Chapter 8

Potential Energy and Conservation of Energy

8.1 Potential Energy

Technically, potential energy is energy that can be associated with the configuration (arrangement) of a system of objects that exert forces on one another.

Some forms of potential energy:

1. Gravitational Potential Energy,
2. Elastic Potential Energy

8.2 Work and Potential Energy

The change ΔU in potential energy (gravitational, elastic, etc) is defined as being equal to the negative of the work done on the object by the force (gravitational, elastic, etc)

$$\Delta U = -W.$$

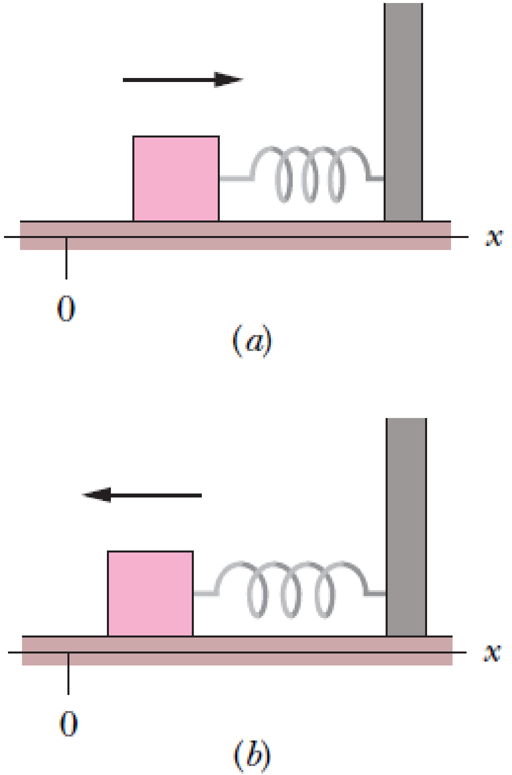


Fig. 8-3 A block, attached to a spring and initially at rest at $x = 0$, is set in motion toward the right. (*a*) As the block moves rightward (as indicated by the arrow), the spring force does negative work on it. (*b*) Then, as the block moves back toward $x = 0$, the spring force does positive work on it.

8.2 Conservative and non-conservative forces

Suppose:

1. A system consists of two or more objects.

2. A force acts between a particle-like object in the system and the rest of the system.

3. When the system configuration changes, the force does work (call it W_1) on the object, transferring energy between the kinetic energy of the object, K, and some other type of energy of the system.

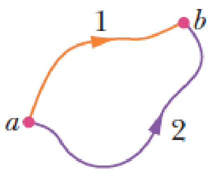
4. When the configuration change is reversed, the force reverses the energy transfer, doing work W_2 in the process.

In a situation in which $W_1=-W_2$ *is always true, the other type of energy is* a potential energy and the force is said to be a conservative force.

A force that is not conservative is called a non-conservative force. The kinetic frictional force and drag force are non-conservative.

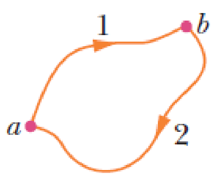
8.3 Path Independence of Conservative Forces

The net work done by a conservative force on a particle moving around any closed path is zero.



(a)

The force is conservative. Any choice of path between the points gives the same amount of work.



(b)

And a round trip gives a total work of zero.

$$W_{ab,1} = W_{ab,2},$$

If the work done from a to b along path 1 as $W_{ab,1}$ and the work done from b back to a along path 2 as $W_{ba,2}$. If the force is conservative, then the net work done during the round trip must be zero

$$W_{ab,1} + W_{ba,2} = 0,$$

$$W_{ab,1} = -W_{ba,2}.$$

If the force is conservative,

$$W_{ab,2} = -W_{ba,2}.$$

$$\longrightarrow W_{ab,1} = W_{ab,2}$$

Sample Problem: Slippery cheese

Figure 8-5*a* shows a 2.0 kg block of slippery cheese that slides along a frictionless track from point *a* to point *b*. The cheese travels through a total distance of 2.0 m along the track, and a net vertical distance of 0.80 m. How much work is done on the cheese by the gravitational force during the slide?

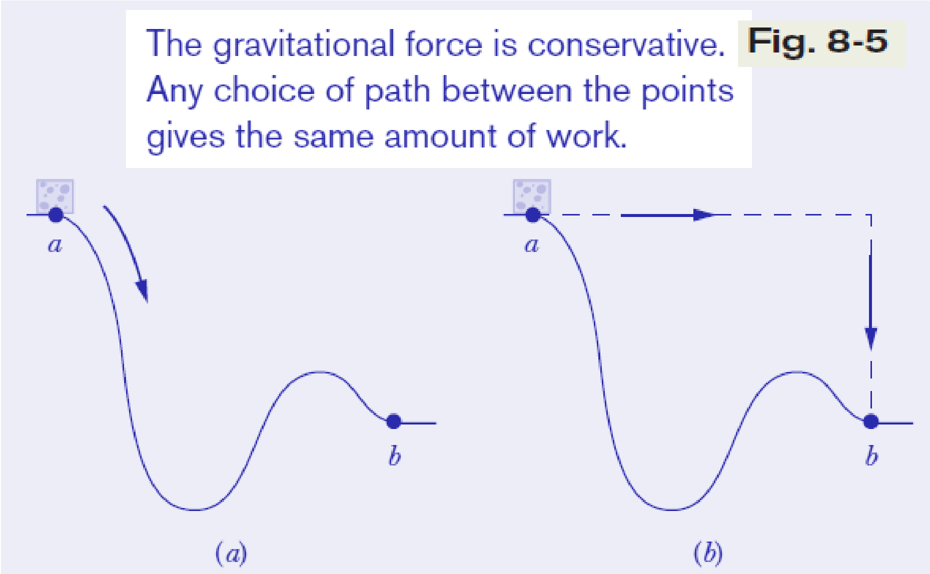


Fig. 8-5

Calculations: Let us choose the dashed path in Fig. 8-5b; it consists of two straight segments. Along the horizontal segment, the angle ϕ is a constant 90°. Even though we do not know the displacement along that horizontal segment, the work W_h done there is

$$W_h = mgd \cos 90^\circ = 0.$$

Along the vertical segment, the displacement *d is 0.80 m* and, with and both downward, the angle is a constant = 0°. Thus, for the work W_v *done along the* vertical part of the dashed path,

$$W_v = mgd \cos 0^\circ = (2.0\ \text{kg})(9.8\ \text{m/s}^2)(0.80\ \text{m})(1) = 15.7\ \text{J}.$$

The total work done on the cheese by $\boldsymbol{F}_g$ as the cheese moves from point *a to point b along the dashed path is then*

$$W = W_h + W_v = 0 + 15.7\ \text{J} \approx 16\ \text{J}. \quad \text{(Answer)}$$

This is also the work done as the cheese slides along the track from *a to b.*

For the most general case, in which the force may vary with position, we may write the work W:

$$W = \int_{x_i}^{x_f} F(x)\, dx.$$

⬇

$$\Delta U = -\int_{x_i}^{x_f} F(x)\, dx.$$

8.4 Determining Potential Energy Values:

Gravitational Potential Energy

A particle with mass *m* moving vertically along a *y* axis (the positive direction is upward). As the particle moves from point y_i to point y_f, the gravitational force does work on it. The corresponding change in the gravitational potential energy of the particle–Earth system is:

$$\Delta U = -\int_{y_i}^{y_f} (-mg)\, dy = mg \int_{y_i}^{y_f} dy = mg\Big[\, y \,\Big]_{y_i}^{y_f},$$

$$\Downarrow$$

$$\Delta U = mg(y_f - y_i) = mg\,\Delta y.$$

$$\Downarrow$$

The gravitational potential energy associated with a particle–Earth system depends only on the vertical position y (or height) of the particle relative to the reference position y =0, not on the horizontal position.

8.4 Determining Potential Energy Values: Elastic Potential Energy

In a block–spring system, the block is moving on the end of a spring of spring constant *k*. As the block moves from point x_i to point x_f , the spring force $F_x = - kx$ does work on the block. The corresponding change in the elastic potential energy of the block–spring system is

$$\Delta U = -\int_{x_i}^{x_f} (-kx)\, dx = k \int_{x_i}^{x_f} x\, dx = \frac{1}{2}k\Big[x^2\Big]_{x_i}^{x_f},$$

$$\Delta U = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2.$$

If the reference configuration is when the spring is at its relaxed length, and the block is at $x_i = 0$.

$$U - 0 = \frac{1}{2}kx^2 - 0,$$

⬇

$$U(x) = \frac{1}{2}kx^2$$

Sample Problem: Gravitational potential energy

A 2.0 kg sloth hangs 5.0 m above the ground (Fig. 8-6).

(a) What is the gravitational potential energy U of the sloth–Earth system if we take the reference point $y = 0$ to be (1) at the ground, (2) at a balcony floor that is 3.0 m above the ground, (3) at the limb, and (4) 1.0 m above the limb? Take the gravitational potential energy to be zero at $y = 0$.

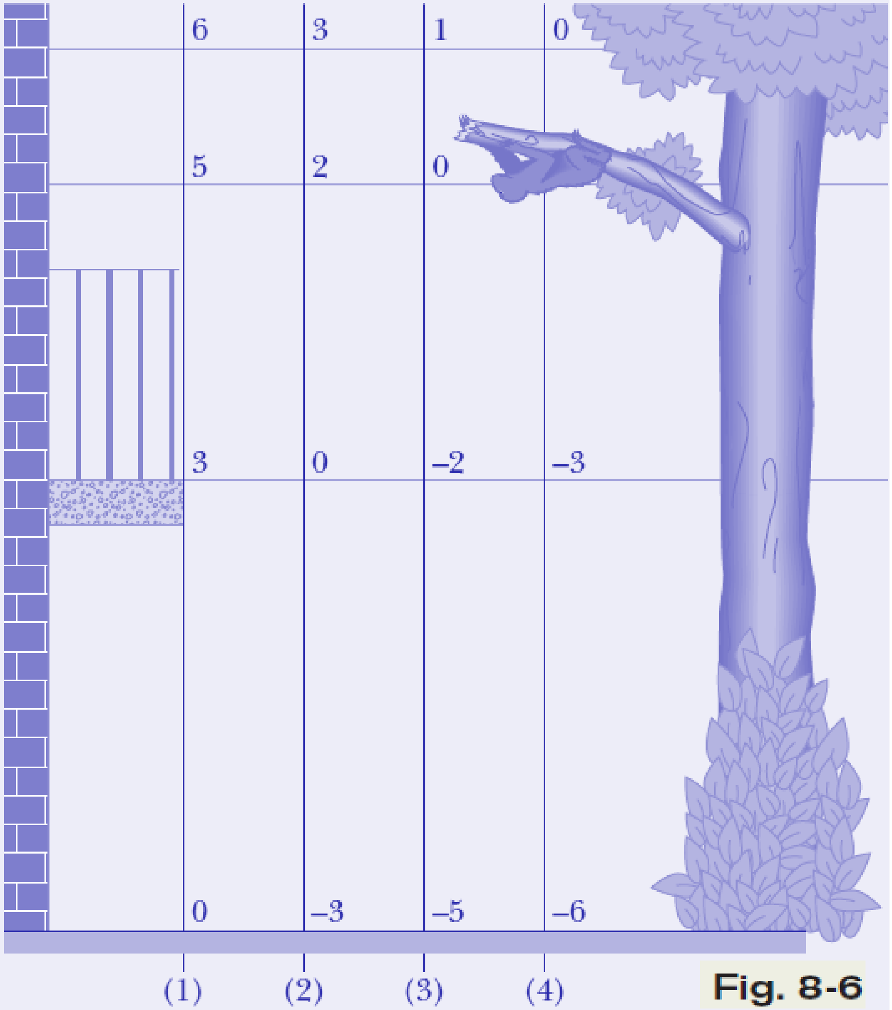


Fig. 8-6

Calculations: For choice (1) the sloth is at $y = 5.0$ m, and

$$U = mgy = (2.0 \text{ kg})(9.8 \text{ m/s}^2)(5.0 \text{ m})$$
$$= 98 \text{ J.} \qquad \text{(Answer)}$$

For the other choices, the values of U are

(2) $U = mgy = mg(2.0 \text{ m}) = 39 \text{ J},$

(3) $U = mgy = mg(0) = 0 \text{ J},$

(4) $U = mgy = mg(-1.0 \text{ m})$
$= -19.6 \text{ J} \approx -20 \text{ J.}$ (Answer)

(b) The sloth drops to the ground. For each choice of reference point, what is the change ΔU in the potential energy of the sloth–Earth system due to the fall?

Calculation: For all four situations, we have the same $\Delta y = -5.0$ m.

$$\Delta U = mg\,\Delta y = (2.0 \text{ kg})(9.8 \text{ m/s}^2)(-5.0 \text{ m})$$
$$= -98 \text{ J.} \qquad \text{(Answer)}$$

8.5 Conservation of Mechanical Energy

Principle of conservation of energy:

In an isolated system where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy E_{mec} of the system, cannot change.

The mechanical energy E_{mec} of a system is the sum of its potential energy U and the kinetic energy K of the objects within it: $E_{mec} = K + U$ (mechanical energy).

With $\Delta K = -\Delta U.$ and $\Delta U = -W.$

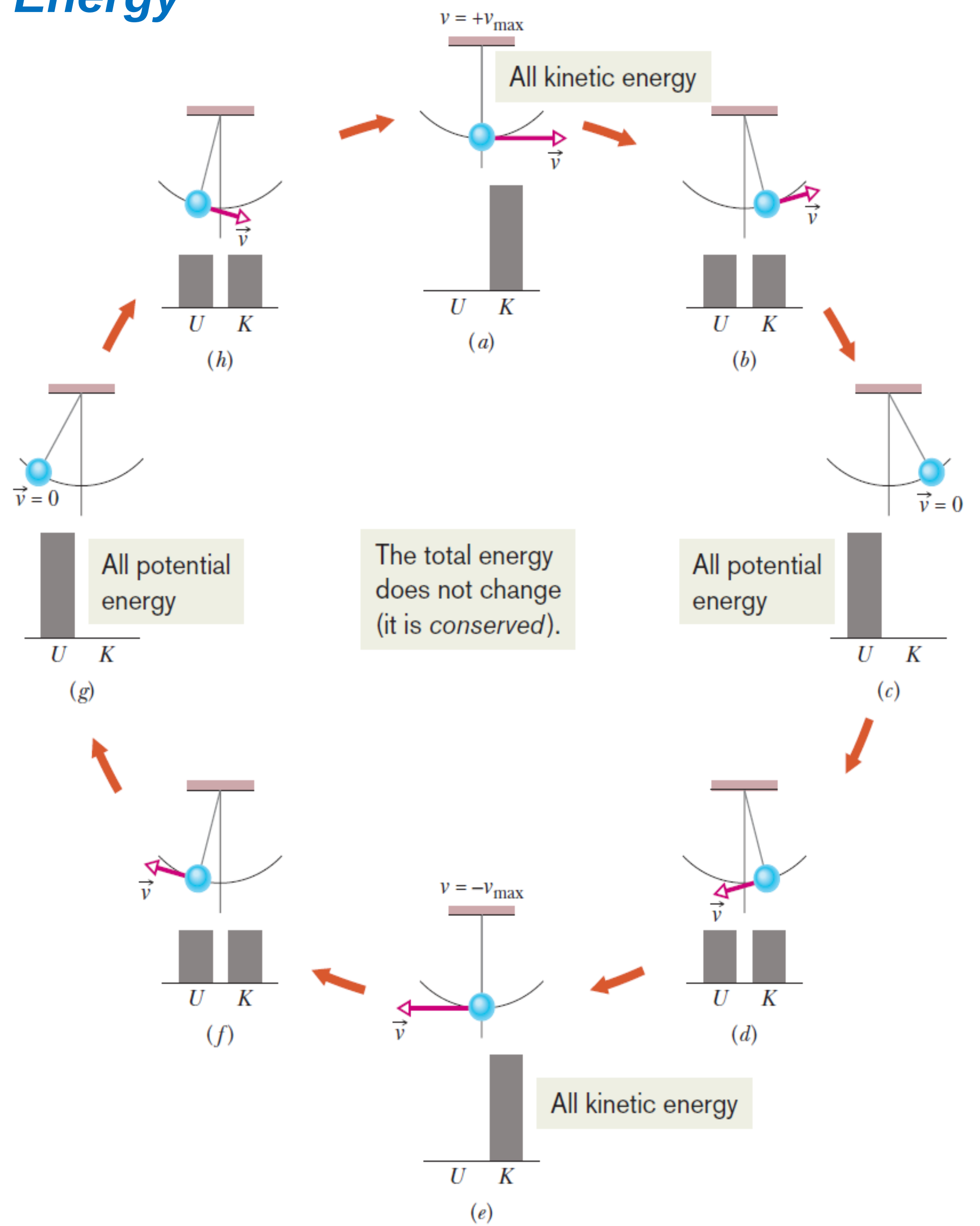
We have: $\Delta K = W$ ➡ $\begin{pmatrix}\text{the sum of } K \text{ and } U \text{ for} \\ \text{any state of a system}\end{pmatrix} = \begin{pmatrix}\text{the sum of } K \text{ and } U \text{ for} \\ \text{any other state of the system}\end{pmatrix}$

$$\Delta E_{mec} = \Delta K + \Delta U = 0.$$

8.5 Conservation of Mechanical Energy

A pendulum swings back and forth. During one full cycle the values of the potential and kinetic energies of the pendulum– Earth system vary as the bob rises and falls, but the mechanical energy E_{mec} of the system remains constant. The energy E_{mec} can be described as continuously shifting between the kinetic and potential forms. In stages (a) and (e), all the energy is kinetic energy. In stages (c) and (g), all the energy is potential energy. In stages (b), (d), (f), and (h), half the energy is kinetic energy and half is potential energy.

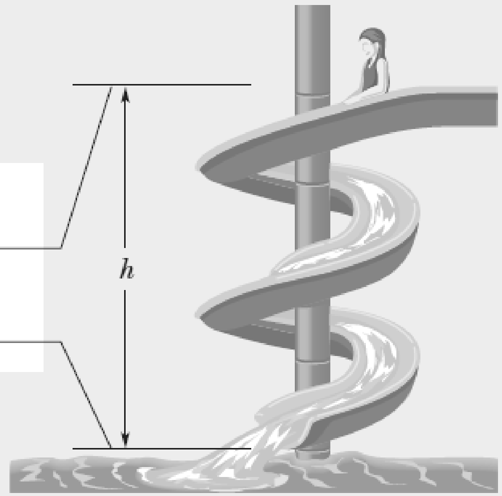
If the swinging involved a frictional force then E_{mec} would not be conserved, and eventually the pendulum would stop.



Sample Problem: Water slide

In Fig. 8-8, a child of mass m is released from rest at the top of a water slide, at height $h = 8.5$ m above the bottom of the slide. Assuming that the slide is frictionless because of the water on it, find the child's speed at the bottom of the slide.

The total mechanical energy at the top is equal to the total at the bottom.



KEY IDEAS

(1) We cannot find her speed at the bottom by using her acceleration along the slide as we might have in earlier chapters because we do not know the slope (angle) of the slide. However, because that speed is related to her kinetic energy, perhaps we can use the principle of conservation of mechanical energy to get the speed. Then we would not need to know the slope. (2) Mechanical energy is conserved in a system *if* the system is isolated and *if* only conservative forces cause energy transfers within it. Let's check.

Forces: Two forces act on the child. The *gravitational force,* a conservative force, does work on her. The *normal force* on her from the slide does no work because its direction at any point during the descent is always perpendicular to the direction in which the child moves.

Calculations: Let the mechanical energy be $E_{mec,t}$ when the child is at the top of the slide and $E_{mec,b}$ when she is at the bottom. Then the conservation principle tells us

$$E_{mec,b} = E_{mec,t}. \tag{8-19}$$

To show both kinds of mechanical energy, we have

$$K_b + U_b = K_t + U_t, \tag{8-20}$$

or

$$\tfrac{1}{2}mv_b^2 + mgy_b = \tfrac{1}{2}mv_t^2 + mgy_t.$$

Dividing by m and rearranging yield

$$v_b^2 = v_t^2 + 2g(y_t - y_b).$$

Putting $v_t = 0$ and $y_t - y_b = h$ leads to

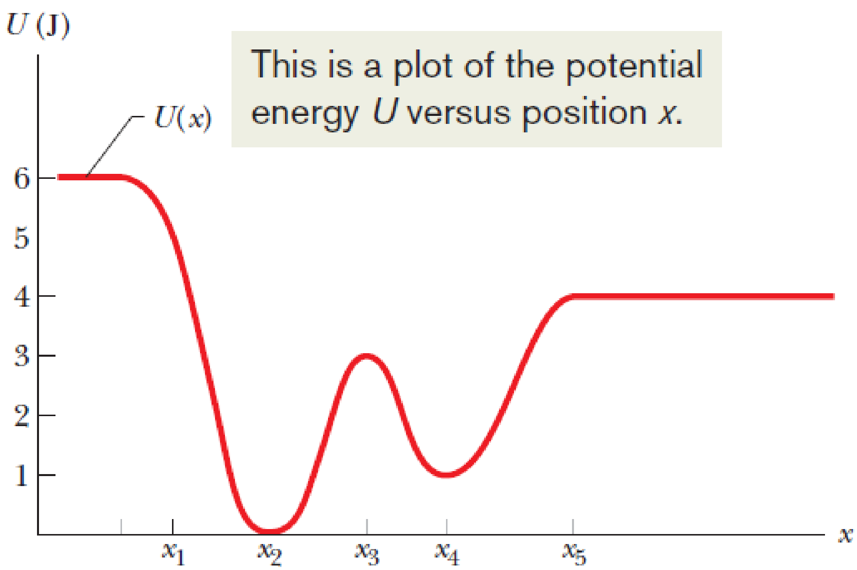
$$v_b = \sqrt{2gh} = \sqrt{(2)(9.8\ \text{m/s}^2)(8.5\ \text{m})}$$
$$= 13\ \text{m/s}. \quad \text{(Answer)}$$

This is the same speed that the child would reach if she fell 8.5 m vertically. On an actual slide, some frictional forces would act and the child would not be moving quite so fast.

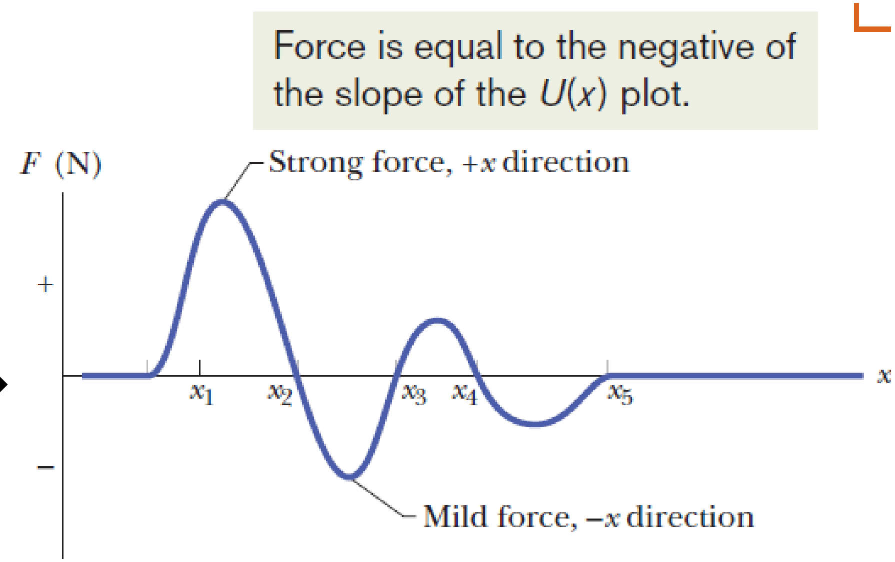
8.6: Reading a Potential Energy Curve

$$\Delta U(x) = -W = -F(x)\,\Delta x.$$

$$F(x) = -\frac{dU(x)}{dx} \qquad \text{(one-dimensional motion),}$$

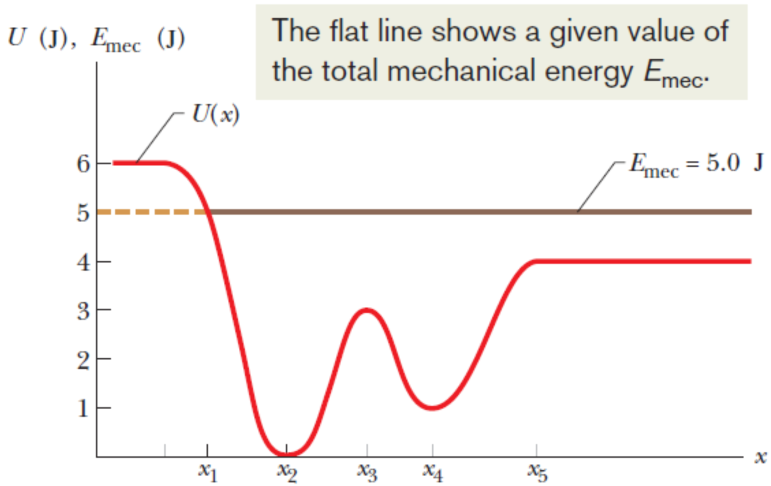


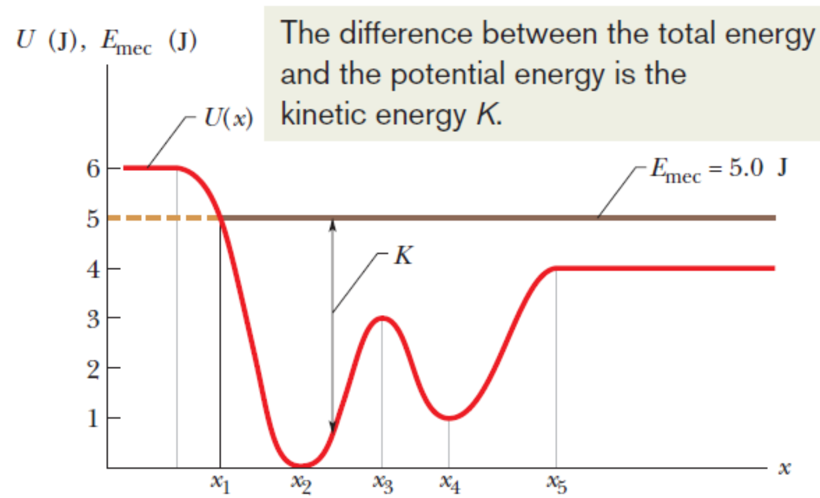
A plot of *U(x)*, the potential energy function of a system containing a particle confined to move along an *x* axis. There is no friction, so mechanical energy is conserved.

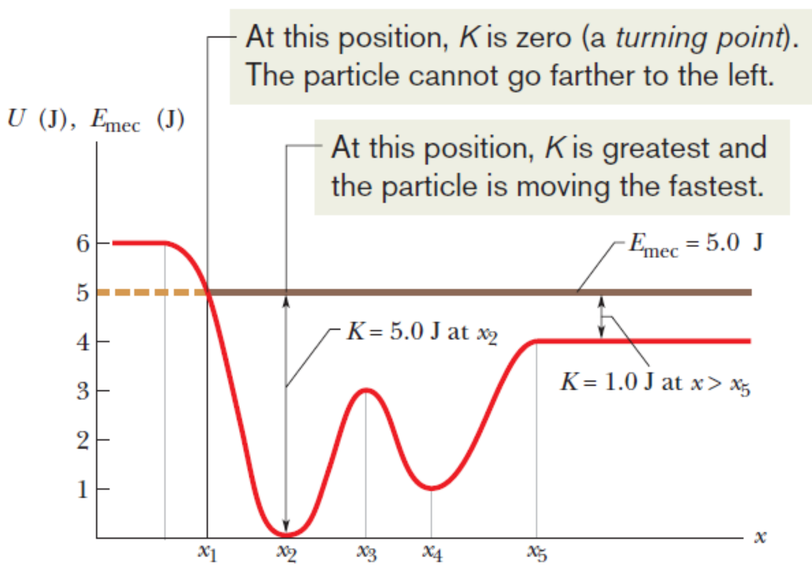


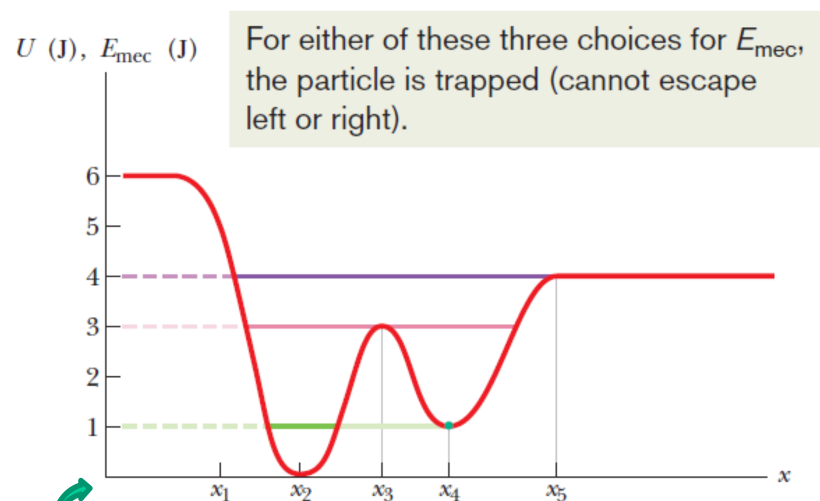
A plot of the force *F(x)* acting on the particle, derived from the potential energy plot by taking its slope at various points.

8.6: Reading a Potential Energy Curve



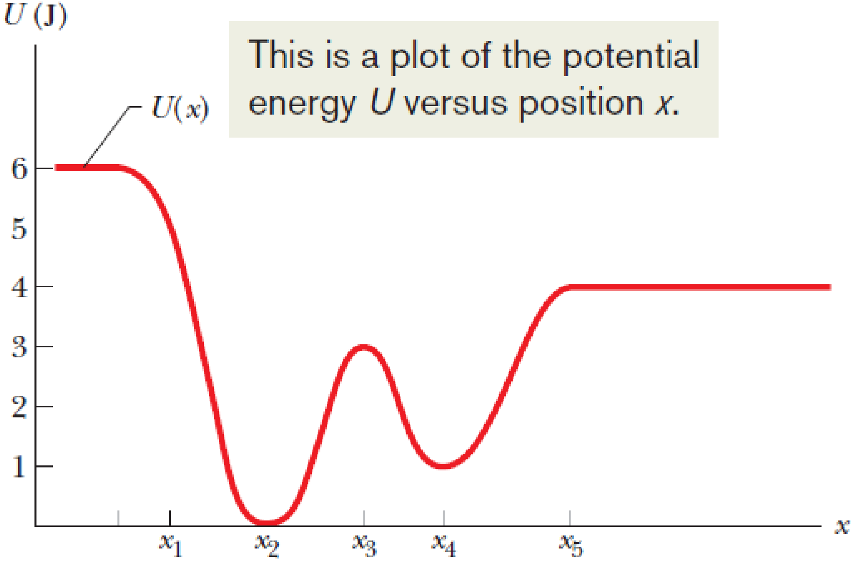






The *U(x) plot with three possible values of* E_{mec} *shown.*

8.6: Reading a Potential Energy Curve, Equilibrium Points



•If we place the object at x_4, it is stuck there. It cannot move left or right on its own because to do so would require a negative kinetic energy. If we push it slightly left or right, a restoring force appears that moves it back to x_4. A particle at such a position is said to be in **stable equilibrium.**

•At any point to the right of x_5, the system's mechanical energy is equal to its potential energy, and so it must be stationary. A particle at such a position is said to be in **neutral equilibrium.**

•x_3 is a point at which K =0. If the particle is located exactly there, the force on it is also zero, and the particle remains stationary. However, if it is displaced even slightly in either direction, a nonzero force pushes it farther in the same direction, and the particle continues to move. A particle at such a position is said to be in **unstable equilibrium.**

Sample Problem: Reading a potential energy graph

A 2.00 kg particle moves along an x axis in one-dimensional motion while a conservative force along that axis acts on it. The potential energy $U(x)$ associated with the force is plotted in Fig. 8-10a. That is, if the particle were placed at any position between $x = 0$ and $x = 7.00$ m, it would have the plotted value of U. At $x = 6.5$ m, the particle has velocity $\vec{v}_0 = (-4.00 \text{ m/s})\hat{\text{i}}$.

(a) From Fig. 8-10a, determine the particle's speed at $x_1 = 4.5$ m.

Calculations: At $x = 6.5$ m, the particle has kinetic energy

$$K_0 = \tfrac{1}{2}mv_0^2 = \tfrac{1}{2}(2.00 \text{ kg})(4.00 \text{ m/s})^2 = 16.0 \text{ J}.$$

Because the potential energy there is $U = 0$, the mechanical energy is

$$E_{\text{mec}} = K_0 + U_0 = 16.0 \text{ J} + 0 = 16.0 \text{ J}.$$

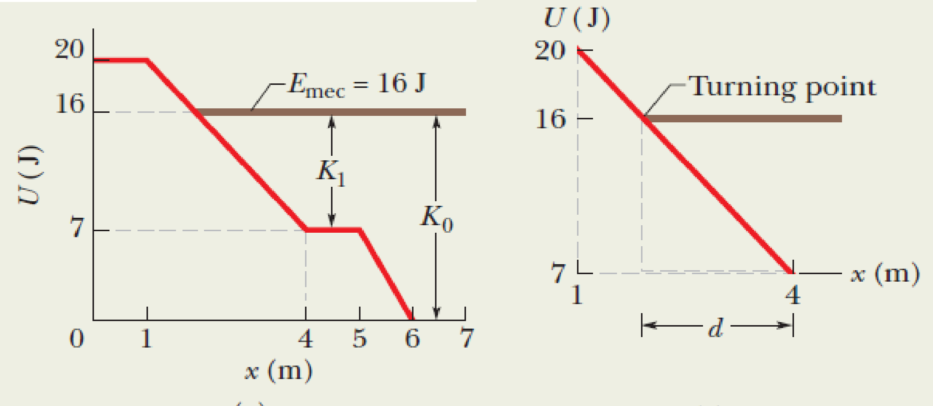
This value for E_{mec} is plotted as a horizontal line in Fig. 8-10a. From that figure we see that at $x = 4.5$ m, the potential energy is $U_1 = 7.0$ J. The kinetic energy K_1 is the difference between E_{mec} and U_1:

$$K_1 = E_{\text{mec}} - U_1 = 16.0 \text{ J} - 7.0 \text{ J} = 9.0 \text{ J}.$$

Because $K_1 = \tfrac{1}{2}mv_1^2$, we find

$$v_1 = 3.0 \text{ m/s}. \qquad \text{(Answer)}$$

(b) Where is the particle's turning point located?



(a) (b)

Calculations: Because K is the difference between E_{mec} and U, we want the point in Fig. 8-10a where the plot of U rises to meet the horizontal line of E_{mec}, as shown in Fig. 8-10b. Because the plot of U is a straight line in Fig. 8-10b, we can draw nested right triangles as shown and then write

$$\frac{16 - 7.0}{d} = \frac{20 - 7.0}{4.0 - 1.0},$$

which gives us $d = 2.08$ m. Thus, the turning point is at

$$x = 4.0 \text{ m} - d = 1.9 \text{ m}. \qquad \text{(Answer)}$$

(c) Evaluate the force acting on the particle when it is in the region $1.9 \text{ m} < x < 4.0 \text{ m}$.

Calculations: For the graph of Fig. 8-10b, we see that for the range $1.0 \text{ m} < x < 4.0 \text{ m}$ the force is

$$F = -\frac{20 \text{ J} - 7.0 \text{ J}}{1.0 \text{ m} - 4.0 \text{ m}} = 4.3 \text{ N}. \qquad \text{(Answer)}$$

Thus, the force has magnitude 4.3 N and is in the positive direction of the x axis. This result is consistent with the fact that the initially leftward-moving particle is stopped by the force and then sent rightward.

8.7: Work Done on a System by an External Force

Work is energy transferred to or from a system by means of an external force acting on that system.

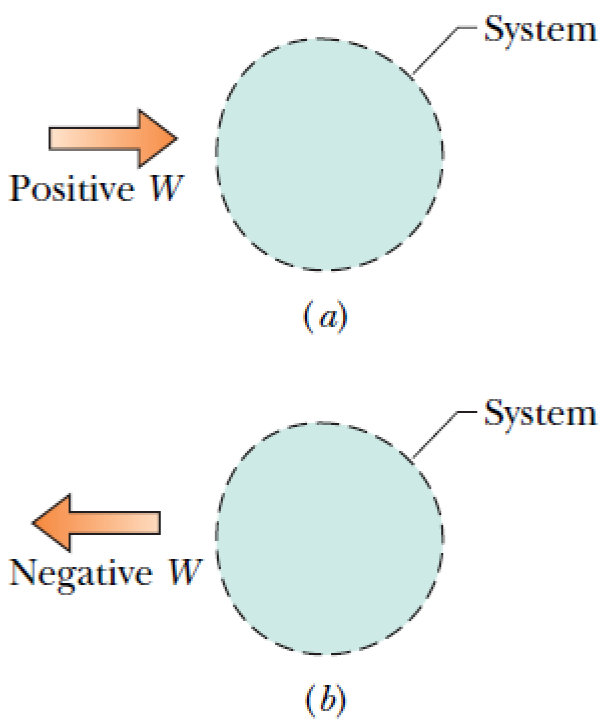


Fig. 8-11 (*a*) Positive work *W* done on an arbitrary system means a transfer of energy to the system. (*b*) Negative work *W* means a transfer of energy from the system.

8.7: Work done on a System by an External Force

FRICTION NOT INVOLVED

$$W = \Delta K + \Delta U,$$

$$W = \Delta E_{mec}$$

Your lifting force transfers energy to kinetic energy and potential energy.

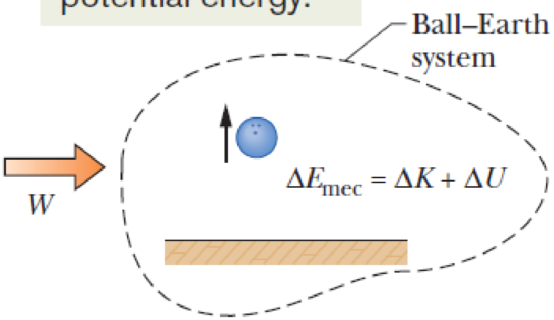


Fig. 8-12 Positive work W is done on a system of a bowling ball and Earth, causing a change ΔE_{mec} in the mechanical energy of the system, a change ΔK in the ball's kinetic energy, and a change ΔU in the system's gravitational potential energy.

FRICTION INVOLVED

$$F - f_k = ma.$$

$$Fd = \Delta K + f_k d.$$

$$Fd = \Delta E_{mec} + f_k d.$$

The applied force supplies energy. The frictional force transfers some of it to thermal energy.

So, the work done by the applied force goes into kinetic energy and also thermal energy.

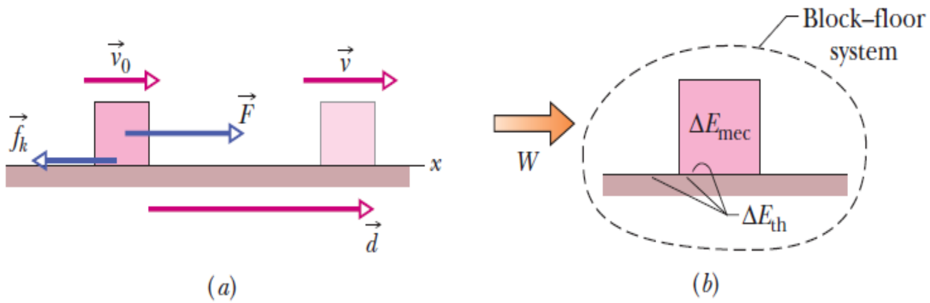


Fig. 8-13 (*a*) A block is pulled across a floor by force $\vec{F}$ while a kinetic frictional force $\vec{f}_k$ opposes the motion. The block has velocity $\vec{v}_0$ at the start of a displacement $\vec{d}$ and velocity $\vec{v}$ at the end of the displacement. (*b*) Positive work W is done on the block–floor system by force $\vec{F}$, resulting in a change ΔE_{mec} in the block's mechanical energy and a change ΔE_{th} in the thermal energy of the block and floor.

Sample Problem: Change in thermal energy

A food shipper pushes a wood crate of cabbage heads (total mass $m = 14$ kg) across a concrete floor with a constant horizontal force $\vec{F}$ of magnitude 40 N. In a straight-line displacement of magnitude $d = 0.50$ m, the speed of the crate decreases from $v_0 = 0.60$ m/s to $v = 0.20$ m/s.

(a) How much work is done by force $\vec{F}$, and on what system does it do the work?

KEY IDEA

Because the applied force $\vec{F}$ is constant, we can calculate the work it does by using Eq. 7-7 ($W = Fd\cos\phi$).

Calculation: Substituting given data, including the fact that force $\vec{F}$ and displacement $\vec{d}$ are in the same direction, we find

$$W = Fd\cos\phi = (40\ \text{N})(0.50\ \text{m})\cos 0^\circ$$
$$= 20\ \text{J}. \qquad \text{(Answer)}$$

Reasoning: We can determine the system on which the work is done to see which energies change. Because the crate's speed changes, there is certainly a change ΔK in the crate's kinetic energy. Is there friction between the floor and the crate, and thus a change in thermal energy? Note that $\vec{F}$ and the crate's velocity have the same direction. Thus, if there is no friction, then $\vec{F}$ should be accelerating the crate to a *greater* speed. However, the crate is *slowing*, so there must be friction and a change ΔE_{th} in thermal energy of the crate and the floor. Therefore, the system on which the work is done is the crate–floor system, because both energy changes occur in that system.

(b) What is the increase ΔE_{th} in the thermal energy of the crate and floor?

KEY IDEA

$$W = \Delta E_{mec} + \Delta E_{th}.$$

Calculations: We know the value of W from (a). The change ΔE_{mec} in the crate's mechanical energy is just the change in its kinetic energy because no potential energy changes occur, so we have

$$\Delta E_{mec} = \Delta K = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2.$$

Substituting this into Eq. 8-34 and solving for ΔE_{th}, we find

$$\Delta E_{th} = W - (\tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2) = W - \tfrac{1}{2}m(v^2 - v_0^2)$$
$$= 20\ \text{J} - \tfrac{1}{2}(14\ \text{kg})[(0.20\ \text{m/s})^2 - (0.60\ \text{m/s})^2]$$
$$= 22.2\ \text{J} \approx 22\ \text{J}. \qquad \text{(Answer)}$$

Law of Conservation of Energy

The total energy E of a system can change only by amounts of energy that are transferred to or from the system.

$$W = \Delta E = \Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}}$$

where E_{mec} *is any change in the mechanical energy of the system*, E_{th} *is any* change in the thermal energy of the system, and E_{int} *is any change in any* other type of internal energy of the system.

The total energy E of an *isolated system* cannot change.

$$\Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}} = 0 \qquad \text{(isolated system)}$$

8.8: Conservation of Energy
External Forces and Internal Energy Transfers

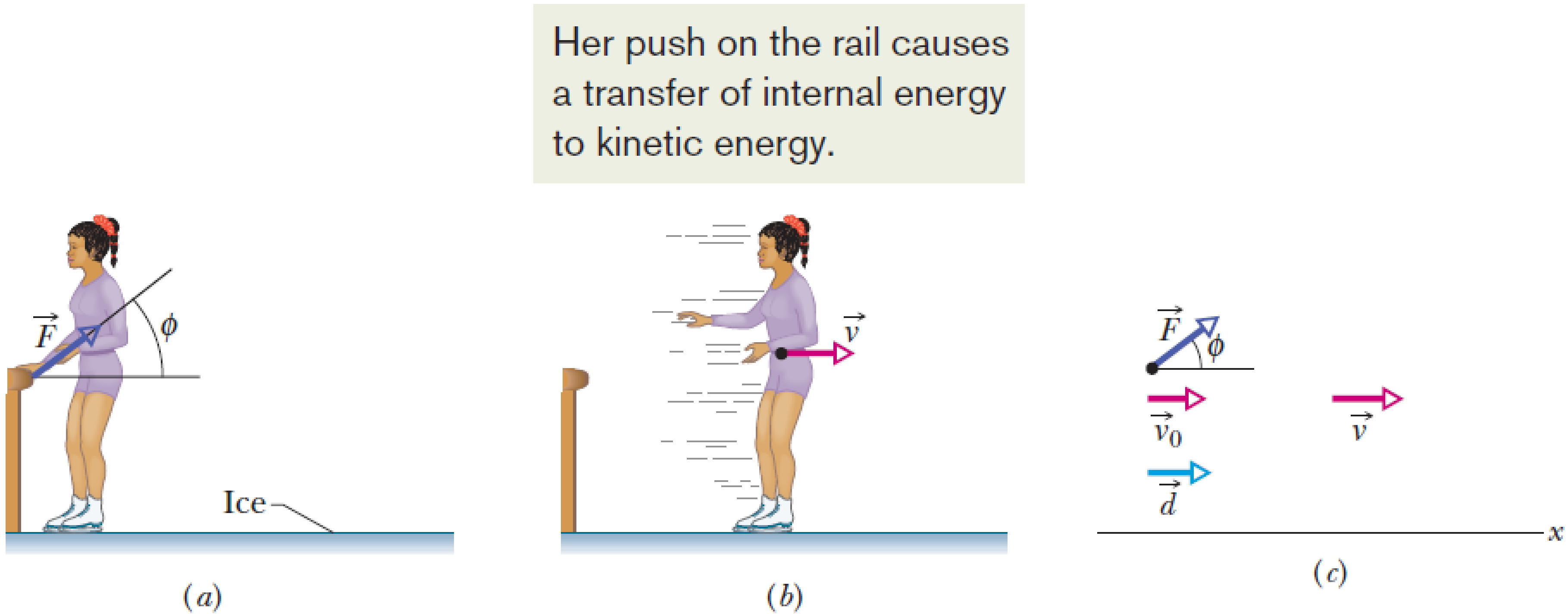


Fig. 8-15 (*a*) As a skater pushes herself away from a railing, the force on her from the railing is $\vec{F}$. (*b*) After the skater leaves the railing, she has velocity $\vec{v}$. (*c*) External force $\vec{F}$ acts on the skater, at angle ϕ with a horizontal x axis. When the skater goes through displacement $\vec{d}$, her velocity is changed from $\vec{v}_0$ (= 0) to $\vec{v}$ by the horizontal component of $\vec{F}$.

An external force can change the kinetic energy or potential energy of an object without doing work on the object—that is, without transferring energy to the object. Instead, the force is responsible for transfers of energy from one type to another inside the object.

8.8: Conservation of Energy: Power

In general, power P is the rate at which energy is transferred by a force from one type to another. If an amount of energy E is transferred in an amount of time t, the average power due to the force is

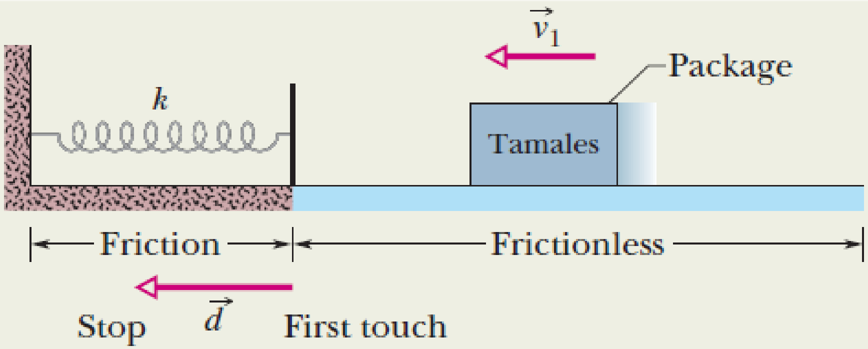
$$P_{\text{avg}} = \frac{\Delta E}{\Delta t}.$$

and the instantaneous power due to the force is

$$P = \frac{dE}{dt}.$$

Sample Problem: energy, friction, spring, and tamales

In Fig. 8-17, a 2.0 kg package of tamales slides along a floor with speed $v_1 = 4.0$ m/s. It then runs into and compresses a spring, until the package momentarily stops. Its path to the initially relaxed spring is frictionless, but as it compresses the spring, a kinetic frictional force from the floor, of magnitude 15 N, acts on the package. If $k = 10\,000$ N/m, by what distance d is the spring compressed when the package stops?



During the rubbing, kinetic energy is transferred to potential energy and thermal energy.

KEY IDEAS

Forces: The normal force on the package from the floor does no work on the package. For the same reason, the gravitational force on the package does no work. As the spring is compressed, a spring force does work on the package. The spring force also pushes against a rigid wall. There is friction between the package and the floor, and the sliding of the package across the floor increases their thermal energies.

System: The package–spring–floor–wall system includes all these forces and energy transfers in one isolated system. From conservation of energy,

$$E_{\text{mec},2} = E_{\text{mec},1} - \Delta E_{\text{th}}. \qquad (8\text{-}42)$$

Calculations: In Eq. 8-42, let subscript 1 correspond to the initial state of the sliding package and subscript 2 correspond to the state in which the package is momentarily stopped and the spring is compressed by distance d. For both states the mechanical energy of the system is the sum of the package's kinetic energy ($K = \frac{1}{2}mv^2$) and the spring's potential energy ($U = \frac{1}{2}kx^2$). For state 1, $U = 0$ (because the spring is not compressed), and the package's speed is v_1. Thus, we have

$$E_{\text{mec},1} = K_1 + U_1 = \tfrac{1}{2}mv_1^2 + 0.$$

For state 2, $K = 0$ (because the package is stopped), and the compression distance is d. Therefore, we have

$$E_{\text{mec},2} = K_2 + U_2 = 0 + \tfrac{1}{2}kd^2.$$

Finally, by Eq. 8-31, we can substitute $f_k d$ for the change ΔE_{th} in the thermal energy of the package and the floor. We can now rewrite Eq. 8-42 as

$$\tfrac{1}{2}kd^2 = \tfrac{1}{2}mv_1^2 - f_k d.$$

Rearranging and substituting known data give us

$$5000d^2 + 15d - 16 = 0.$$

Solving this quadratic equation yields

$$d = 0.055 \text{ m} = 5.5 \text{ cm}. \qquad \text{(Answer)}$$