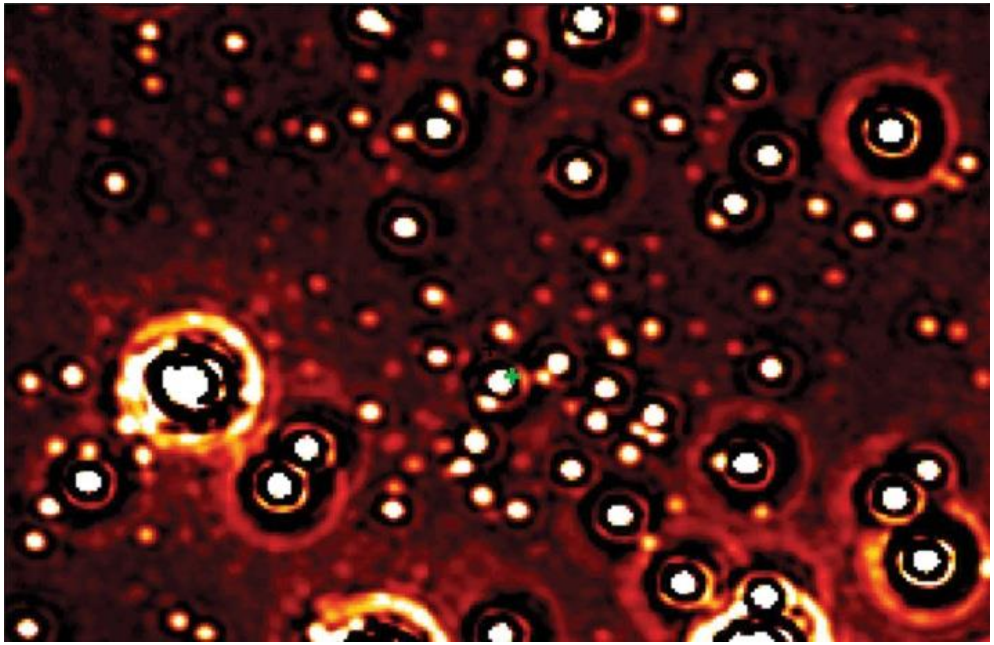
Chapter 13

Gravitation



13.2 Newton's Law of Gravitation

$$F = G\frac{m_1 m_2}{r^2} \qquad \text{(Newton's law of gravitation).}$$

Here m_1 and m_2 are the masses of the particles, r is the distance between them, and G is the gravitational constant.

$$G = 6.67 \text{ x}10^{11} \text{ Nm}^2/\text{kg}^2$$
$$= 6.67 \text{ x}10^{11} \text{ m}^3/\text{kg s}^2.$$

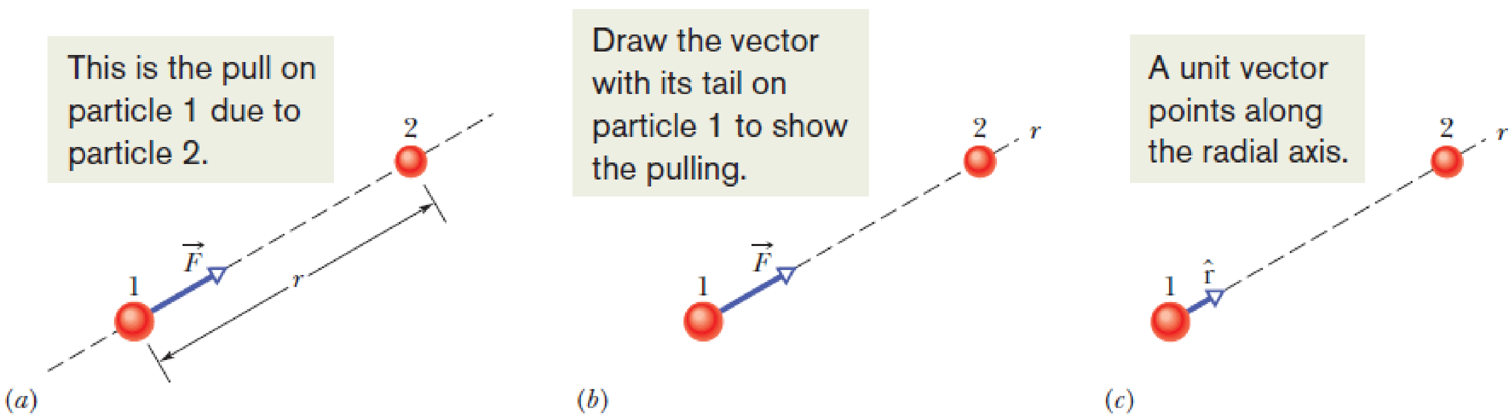


Fig. 13-2 (a) The gravitational force on particle 1 due to particle 2 is an attractive force because particle 1 is attracted to particle 2. (b) Force is directed along a radial coordinate axis r extending from particle 1 through particle 2. (c) is in the direction of a unit vector $\hat{r}$ along the r axis.

13.2 Newton's Law of Gravitation

A uniform spherical shell of matter attracts a particle that is outside the shell as if all the shell's mass were concentrated at its center.

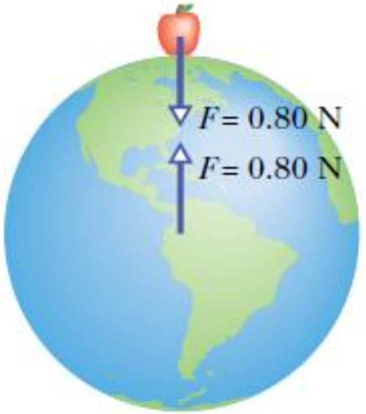


Fig. 13-3 The apple pulls up on Earth just as hard as Earth pulls down on the apple.

12.3 Gravitation and the Principle of Superposition

For n interacting particles, we can write the principle of superposition for the gravitational forces on particle 1 as

$$\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15} + \cdots + \vec{F}_{1n}.$$

Here $\mathbf{F}_{1,\text{net}}$ is the net force on particle 1 due to the other particles and, for example, $\mathbf{F}_{13}$ is the force on particle 1 from particle 3, etc. Therefore,

$$\vec{F}_{1,\text{net}} = \sum_{i=2}^{n} \vec{F}_{1i}.$$

The gravitational force on a particle from a real (extended) object can be expressed as:

$$\vec{F}_1 = \int d\vec{F},$$

Here the integral is taken over the entire extended object .

Example, Net Gravitational Force:

Figure 13-4a shows an arrangement of three particles, particle 1 of mass m_1= 6.0 kg and particles 2 and 3 of mass m_2=m_3=4.0 kg, and distance a =2.0 cm. What is the net gravitational force 1,net on particle 1 due to the other particles?

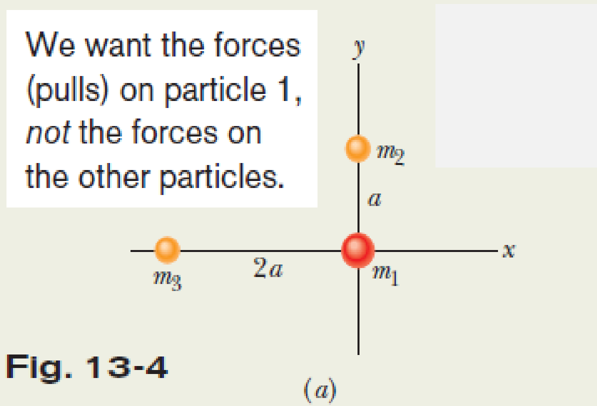


Fig. 13-4

Calculations:

$$F_{12} = \frac{Gm_1m_2}{a^2}$$

$$= \frac{(6.67 \times 10^{-11}\ \mathrm{m^3/kg\cdot s^2})(6.0\ \mathrm{kg})(4.0\ \mathrm{kg})}{(0.020\ \mathrm{m})^2}$$

$$= 4.00 \times 10^{-6}\ \mathrm{N}.$$

$$F_{13} = \frac{Gm_1m_3}{(2a)^2}$$

$$= \frac{(6.67 \times 10^{-11}\ \mathrm{m^3/kg\cdot s^2})(6.0\ \mathrm{kg})(4.0\ \mathrm{kg})}{(0.040\ \mathrm{m})^2}$$

$$= 1.00 \times 10^{-6}\ \mathrm{N}.$$

$$F_{1,net} = \sqrt{(F_{12})^2 + (-F_{13})^2}$$

$$= \sqrt{(4.00 \times 10^{-6}\ \mathrm{N})^2 + (-1.00 \times 10^{-6}\ \mathrm{N})^2}$$

$$= 4.1 \times 10^{-6}\ \mathrm{N}. \qquad \text{(Answer)}$$

Relative to the positive direction of the x axis, the direction of $F_{1,net}$ is:

$$\theta = \tan^{-1}\frac{F_{12}}{-F_{13}} = \tan^{-1}\frac{4.00 \times 10^{-6}\ \mathrm{N}}{-1.00 \times 10^{-6}\ \mathrm{N}} = -76°.$$

13.4: Gravitation Near Earth's Surface

If the particle is released, it will fall toward the center of Earth, as a result of the gravitational force , with an acceleration we shall call the gravitational acceleration a_g. Newton's second law tells us that magnitudes F and a_g are related by

$$F = ma_g.$$

If the Earth is a uniform sphere of mass M, the magnitude of the gravitational force from Earth on a particle of mass m, located outside Earth a distance r from Earth's center, is

$$F = G\frac{Mm}{r^2}.$$

Therefore,

$$a_g = \frac{GM}{r^2}.$$

Table 13-1

Variation of a_g with Altitude

Altitude (km)	a_g (m/s^2)	Altitude Example
0	9.83	Mean Earth surface
8.8	9.80	Mt. Everest
36.6	9.71	Highest crewed balloon
400	8.70	Space shuttle orbit
35 700	0.225	Communications satellite

13.4: Gravitation Near Earth's Surface

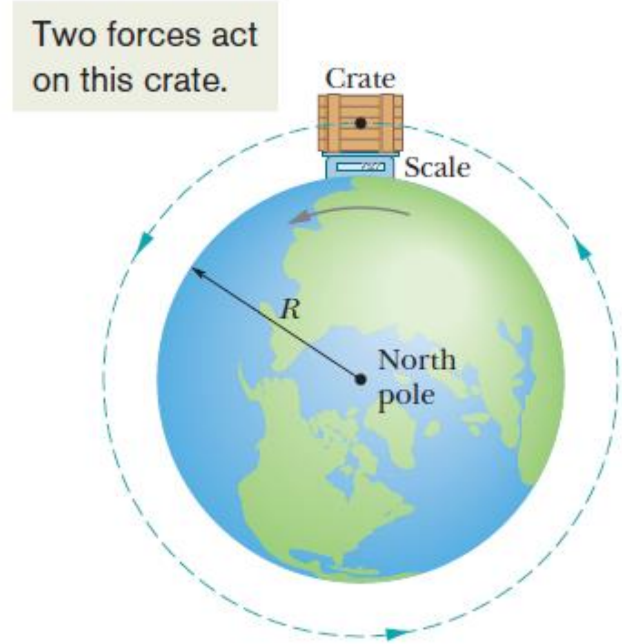
Any g value measured at a given location will differ from the a_g value given before for that location for three reasons:

(1) Earth's mass is not distributed uniformly,

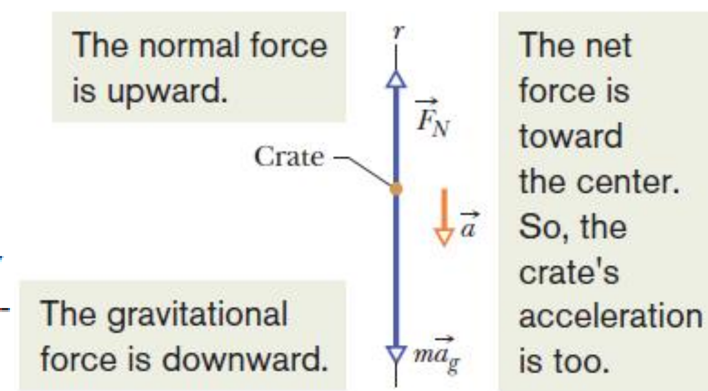
(2) Earth is not a perfect sphere, and

(3) Earth rotates.

For the same three reasons, the measured weight mg of a particle also differs from The magnitude of the gravitational force on the particle.



(a)



(b)

Fig. 13-6 (*a*) A crate sitting on a scale at Earth's equator, as seen by an observer positioned on Earth's rotation axis at some point above the north pole. (*b*) A free-body diagram for the crate, with a radial *r* axis extending from Earth's center. The gravitational force on the crate is represented with its equivalent $m\vec{a}_g$. The normal force on the crate from the scale is $\vec{F}_N$. Because of Earth's rotation, the crate has a centripetal acceleration $\vec{a}$ that is directed toward Earth's center.

Example, Difference in Accelerations

(a) An astronaut whose height h is 1.70 m floats "feet down" in an orbiting space shuttle at distance $r = 6.77 \times 10^6$ m away from the center of Earth. What is the difference between the gravitational acceleration at her feet and at her head?

KEY IDEAS

We can approximate Earth as a uniform sphere of mass M_E. Then, from Eq. 13-11, the gravitational acceleration at any distance r from the center of Earth is

$$a_g = \frac{GM_E}{r^2}. \qquad (13\text{-}15)$$

We might simply apply this equation twice, first with $r = 6.77 \times 10^6$ m for the location of the feet and then with $r = 6.77 \times 10^6 \text{ m} + 1.70 \text{ m}$ for the location of the head. However, a calculator may give us the same value for a_g twice, and thus a difference of zero, because h is so much smaller than r. Here's a more promising approach: Because we have a differential change dr in r between the astronaut's feet and head, we should differentiate Eq. 13-15 with respect to r.

Calculations: The differentiation gives us

$$da_g = -2\frac{GM_E}{r^3}\,dr, \qquad (13\text{-}16)$$

where da_g is the differential change in the gravitational acceleration due to the differential change dr in r. For the astronaut, $dr = h$ and $r = 6.77 \times 10^6$ m. Substituting data into Eq. 13-16, we find

$$da_g = -2\frac{(6.67 \times 10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(5.98 \times 10^{24}\ \text{kg})}{(6.77 \times 10^6\ \text{m})^3}(1.70\ \text{m})$$

$$= -4.37 \times 10^{-6}\ \text{m/s}^2, \qquad \text{(Answer)}$$

where the M_E value is taken from Appendix C. This result means that the gravitational acceleration of the astronaut's feet toward Earth is slightly greater than the gravitational acceleration of her head toward Earth. This difference in acceleration (often called a *tidal effect*) tends to stretch her body, but the difference is so small that she would never even sense the stretching, much less suffer pain from it.

(b) If the astronaut is now "feet down" at the same orbital radius $r = 6.77 \times 10^6$ m about a black hole of mass $M_h = 1.99 \times 10^{31}$ kg (10 times our Sun's mass), what is the difference between the gravitational acceleration at her feet and at her head? The black hole has a mathematical surface (*event horizon*) of radius $R_h = 2.95 \times 10^4$ m. Nothing, not even light, can escape from that surface or anywhere inside it. Note that the astronaut is well outside the surface (at $r = 229R_h$).

Calculations: We again have a differential change dr in r between the astronaut's feet and head, so we can again use Eq. 13-16. However, now we substitute $M_h = 1.99 \times 10^{31}$ kg for M_E. We find

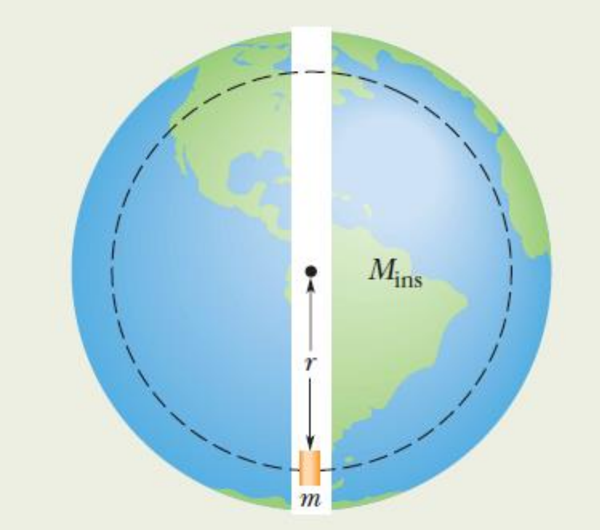
$$da_g = -2\frac{(6.67 \times 10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(1.99 \times 10^{31}\ \text{kg})}{(6.77 \times 10^6\ \text{m})^3}(1.70\ \text{m})$$

$$= -14.5\ \text{m/s}^2. \qquad \text{(Answer)}$$

This means that the gravitational acceleration of the astronaut's feet toward the black hole is noticeably larger than that of her head. The resulting tendency to stretch her body would be bearable but quite painful. If she drifted closer to the black hole, the stretching tendency would increase drastically.

13.5: Gravitation Inside Earth

A uniform shell of matter exerts no net gravitational force on a particle located inside it.



Sample Problem

Three explorers attempt to travel by capsule through a tunnel directly from the south pole to the north pole. According to the story, as the capsule approaches Earth's center, the gravitational force on the explorers becomes alarmingly large and then, exactly at the center, it suddenly but only momentarily disappears. Then the capsule travels through the second half of the tunnel, to the north pole.

Check this story by finding the gravitational force on the capsule of mass *m when it reaches a distance r* from Earth's center. Assume that Earth is a sphere of uniform density ρ (mass per unit volume).

Calculations:

$$F = \frac{GmM_{ins}}{r^2}.$$

$$M_{ins} = \rho V_{ins} = \rho \frac{4\pi r^3}{3}$$

$$F = \frac{4\pi Gm\rho}{3} r.$$

The force magnitude depends linearly on the capsule's distance r from Earth's center. Thus, as r decreases, F also decreases, until it is zero at Earth's center.

13.6: Gravitational Potential Energy

The gravitational potential energy of the two-particle system is:

$$U = -\frac{GMm}{r}$$

U(r) approaches zero as r approaches infinity and that for any finite value of r, the value of U(r) is negative.

If the system contains more than two particles, consider each pair of particles in turn, calculate the gravitational potential energy of that pair with the above relation, as if the other particles were not there, and then algebraically sum the results. That is,

$$U = -\left(\frac{Gm_1m_2}{r_{12}} + \frac{Gm_1m_3}{r_{13}} + \frac{Gm_2m_3}{r_{23}}\right).$$

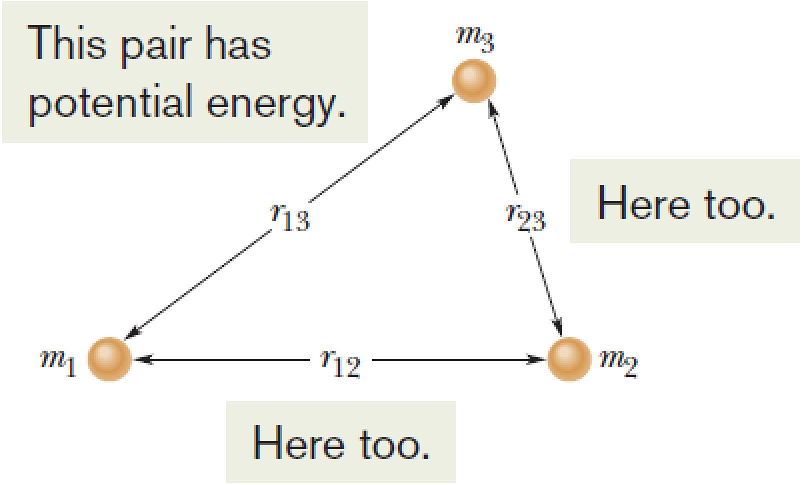
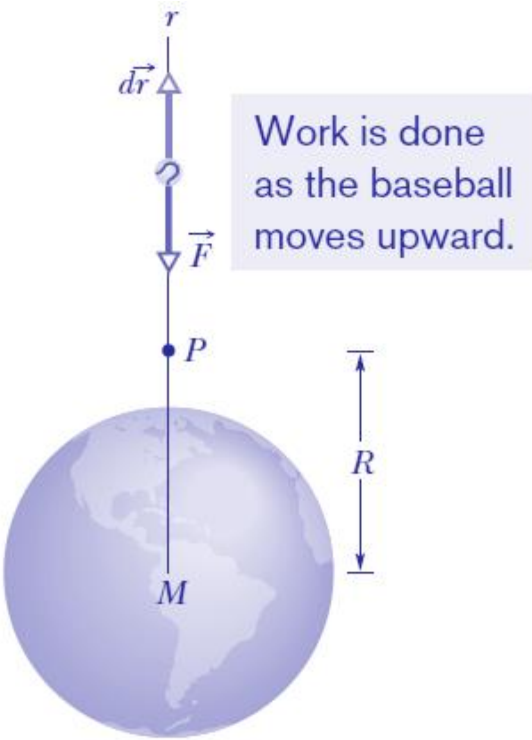


Fig. 13-8 A system consisting of three particles. The gravitational potential energy *of the system* is the sum of the gravitational potential energies of all three pairs of particles.

13.6: Gravitational Potential Energy



Let us shoot a baseball directly away from Earth along the path in the figure. We want to find the gravitational potential energy U of the ball at point P along its path, at radial distance R from Earth's center.

The work W done on the ball by the gravitational force as the ball travels from point P to a great (infinite) distance from Earth is:

$$W = \int_R^\infty \vec{F}(r) \cdot d\vec{r}.$$

$$\vec{F}(r) \cdot d\vec{r} = F(r)\, dr \cos\phi = -\frac{GMm}{r^2}\, dr,$$

$$W = -GMm \int_R^\infty \frac{1}{r^2}\, dr = \left[\frac{GMm}{r}\right]_R^\infty$$

$$= 0 - \frac{GMm}{R} = -\frac{GMm}{R},$$

where W is the work required to move the ball from point P (at distance R) to infinity.

Work can also be expressed in terms of potential energies as

$$U_\infty - U = -W. \Rightarrow U = W = -\frac{GMm}{R}.$$

13.6: Gravitational Potential Energy
Path Independence

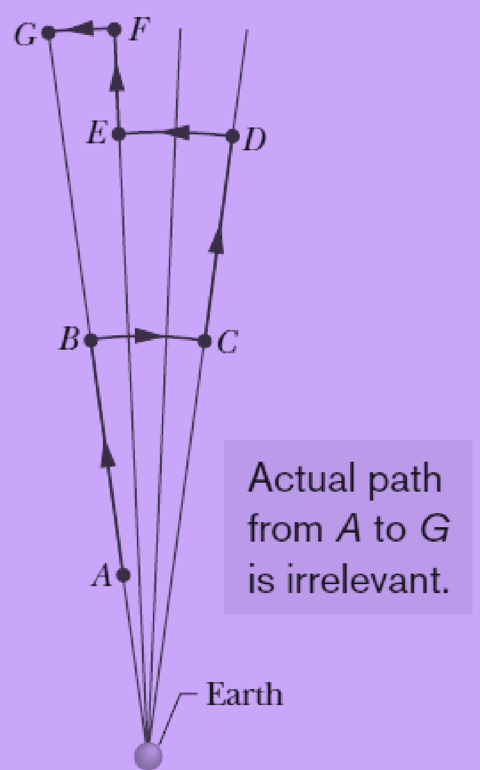


Fig. 13-10 Near Earth, a baseball is moved from point *A* to point *G* along a path consisting of radial lengths and circular arcs.

The work done along each circular arc is zero, because the direction of **F** is perpendicular to the arc at every point. Thus, W is the sum of only the works done by **F** along the three radial lengths.

The gravitational force is a conservative force. Thus, the work done by the gravitational force on a particle moving from an initial point *i* to a final point *f* is independent of the path taken between the points. The change ΔU in the gravitational potential energy from point *i* to point *f* is given by

$$\Delta U = U_f - U_i = -W.$$

Since the work W done by a conservative force is independent of the actual path taken, the change ΔU in gravitational potential energy is **also independent** of the path taken.

13.6: Gravitational Potential Energy: Potential Energy and Force

$$(F(x) = -dU(x)/dx)$$

$$F = -\frac{dU}{dr} = -\frac{d}{dr}\left(-\frac{GMm}{r}\right)$$

$$= -\frac{GMm}{r^2}.$$

The minus sign indicates that the force on mass m points radially inward, toward mass M.

13.6: Gravitational Potential Energy: Escape Speed

If you fire a projectile upward, there is a certain minimum initial speed that will cause it to move upward forever, theoretically coming to rest only at infinity.

This minimum initial speed is called the (Earth) **escape speed.**

Consider a projectile of mass m, leaving the surface of a planet (mass M, radius R) with escape speed v. The projectile has a kinetic energy K given by ½ mv^2, and a potential energy U given by:

$$U = -\frac{GMm}{R}$$

When the projectile reaches infinity, it stops and thus has no kinetic energy. It also has no potential energy because an infinite separation between two bodies is our zero-potential-energy configuration. Its total energy at infinity is therefore zero. From the principle of conservation of energy, its total energy at the planet's surface must also have been zero, and so

$$K + U = \tfrac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right) = 0.$$

This gives the escape speed

$$v = \sqrt{\frac{2GM}{R}}.$$

13.6: Gravitational Potential Energy: Escape Speed

Table 13-2

Some Escape Speeds

Body	Mass (kg)	Radius (m)	Escape Speed (km/s)
Ceres[a]	1.17×10^{21}	3.8×10^{5}	0.64
Earth's moon[a]	7.36×10^{22}	1.74×10^{6}	2.38
Earth	5.98×10^{24}	6.37×10^{6}	11.2
Jupiter	1.90×10^{27}	7.15×10^{7}	59.5
Sun	1.99×10^{30}	6.96×10^{8}	618
Sirius B[b]	2×10^{30}	1×10^{7}	5200
Neutron star[c]	2×10^{30}	1×10^{4}	2×10^{5}

[a] The most massive of the asteroids.
[b] A *white dwarf* (a star in a final stage of evolution) that is a companion of the bright star Sirius.
[c] The collapsed core of a star that remains after that star has exploded in a *supernova* event.

Example:

An asteroid, headed directly toward Earth, has a speed of 12 km/s relative to the planet when the asteroid is 10 Earth radii from Earth's center. Neglecting the effects of Earth's atmosphere on the asteroid, find the asteroid's speed v_f when it reaches Earth's surface.

KEY IDEAS

Because we are to neglect the effects of the atmosphere on the asteroid, the mechanical energy of the asteroid–Earth system is conserved during the fall. Thus, the final mechanical energy (when the asteroid reaches Earth's surface) is equal to the initial mechanical energy. With kinetic energy K and gravitational potential energy U, we can write this as

$$K_f + U_f = K_i + U_i. \tag{13-29}$$

Also, if we assume the system is isolated, the system's linear momentum must be conserved during the fall. Therefore, the momentum change of the asteroid and that of Earth must be equal in magnitude and opposite in sign. However, because Earth's mass is so much greater than the asteroid's mass, the change in Earth's speed is negligible relative to the change in the asteroid's speed. So, the change in Earth's kinetic energy is also negligible. Thus, we can assume that the kinetic energies in Eq. 13-29 are those of the asteroid alone.

Calculations: Let m represent the asteroid's mass and M represent Earth's mass (5.98×10^{24} kg). The asteroid is initially at distance $10R_E$ and finally at distance R_E, where R_E is Earth's radius (6.37×10^6 m). Substituting Eq. 13-21 for U and $\frac{1}{2}mv^2$ for K, we rewrite Eq. 13-29 as

$$\frac{1}{2}mv_f^2 - \frac{GMm}{R_E} = \frac{1}{2}mv_i^2 - \frac{GMm}{10R_E}.$$

Rearranging and substituting known values, we find

$$\begin{aligned} v_f^2 &= v_i^2 + \frac{2GM}{R_E}\left(1 - \frac{1}{10}\right) \\ &= (12 \times 10^3 \text{ m/s})^2 \\ &\quad + \frac{2(6.67 \times 10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2)(5.98 \times 10^{24} \text{ kg})}{6.37 \times 10^6 \text{ m}} 0.9 \\ &= 2.567 \times 10^8 \text{ m}^2/\text{s}^2, \end{aligned}$$

and

$$v_f = 1.60 \times 10^4 \text{ m/s} = 16 \text{ km/s}. \qquad \text{(Answer)}$$

At this speed, the asteroid would not have to be particularly large to do considerable damage at impact. If it were only 5 m across, the impact could release about as much energy as the nuclear explosion at Hiroshima. Alarmingly, about 500 million asteroids of this size are near Earth's orbit, and in 1994 one of them apparently penetrated Earth's atmosphere and exploded 20 km above the South Pacific (setting off nuclear-explosion warnings on six military satellites). The impact of an asteroid 500 m across (there may be a million of them near Earth's orbit) could end modern civilization and almost eliminate humans worldwide.

13.7: Planets and Satellites: Kepler's Laws

1. THE LAW OF ORBITS: All planets move in elliptical orbits, with the Sun at one focus.

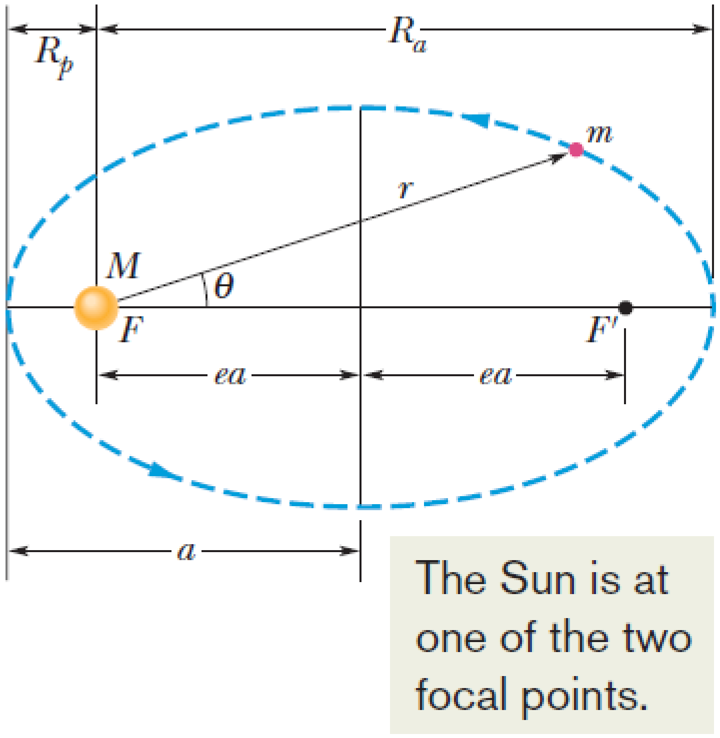


Fig. 13-12 A planet of mass m moving in an elliptical orbit around the Sun. The Sun, of mass M, is at one focus F of the ellipse. The other focus is F', which is located in empty space. Each focus is a distance ea from the ellipse's center, with e being the eccentricity of the ellipse. The semimajor axis a of the ellipse, the perihelion (nearest the Sun) distance R_p, and the aphelion (farthest from the Sun) distance R_a are also shown.

13.7: Planets and Satellites: Kepler's Laws

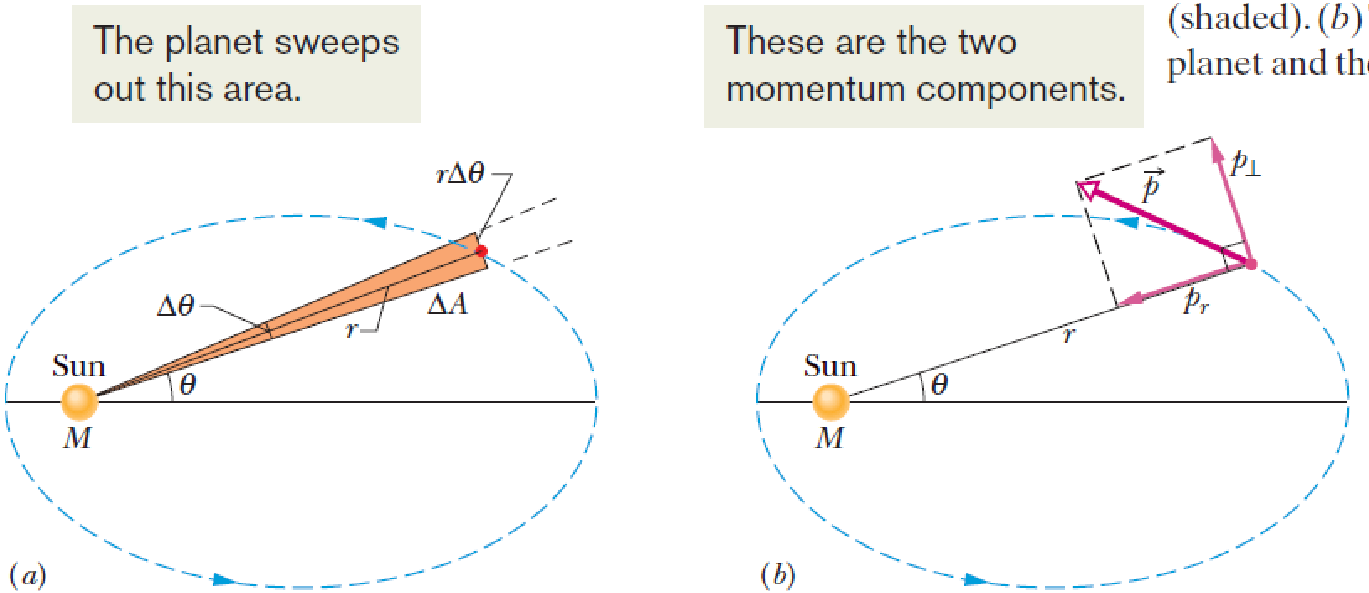


Fig. 13-13 (*a*) In time Δt, the line r connecting the planet to the Sun moves through an angle $\Delta\theta$, sweeping out an area ΔA (shaded). (*b*) The linear momentum $\vec{p}$ of the planet and the components of $\vec{p}$.

2. THE LAW OF AREAS: A line that connects a planet to the Sun sweeps out equal areas in the plane of the planet's orbit in equal time intervals; that is, the rate dA/dt at which it sweeps out area A is constant.

$$\frac{dA}{dt} = \tfrac{1}{2}r^2\frac{d\theta}{dt} = \tfrac{1}{2}r^2\omega,$$

Angular momentum, L:

$$L = rp_\perp = (r)(mv_\perp) = (r)(m\omega r) = mr^2\omega,$$

$$\frac{dA}{dt} = \frac{L}{2m}.$$

3. THE LAW OF PERIODS: The square of the period of any planet is proportional to the cube of the semimajor axis of its orbit.

Consider a circular orbit with radius r (the radius of a circle is equivalent to the semimajor axis of an ellipse). Applying Newton's second law to the orbiting planet yields

$$\frac{GMm}{r^2} = (m)(\omega^2 r).$$

Using the relation of the angular velocity, ω, and the period, T, one gets:

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3 \qquad \text{(law of periods).}$$

Table 13-3

Kepler's Law of Periods for the Solar System

Planet	Semimajor Axis a (10^{10} m)	Period T (y)	T^2/a^3 (10^{-34} y^2/m^3)
Mercury	5.79	0.241	2.99
Venus	10.8	0.615	3.00
Earth	15.0	1.00	2.96
Mars	22.8	1.88	2.98
Jupiter	77.8	11.9	3.01
Saturn	143	29.5	2.98
Uranus	287	84.0	2.98
Neptune	450	165	2.99
Pluto	590	248	2.99

Example, Halley's Comet

Comet Halley orbits the Sun with a period of 76 years and, in 1986, had a distance of closest approach to the Sun, its *perihelion distance* R_p, of 8.9×10^{10} m. Table 13-3 shows that this is between the orbits of Mercury and Venus.

(a) What is the comet's farthest distance from the Sun, which is called its *aphelion distance* R_a?

KEY IDEAS

From Fig. 13-12, we see that $R_a + R_p = 2a$, where a is the semimajor axis of the orbit. Thus, we can find R_a if we first find a. We can relate a to the given period via the law of periods (Eq. 13-34) if we simply substitute the semimajor axis a for r.

Calculations: Making that substitution and then solving for a, we have

$$a = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3}. \qquad (13\text{-}35)$$

If we substitute the mass M of the Sun, 1.99×10^{30} kg, and the period T of the comet, 76 years or 2.4×10^9 s, into Eq. 13-35, we find that $a = 2.7 \times 10^{12}$ m. Now we have

$$\begin{aligned} R_a &= 2a - R_p \\ &= (2)(2.7 \times 10^{12}\text{ m}) - 8.9 \times 10^{10}\text{ m} \\ &= 5.3 \times 10^{12}\text{ m}. \qquad \text{(Answer)} \end{aligned}$$

Table 13-3 shows that this is a little less than the semimajor axis of the orbit of Pluto. Thus, the comet does not get farther from the Sun than Pluto.

(b) What is the eccentricity e of the orbit of comet Halley?

KEY IDEA

We can relate e, a, and R_p via Fig. 13-12, in which we see that $ea = a - R_p$.

Calculation: We have

$$e = \frac{a - R_p}{a} = 1 - \frac{R_p}{a} \qquad (13\text{-}36)$$

$$= 1 - \frac{8.9 \times 10^{10}\text{ m}}{2.7 \times 10^{12}\text{ m}} = 0.97. \qquad \text{(Answer)}$$

This tells us that, with an eccentricity approaching unity, this orbit must be a long thin ellipse.

13.8: Satellites: Orbits and Energy

As a satellite orbits Earth in an elliptical path, the mechanical energy E of the satellite remains constant. Assume that the satellite's mass is so much smaller than Earth's mass.

The potential energy of the system is given by

$$U = -\frac{GMm}{r}$$

For a satellite in a circular orbit,

$$\frac{GMm}{r^2} = m\frac{v^2}{r},$$

Thus, one gets:

$$E = K + U = \frac{GMm}{2r} - \frac{GMm}{r}$$

$$E = -\frac{GMm}{2r} \qquad \text{(circular orbit).}$$

For an elliptical orbit (semimajor axis a),

$$E = -\frac{GMm}{2a}$$

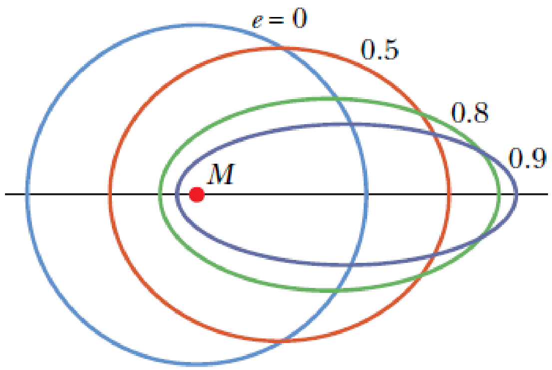
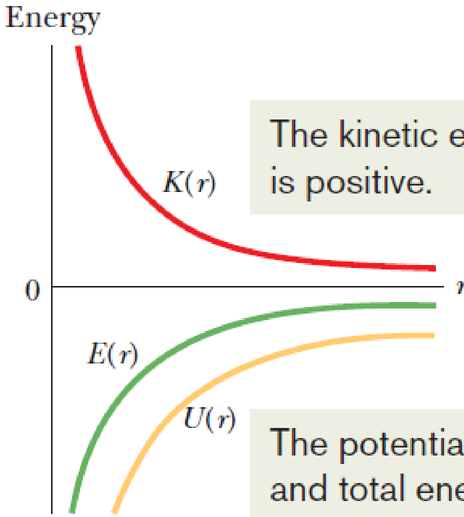


Fig. 13-15 Four orbits with different eccentricities e about an object of mass M. All four orbits have the same semimajor axis a and thus correspond to the same total mechanical energy E.

This is a plot of a satellite's energies versus orbit radius.



The kinetic energy is positive.

The potential energy and total energy are negative.

Example, Mechanical Energy of a Bowling Ball

A playful astronaut releases a bowling ball, of mass $m = 7.20$ kg, into circular orbit about Earth at an altitude h of 350 km.

(a) What is the mechanical energy E of the ball in its orbit?

KEY IDEA

We can get E from the orbital energy, given by Eq. 13-40 ($E = -GMm/2r$), if we first find the orbital radius r. (It is *not* simply the given altitude.)

Calculations: The orbital radius must be

$$r = R + h = 6370\text{ km} + 350\text{ km} = 6.72 \times 10^6\text{ m},$$

in which R is the radius of Earth. Then, from Eq. 13-40, the mechanical energy is

$$E = -\frac{GMm}{2r}$$
$$= -\frac{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\text{ kg})(7.20\text{ kg})}{(2)(6.72 \times 10^6\text{ m})}$$
$$= -2.14 \times 10^8\text{ J} = -214\text{ MJ.} \qquad \text{(Answer)}$$

(b) What is the mechanical energy E_0 of the ball on the launchpad at Cape Canaveral (before it, the astronaut, and the spacecraft are launched)? From there to the orbit, what is the change ΔE in the ball's mechanical energy?

KEY IDEA

On the launchpad, the ball is *not* in orbit and thus Eq. 13-40 does *not* apply. Instead, we must find $E_0 = K_0 + U_0$, where K_0 is the ball's kinetic energy and U_0 is the gravitational potential energy of the ball–Earth system.

Calculations: To find U_0, we use Eq. 13-21 to write

$$U_0 = -\frac{GMm}{R}$$
$$= -\frac{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\text{ kg})(7.20\text{ kg})}{6.37 \times 10^6\text{ m}}$$
$$= -4.51 \times 10^8\text{ J} = -451\text{ MJ.}$$

The kinetic energy K_0 of the ball is due to the ball's motion with Earth's rotation. You can show that K_0 is less than 1 MJ, which is negligible relative to U_0. Thus, the mechanical energy of the ball on the launchpad is

$$E_0 = K_0 + U_0 \approx 0 - 451\text{ MJ} = -451\text{ MJ.} \qquad \text{(Answer)}$$

The *increase* in the mechanical energy of the ball from launchpad to orbit is

$$\Delta E = E - E_0 = (-214\text{ MJ}) - (-451\text{ MJ})$$
$$= 237\text{ MJ.} \qquad \text{(Answer)}$$

This is worth a few dollars at your utility company. Obviously the high cost of placing objects into orbit is not due to their required mechanical energy.

13.9: Einstein and Gravitation

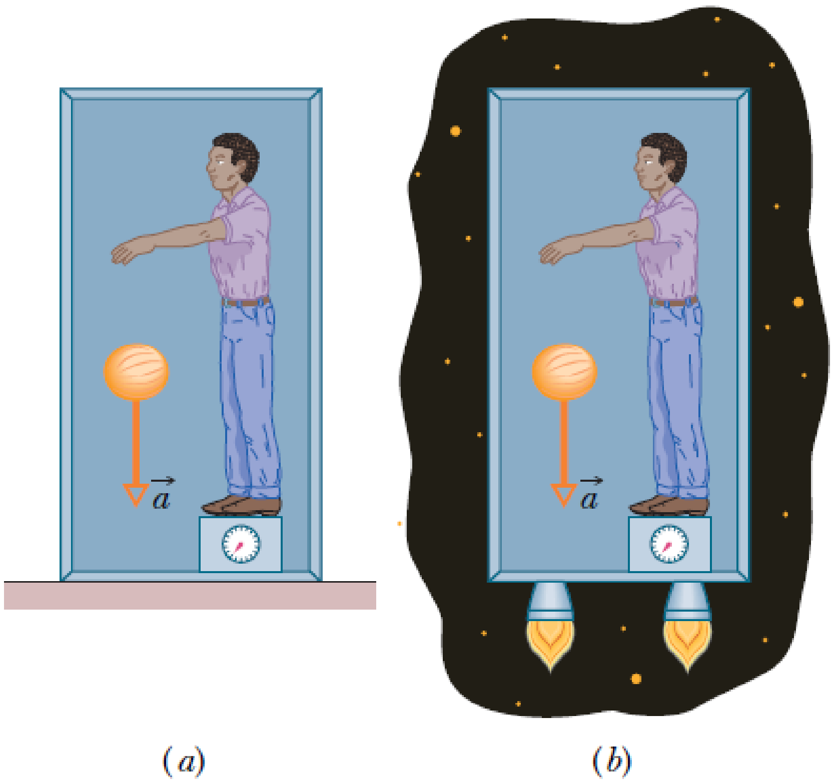


Fig. 13-17 (*a*) A physicist in a box resting on Earth sees a cantaloupe falling with acceleration $a = 9.8\ m/s^2$. (*b*) If he and the box accelerate in deep space at $9.8\ m/s^2$, the cantaloupe has the same acceleration relative to him. It is not possible, by doing experiments within the box, for the physicist to tell which situation he is in. For example, the platform scale on which he stands reads the same weight in both situations.

The fundamental postulate of Einstein's general theory of relativity about gravitation (the gravitating of objects toward each other) is called the **principle of equivalence,** which says that gravitation and acceleration are equivalent.

13.9: Einstein and Gravitation: Curvature of Space

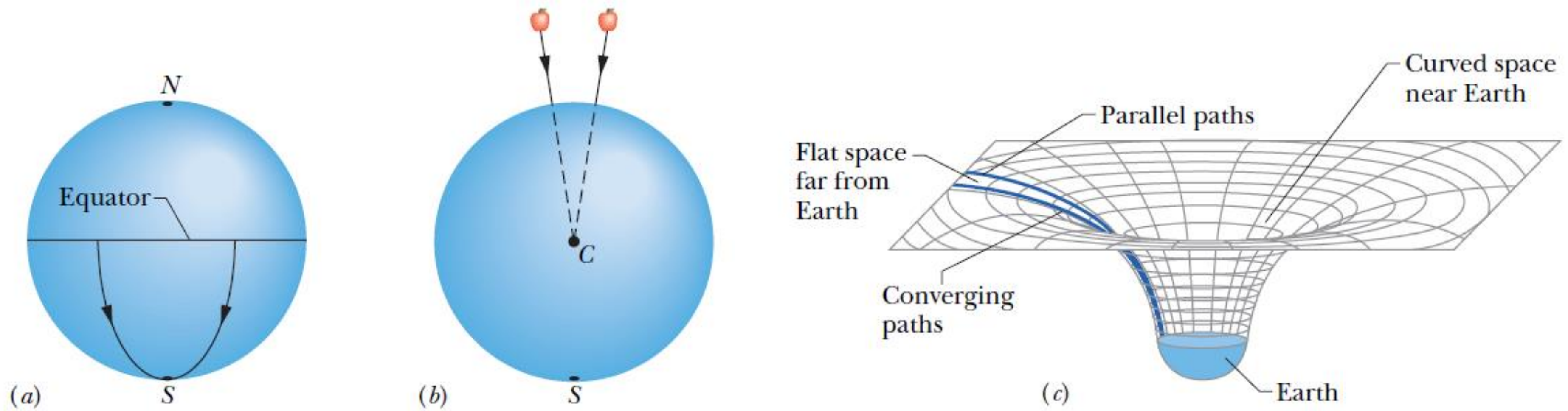
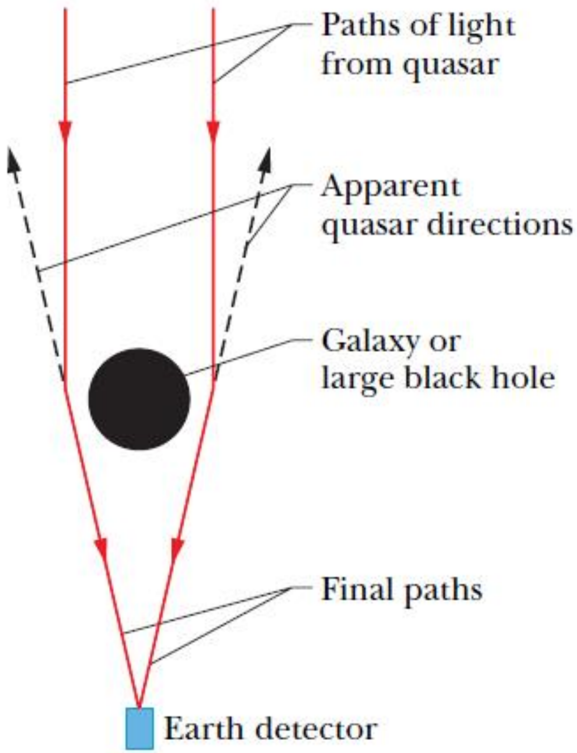


Fig. 13-18 (*a*) Two objects moving along lines of longitude toward the south pole converge because of the curvature of Earth's surface. (*b*) Two objects falling freely near Earth move along lines that converge toward the center of Earth because of the curvature of space near Earth. (*c*) Far from Earth (and other masses), space is flat and parallel paths remain parallel. Close to Earth, the parallel paths begin to converge because space is curved by Earth's mass.

13.9: Einstein and Gravitation: Curvature of Space



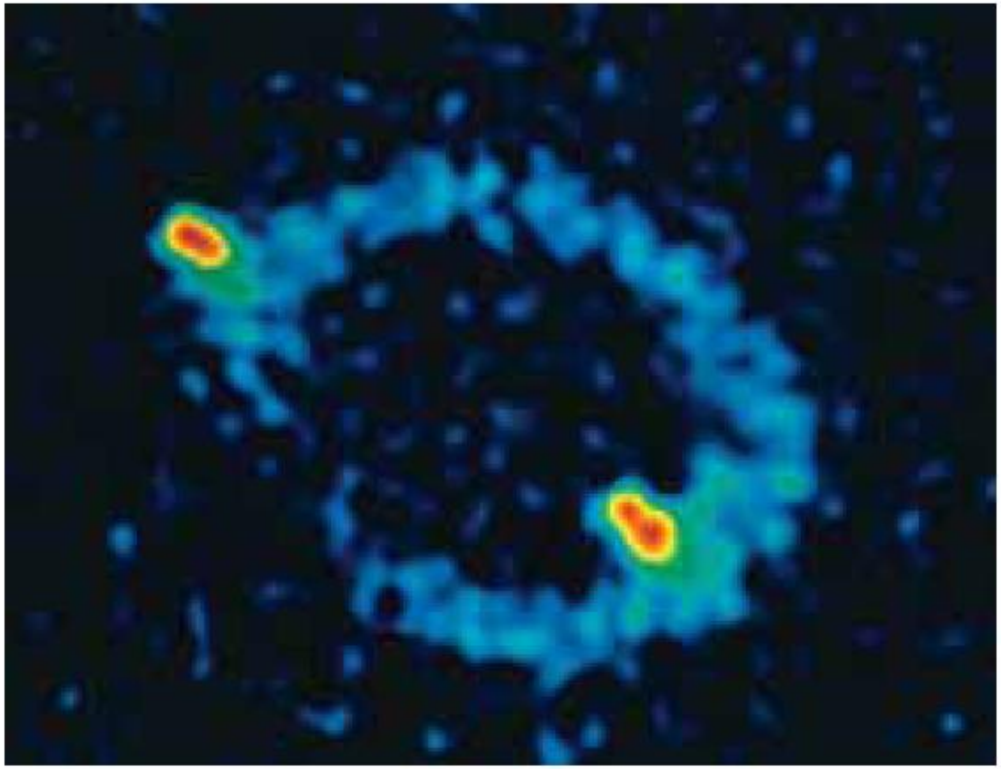


Fig. 13-19 (*a*) Light from a distant quasar follows curved paths around a galaxy or a large black hole because the mass of the galaxy or black hole has curved the adjacent space. If the light is detected, it appears to have originated along the backward extensions of the final paths (dashed lines). (*b*) The Einstein ring known as MG1131+0456 on the computer screen of a telescope. The source of the light (actually, radio waves, which are a form of invisible light) is far behind the large, unseen galaxy that produces the ring; a portion of the source appears as the two bright spots seen along the ring. (*Courtesy National Radio Astronomy Observatory*)