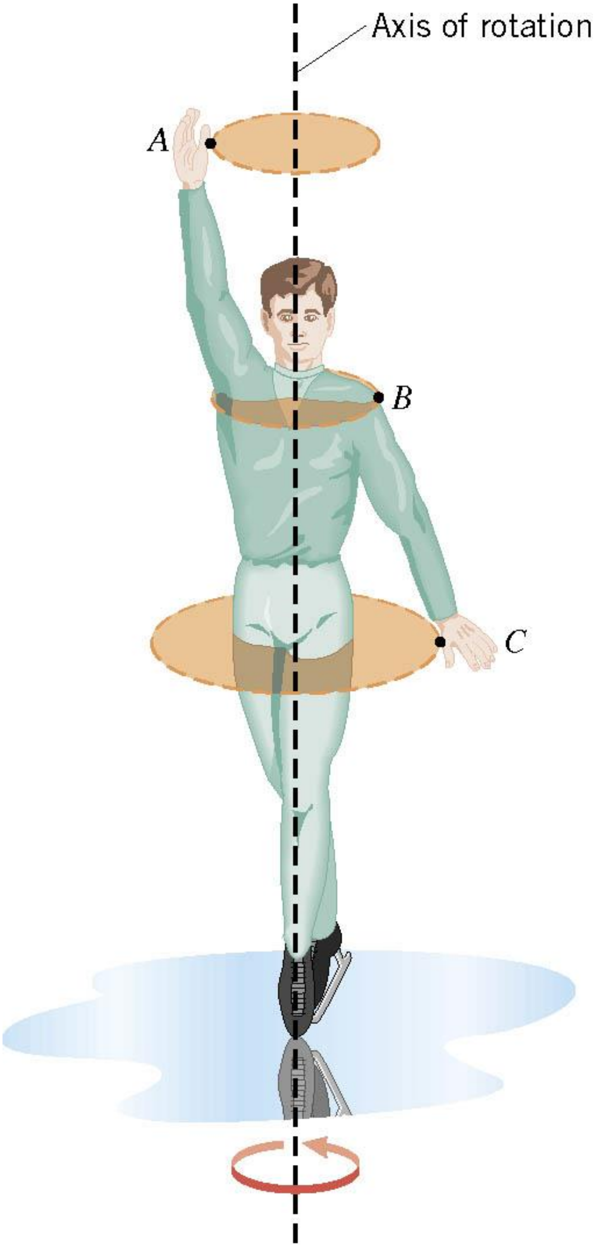
Chapter 8

Rotational Kinematics

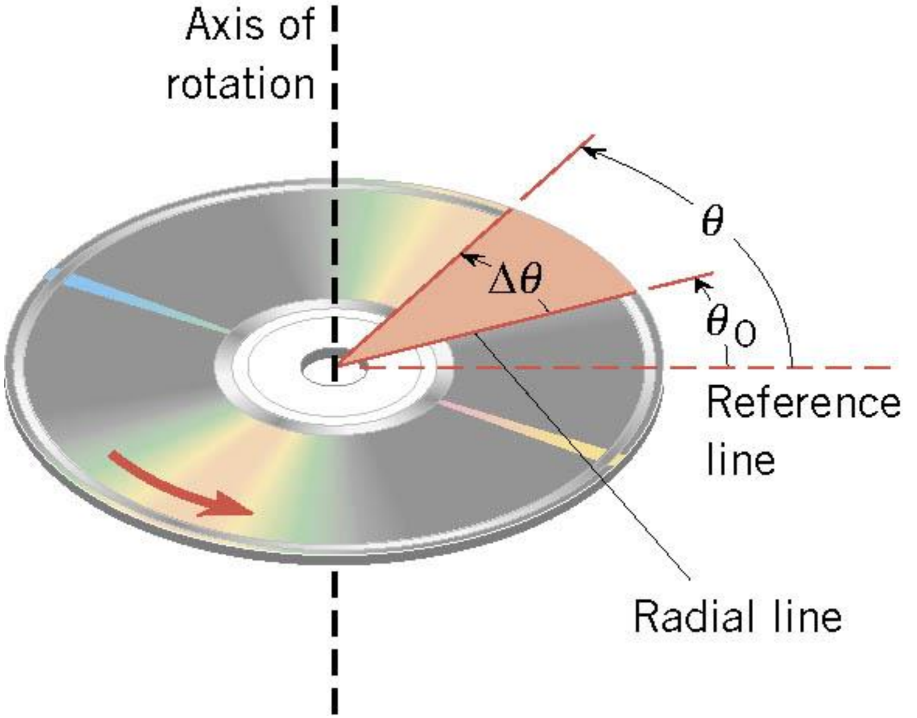
8.1 Rotational Motion and Angular Displacement

In the simplest kind of rotation, points on a rigid object move on circular paths around an ***axis of rotation.***



The angle through which the object rotates is called the ***angular displacement.***

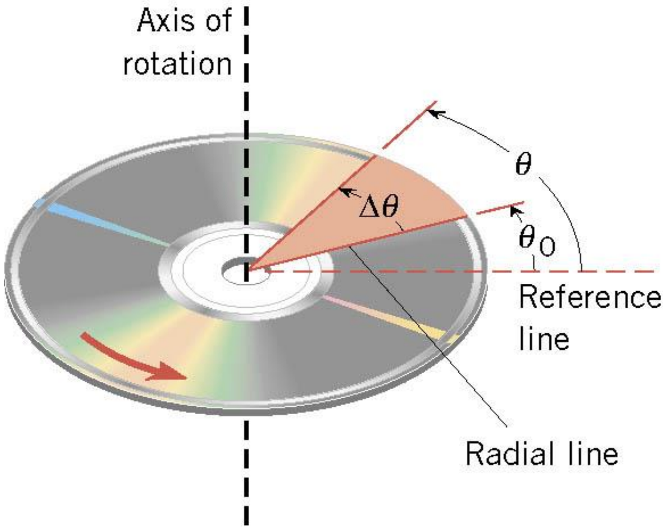
$$\Delta\theta = \theta - \theta_o$$



DEFINITION OF ANGULAR DISPLACEMENT

When a rigid body rotates about a fixed axis, the angular displacement is the angle swept out by a line passing through any point on the body and intersecting the axis of rotation perpendicularly.

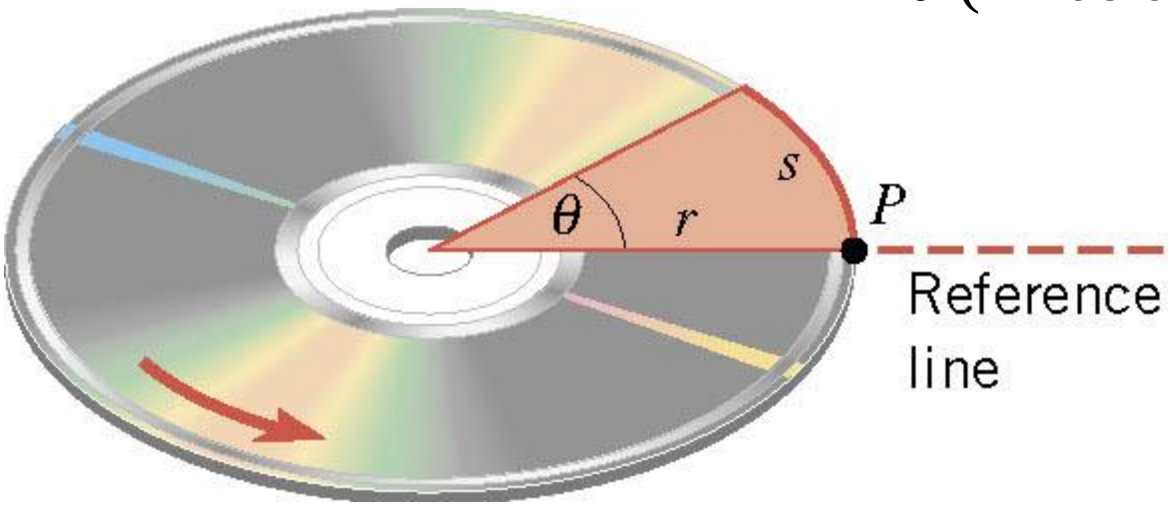
By convention, the angular displacement is positive if it is counterclockwise and negative if it is clockwise.



SI Unit of Angular Displacement: radian (rad)

8.1 Rotational Motion and Angular Displacement

$$\theta \text{ (in radians)} = \frac{\text{Arc length}}{\text{Radius}} = \frac{s}{r}$$



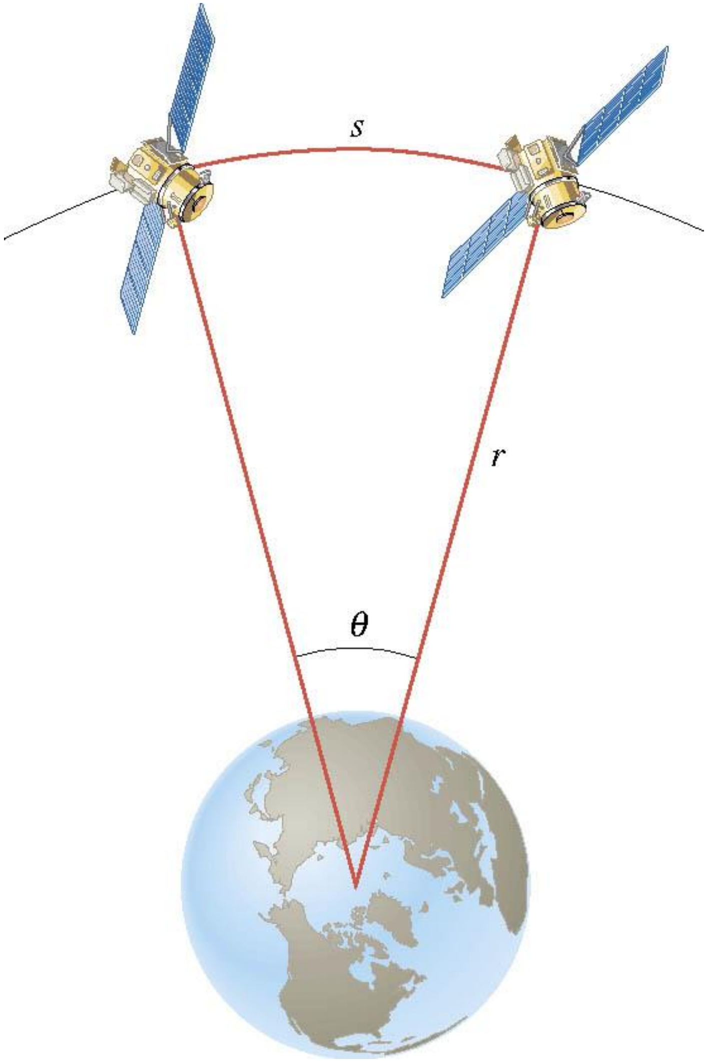
For a full revolution:

$$\theta = \frac{2\pi r}{r} = 2\pi \text{ rad} \quad \Rightarrow \quad 2\pi \text{ rad} = 360^\circ$$

Example 1 Adjacent Synchronous Satellites

Synchronous satellites are put into an orbit whose radius is 4.23×10^7 m.

If the angular separation of the two satellites is 2.00 degrees, find the arc length that separates them.



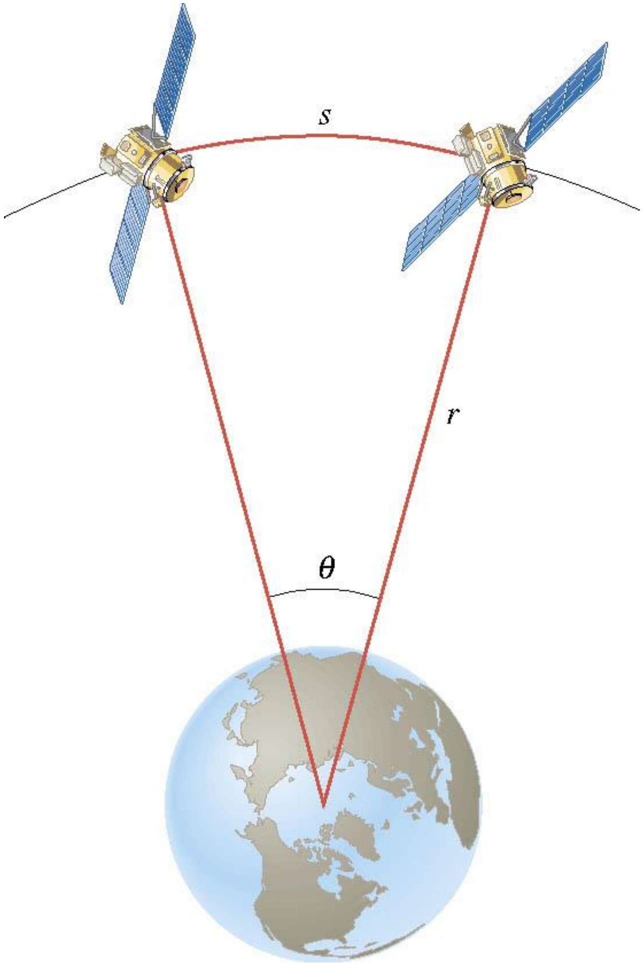
8.1 Rotational Motion and Angular Displacement

$$\theta \text{ (in radians)} = \frac{\text{Arc length}}{\text{Radius}} = \frac{s}{r}$$

$$2.00\,\text{deg}\left(\frac{2\pi\,\text{rad}}{360\,\text{deg}}\right) = 0.0349\,\text{rad}$$

$$s = r\theta = \left(4.23\times10^{7}\,\text{m}\right)\left(0.0349\,\text{rad}\right)$$

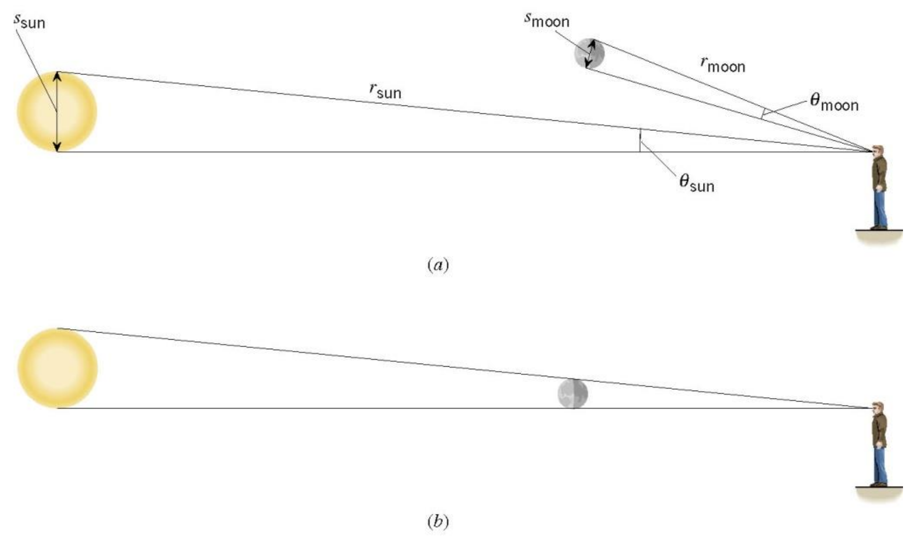
$$= 1.48\times10^{6}\,\text{m} \;\; (920\,\text{miles})$$



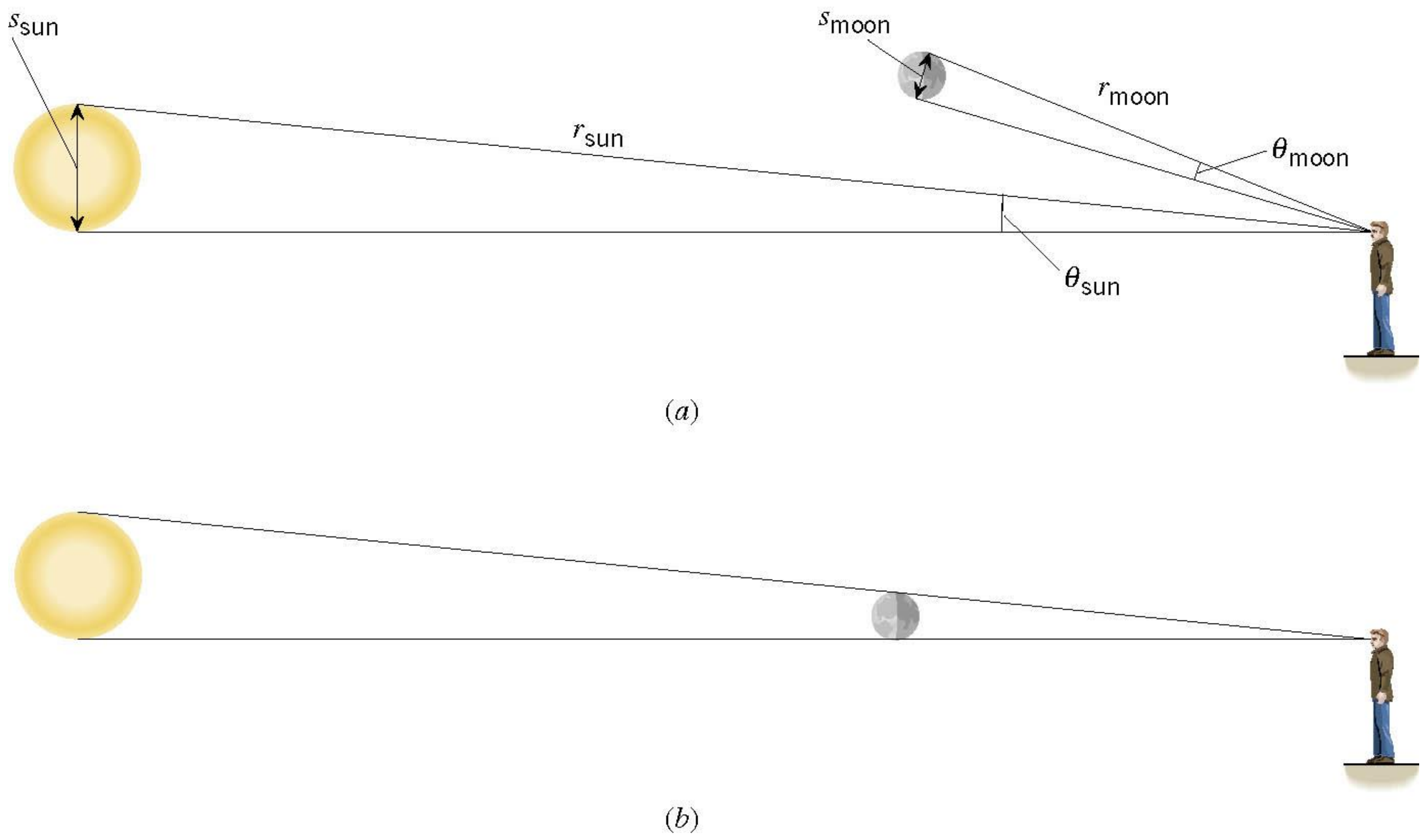
Conceptual Example 2 A Total Eclipse of the Sun

The diameter of the sun is about 400 times greater than that of the moon. By coincidence, the sun is also about 400 times farther from the earth than is the moon.

For an observer on the earth, compare the angle subtended by the moon to the angle subtended by the sun and explain why this result leads to a total solar eclipse.



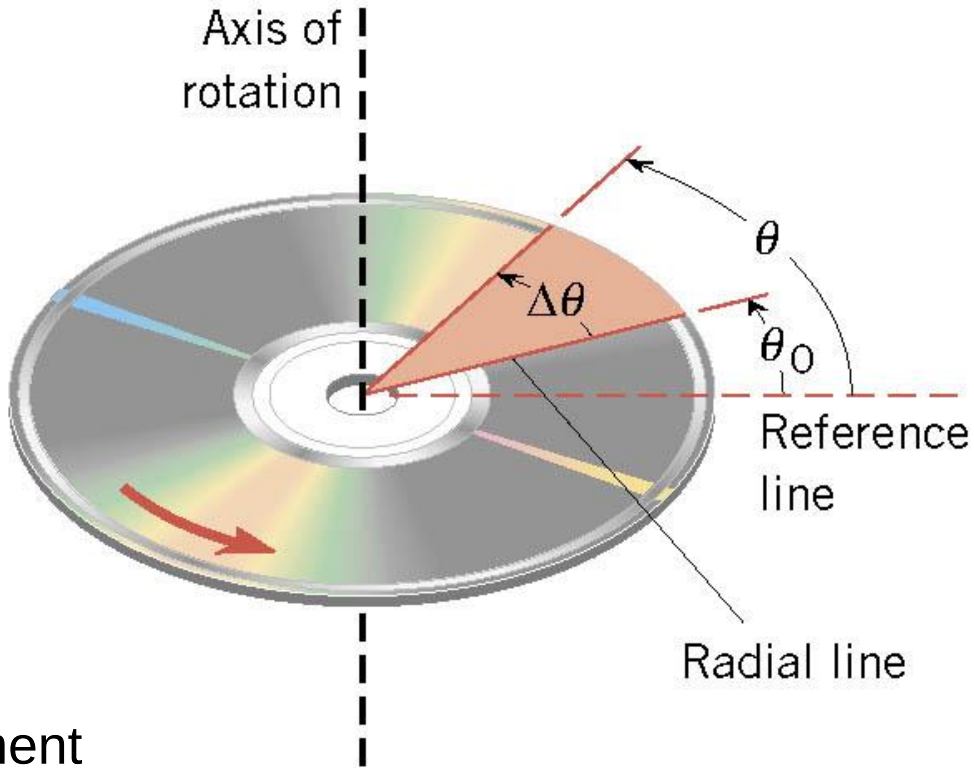
8.1 Rotational Motion and Angular Displacement



$$\theta\,(\text{in radians}) = \frac{\text{Arc length}}{\text{Radius}} = \frac{s}{r}$$

8.2 Angular Velocity and Angular Acceleration

$$\Delta\theta = \theta - \theta_o$$



How do we describe the rate at which the angular displacement is changing?

DEFINITION OF AVERAGE ANGULAR VELOCITY

$$\text{Average angular velocity} = \frac{\text{Angular displacement}}{\text{Elapsed time}}$$

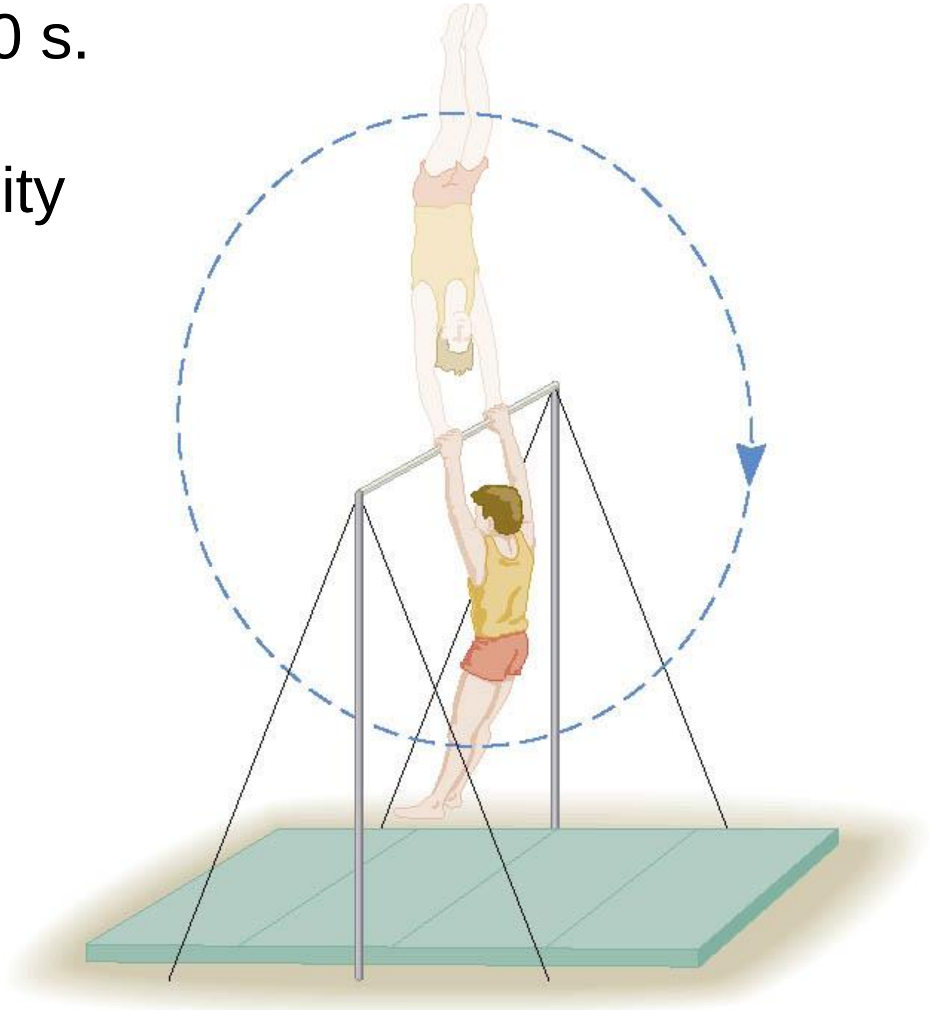
$$\overline{\omega} = \frac{\theta - \theta_o}{t - t_o} = \frac{\Delta\theta}{\Delta t}$$

SI Unit of Angular Velocity: radian per second (rad/s)

Example 3 Gymnast on a High Bar

A gymnast on a high bar swings through two revolutions in a time of 1.90 s.

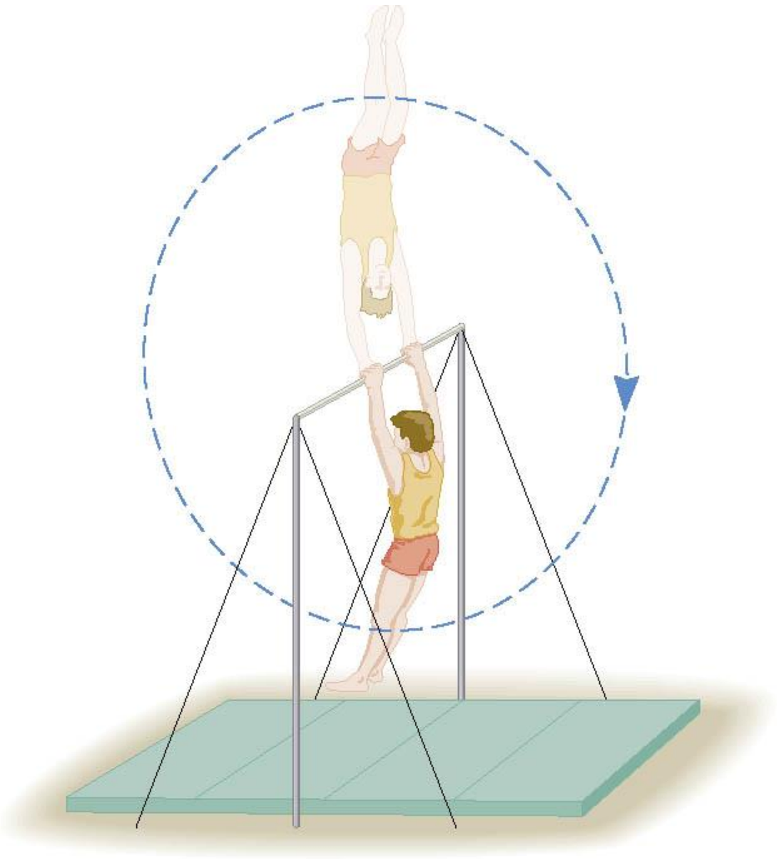
Find the average angular velocity of the gymnast.



8.2 Angular Velocity and Angular Acceleration

$$\Delta\theta = -2.00\,\text{rev}\left(\frac{2\pi\,\text{rad}}{1\,\text{rev}}\right) = -12.6\,\text{rad}$$

$$\overline{\omega} = \frac{-12.6\,\text{rad}}{1.90\,\text{s}} = -6.63\,\text{rad/s}$$



INSTANTANEOUS ANGULAR VELOCITY

$$\omega = \lim_{\Delta t \to 0} \overline{\omega} = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t}$$

Changing angular velocity means that an ***angular acceleration*** is occurring.

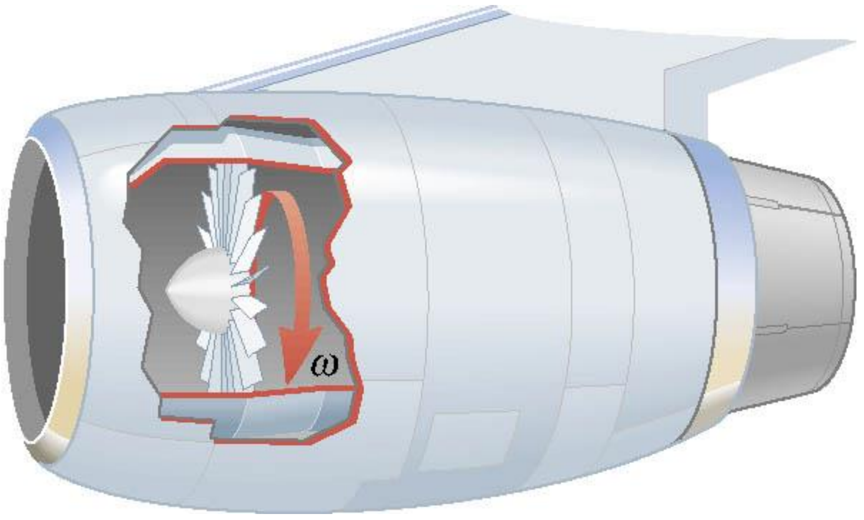
DEFINITION OF AVERAGE ANGULAR ACCELERATION

$$\text{Average angular acceleration} = \frac{\text{Change in angular velocity}}{\text{Elapsed time}}$$

$$\overline{\alpha} = \frac{\omega - \omega_o}{t - t_o} = \frac{\Delta\omega}{\Delta t}$$

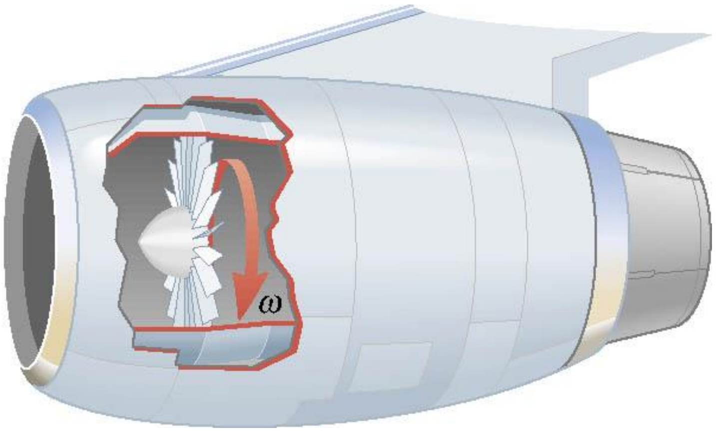
SI Unit of Angular acceleration: radian per second squared (rad/s^2)

Example 4 A Jet Revving Its Engines



As seen from the front of the engine, the fan blades are rotating with an angular speed of -110 rad/s. As the plane takes off, the angular velocity of the blades reaches -330 rad/s in a time of 14 s.

Find the angular acceleration, assuming it to be constant.



$$\bar{\alpha} = \frac{(-330\,\mathrm{rad/s}) - (-110\,\mathrm{rad/s})}{14\,\mathrm{s}} = -16\,\mathrm{rad/s^2}$$

Recall the equations of kinematics for constant acceleration.

Five kinematic variables:

1. displacement, *x*
2. acceleration (constant), *a*
3. final velocity (at time *t*), *v*
4. initial velocity, v_o
5. elapsed time, t

$$v = v_o + at$$

$$x = \tfrac{1}{2}\left(v_o + v\right)t$$

$$v^2 = v_o^2 + 2ax$$

$$x = v_o t + \tfrac{1}{2}at^2$$

8.3 The Equations of Rotational Kinematics

The equations of rotational kinematics for constant angular acceleration:

ANGULAR ACCELERATION

ANGULAR VELOCITY

$$\omega = \omega_o + \alpha t$$

$$\theta = \tfrac{1}{2}\left(\omega_o + \omega\right)t$$

TIME

ANGULAR DISPLACEMENT

$$\omega^2 = \omega_o^2 + 2\alpha\theta$$

$$\theta = \omega_o t + \tfrac{1}{2}\alpha t^2$$

8.3 The Equations of Rotational Kinematics

Table 8.2 Symbols Used in Rotational and Linear Kinematics

Rotational Motion	Quantity	Linear Motion
θ	Displacement	x
ω_0	Initial velocity	v_0
ω	Final velocity	v
α	Acceleration	a
t	Time	t

Table 8.1 The Equations of Kinematics for Rotational and Linear Motion

Rotational Motion (α = constant)		Linear Motion (a = constant)	
$\omega = \omega_0 + \alpha t$	(8.4)	$v = v_0 + at$	(2.4)
$\theta = \frac{1}{2}(\omega_0 + \omega)t$	(8.6)	$x = \frac{1}{2}(v_0 + v)t$	(2.7)
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	(8.7)	$x = v_0 t + \frac{1}{2}at^2$	(2.8)
$\omega^2 = \omega_0^2 + 2\alpha\theta$	(8.8)	$v^2 = v_0^2 + 2ax$	(2.9)

Reasoning Strategy

1. Make a drawing.

2. Decide which directions are to be called positive (+) and negative (-). (The text uses CCW to be positive.)

3. Write down the values that are given for any of the five kinematic variables.

4. Verify that the information contains values for at least three of the five kinematic variables. Select the appropriate equation.

5. When the motion is divided into segments, remember that the final angular velocity of one segment is the initial velocity for the next.

6. Keep in mind that there may be two possible answers to a kinematics problem.

Example 5 Blending with a Blender

The blades are whirling with an angular velocity of +375 rad/s when the “puree” button is pushed in.

When the “blend” button is pushed, the blades accelerate and reach a greater angular velocity after the blades have rotated through an angular displacement of +44.0 rad.

The angular acceleration has a constant value of +1740 rad/s^2.

Find the final angular velocity of the blades.

θ	α	ω	ω_o	t
+44.0 rad	+1740 rad/s^2	?	+375 rad/s	

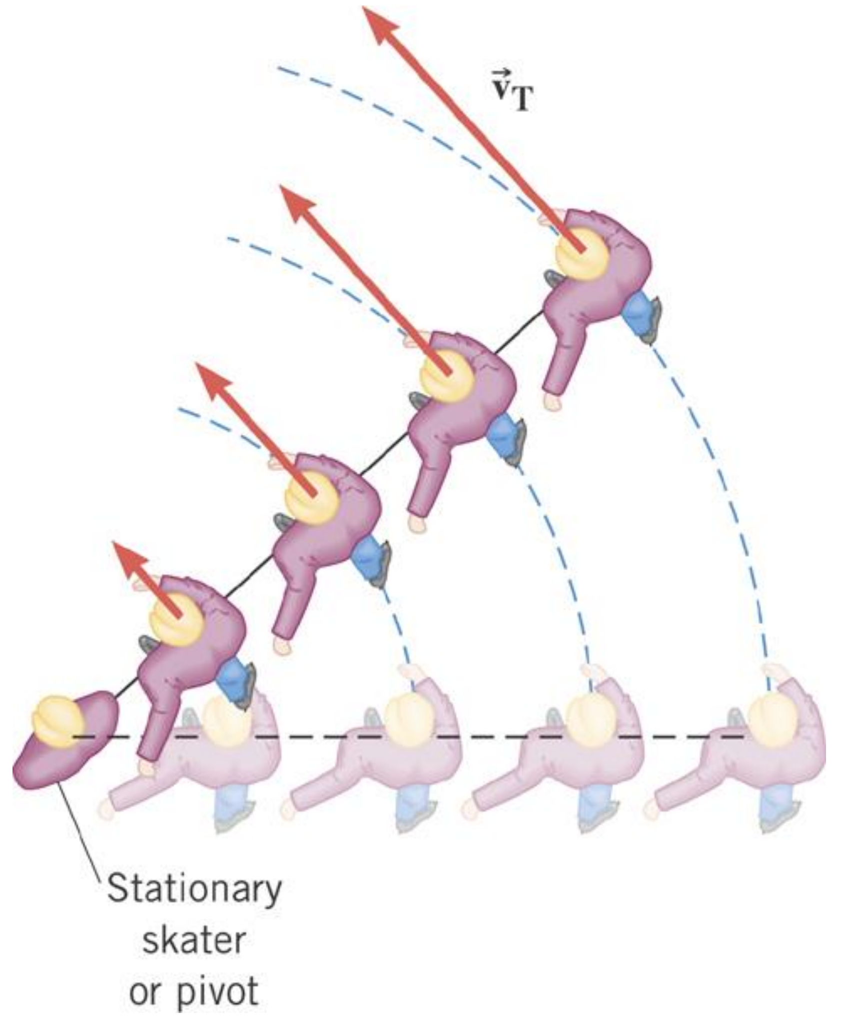
$$\omega^2 = \omega_o^2 + 2\alpha\theta$$

$$\omega = \sqrt{\omega_o^2 + 2\alpha\theta}$$

$$= \sqrt{(375\,\text{rad/s})^2 + 2(1740\,\text{rad/s}^2)(44.0\,\text{rad})} = +542\,\text{rad/s}$$

$$\vec{\mathbf{v}}_{\mathbf{T}} = \text{tangential velocity}$$

$$v_T = \text{tangential speed}$$

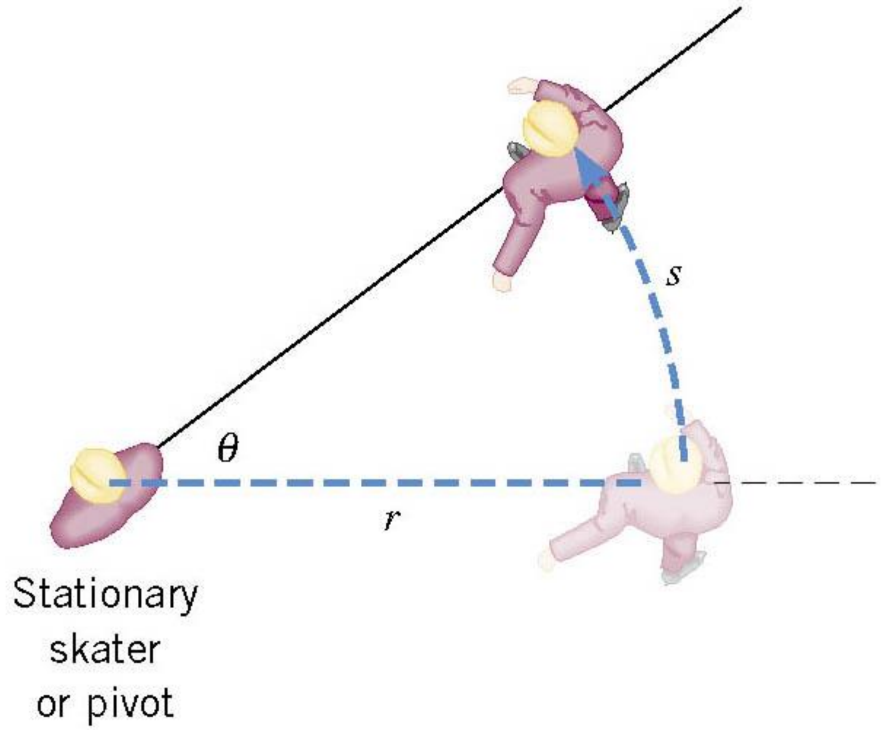


8.4 Angular Variables and Tangential Variables

$$v_T = \frac{s}{t} = \frac{r\theta}{t} = r\left(\frac{\theta}{t}\right)$$

$$\omega = \frac{\theta}{t}$$

$$v_T = r\omega \quad (\omega \,\text{in rad/s})$$

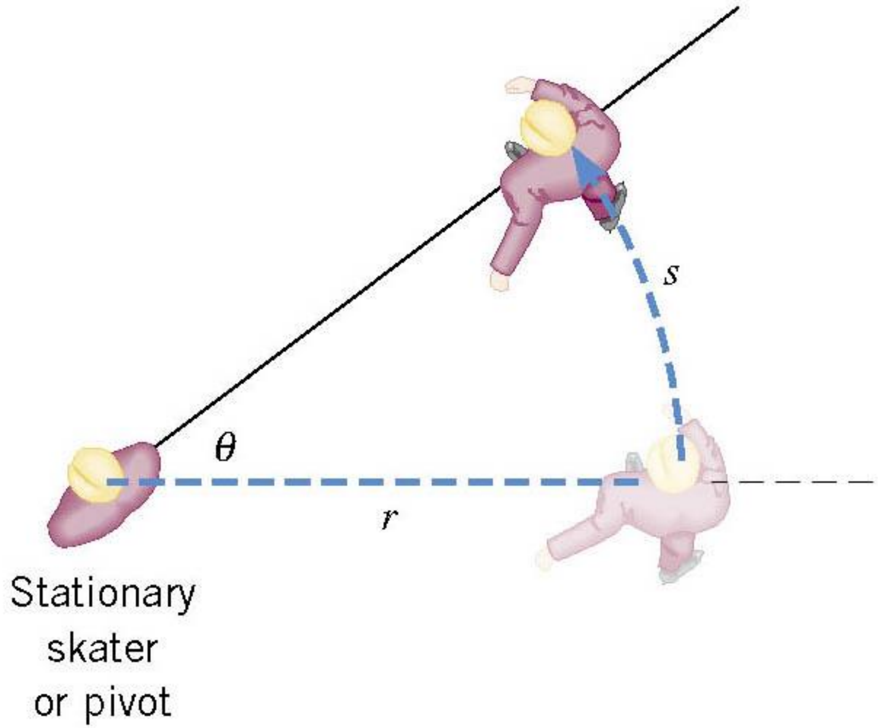


8.4 Angular Variables and Tangential Variables

$$\alpha = \frac{\omega - \omega_o}{t}$$

$$a_T = \frac{v_T - v_{To}}{t} = \frac{(r\omega) - (r\omega_o)}{t} = r\frac{\omega - \omega_o}{t}$$

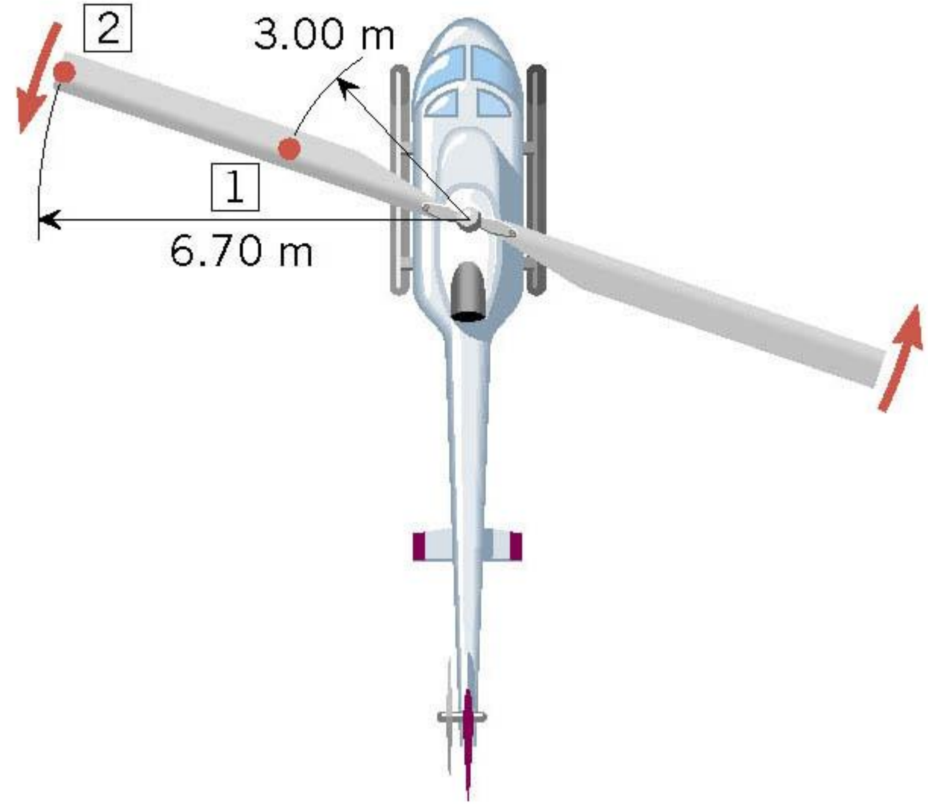
$$a_T = r\alpha \qquad (\alpha \text{ in rad/s}^2)$$



Example 6 A Helicopter Blade

A helicopter blade has an angular speed of 6.50 rev/s and an angular acceleration of 1.30 rev/s^2.

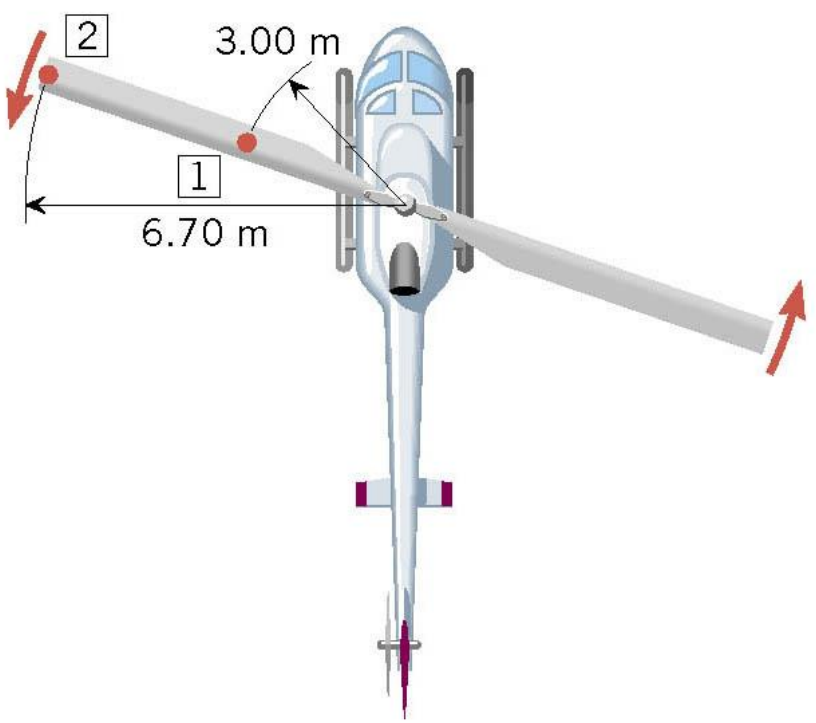
For point 1 on the blade, find the magnitude of (a) the tangential speed and (b) the tangential acceleration.



8.4 Angular Variables and Tangential Variables

$$\omega = \left(6.50\,\frac{\text{rev}}{\text{s}}\right)\left(\frac{2\pi\ \text{rad}}{1\,\text{rev}}\right) = 40.8\,\text{rad/s}$$

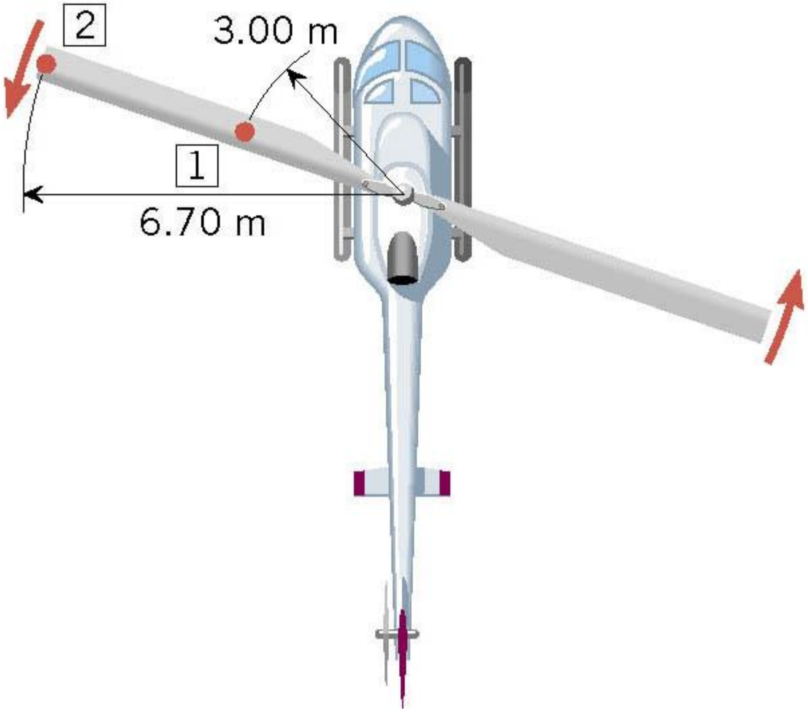
$$v_T = r\alpha = (3.00\,\text{m})(40.8\,\text{rad/s}) = 122\,\text{m/s}$$



8.4 Angular Variables and Tangential Variables

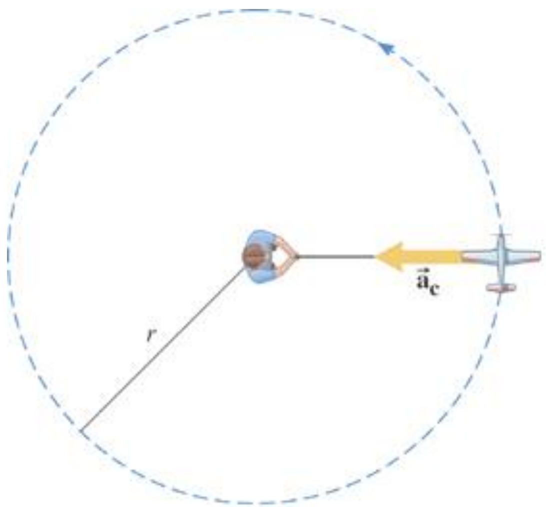
$$\alpha = \left(1.30\,\frac{\text{rev}}{\text{s}^2}\right)\left(\frac{2\pi\ \text{rad}}{1\,\text{rev}}\right) = 8.17\ \text{rad/s}^2$$

$$a_T = r\alpha = (3.00\ \text{m})\left(8.17\ \text{rad/s}^2\right) = 24.5\ \text{m/s}^2$$

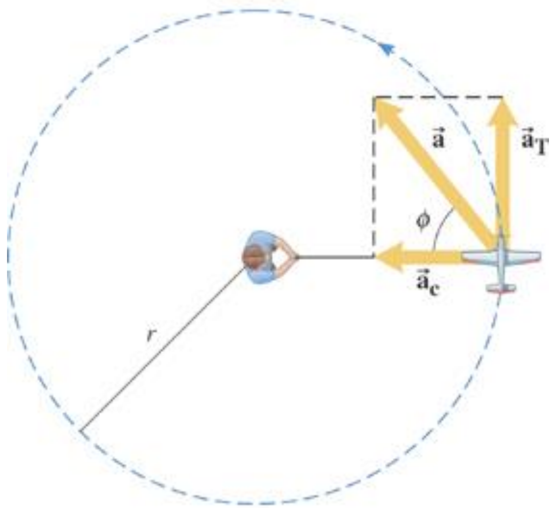


8.5 Centripetal Acceleration and Tangential Acceleration

$$a_c = \frac{v_T^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2 \quad (\omega \text{ in rad/s})$$



(a) Uniform circular motion

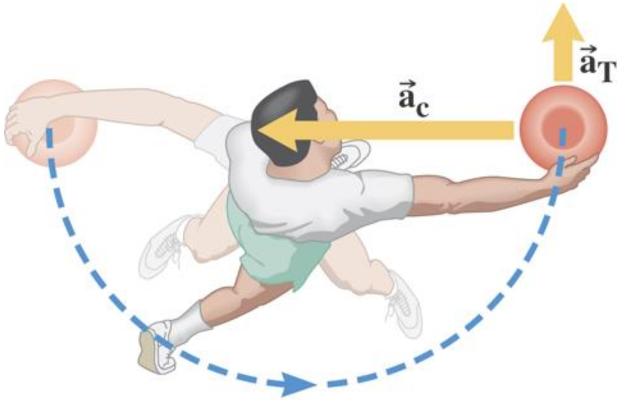


(b) Nonuniform circular motion

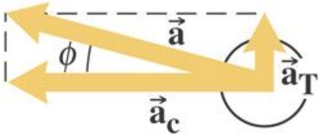
Example 7 A Discus Thrower

Starting from rest, the thrower accelerates the discus to a final angular speed of +15.0 rad/s in a time of 0.270 s before releasing it. During the acceleration, the discus moves in a circular arc of radius 0.810 m.

Find the magnitude of the total acceleration.



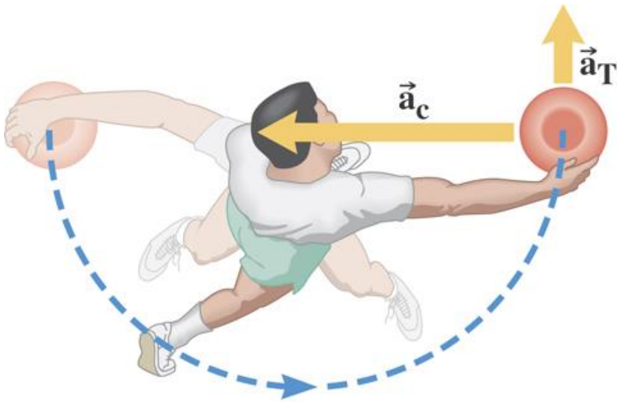
(a)



(b)

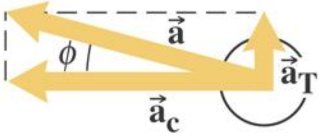
8.5 Centripetal Acceleration and Tangential Acceleration

$$a_c = r\omega^2 = (0.810\,\text{m})(15.0\,\text{rad/s})^2$$
$$= 182\,\text{m/s}^2$$



(a)

$$a_T = r\alpha = r\frac{\omega\text{-}\omega_o}{t} = (0.810\,\text{m})\left(\frac{15.0\,\text{rad/s}}{0.270\,\text{s}}\right)$$
$$= 45.0\,\text{m/s}^2$$



(b)

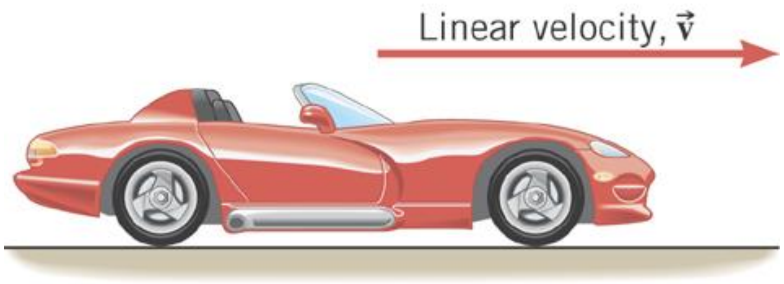
$$a = \sqrt{a_T^2 + a_c^2} = \sqrt{(182\,\text{m/s}^2) + (45.0\,\text{m/s}^2)} = 187\,\text{m/s}^2$$

8.6 Rolling Motion

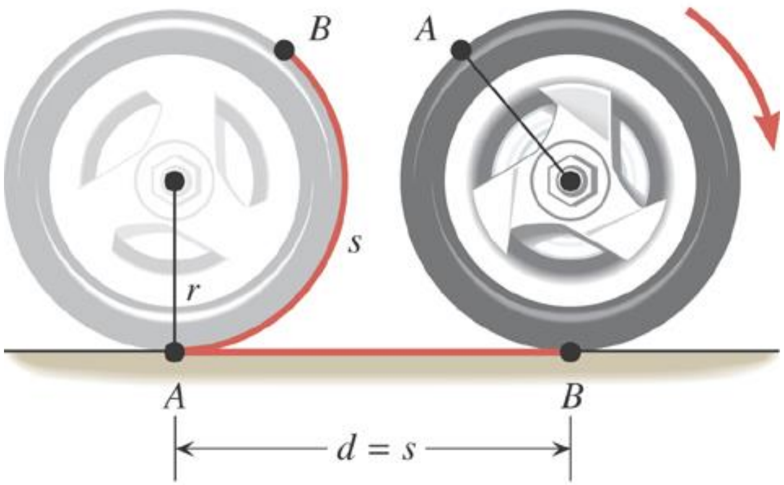
The tangential speed of a point on the outer edge of the tire is equal to the speed of the car over the ground.

$$v = r\omega$$

$$a = r\alpha$$



(a)

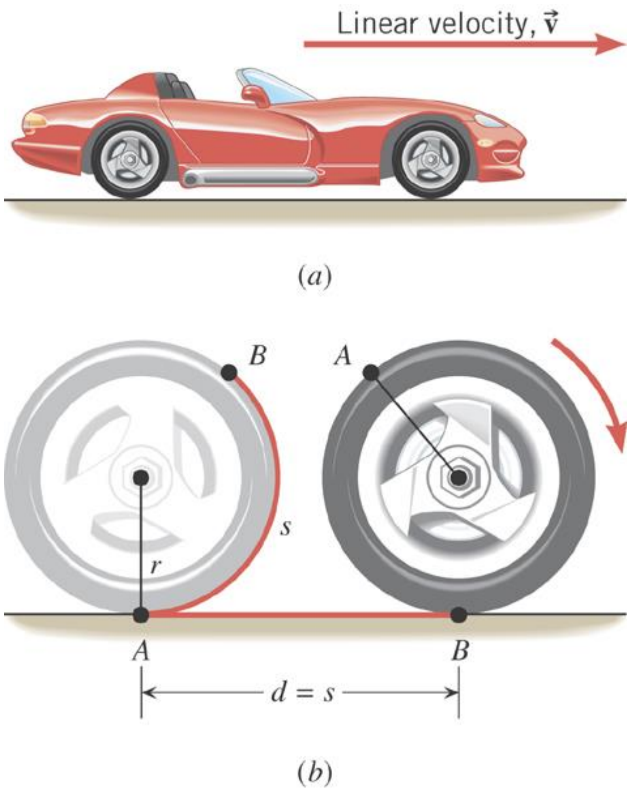


(b)

Example 8 An Accelerating Car

Starting from rest, the car accelerates for 20.0 s with a constant linear acceleration of 0.800 m/s^2. The radius of the tires is 0.330 m.

What is the angle through which each wheel has rotated?



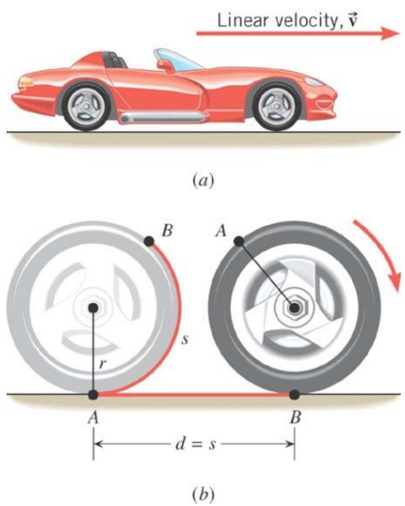
8.6 Rolling Motion

$$\alpha = \frac{a}{r} = \frac{0.800\,\text{m/s}^2}{0.330\,\text{m}} = 2.42\,\text{rad/s}^2$$

θ	α	ω	ω_o	t
?	-2.42 rad/s^2		0 rad/s	20.0 s

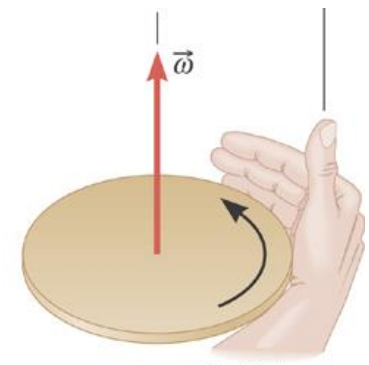
$$\theta = \omega_o t + \tfrac{1}{2}\alpha t^2$$

$$\theta = \tfrac{1}{2}\left(-2.42\,\text{rad/s}^2\right)^2\left(20.0\,\text{s}\right)^2 = -484\,\text{rad}$$



8.7 The Vector Nature of Angular Variables

Right-Hand Rule: Grasp the axis of rotation with your right hand, so that your fingers circle the axis in the same sense as the rotation.



Your extended thumb points along the axis in the direction of the angular velocity.