Chapter 11

Fluids

DEFINITION OF MASS DENSITY

The mass density of a substance is the mass of a substance divided by its volume:

$$\rho = \frac{m}{V}$$

SI Unit of Mass Density: kg/m^3

11.1 Mass Density

Table 11.1 Mass Densities[a] of Common Substances

Substance	Mass Density ρ (kg/m^3)
Solids	
Aluminum	2700
Brass	8470
Concrete	2200
Copper	8890
Diamond	3520
Gold	19 300
Ice	917
Iron (steel)	7860
Lead	11 300
Quartz	2660
Silver	10 500
Wood (yellow pine)	550
Liquids	
Blood (whole, 37 °C)	1060
Ethyl alcohol	806
Mercury	13 600
Oil (hydraulic)	800
Water (4 °C)	1.000×10^3
Gases	
Air	1.29
Carbon dioxide	1.98
Helium	0.179
Hydrogen	0.0899
Nitrogen	1.25
Oxygen	1.43

[a] Unless otherwise noted, densities are given at 0 °C and 1 atm pressure.

Example 1 **Blood as a Fraction of Body Weight**

The body of a man whose weight is 690 N contains about $5.2\text{x}10^{-3}\ \text{m}^3$ of blood.

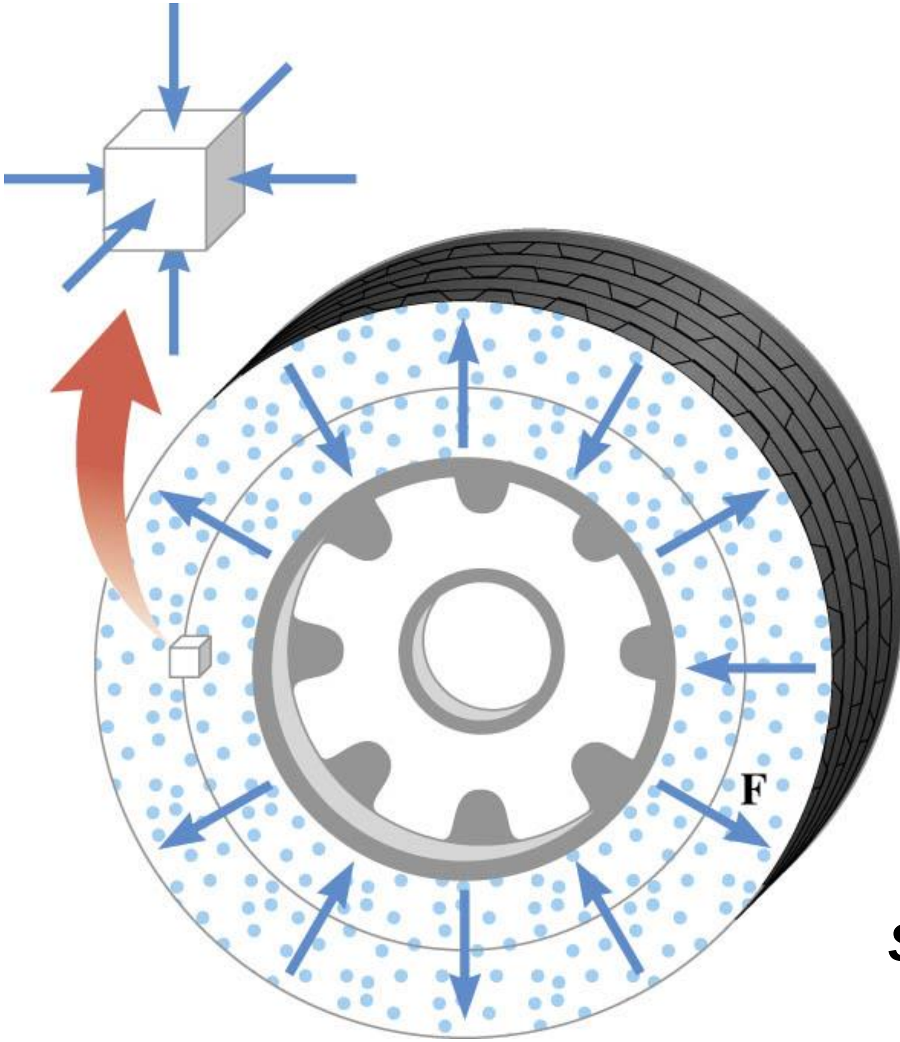
(a) Find the blood's weight and (b) express it as a percentage of the body weight.

$$m = V\rho = \left(5.2\times10^{-3}\,\text{m}^3\right)\left(1060\,\text{kg/m}^3\right) = 5.5\,\text{kg}$$

(a) $$W = mg = (5.5\,\mathrm{kg})(9.80\,\mathrm{m/s^2}) = 54\,\mathrm{N}$$

(b) $$\mathrm{Percentage} = \frac{54\,\mathrm{N}}{690\,\mathrm{N}} \times 100\% = 7.8\%$$

11.2 Pressure



$$P = \frac{F}{A}$$

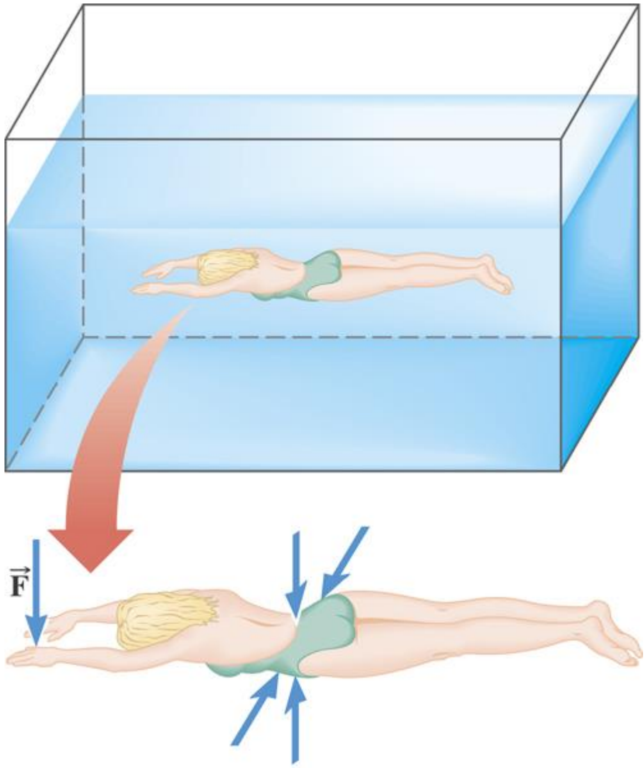
SI Unit of Pressure: $1\ N/m^2 = 1Pa$

Pascal

Example 2 The Force on a Swimmer

Suppose the pressure acting on the back of a swimmer's hand is 1.2×10^{5} Pa. The surface area of the back of the hand is $8.4\times10^{-3}\,m^{2}$.

(a) Determine the magnitude of the force that acts on it.
(b) Discuss the direction of the force.

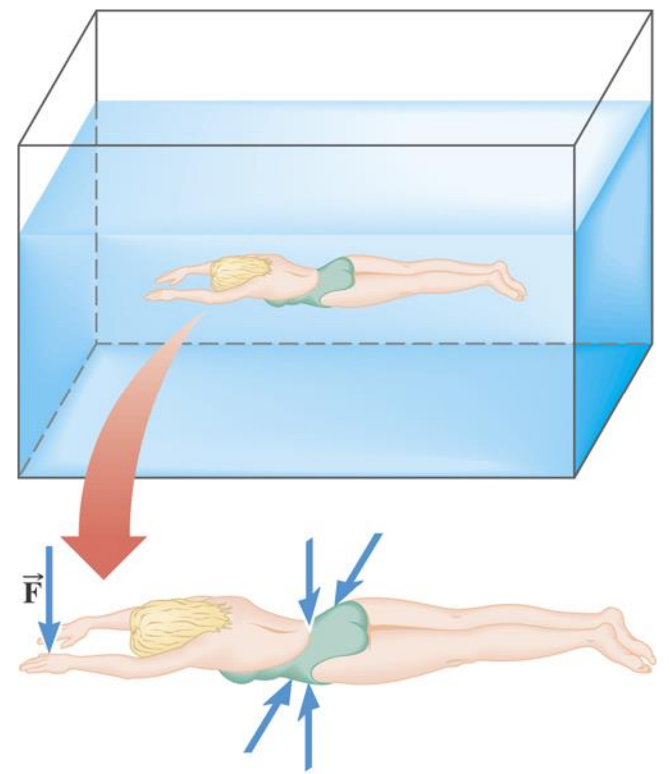


$$P = \frac{F}{A}$$

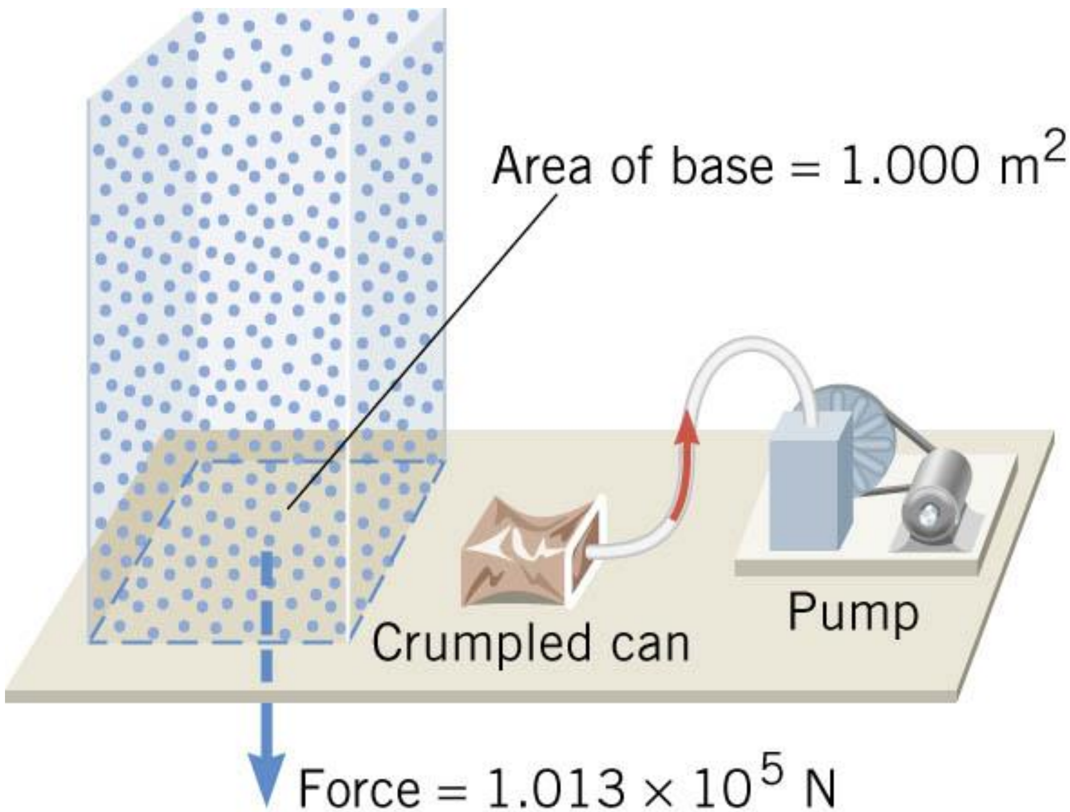
$$F = PA = \left(1.2\times10^{5}\ \mathrm{N/m^{2}}\right)\left(8.4\times10^{-3}\,\mathrm{m^{2}}\right)$$

$$= 1.0\times10^{3}\ \mathrm{N}$$

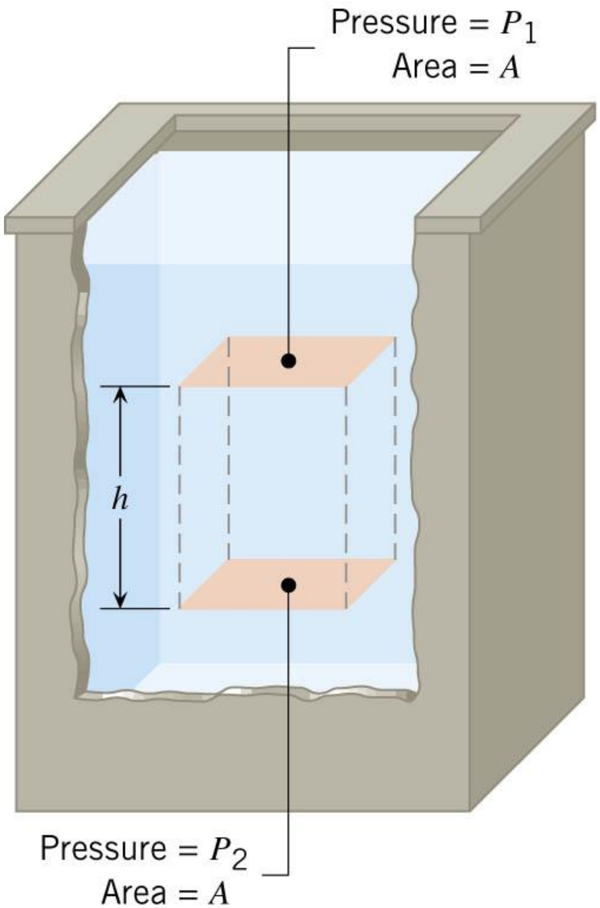
Since the water pushes perpendicularly against the back of the hand, the force is directed downward in the drawing.



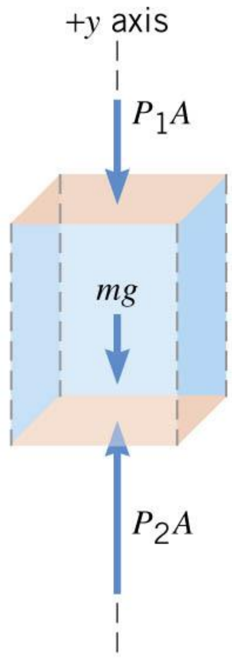
Atmospheric Pressure at Sea Level: 1.013×10^5 Pa = 1 atmosphere



11.3 Pressure and Depth in a Static Fluid



(a)

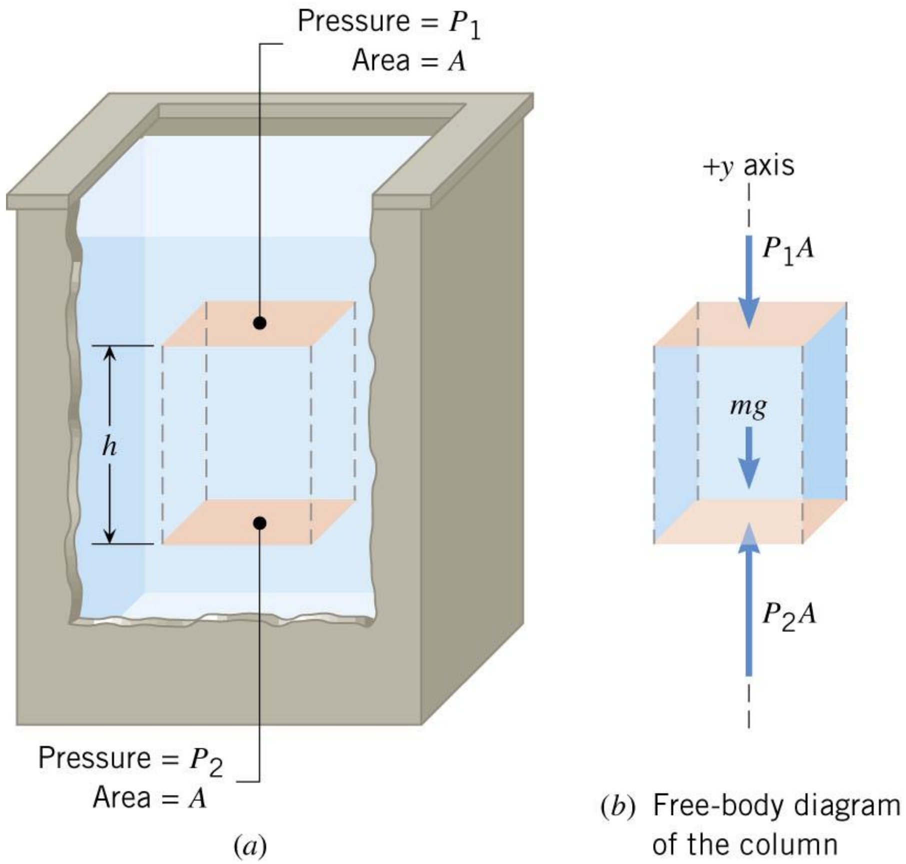


(b) Free-body diagram of the column

$$\sum F_y = P_2A - P_1A - mg = 0$$

$$P_2A = P_1A + mg$$

$$m = V\rho$$



$$V = Ah$$

$$P_2 A = P_1 A + \rho V g$$

$$P_2 A = P_1 A + \rho A h g$$

$$P_2 = P_1 + \rho h g$$

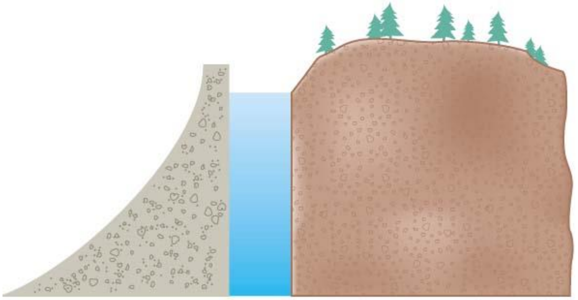
Conceptual Example 3 The Hoover Dam

Lake Mead is the largest wholly artificial reservoir in the United States. The water in the reservoir backs up behind the dam for a considerable distance (120 miles).


(a)

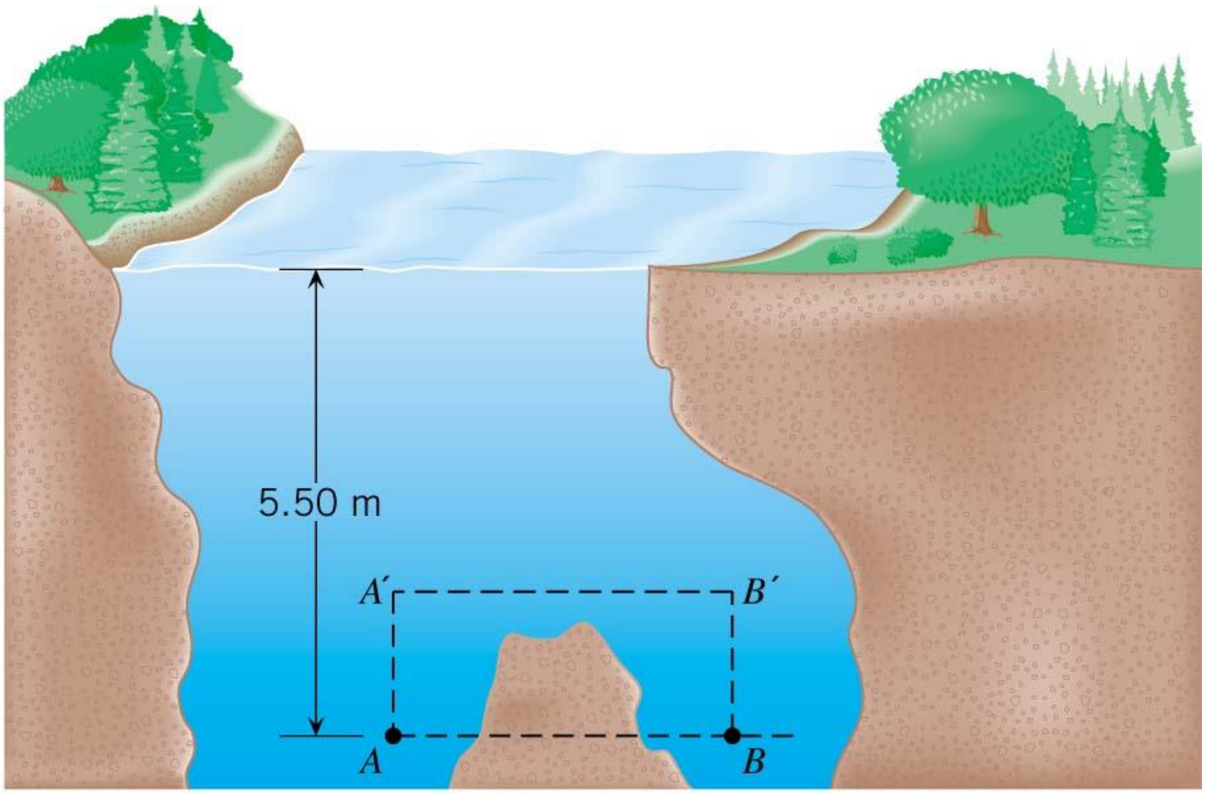
Suppose that all the water in Lake Mead were removed except a relatively narrow vertical column.

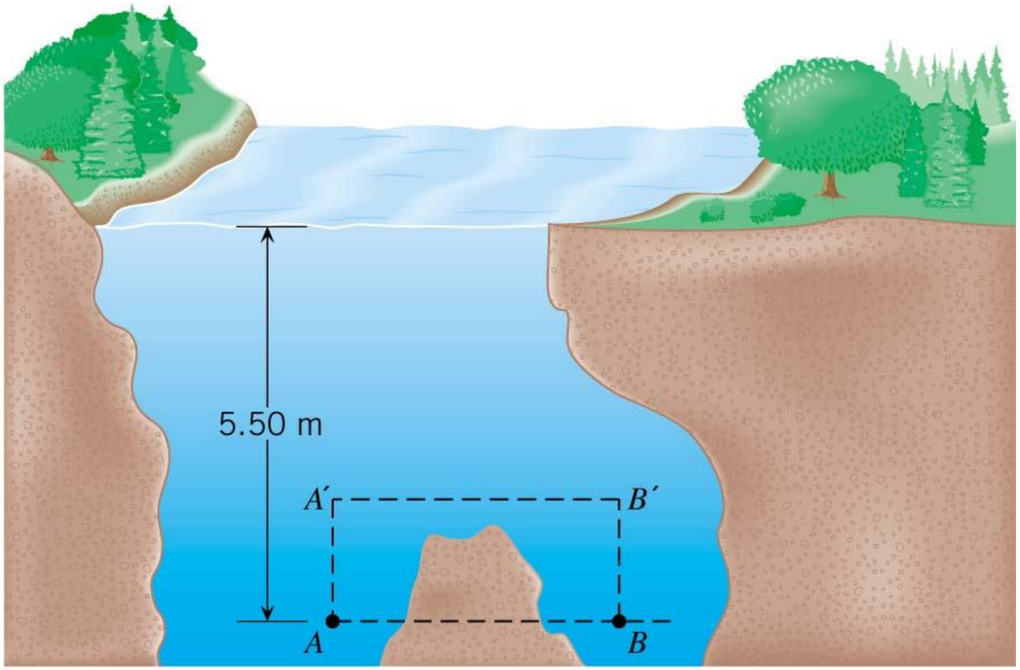
Would the Hoover Same still be needed to contain the water, or could a much less massive structure do the job?


(b)

Example 4 The Swimming Hole

Points A and B are located a distance of 5.50 m beneath the surface of the water. Find the pressure at each of these two locations.



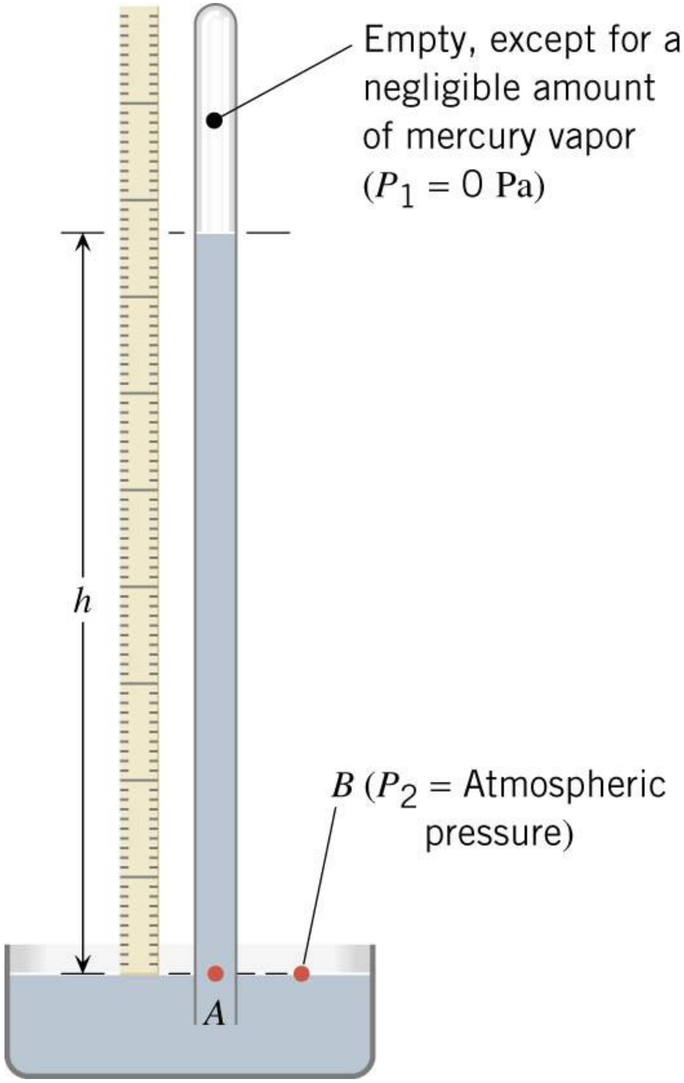


$$P_2 = P_1 + \rho g h$$

$$P_2 = \overbrace{\left(1.01\times10^5\ \text{Pa}\right)}^{\text{atmospheric pressure}} + \left(1.00\times10^3\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\left(5.50\ \text{m}\right)$$

$$= 1.55\times10^5\ \text{Pa}$$

11.4 Pressure Gauges



$$P_2 = P_1 + \rho g h$$

$$\Downarrow$$

$$P_{atm} = \rho g h$$

$$\Downarrow$$

$$h = \frac{P_{atm}}{\rho g} = \frac{\left(1.01\times 10^5 \text{ Pa}\right)}{\left(13.6\times 10^3 \text{ kg/m}^3\right)\left(9.80 \text{ m/s}^2\right)}$$

$$= 0.760 \text{ m} = 760 \text{ mm}$$

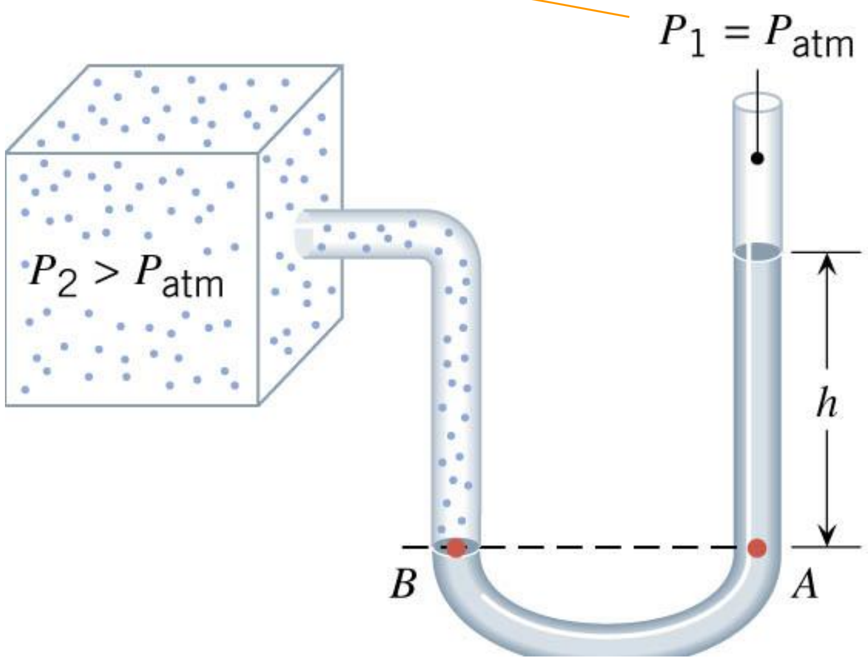
11.4 Pressure Gauges

$$P_2 = P_B = P_A$$

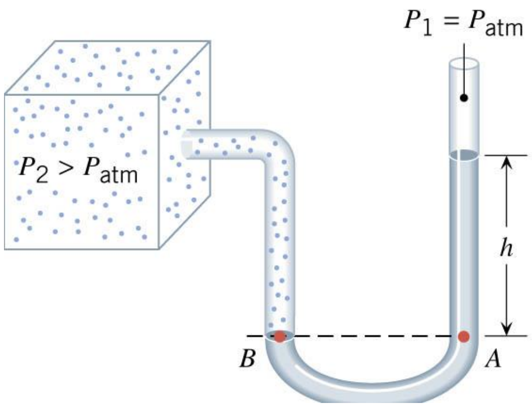
$$P_A = P_1 + \rho g h$$

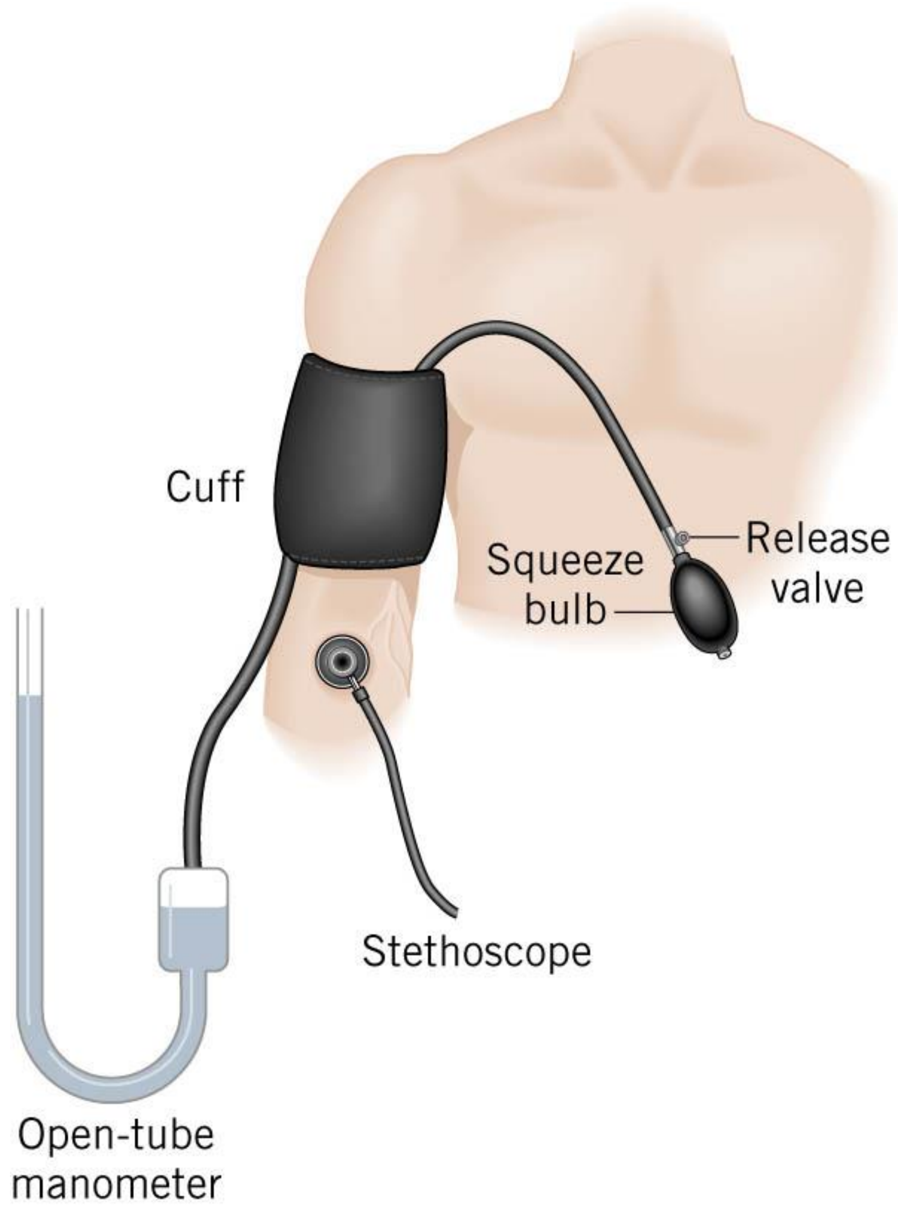
absolute pressure

$$\underbrace{P_2 - P_{atm}}_{\text{gauge pressure}} = \rho g h$$



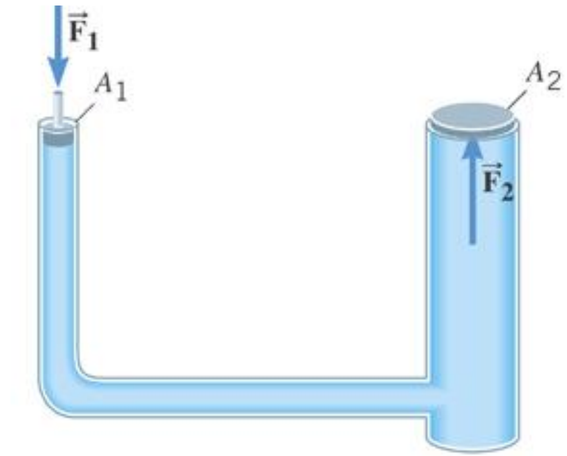
11.4 Pressure Gauges



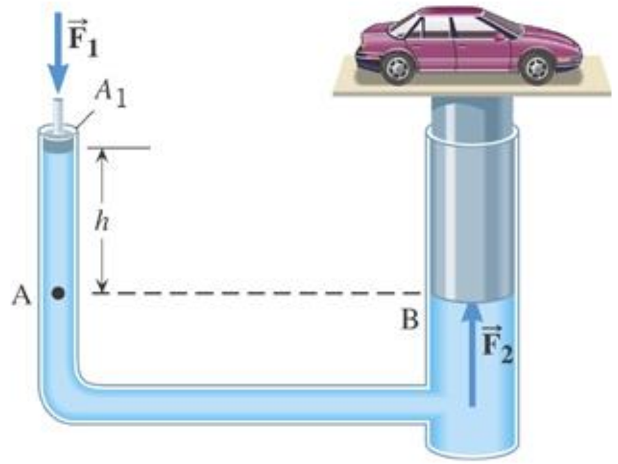


PASCAL'S PRINCIPLE

Any change in the pressure applied to a completely enclosed fluid is transmitted undiminished to all parts of the fluid and enclosing walls.

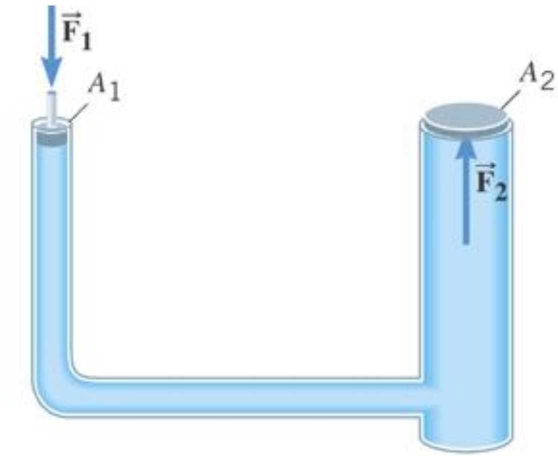


(a)

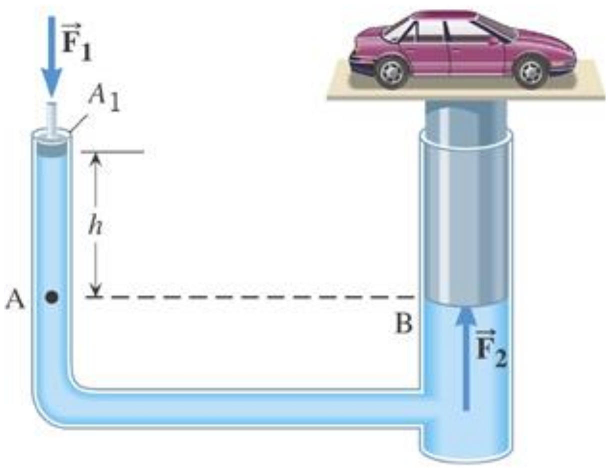


(b)

11.5 Pascal's Principle



(a)



(b)

$$P_2 = P_1 + \rho g (0\,\mathrm{m})$$

$$\Downarrow$$

$$\frac{F_2}{A_2} = \frac{F_1}{A_1}$$

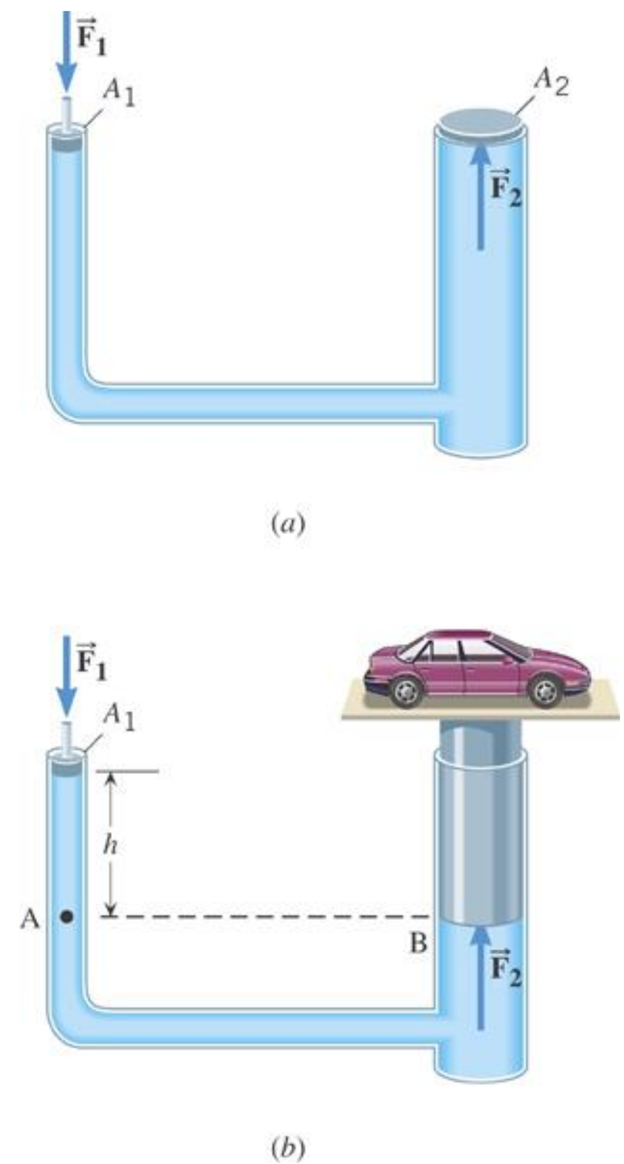
$$\Downarrow$$

$$F_2 = F_1 \left(\frac{A_2}{A_1} \right)$$

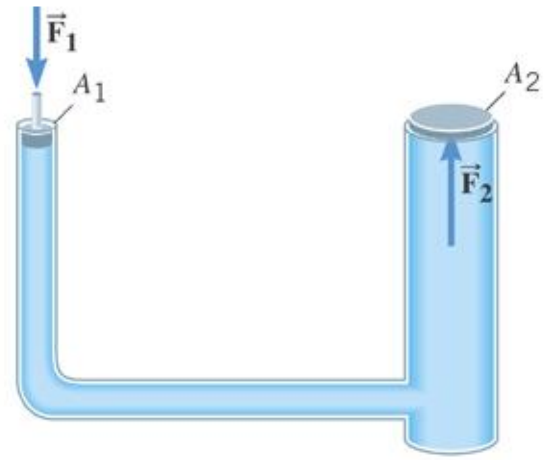
Example 7 A Car Lift

The input piston has a radius of 0.0120 m and the output plunger has a radius of 0.150 m.

The combined weight of the car and the plunger is 20500 N. Suppose that the input piston has a negligible weight and the bottom surfaces of the piston and plunger are at the same level. What is the required input force?

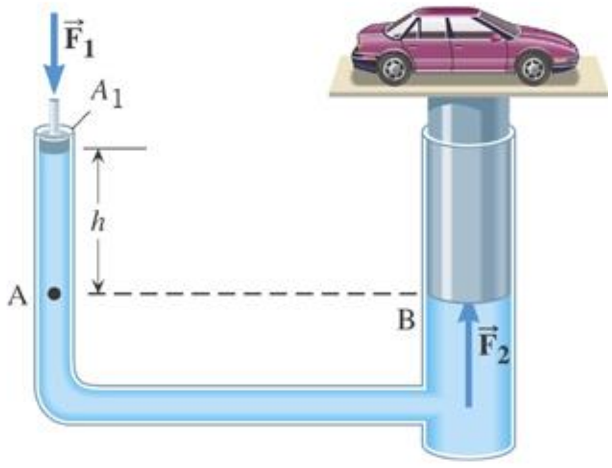


$$F_2 = F_1\left(\frac{A_2}{A_1}\right)$$



(a)

$$F_2 = \left(20500\ \text{N}\right)\frac{\pi\left(0.0120\ \text{m}\right)^2}{\pi\left(0.150\ \text{m}\right)^2} = 131\ \text{N}$$



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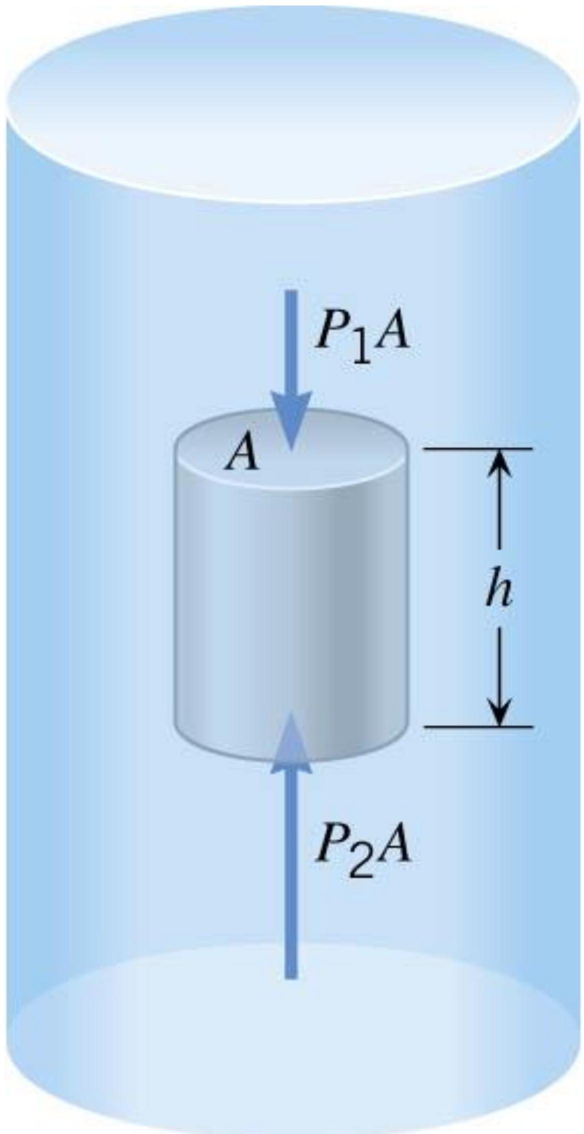


$$P_2 - P_1 = \rho g h$$

$$F_B = P_2 A - P_1 A = \left(P_2 - P_1\right) A$$

$$\Downarrow$$

$$V = hA$$

$$F_B = \rho g h A$$

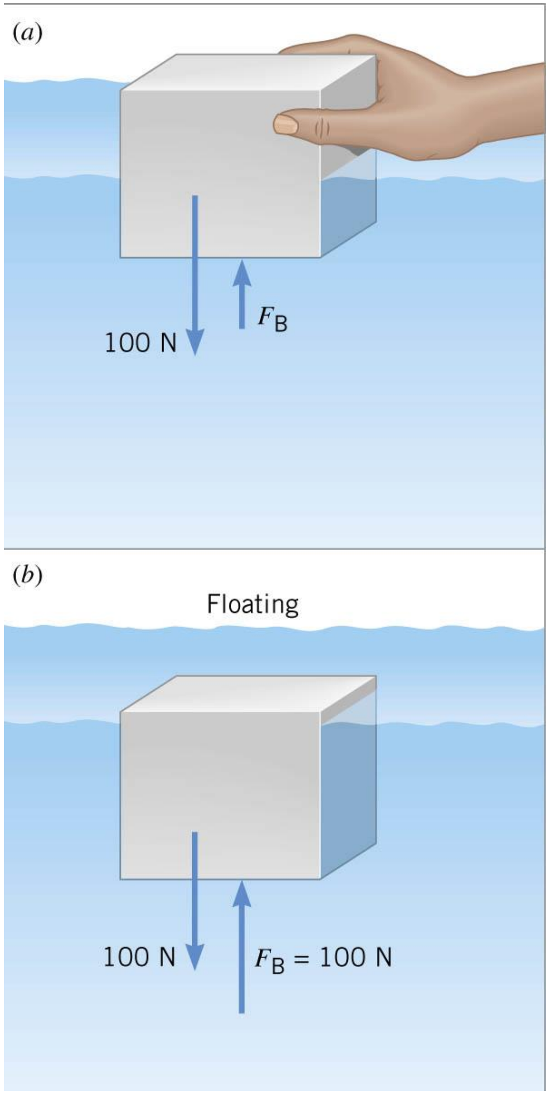
$$\Downarrow$$

$$F_B = \underbrace{\rho V}_{\text{mass of displaced fluid}} g$$

ARCHIMEDES' PRINCIPLE

Any fluid applies a buoyant force to an object that is partially or completely immersed in it; the magnitude of the buoyant force equals the weight of the fluid that the object displaces:

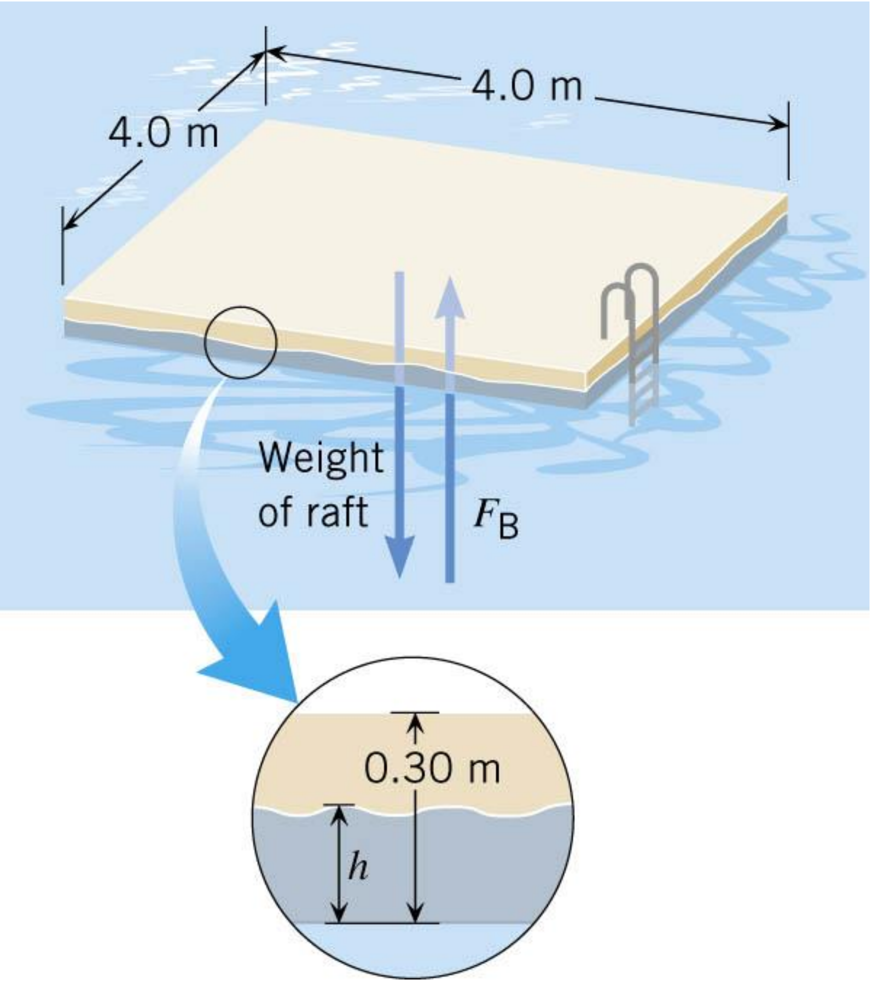
$$\underbrace{F_B}_{\substack{\text{Magnitude of} \\ \text{buoyant force}}} = \underbrace{W_{\text{fluid}}}_{\substack{\text{Weight of} \\ \text{displaced fluid}}}$$



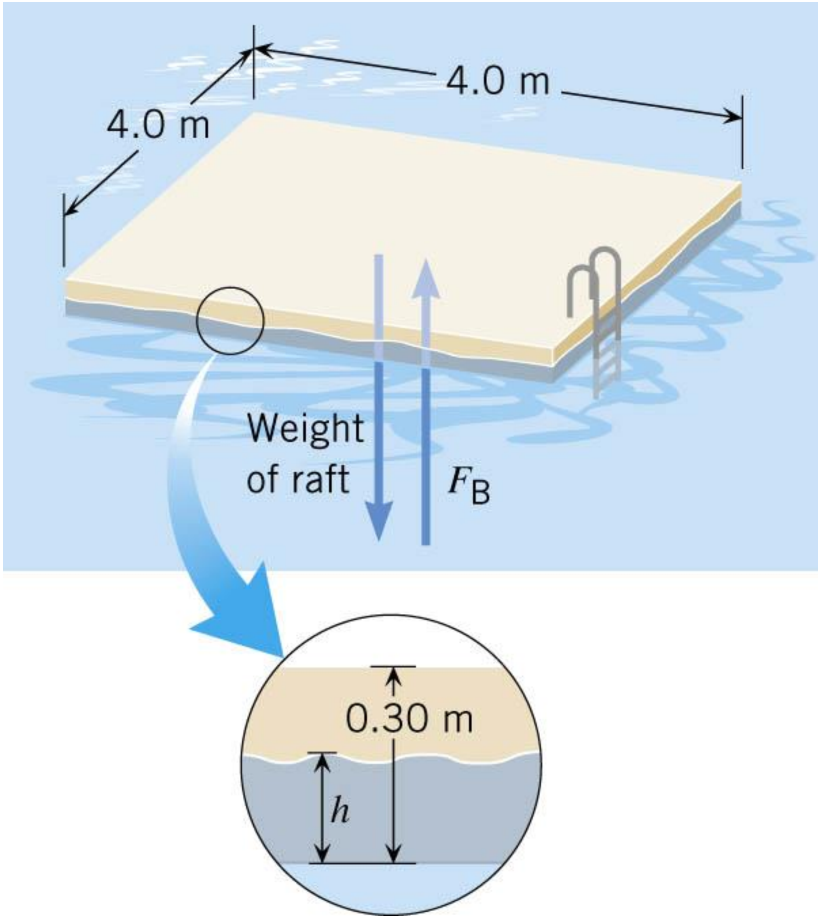
If the object is floating then the magnitude of the buoyant force is equal to the magnitude of its weight.

Example 9 A Swimming Raft

The raft is made of solid square pinewood. Determine whether the raft floats in water and if so, how much of the raft is beneath the surface.



$$V_{raft} = (4.0\,\text{m})(4.0\,\text{m})(0.30\,\text{m}) = 4.8\,\text{m}^3$$



$$F_B^{max} = \rho V g = \rho_{water} V_{water} g$$

$$= (1000\,\text{kg/m}^3)(4.8\text{m}^3)(9.80\,\text{m/s}^2)$$

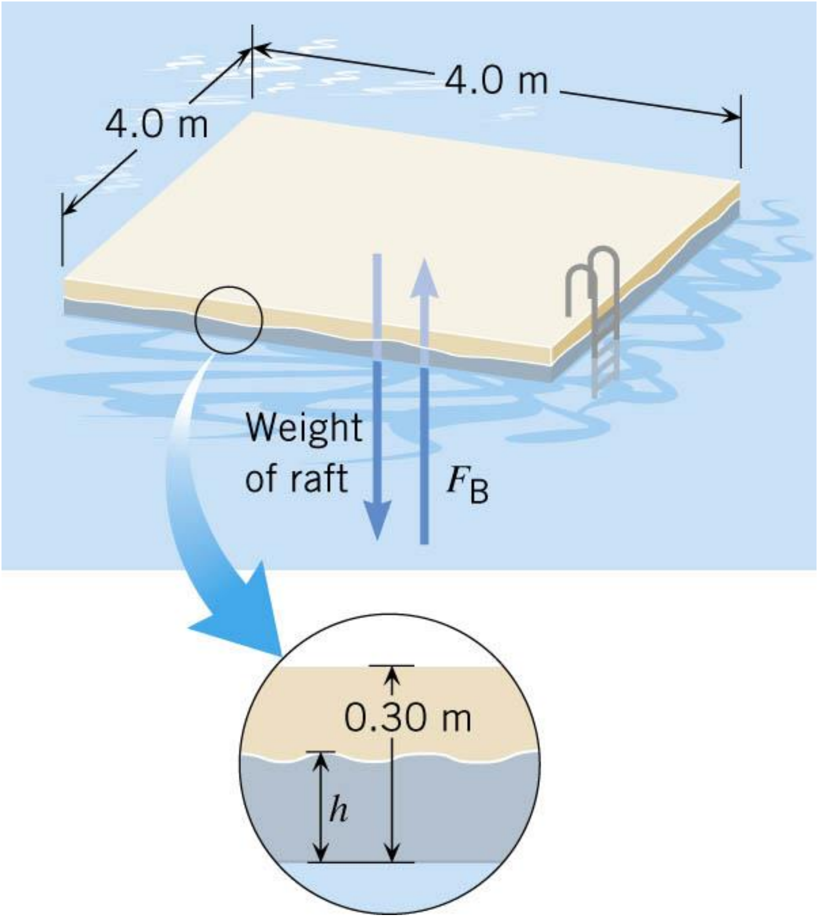
$$= 47000\ \text{N}$$

$$W_{raft} = m_{raft}g = \rho_{pine}V_{raft}g$$

$$= (550\,\mathrm{kg/m^3})(4.8\,\mathrm{m^3})(9.80\,\mathrm{m/s^2})$$

$$= 26000\ \mathrm{N} < 47000\ \mathrm{N}$$

The raft floats!



If the raft is floating:

$$W_{raft} = F_B$$

⇓

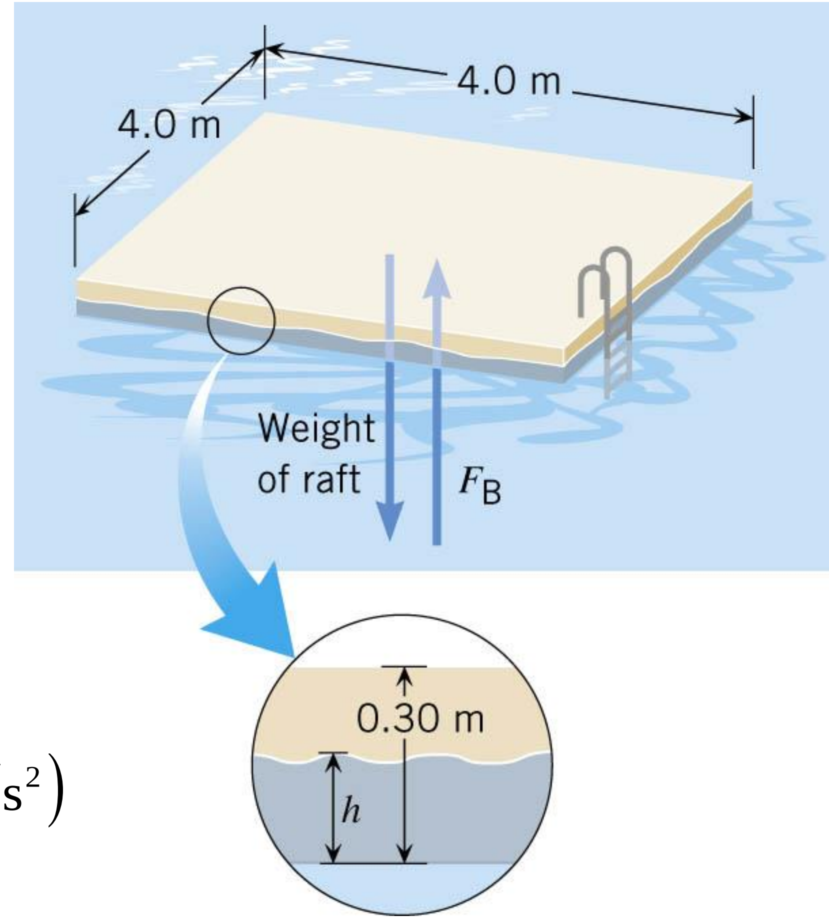
$$26000\ \text{N} = \rho_{water} V_{water} g$$

⇓

$$26000\ \text{N} = \left(1000\ \text{kg/m}^3\right)\left(4.0\ \text{m}\right)\left(4.0\ \text{m}\right) h \left(9.80\ \text{m/s}^2\right)$$

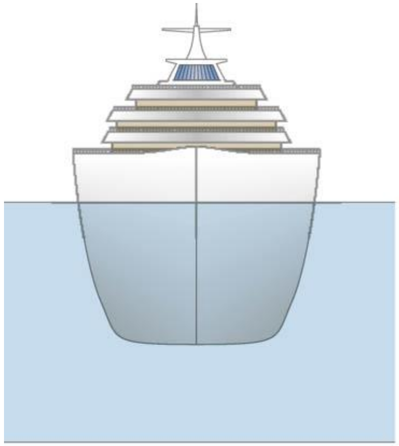
⇓

$$h = \frac{26000\ \text{N}}{\left(1000\ \text{kg/m}^3\right)\left(4.0\ \text{m}\right)\left(4.0\ \text{m}\right)\left(9.80\ \text{m/s}^2\right)} = 0.17\ \text{m}$$



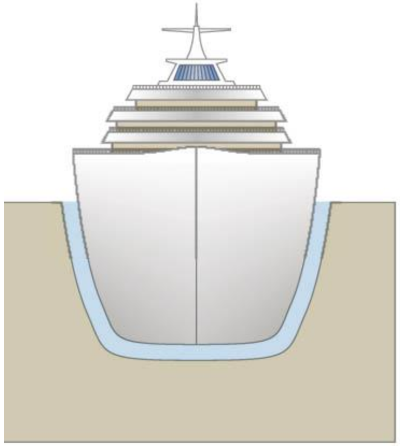
Conceptual Example 10 How Much Water is Needed to Float a Ship?

A ship floating in the ocean is a familiar sight. But is all that water really necessary? Can an ocean vessel float in the amount of water than a swimming pool contains?



(a)

(b)



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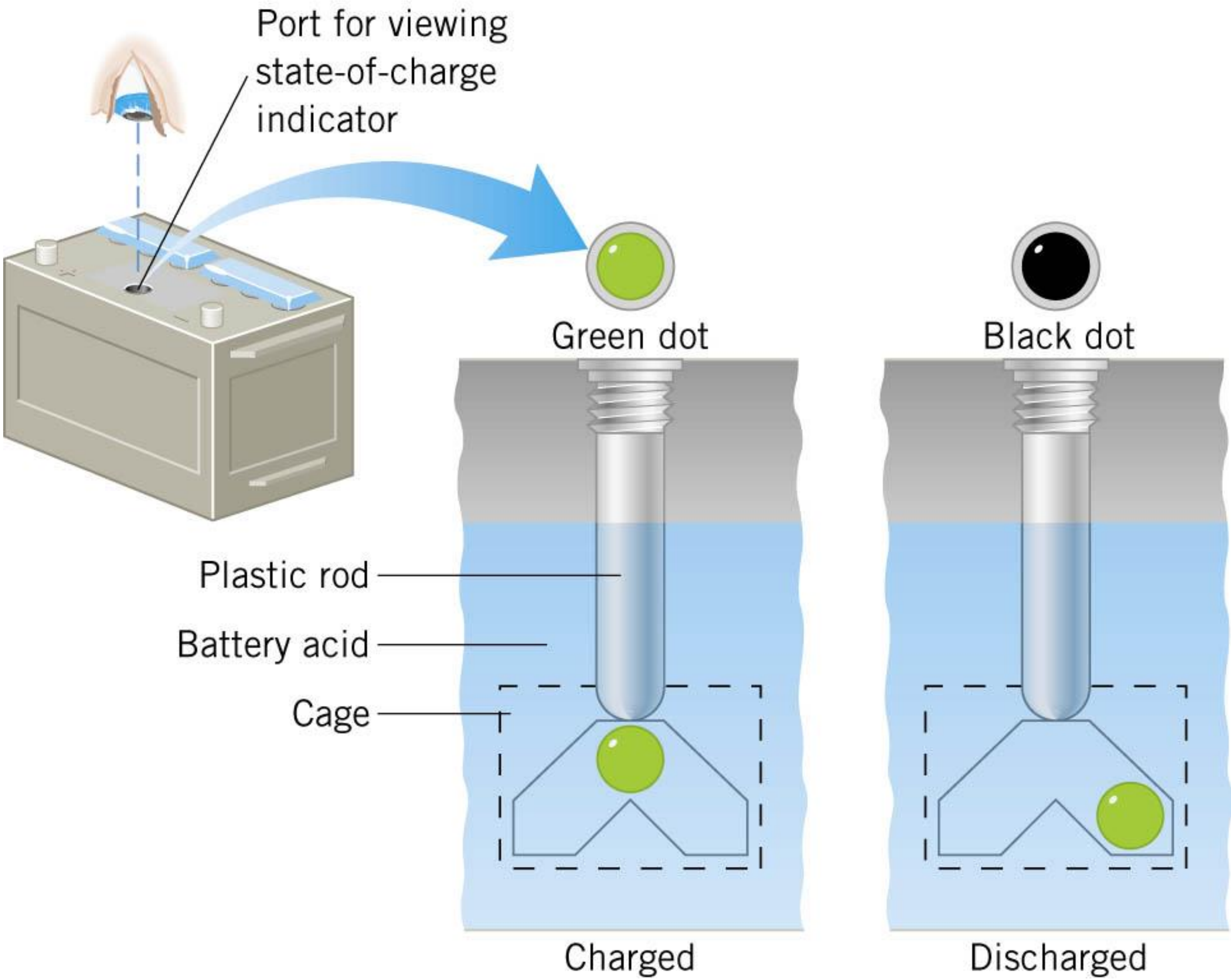
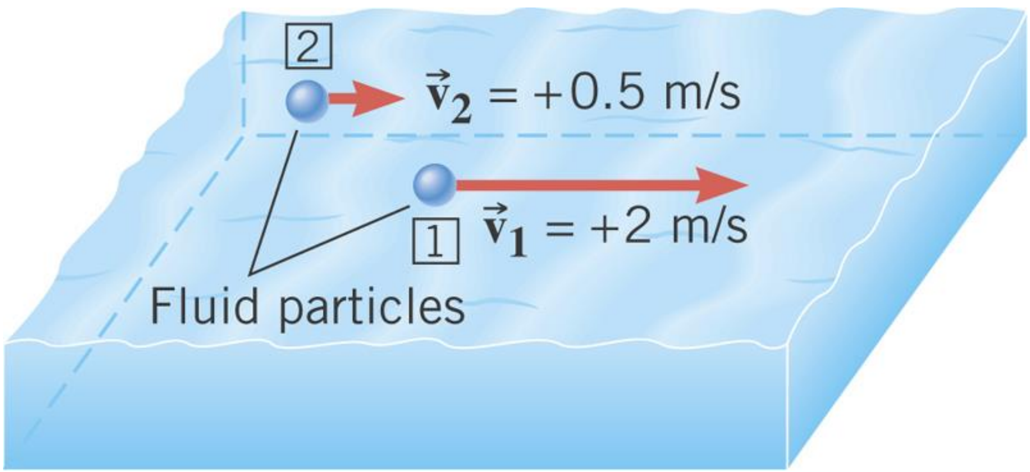

Port for viewing
state-of-charge
indicator
Green dot
Black dot
Plastic rod
Battery acid
Cage
Charged
Discharged

In ***steady flow*** the velocity of the fluid particles at any point is constant as time passes.

Unsteady flow exists whenever the velocity of the fluid particles at a point changes as time passes.



Turbulent flow is an extreme kind of unsteady flow in which the velocity of the fluid particles at a point change erratically in both magnitude and direction.

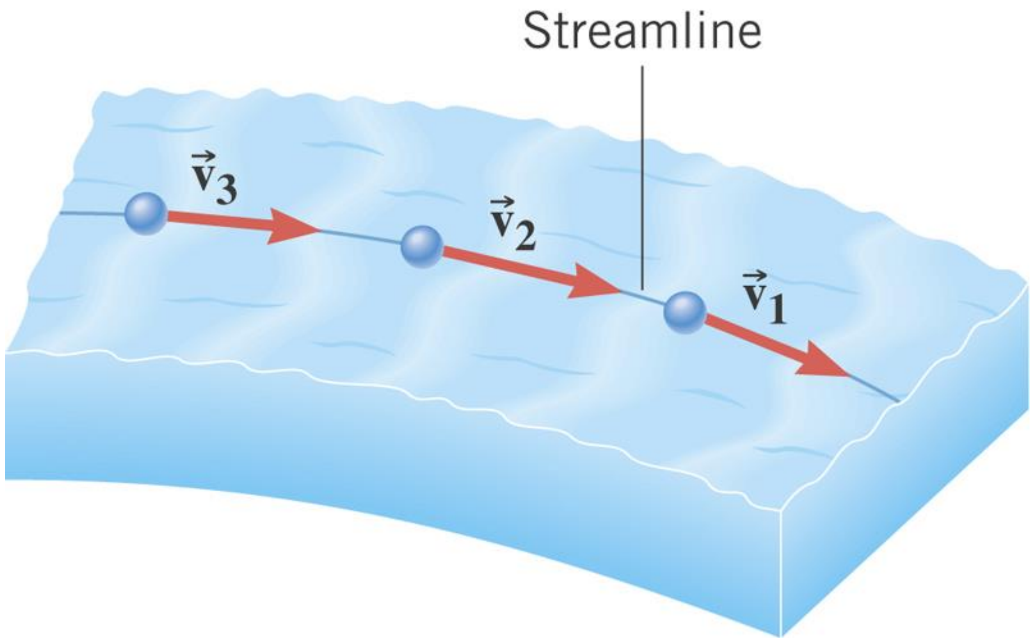
11.7 Fluids in Motion

Fluid flow can be ***compressible*** or ***incompressible***. Most liquids are nearly incompressible.

Fluid flow can be ***viscous*** or ***nonviscous***.

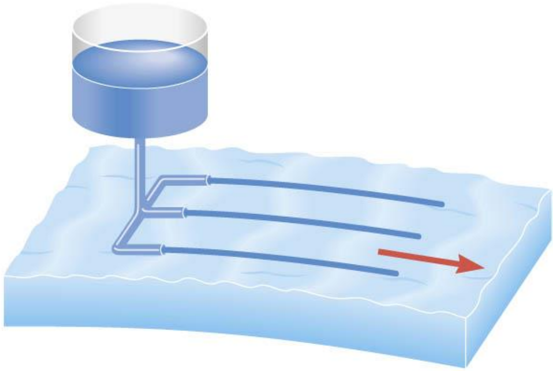
An incompressible, nonviscous fluid is called an ***ideal fluid***.

When the flow is steady, ***streamlines*** are often used to represent the trajectories of the fluid particles.

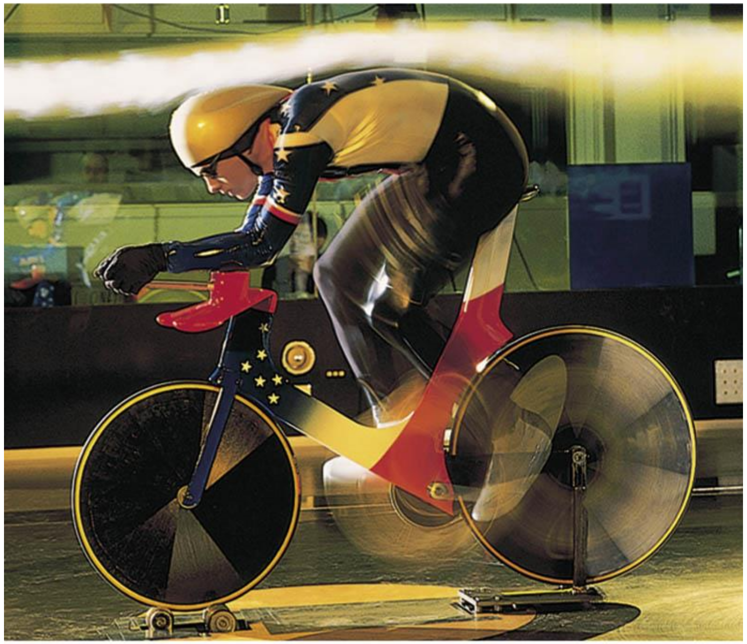


11.7 Fluids in Motion

Making streamlines with dye and smoke.



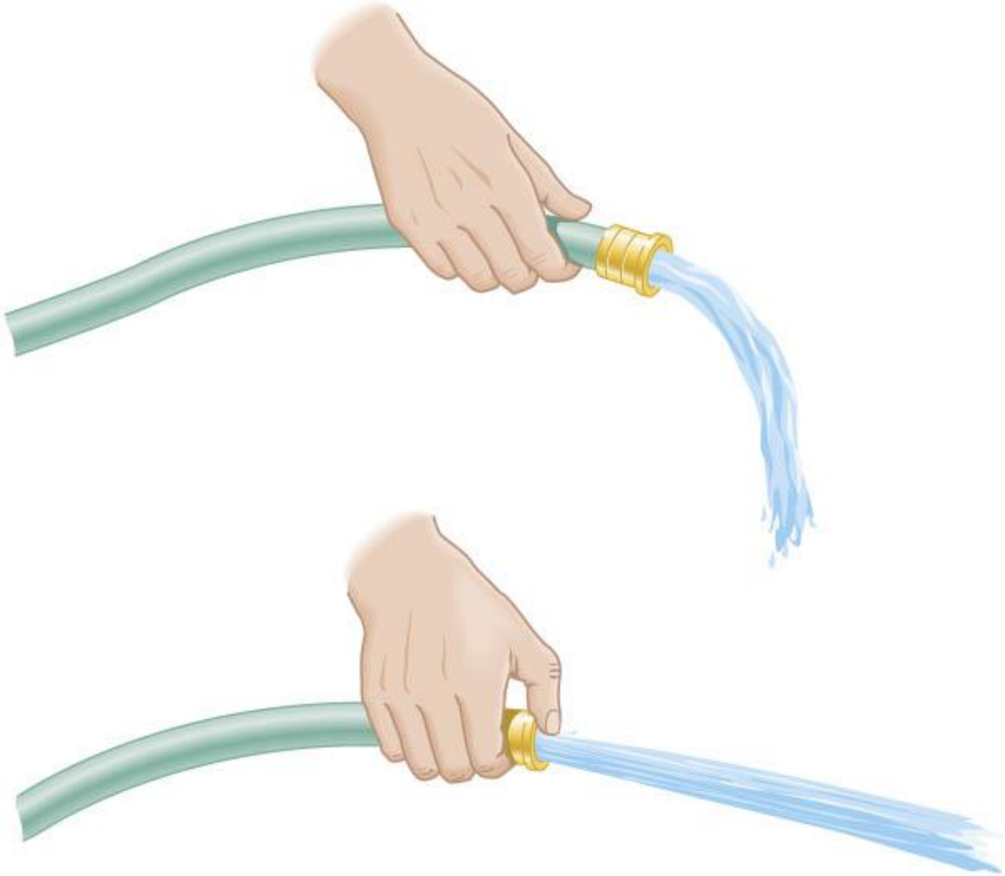
(a)



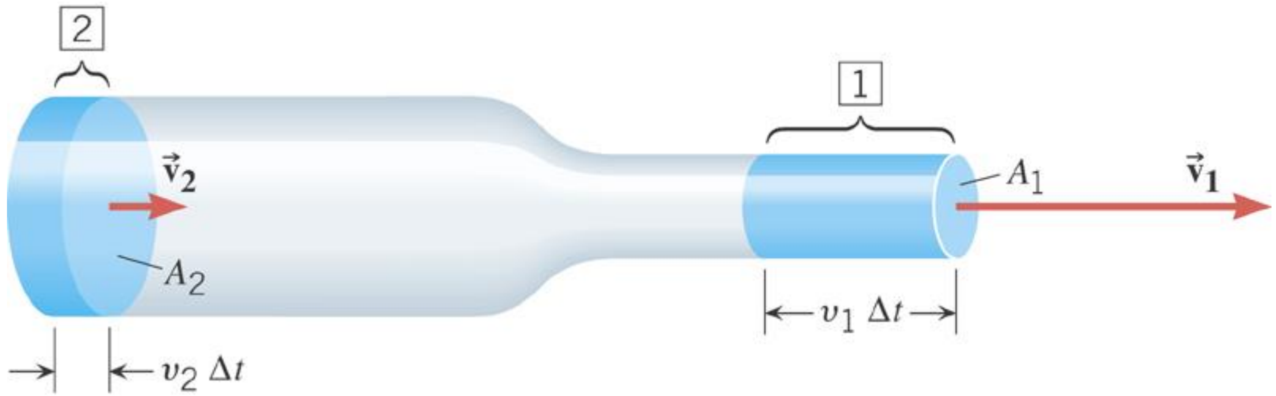
(b)

11.8 The Equation of Continuity

The mass of fluid per second that flows through a tube is called the ***mass flow rate***.



$$\Delta m = \rho V = \rho A \underbrace{v\Delta t}_{\text{distance}}$$



$$\frac{\Delta m_2}{\Delta t} = \rho_2 A_2 v_2$$

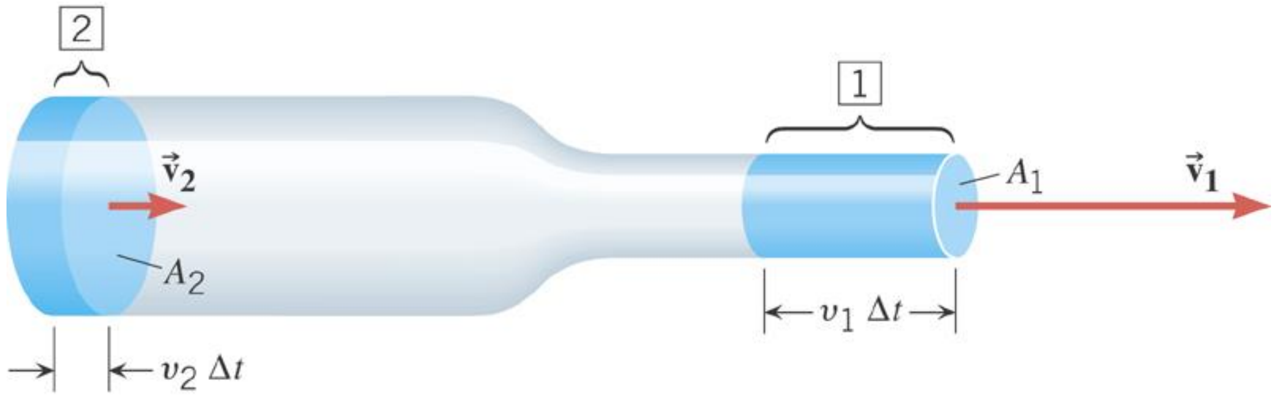
$$\frac{\Delta m_1}{\Delta t} = \rho_1 A_1 v_1$$

EQUATION OF CONTINUITY

The mass flow rate has the same value at every position along a tube that has a single entry and a single exit for fluid flow.

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

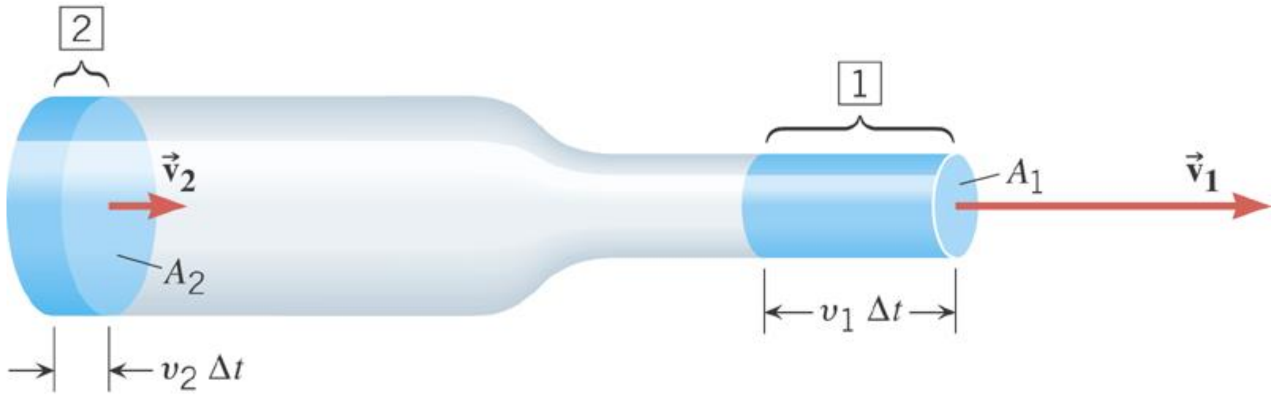
SI Unit of Mass Flow Rate: kg/s

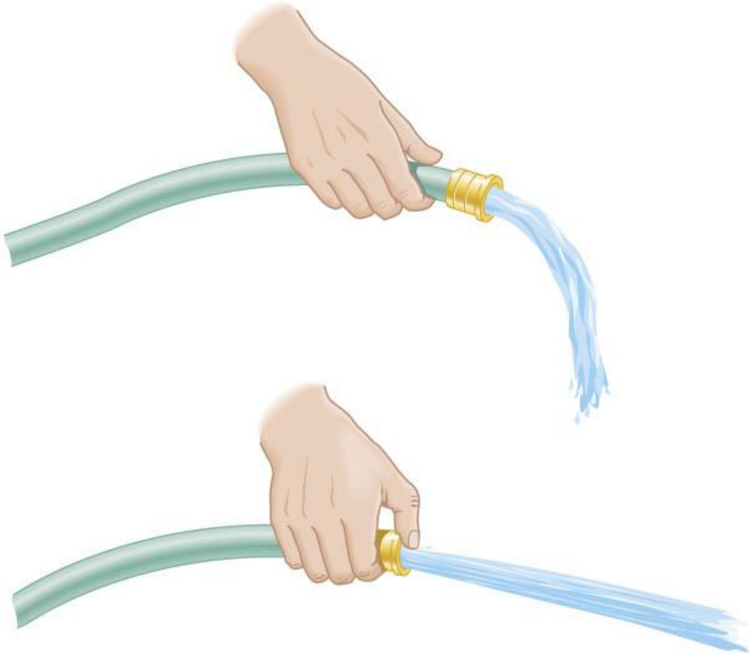


11.8 The Equation of Continuity

Incompressible fluid: $$A_1 v_1 = A_2 v_2$$

Volume flow rate Q: $$Q = Av$$

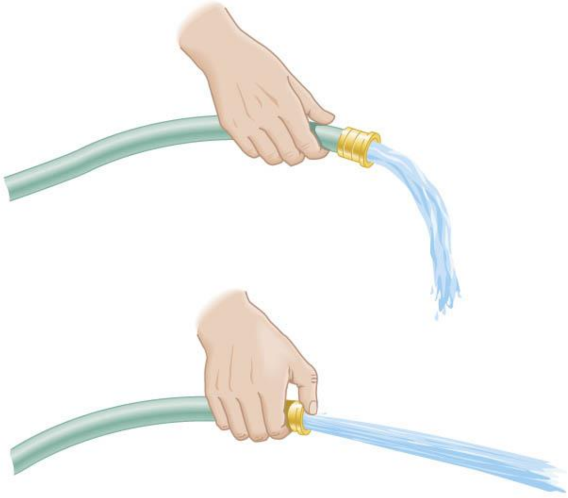




Example 12 A Garden Hose

A garden hose has an unobstructed opening with a cross sectional area of $2.85 \times 10^{-4} m^2$. It fills a bucket with a volume of $8.00 \times 10^{-3} m^3$ in 30 seconds.

Find the speed of the water that leaves the hose through (a) the unobstructed opening and (b) an obstructed opening with half as much area.



(a)

$$Q = Av$$

$$v = \frac{Q}{A} = \frac{\left(8.00\times10^{-3}\,\mathrm{m}^3\right)/\left(30.0\,\mathrm{s}\right)}{2.85\times10^{-4}\,\mathrm{m}^2} = 0.936\,\mathrm{m/s}$$

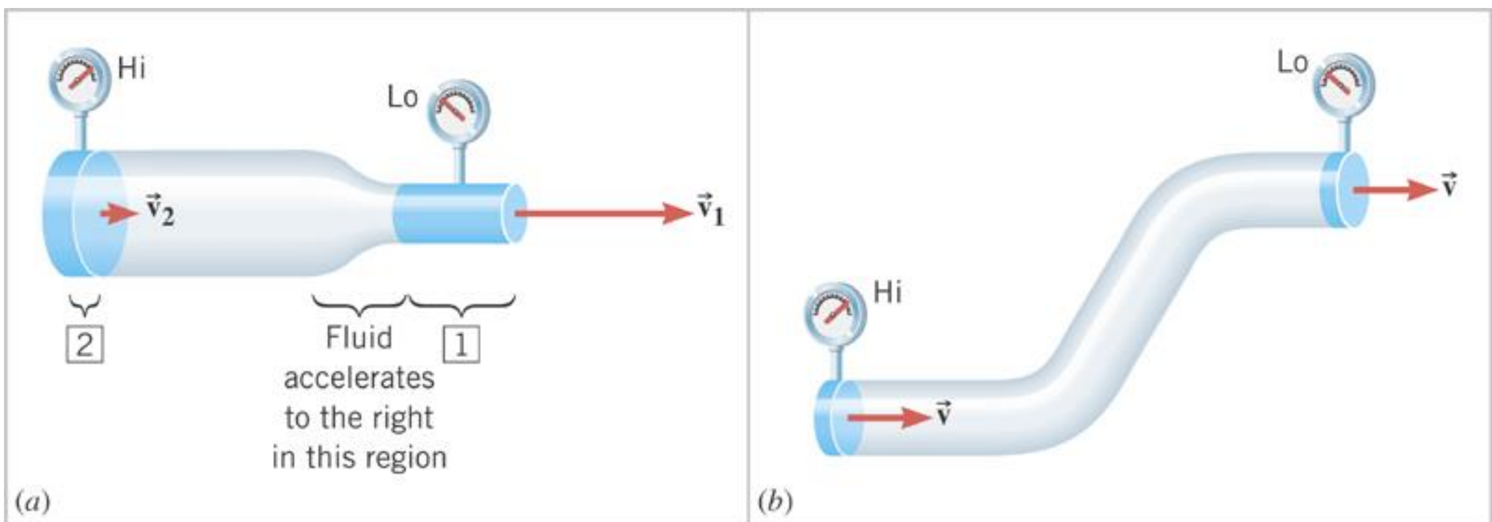
(b)

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1}{A_2} v_1 = (2)(0.936\,\mathrm{m/s}) = 1.87\,\mathrm{m/s}$$

The fluid accelerates toward the lower pressure regions.

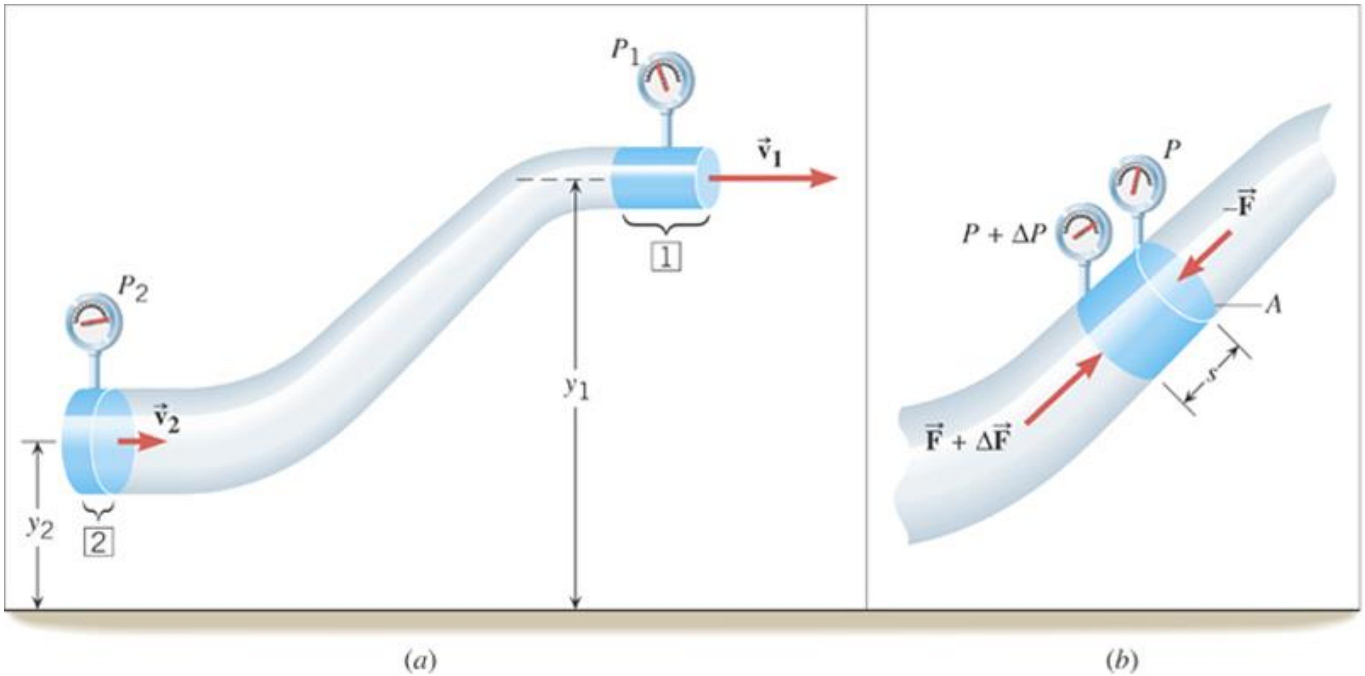
According to the pressure-depth relationship, the pressure is lower at higher levels, provided the area of the pipe does not change.



11.9 Bernoulli's Equation

$$W = \left(\sum F\right)s = (\Delta F)s = (\Delta P)As = (P_2 - P_1)V$$

$$W_{nc} = \left(\tfrac{1}{2}mv_1^2 + mgy_1\right) - \left(\tfrac{1}{2}mv_2^2 + mgy_2\right)$$



11.9 Bernoulli's Equation

$$\left(P_2 - P_1\right)V = \left(\tfrac{1}{2}mv_1^2 + mgy_1\right) - \left(\tfrac{1}{2}mv_2^2 + mgy_2\right)$$

$$\Downarrow$$

$$\left(P_2 - P_1\right) = \left(\tfrac{1}{2}\rho v_1^2 + \rho g y_1\right) - \left(\tfrac{1}{2}\rho v_2^2 + \rho g y_2\right)$$

BERNOULLI'S EQUATION

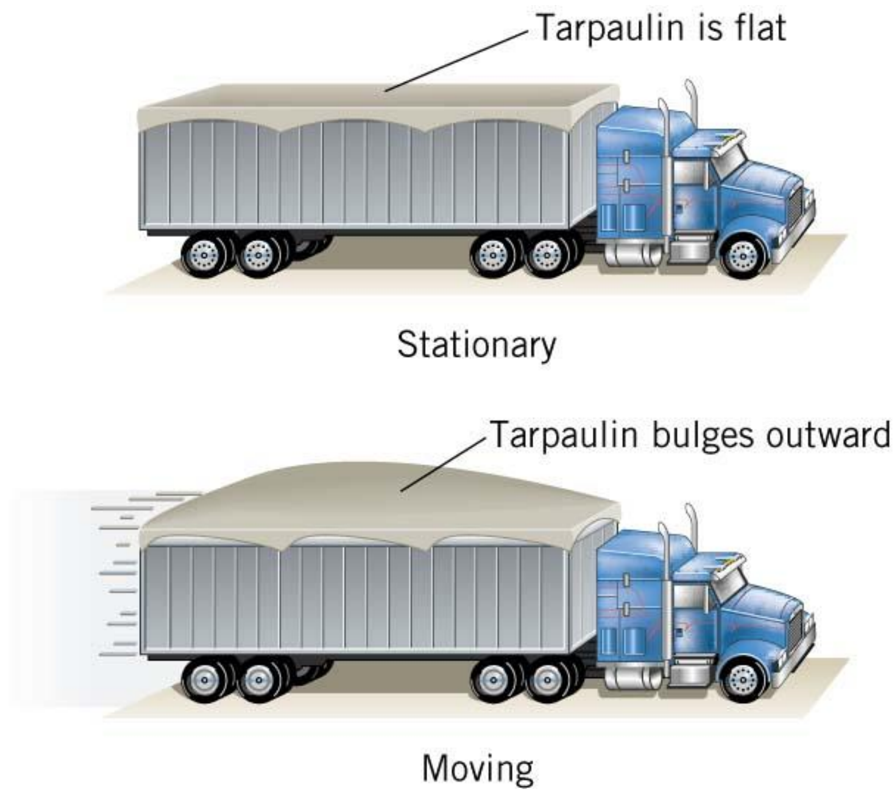
In steady flow of a nonviscous, incompressible fluid, the pressure, the fluid speed, and the elevation at two points are related by:

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

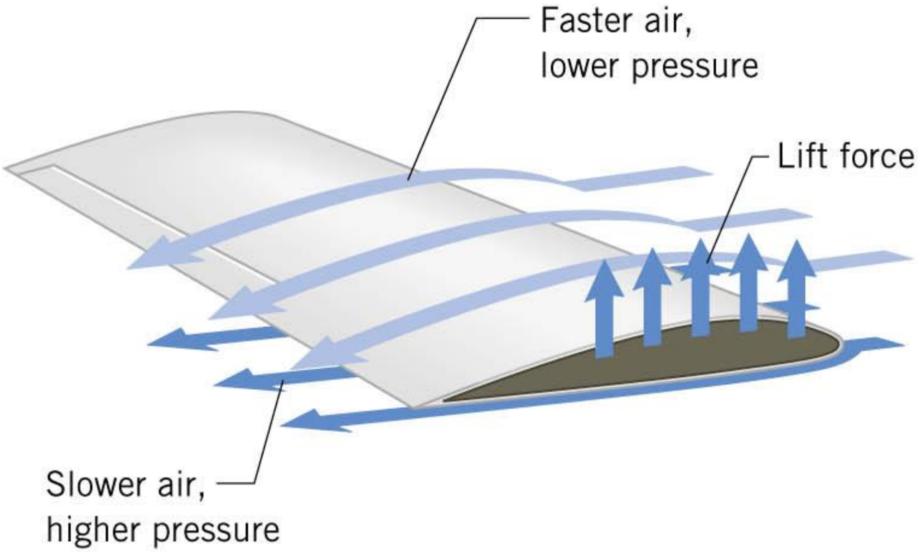
Conceptual Example 14 Tarpaulins and Bernoulli's Equation

When the truck is stationary, the tarpaulin lies flat, but it bulges outward when the truck is speeding down the highway.

Account for this behavior.



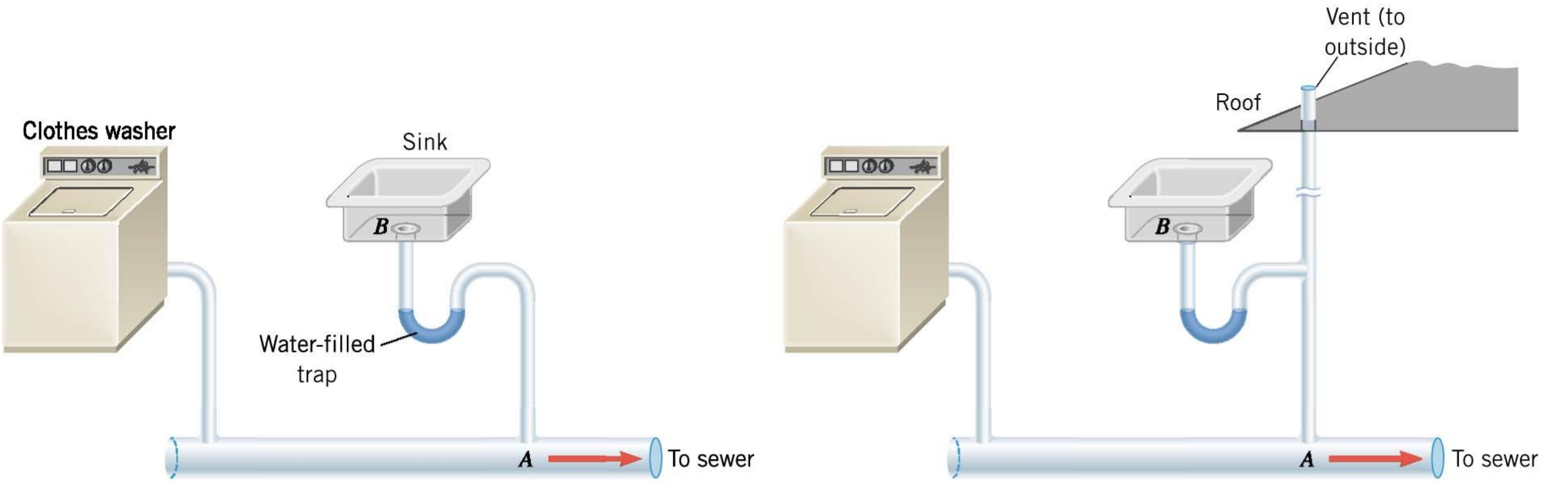
11.10 Applications of Bernoulli's Equation



(*a*)

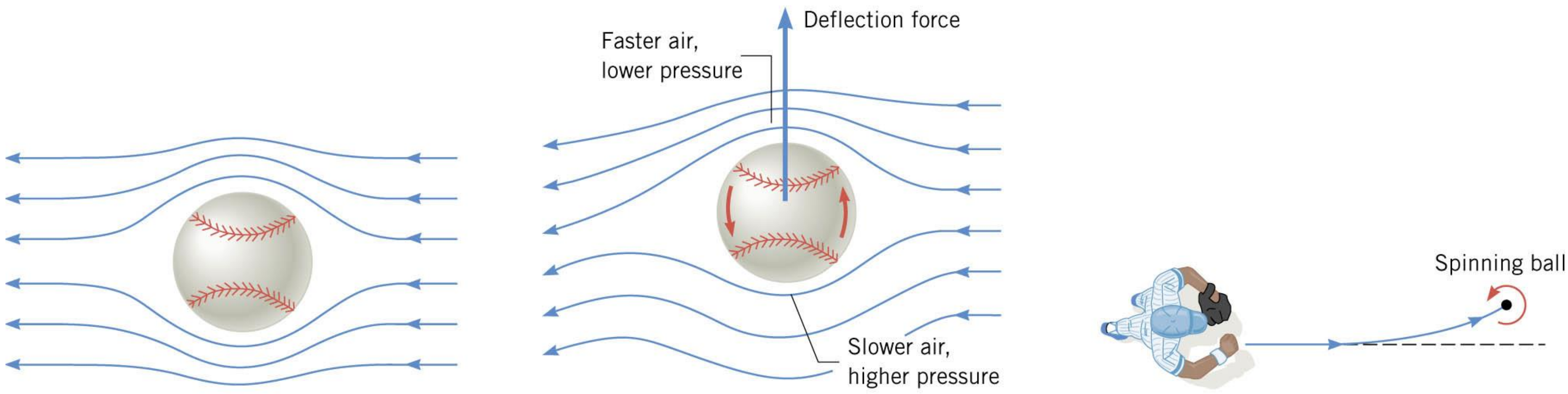
(*b*)

11.10 Applications of Bernoulli's Equation



(*a*) Without vent (*b*) With vent

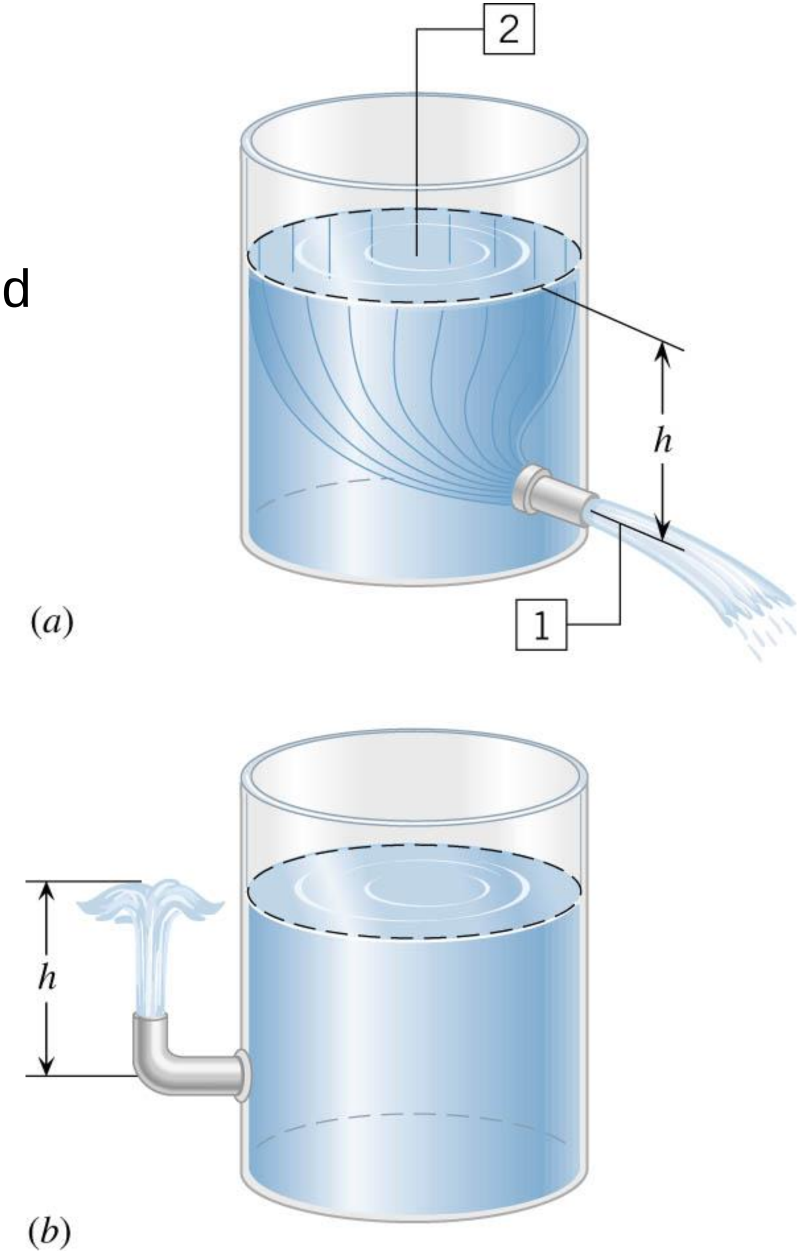
11.10 Applications of Bernoulli's Equation



(*a*) Without spin (*b*) With spin (*c*)

Example 16 Efflux Speed

The tank is open to the atmosphere at the top. Find an expression for the speed of the liquid leaving the pipe at the bottom.



11.10 Applications of Bernoulli's Equation

$$P_1 = P_2 = P_{atm}$$

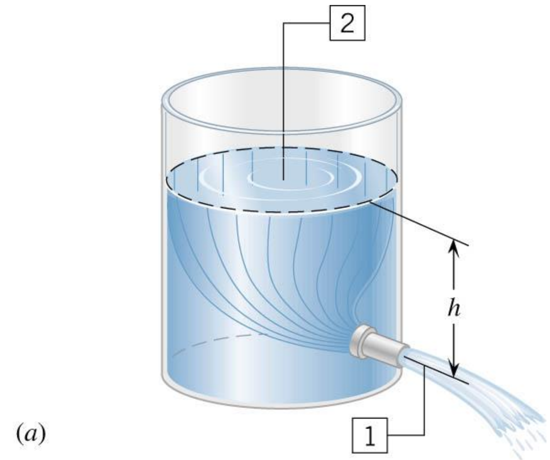
$$v_2 \approx 0$$

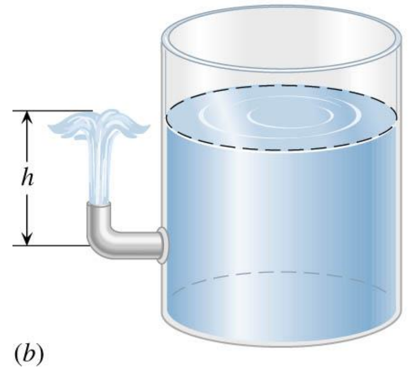
$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

$$y_2 - y_1 = h$$

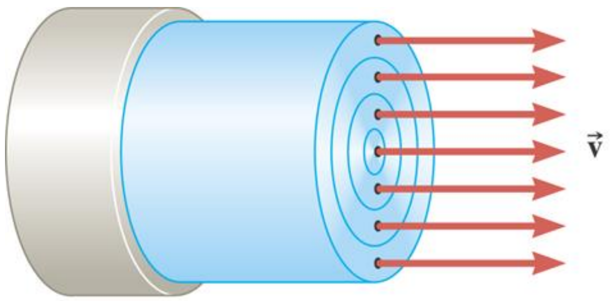
$$\tfrac{1}{2}\rho v_1^2 = \rho g h$$

$$v_1 = \sqrt{2gh}$$



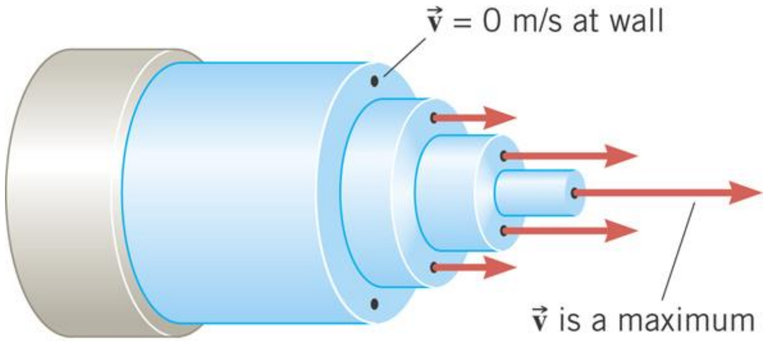


11.11 Viscous Flow



(a)

Flow of an ideal fluid.



(b)

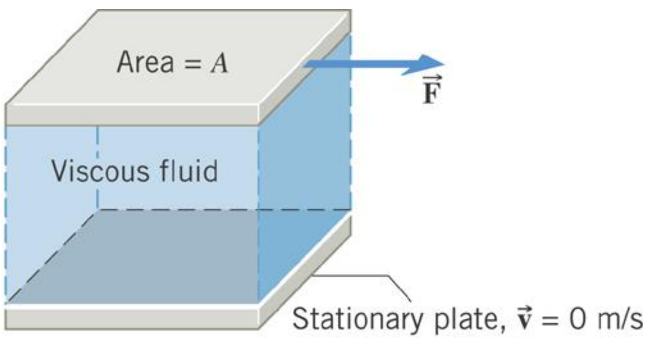
Flow of a viscous fluid.

FORCE NEEDED TO MOVE A LAYER OF VISCOUS FLUID WITH CONSTANT VELOCITY

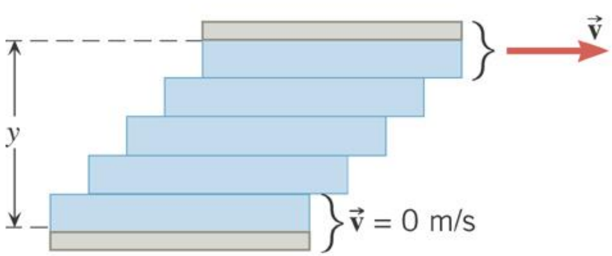
The magnitude of the tangential force required to move a fluid layer at a constant speed is given by:

$$F = \frac{\eta A v}{y}$$

coefficient of viscosity



(a)



(b)

SI Unit of Viscosity: Pa·s

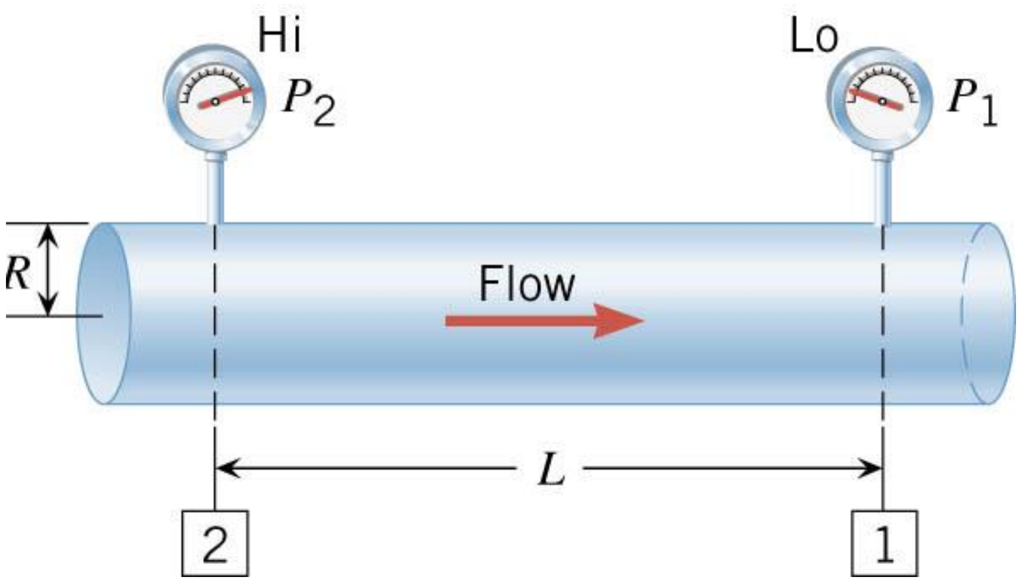
Common Unit of Viscosity: poise (P)

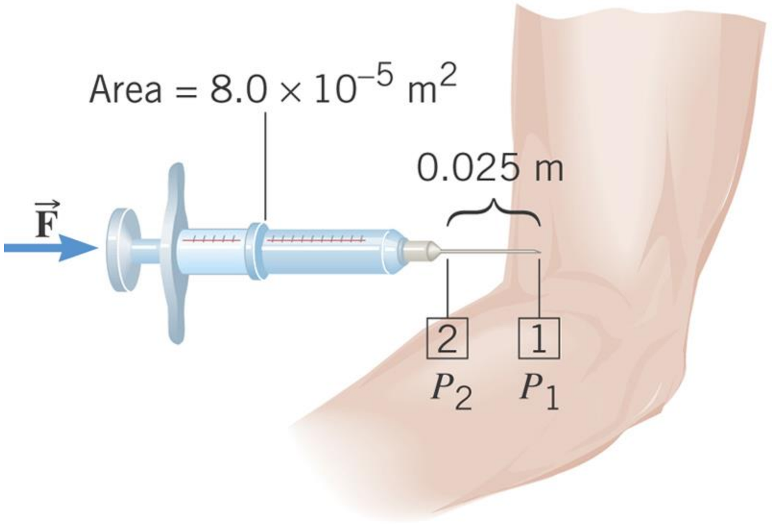
1 poise (P) = 0.1 Pa·s

POISEUILLE'S LAW

The volume flow rate is given by:

$$Q = \frac{\pi R^4 (P_2 - P_1)}{8\eta L}$$

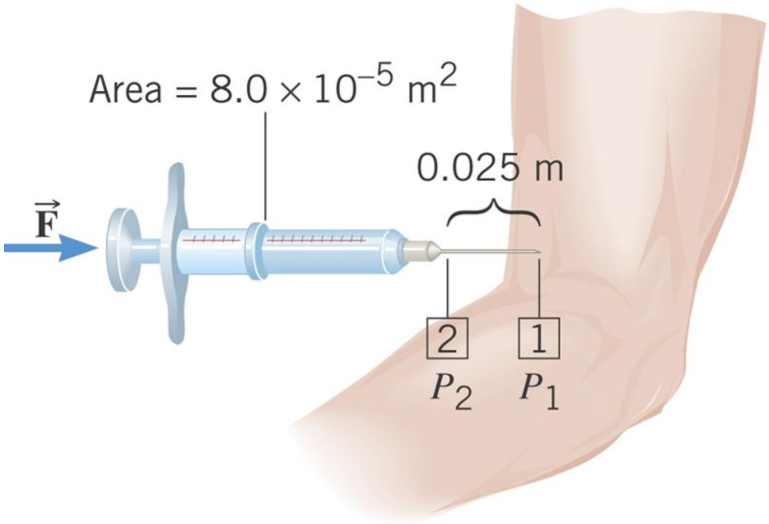




Example 17 Giving and Injection

A syringe is filled with a solution whose viscosity is $1.5x10^{-3}$ Pa·s. The internal radius of the needle is $4.0x10^{-4}$m.

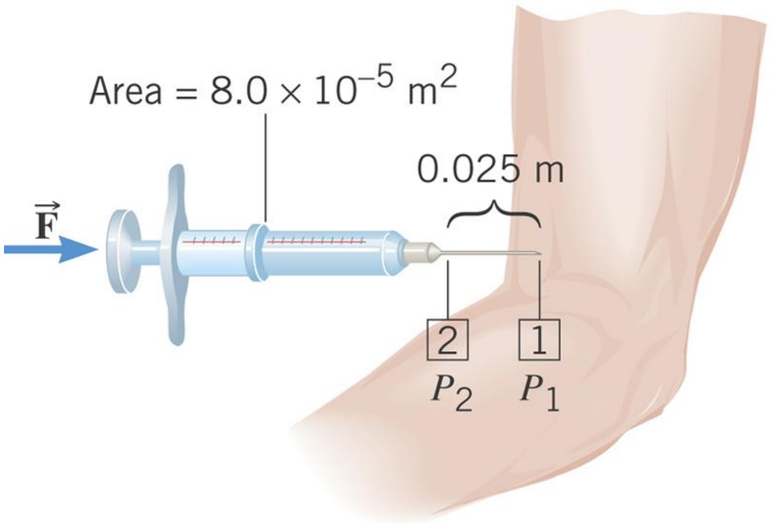
The gauge pressure in the vein is 1900 Pa. What force must be applied to the plunger, so that $1.0x10^{-6}m^3$ of fluid can be injected in 3.0 s?



$$P_2 - P = \frac{8\eta LQ}{\pi R^4}$$

$$= \frac{8\left(1.5\times10^{-3}\,\text{Pa}\cdot\text{s}\right)\left(0.025\,\text{m}\right)\left(1.0\times10^{-6}\,\text{m}^3/3.0\,\text{s}\right)}{\pi\left(4.0\times10^{-4}\,\text{m}\right)^4}$$

$$= 1200\,\text{Pa}$$



$$P_1 = 1900\ \text{Pa}$$

$$P_2 - P_1 = 1200\ \text{Pa}$$

$$\Downarrow$$

$$P_2 = 3100\ \text{Pa}$$

$$F = P_2 A = (3100\ \text{Pa})(8.0\times10^{-5}\ \text{m}^2) = 0.25\text{N}$$