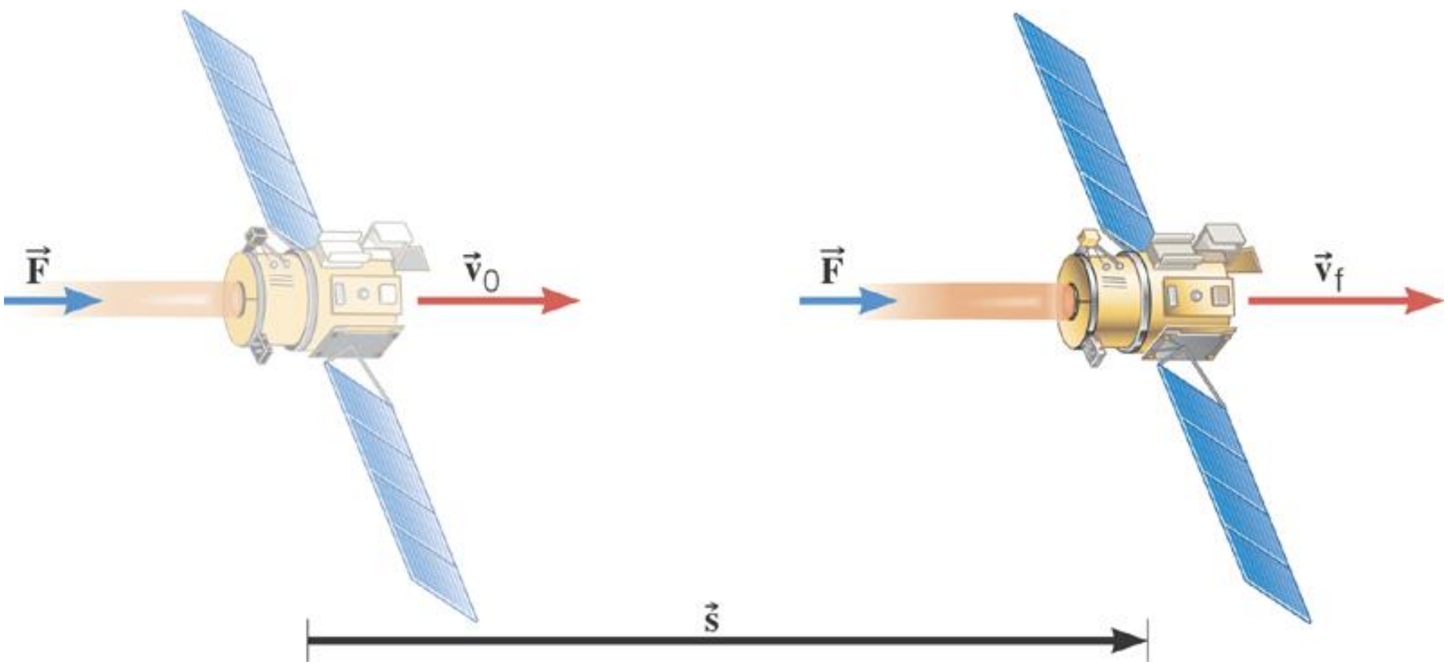
The mass of the space probe is 474-kg and its initial velocity is 275 m/s. If the 56.0-mN force acts on the probe through a displacement of 2.42×10^{9}m, what is its final speed?



6.2 The Work-Energy Theorem and Kinetic Energy

$$W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_o^2$$

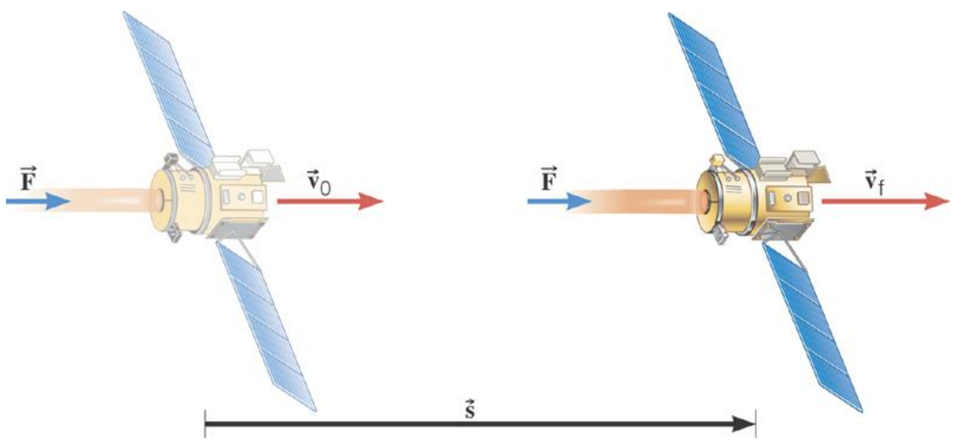
$$W = \left[\left(\sum F\right)\cos\theta\right] s$$

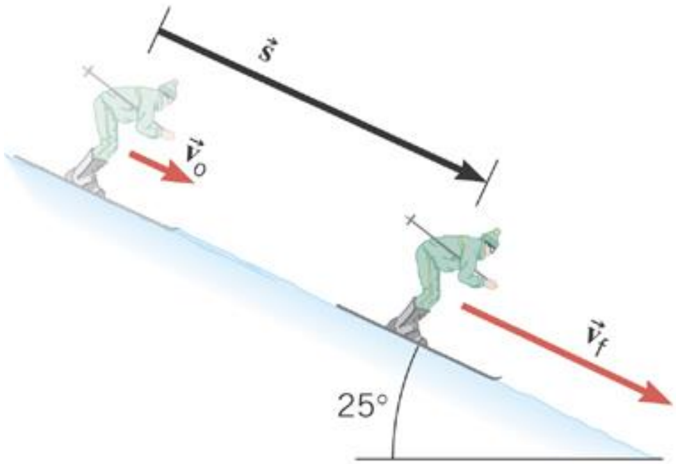


$$\left[\left(\sum \mathrm{F}\right)\cos\theta\right]s = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_o^2$$

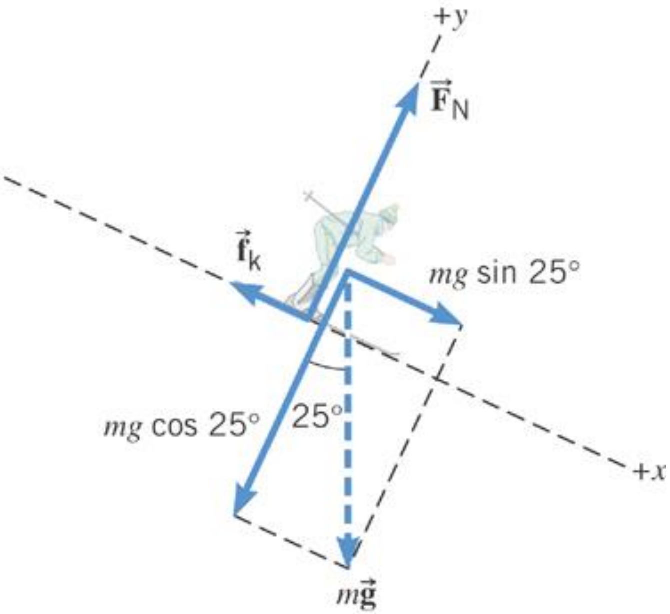
$$\left(5.60\times10^{-2}\,\mathrm{N}\right)\cos 0^\circ\left(2.42\times10^{9}\,\mathrm{m}\right) = \tfrac{1}{2}\left(474\,\mathrm{kg}\right)v_f^2 - \tfrac{1}{2}\left(474\,\mathrm{kg}\right)\left(275\,\mathrm{m/s}\right)^2$$

$$v_f = 805\,\mathrm{m/s}$$





(a)

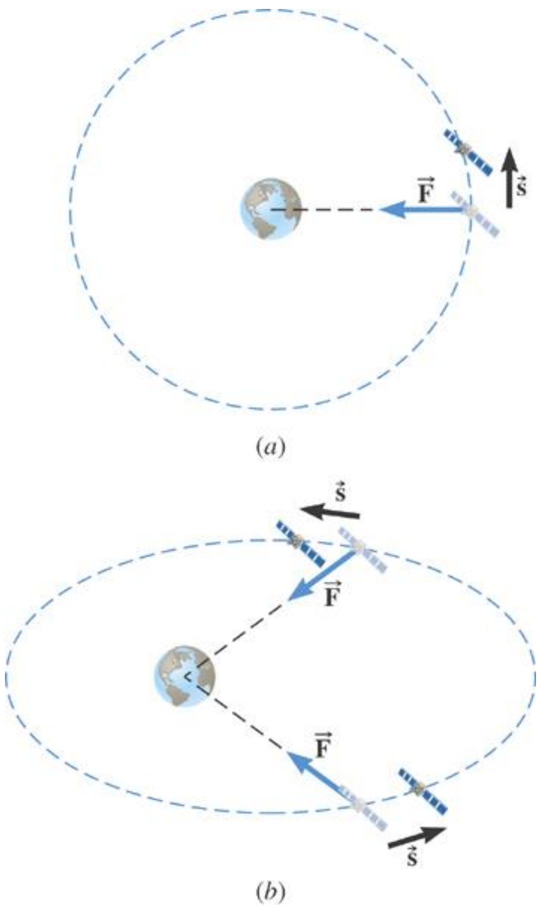


(b) Free-body diagram for the skier

In this case the net force is $\sum F = mg\sin 25^\circ - f_k$

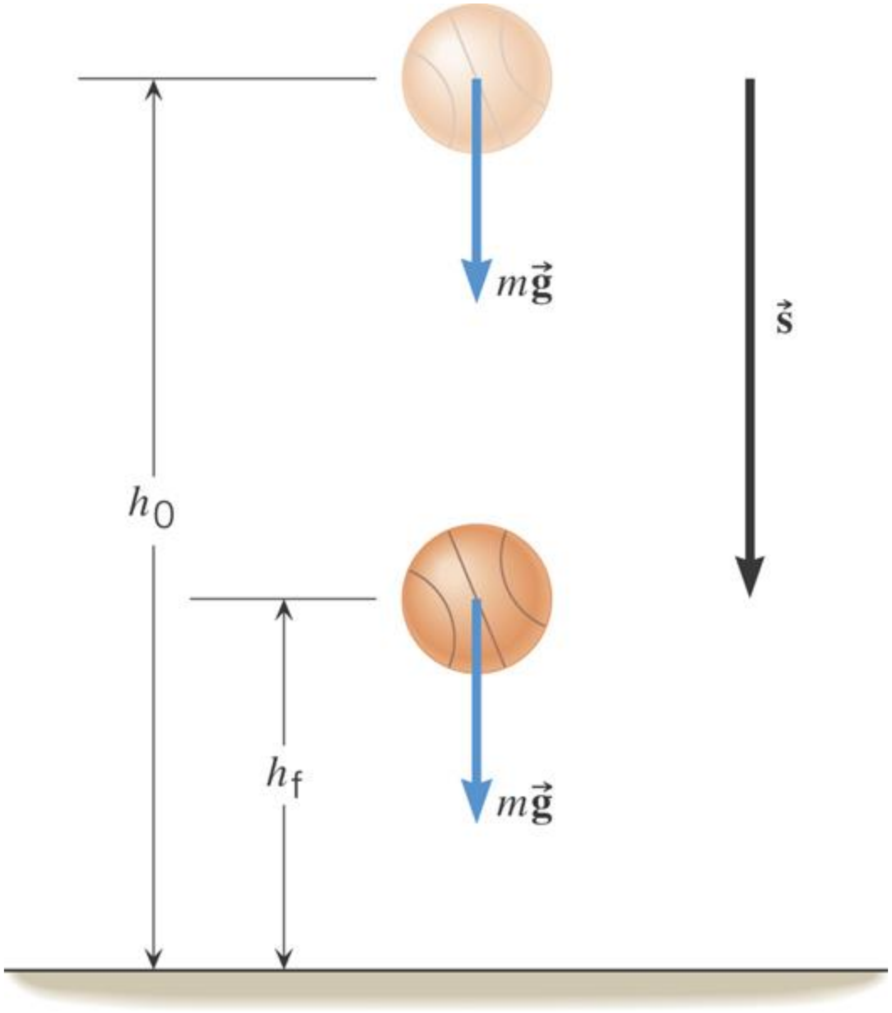
Conceptual Example 6 Work and Kinetic Energy

A satellite is moving about the earth in a circular orbit and an elliptical orbit. For these two orbits, determine whether the kinetic energy of the satellite changes during the motion.

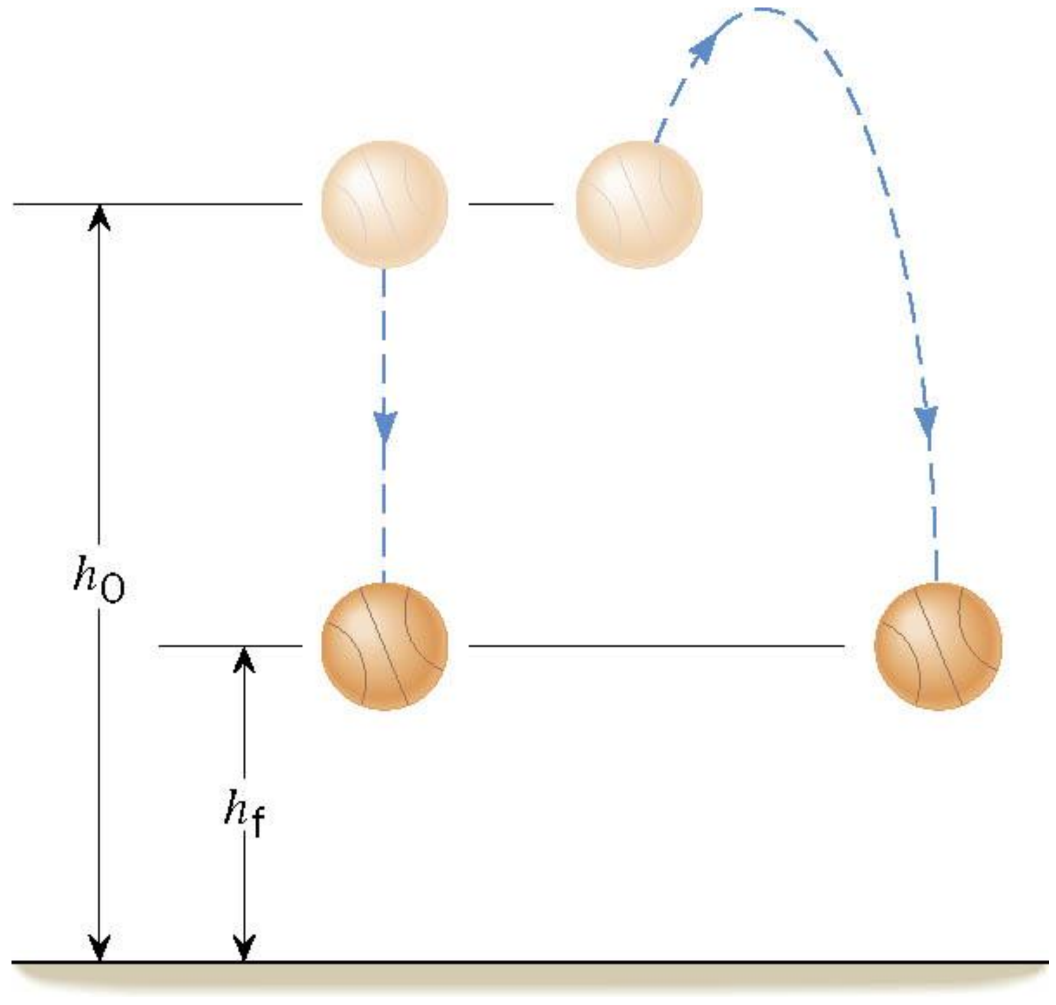


$$W = (F\cos\theta)s$$

$$W_{\text{gravity}} = mg(h_o - h_f)$$

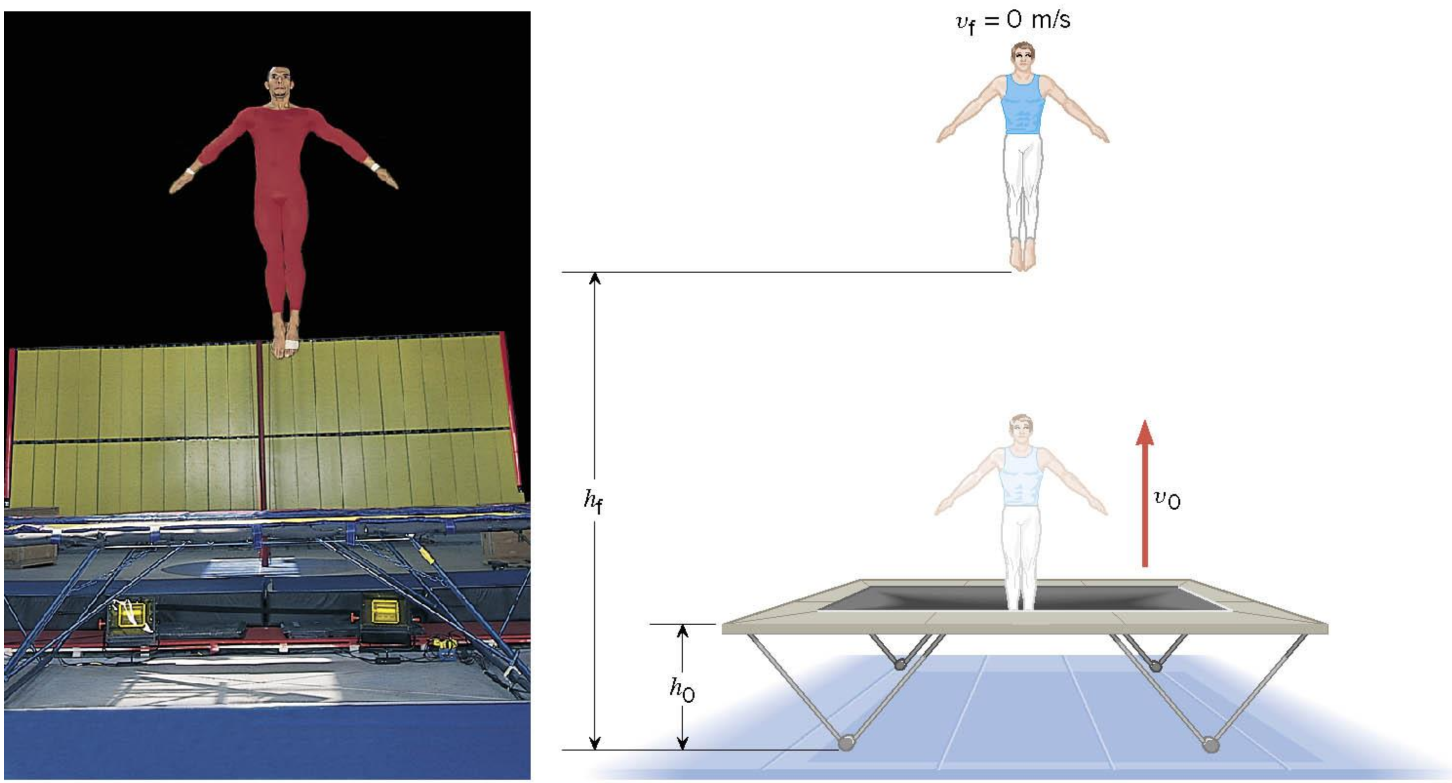


$$W_{\text{gravity}} = mg\left(h_o - h_f\right)$$



Example 7 A Gymnast on a Trampoline

The gymnast leaves the trampoline at an initial height of 1.20 m and reaches a maximum height of 4.80 m before falling back down. What was the initial speed of the gymnast?



(*a*) (*b*)

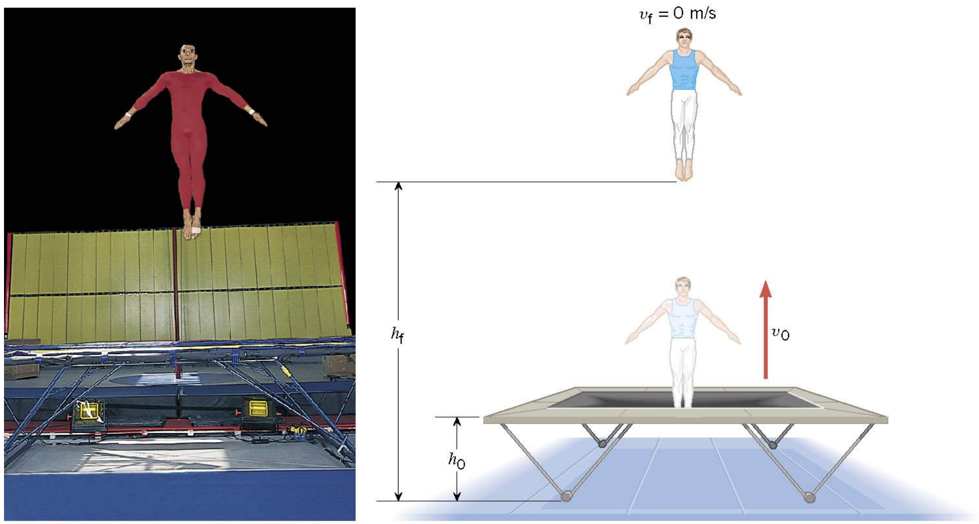
6.3 Gravitational Potential Energy

$$W = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_o^2$$

$$W_{\text{gravity}} = mg(h_o - h_f)$$

$$mg(h_o - h_f) = -\tfrac{1}{2}mv_o^2$$

$$v_o = \sqrt{-2g(h_o - h_f)}$$



(a) (b)

$$v_o = \sqrt{-2\left(9.80\,\text{m/s}^2\right)\left(1.20\,\text{m} - 4.80\,\text{m}\right)} = 8.40\,\text{m/s}$$

$$W_{\text{gravity}} = mgh_o - mgh_f$$

DEFINITION OF GRAVITATIONAL POTENTIAL ENERGY

The gravitational potential energy PE is the energy that an object of mass *m* has by virtue of its position relative to the surface of the earth. That position is measured by the height *h* of the object relative to an arbitrary zero level:

$$\text{PE} = mgh$$

$$1\,\text{N}\cdot\text{m} = 1\,\text{joule}\,(\text{J})$$

DEFINITION OF A CONSERVATIVE FORCE

Version 1 A force is conservative when the work it does on a moving object is independent of the path between the object's initial and final positions.

Version 2 A force is conservative when it does no work on an object moving around a closed path, starting and finishing at the same point.

Table 6.2 Some Conservative and Nonconservative Forces

Conservative Forces

Gravitational force (Ch. 4)

Elastic spring force (Ch. 10)

Electric force (Ch. 18, 19)

Nonconservative Forces

Static and kinetic frictional forces

Air resistance

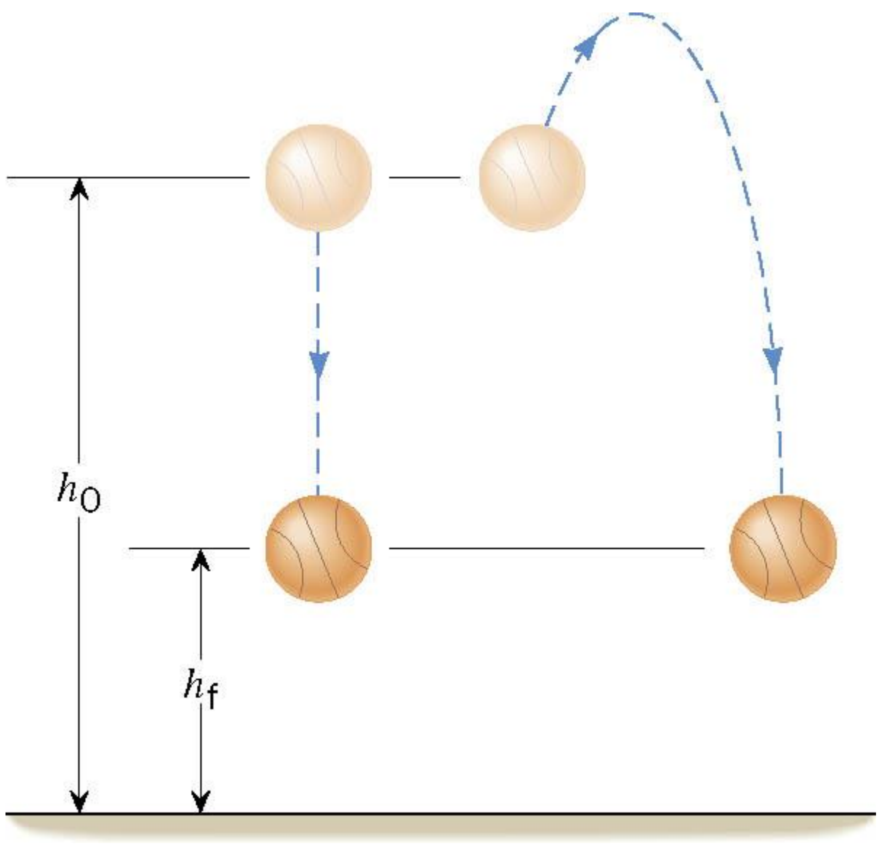
Tension

Normal force

Propulsion force of a rocket

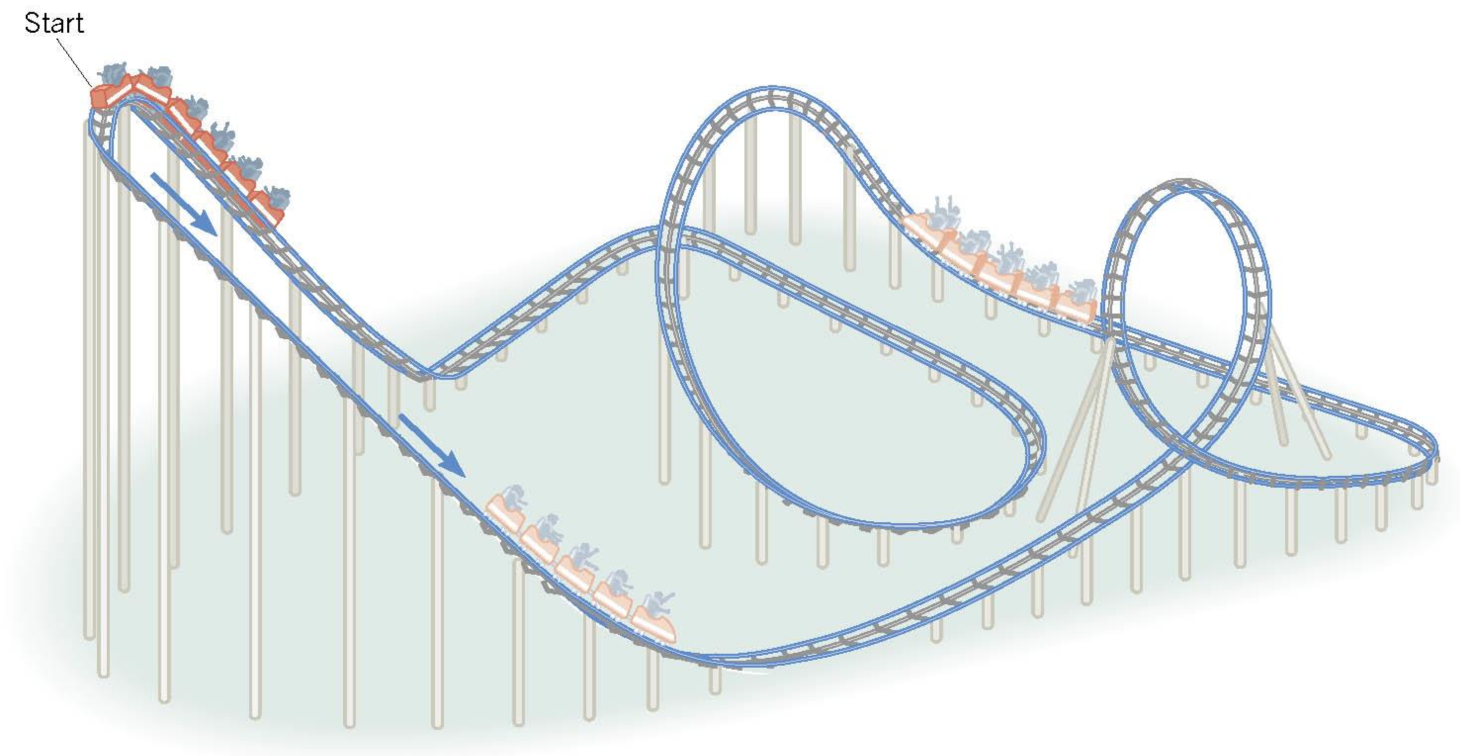
Version 1 A force is conservative when the work it does on a moving object is independent of the path between the object's initial and final positions.

$$W_{\text{gravity}} = mg\left(h_o - h_f\right)$$



Version 2 A force is conservative when it does no work on an object moving around a closed path, starting and finishing at the same point.

$$W_{\text{gravity}} = mg(h_o - h_f) \qquad h_o = h_f$$



An example of a nonconservative force is the kinetic frictional force.

$$W = (F\cos\theta)s = f_k \cos 180^\circ s = -f_k s$$

The work done by the kinetic frictional force is always negative. Thus, it is impossible for the work it does on an object that moves around a closed path to be zero.

The concept of potential energy is not defined for a nonconservative force.

6.4 Conservative Versus Nonconservative Forces

In normal situations both conservative and nonconservative forces act simultaneously on an object, so the work done by the net external force can be written as

$$W = W_c + W_{nc}$$

$$W = \mathrm{KE_f} - \mathrm{KE_o} = \Delta\mathrm{KE}$$

$$W_c = W_{\mathrm{gravity}} = mgh_o - mgh_f = \mathrm{PE_o} - \mathrm{PE_f} = -\Delta\mathrm{PE}$$

6.4 Conservative Versus Nonconservative Forces

$$W = W_c + W_{nc}$$

$$\Delta \mathrm{KE} = -\Delta \mathrm{PE} + W_{nc}$$

THE WORK-ENERGY THEOREM

$$W_{nc} = \Delta \mathrm{KE} + \Delta \mathrm{PE}$$

6.5 *The Conservation of Mechanical Energy*

$$W_{nc} = \Delta\mathrm{KE} + \Delta\mathrm{PE} = \left(\mathrm{KE_f} - \mathrm{KE_o}\right) + \left(\mathrm{PE_f} - \mathrm{PE_o}\right)$$

$$W_{nc} = \left(\mathrm{KE_f} + \mathrm{PE_f}\right) + \left(\mathrm{KE_o} + \mathrm{PE_o}\right)$$

$$W_{nc} = \mathrm{E_f} - \mathrm{E_o}$$

If the net work on an object by nonconservative forces is zero, then its energy does not change:

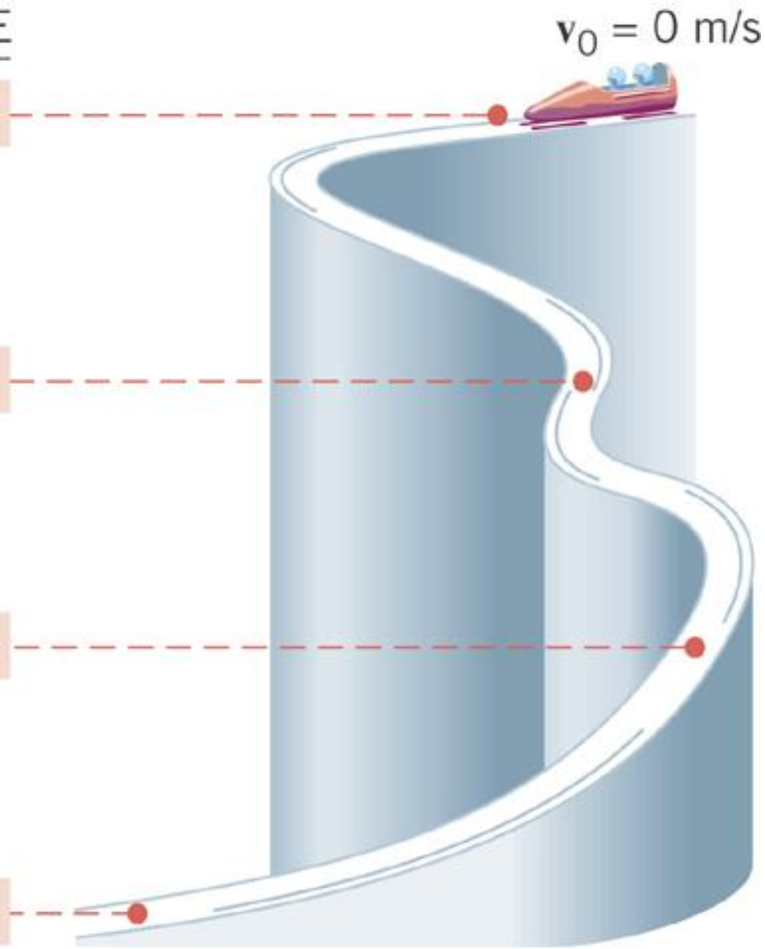
$$\mathrm{E_f} = \mathrm{E_o}$$

THE PRINCIPLE OF CONSERVATION OF MECHANICAL ENERGY

The total mechanical energy (E = KE + PE) of an object remains constant as the object moves, provided that the net work done by external nononservative forces is zero.

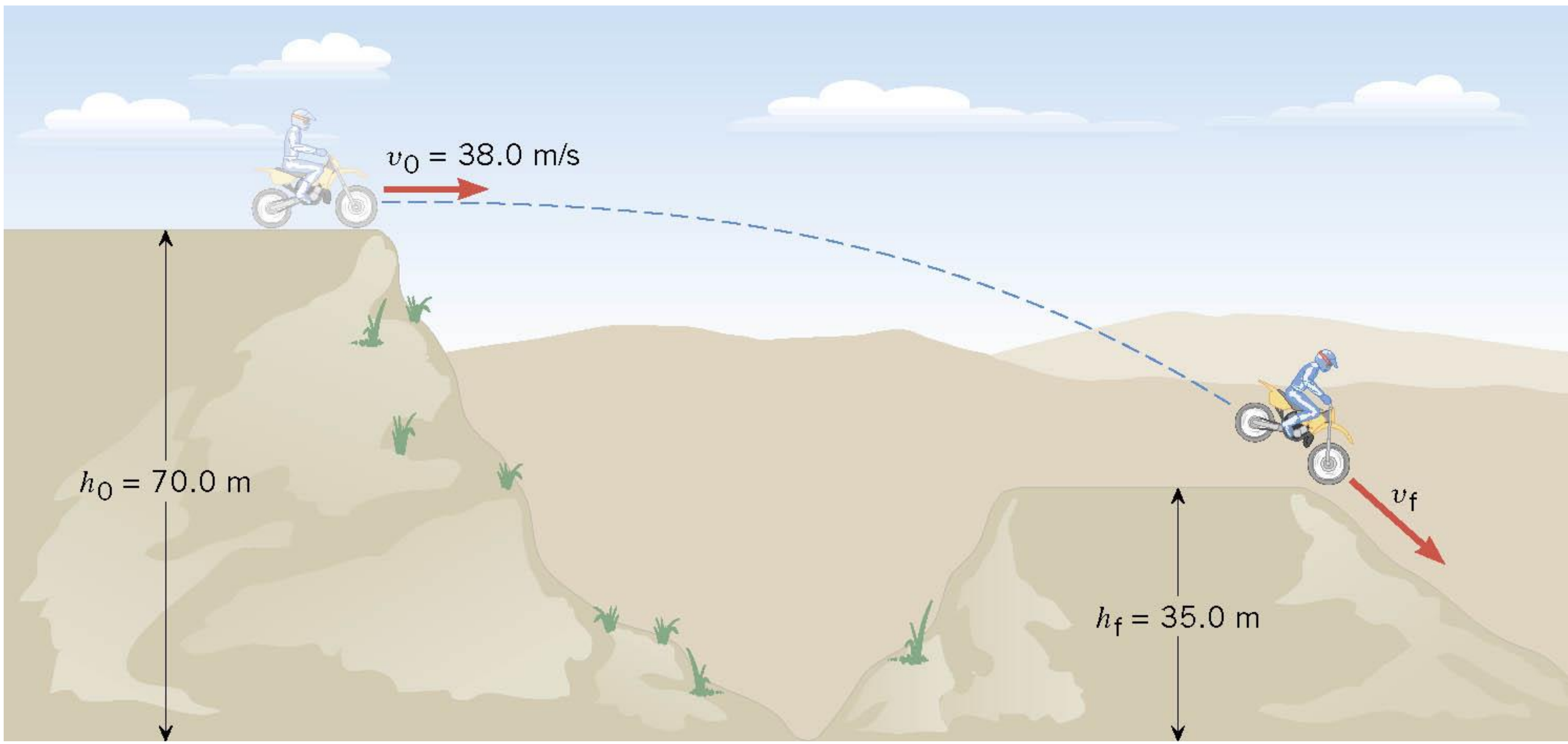
6.5 The Conservation of Mechanical Energy

KE	PE	E = KE + PE
0 J	600 000 J	600 000 J
200 000 J	400 000 J	600 000 J
400 000 J	200 000 J	600 000 J
600 000 J	0 J	600 000 J

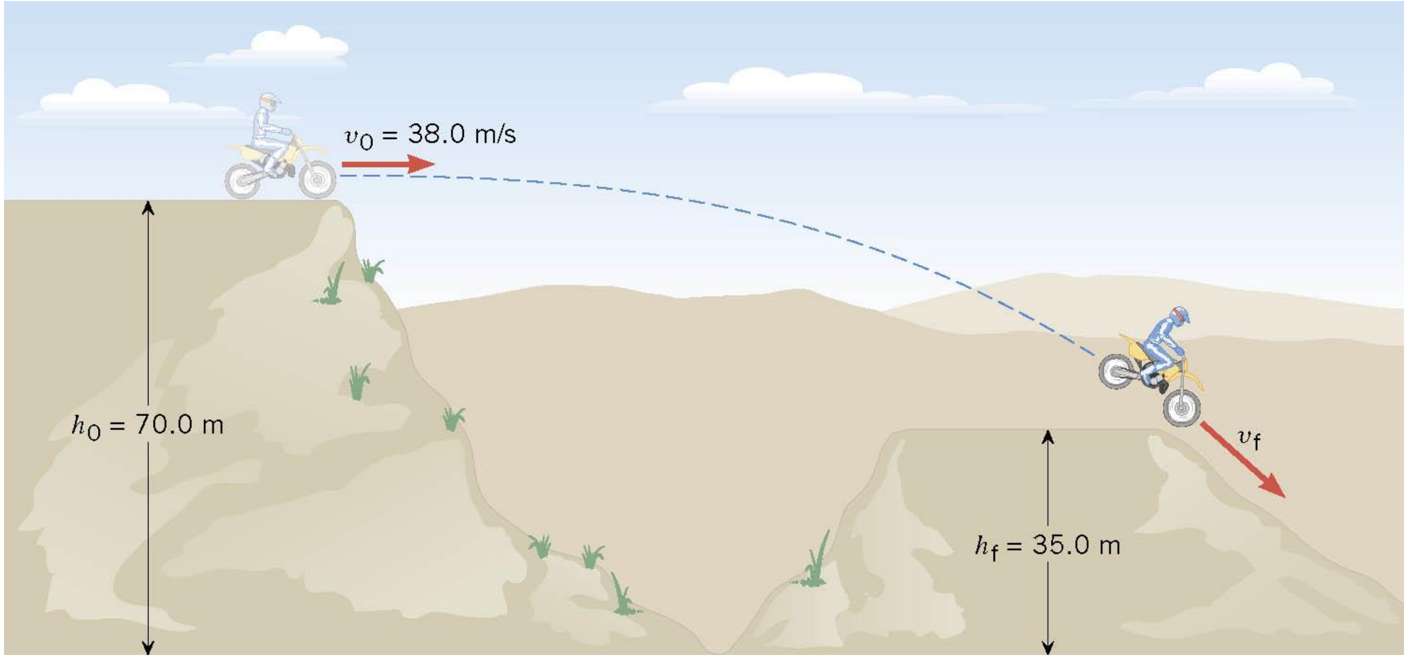


Example 8 A Daredevil Motorcyclist

A motorcyclist is trying to leap across the canyon by driving horizontally off a cliff 38.0 m/s. Ignoring air resistance, find the speed with which the cycle strikes the ground on the other side.



6.5 The Conservation of Mechanical Energy

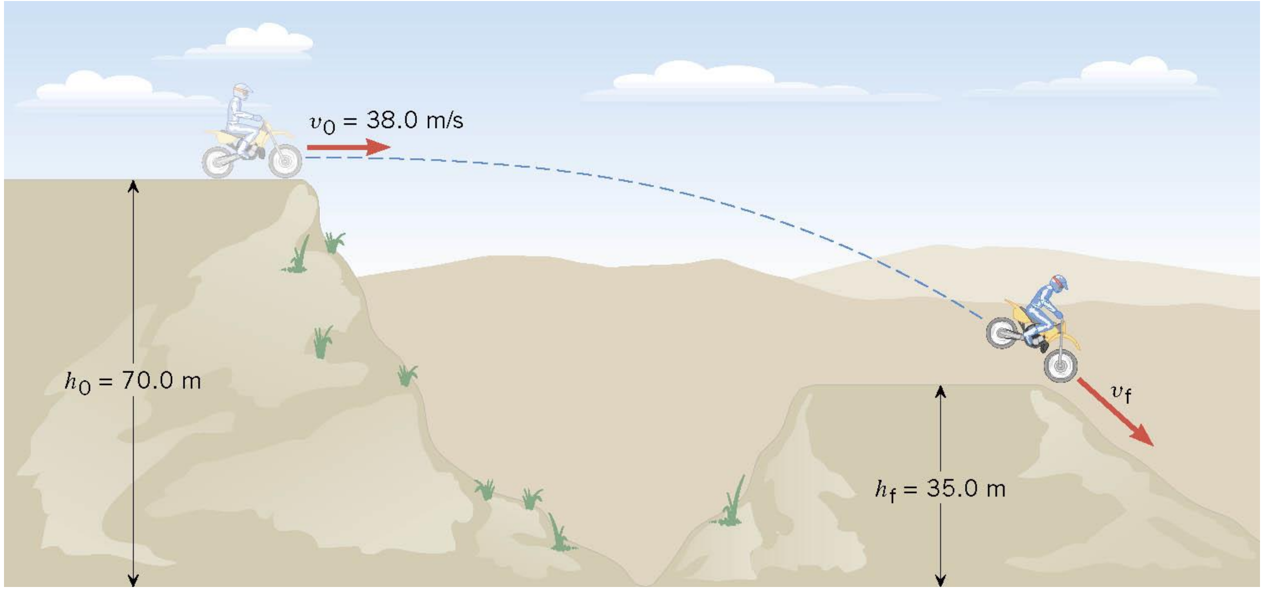


$$E_f = E_o$$

$$mgh_f + \tfrac{1}{2}mv_f^2 = mgh_o + \tfrac{1}{2}mv_o^2$$

$$gh_f + \tfrac{1}{2}v_f^2 = gh_o + \tfrac{1}{2}v_o^2$$

6.5 The Conservation of Mechanical Energy



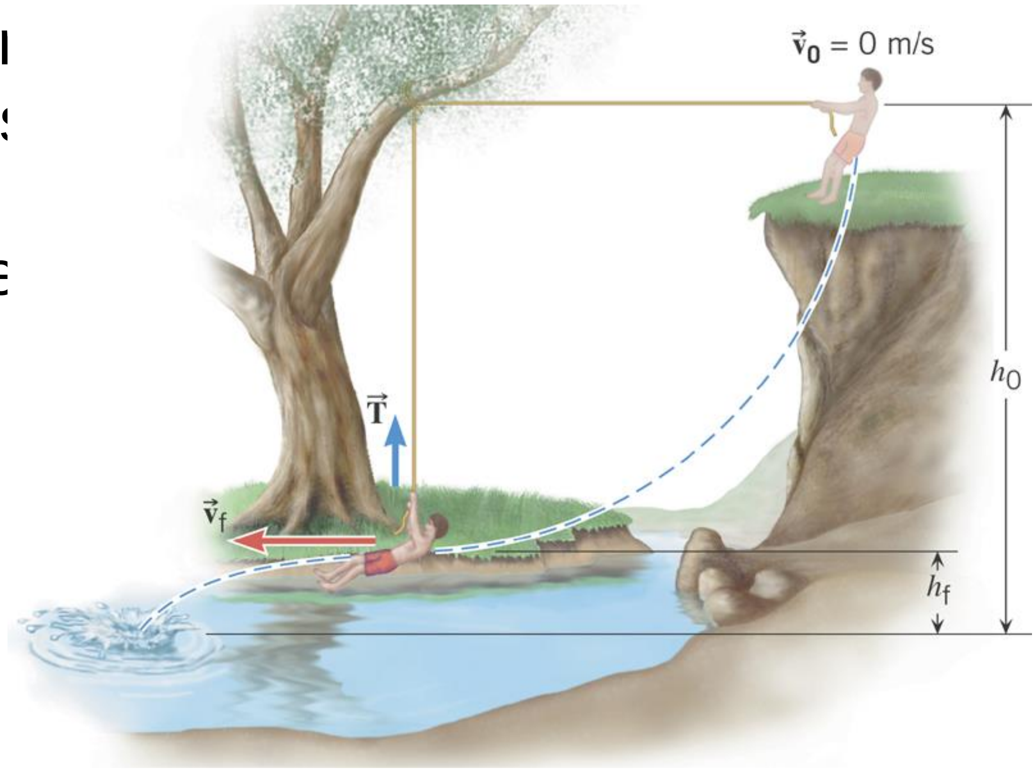
$$gh_f + \tfrac{1}{2}v_f^2 = gh_o + \tfrac{1}{2}v_o^2$$

$$v_f = \sqrt{2g\left(h_o - h_f\right) + v_o^2}$$

$$v_f = \sqrt{2\left(9.8\,\text{m/s}^2\right)\left(35.0\text{m}\right) + \left(38.0\,\text{m/s}\right)^2} = 46.2\,\text{m/s}$$

Conceptual Example 9 The Favorite Swimming Hole

The person starts from rest, with the rope held in the horizontal position, swings downward, and then lets go of the rope. Three forces act on him: his weight, the tension in the rope, and the force of air resistance.

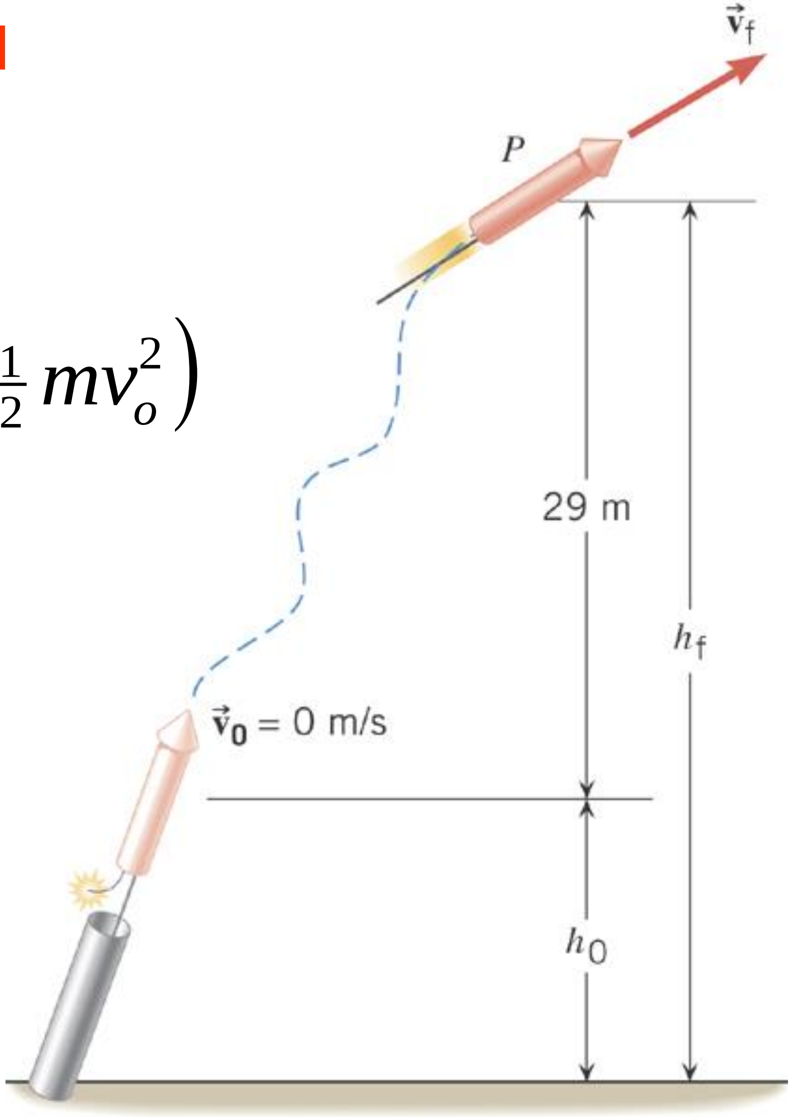


Can the principle of conservation of energy be used to calculate his final speed?

THE WORK-ENERGY THEOREM

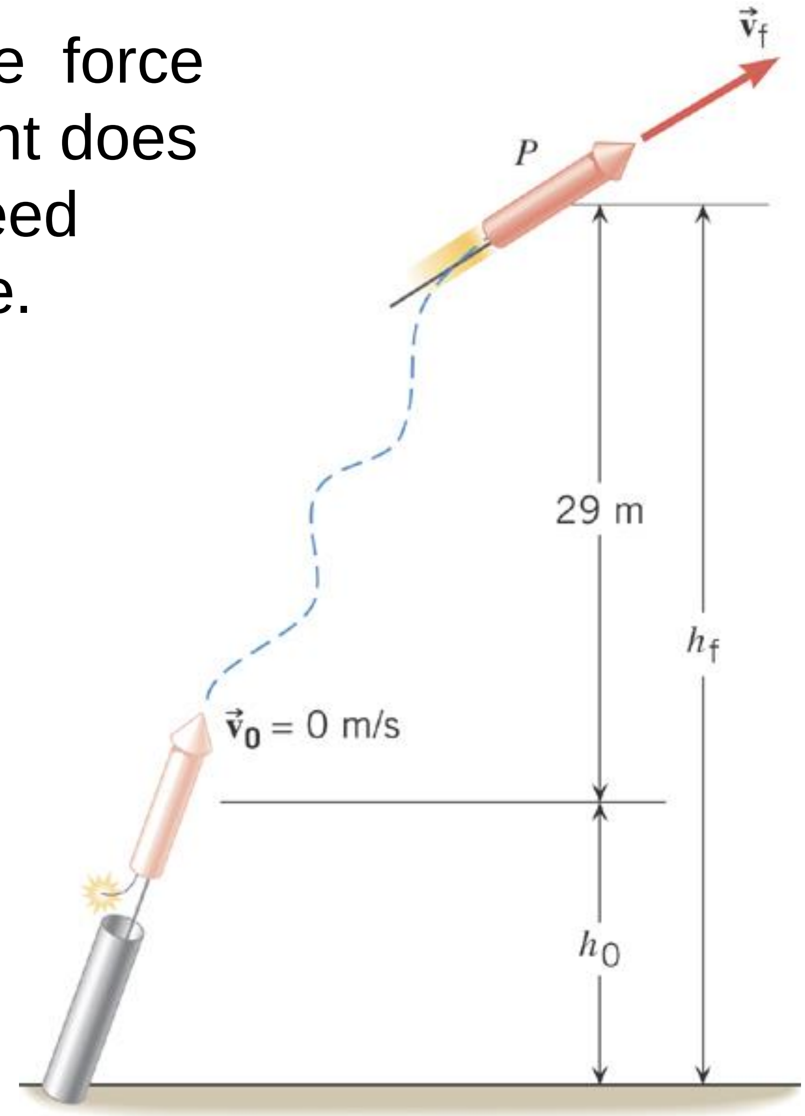
$$W_{nc} = \mathrm{E}_{\mathrm{f}} - \mathrm{E}_{\mathrm{o}}$$

$$W_{nc} = \left(mgh_f + \tfrac{1}{2}mv_f^2\right) - \left(mgh_o + \tfrac{1}{2}mv_o^2\right)$$



Example 11 Fireworks

Assuming that the nonconservative force generated by the burning propellant does 425 J of work, what is the final speed of the rocket. Ignore air resistance.



$$W_{nc} = \left(mgh_f + \tfrac{1}{2}mv_f^2\right) - \left(mgh_o + \tfrac{1}{2}mv_o^2\right)$$

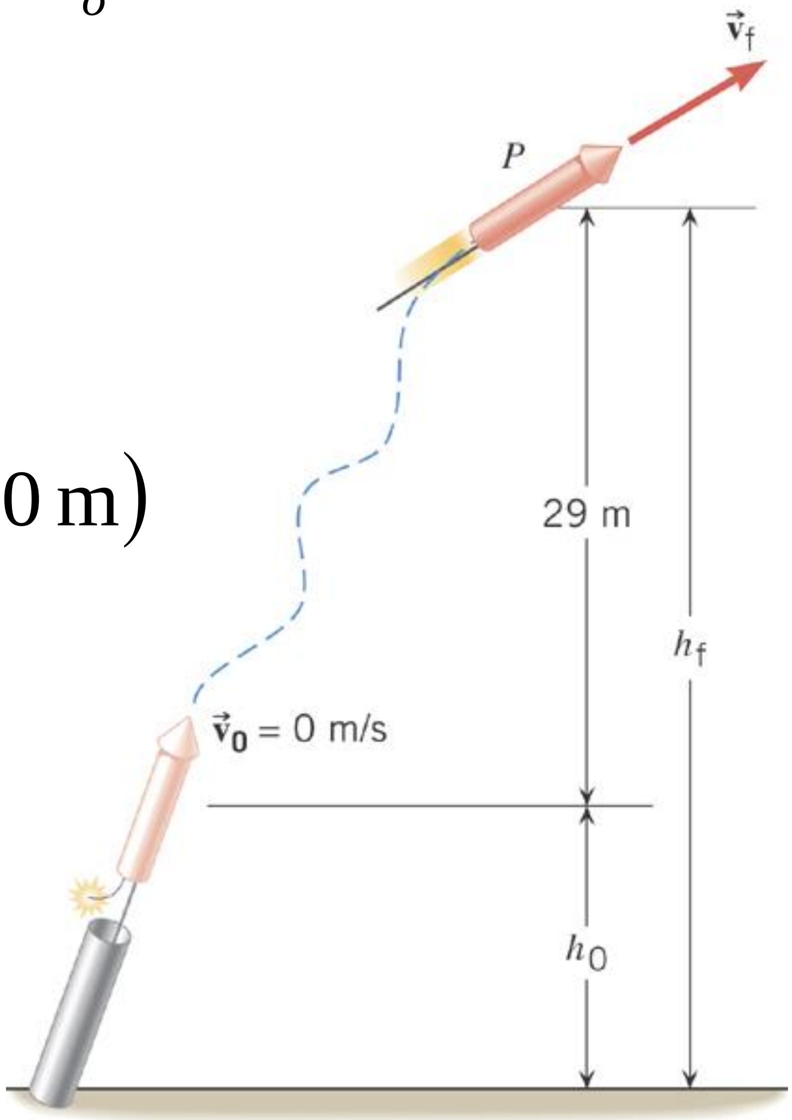
6.6 Nonconservative Forces and the Work-Energy Theorem

$$W_{nc} = mgh_f - mgh_o + \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_o^2$$

$$W_{nc} = mg(h_f - h_o) + \tfrac{1}{2}mv_f^2$$

$$425\,\text{J} = (0.20\,\text{kg})(9.80\,\text{m/s}^2)(29.0\,\text{m}) + \tfrac{1}{2}(0.20\,\text{kg})v_f^2$$

$$v_f = 61\,\text{m/s}$$



DEFINITION OF AVERAGE POWER

Average power is the rate at which work is done, and it is obtained by dividing the work by the time required to perform the work.

$$\overline{P} = \frac{\text{Work}}{\text{Time}} = \frac{W}{t}$$

$$\text{joule/s} = \text{watt (W)}$$

6.7 Power

$$\overline{P} = \frac{\text{Change in energy}}{\text{Time}}$$

$$1\,\text{horsepower} = 550\,\text{foot}\cdot\text{pounds/second} = 745.7\,\text{watts}$$

$$\overline{P} = F\,\overline{v}$$

Table 6.4 Human Metabolic Rates[a]

Activity	Rate (watts)
Running (15 km/h)	1340 W
Skiing	1050 W
Biking	530 W
Walking (5 km/h)	280 W
Sleeping	77 W

[a]For a young 70-kg male.

THE PRINCIPLE OF CONSERVATION OF ENERGY

Energy can neither be created not destroyed, but can only be converted from one form to another.

6.9 Work Done by a Variable Force

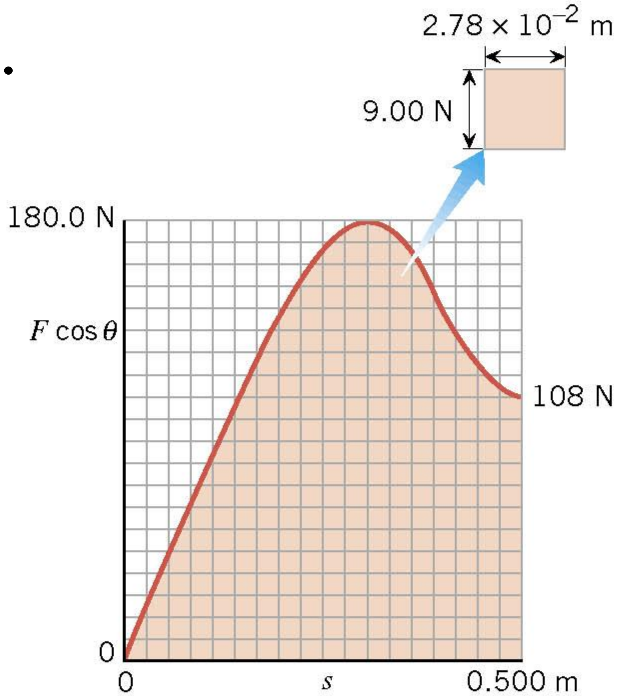
Constant Force

$$W = (F\cos\theta)s$$

Variable Force

$$W \approx (F\cos\theta)_1 \Delta s_1 + (F\cos\theta)_2 \Delta s_2 + \cdots$$

(a)



(b)