Chapter 14

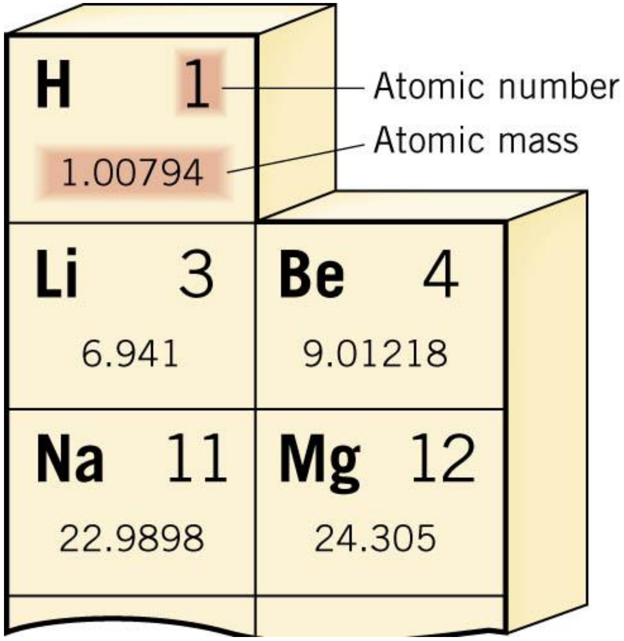
The Ideal Gas Law and Kinetic Theory

14.1 Molecular Mass, the Mole, and Avogadro's Number

To facilitate comparison of the mass of one atom with another, a mass scale know as the ***atomic mass scale*** has been established.

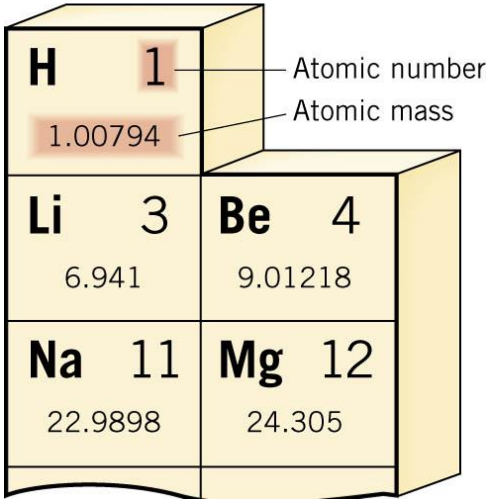
The unit is called the ***atomic mass unit*** (symbol u). The reference element is chosen to be the most abundant isotope of carbon, which is called carbon-12.

$$1\,\mathrm{u} = 1.6605\times10^{-27}\,\mathrm{kg}$$



The atomic mass is given in atomic mass units. For example, a Li atom has a mass of 6.941u.

14.1 Molecular Mass, the Mole, and Avogadro's Number



One ***mole*** of a substance contains as many particles as there are atoms in 12 grams of the isotope carbon-12.

The number of atoms per mole is known as Avogadro's number, N_A.

$$N_A = 6.022 \times 10^{23}\,\text{mol}^{-1}$$

$$n = \frac{N}{N_A}$$

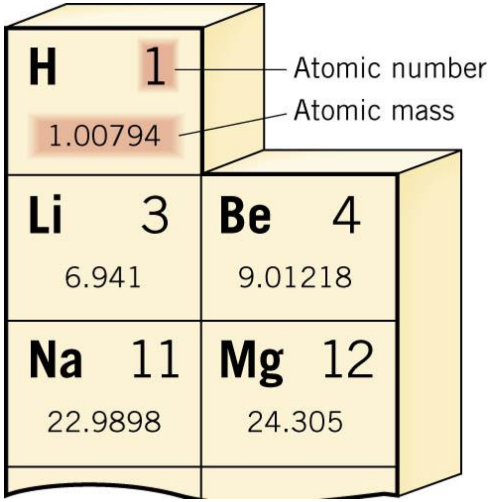
number of moles

number of atoms

$$n = \frac{m_{\text{particle}} N}{m_{\text{particle}} N_A} = \frac{m}{\text{Mass per mole}}$$

The mass per mole (in g/mol) of a substance has the same numerical value as the atomic or molecular mass of the substance (in atomic mass units).

For example Hydrogen has an atomic mass of 1.00794 g/mol, while the mass of a single hydrogen atom is 1.00794 u.



Example 1 The Hope Diamond and the Rosser Reeves Ruby

The Hope diamond (44.5 carats) is almost pure carbon. The Rosser Reeves ruby (138 carats) is primarily aluminum oxide (Al_2O_3). One carat is equivalent to a mass of 0.200 g. Determine (a) the number of carbon atoms in the Hope diamond and (b) the number of Al_2O_3 molecules in the ruby.

(a)

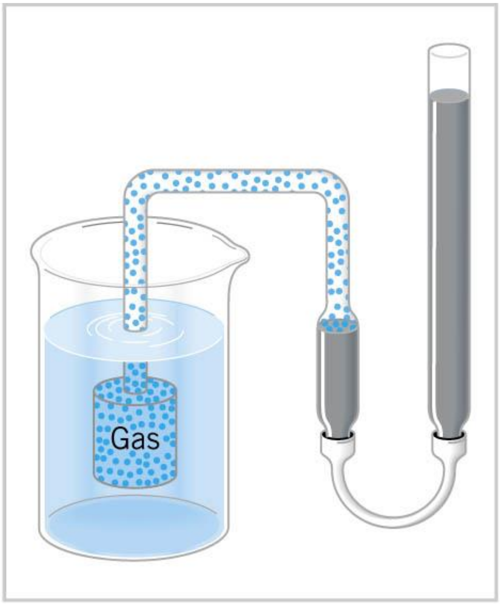
$$n = \frac{m}{\text{Mass per mole}} = \frac{(44.5\text{ carats})[(0.200\text{ g})/(1\text{ carat})]}{12.011\text{ g/mol}} = 0.741\text{ mol}$$

$$N = nN_A = (0.741\text{ mol})(6.022\times10^{23}\text{ mol}^{-1}) = 4.46\times10^{23}\text{ atoms}$$

(b)

$$n = \frac{m}{\text{Mass per mole}} = \frac{(138\text{ carats})[(0.200\text{ g})/(1\text{ carat})]}{\underbrace{101.96}_{2(26.98)+3(15.99)}\text{ g/mol}} = 0.271\text{ molecules}$$

$$N = nN_A = (0.271\text{ mol})(6.022\times10^{23}\text{ mol}^{-1}) = 1.63\times10^{23}\text{ atoms}$$

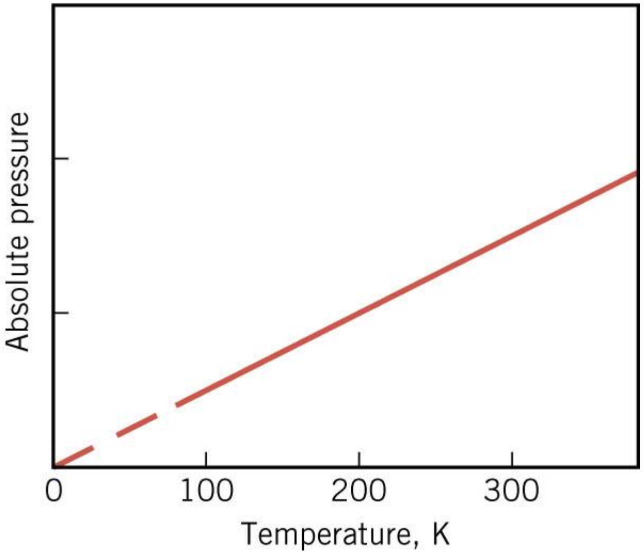


An ***ideal gas*** is an idealized model for real gases that have sufficiently low densities.

The condition of low density means that the molecules are so far apart that they do not interact except during collisions, which are effectively elastic.

At constant volume the pressure is proportional to the temperature.

$$P \propto T$$

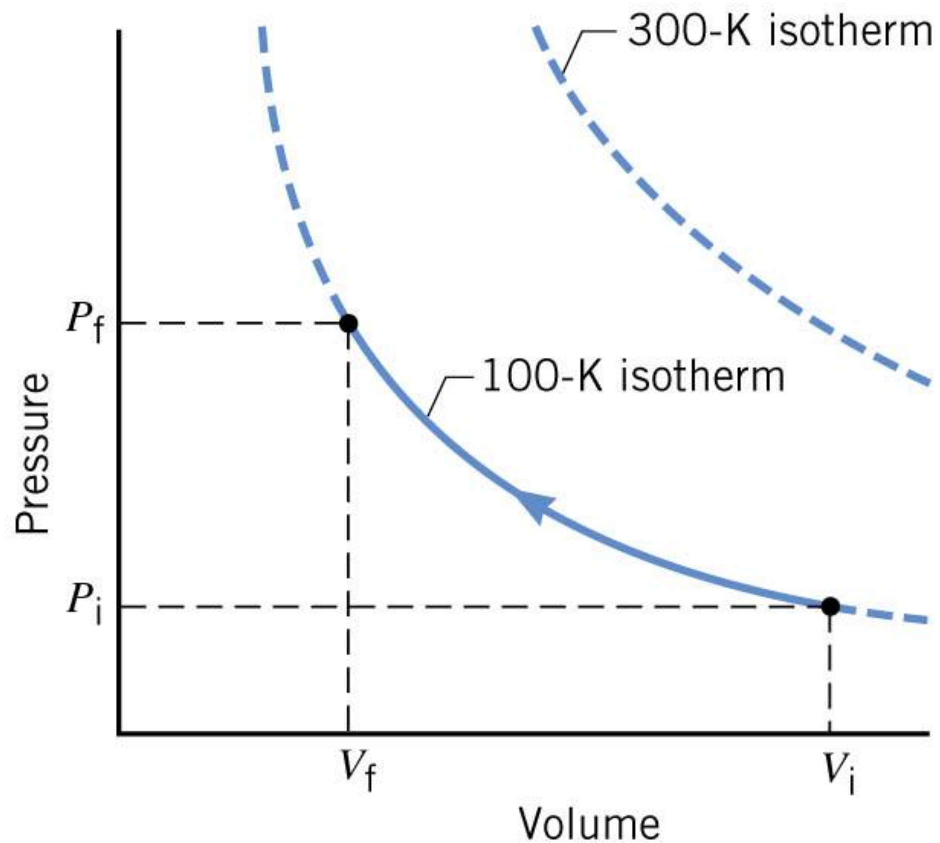


At constant temperature, the pressure is inversely proportional to the volume.

$$P \propto 1/V$$

The pressure is also proportional to the amount of gas.

$$P \propto n$$



THE IDEAL GAS LAW

The absolute pressure of an ideal gas is directly proportional to the Kelvin temperature and the number of moles of the gas and is inversely proportional to the volume of the gas.

$$P = \frac{nRT}{V}$$

$$PV = nRT$$

$$R = 8.31\,\mathrm{J}/(\mathrm{mol}\cdot\mathrm{K})$$

$$n = \frac{N}{N_A}$$

$$PV = nRT = N\left(\frac{R}{N_A}\right)T = NkT$$

$$k = \frac{R}{N_A} = \frac{8.31\,\mathrm{J}/(\mathrm{mol}\cdot\mathrm{K})}{6.022\times10^{23}\,\mathrm{mol}^{-1}} = 1.38\times10^{-23}\,\mathrm{J/K}$$

Example 2 Oxygen in the Lungs

In the lungs, the respiratory membrane separates tiny sacs of air (pressure $1.00\text{x}10^5$Pa) from the blood in the capillaries. These sacs are called alveoli. The average radius of the alveoli is 0.125 mm, and the air inside contains 14% oxygen. Assuming that the air behaves as an ideal gas at 310K, find the number of oxygen molecules in one of these sacs.

$$PV = NkT$$

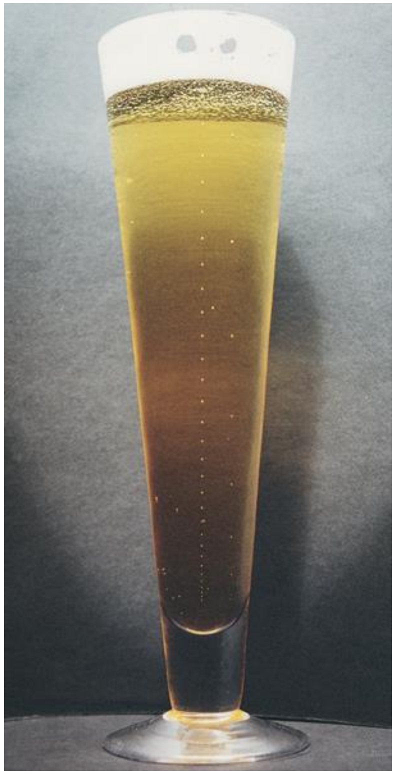
$$N = \frac{PV}{kT} = \frac{\left(1.00\times10^{5}\,\text{Pa}\right)\left[\frac{4}{3}\pi\left(0.125\times10^{-3}\,\text{m}\right)^{3}\right]}{\left(1.38\times10^{-23}\,\text{J/K}\right)\left(310\,\text{K}\right)}$$

$$= 1.9\times10^{14}\ \text{molecules of air}$$

$$\text{Number of molecules of oxygen in one sac} =$$
$$\left(1.9\times10^{14}\right)\times\left(0.14\right) = 2.7\times10^{13}\,\text{molecules}$$

Conceptual Example 3 **Beer Bubbles on the Rise**

Watch the bubbles rise in a glass of beer. If you look carefully, you'll see them grow in size as they move upward, often doubling in volume by the time they reach the surface. Why does the bubble grow as it ascends?



Consider a sample of an ideal gas that is taken from an initial to a final state, with the amount of the gas remaining constant.

$$PV = nRT \Longrightarrow \frac{PV}{T} = nR = \text{constant}$$

$$\Downarrow$$

$$\frac{P_f V_f}{T_f} = \frac{P_i V_i}{T_i}$$

$$\frac{P_f V_f}{T_f} = \frac{P_i V_i}{T_i}$$

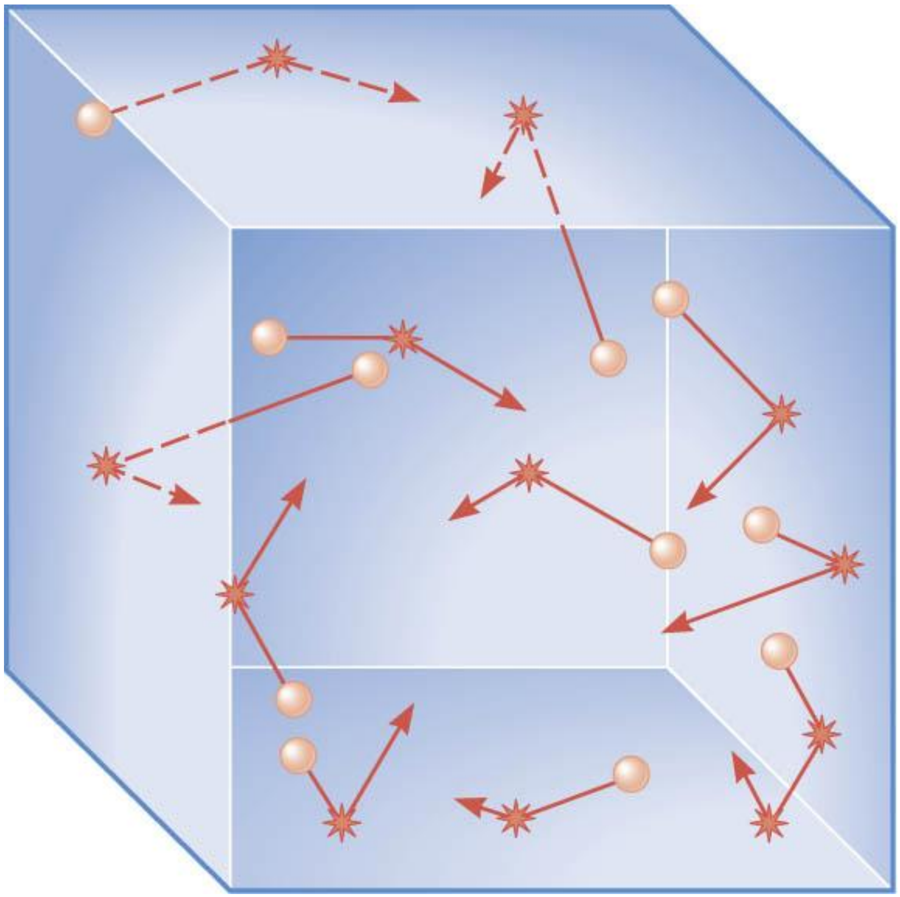
Constant T, constant n: $$P_f V_f = P_i V_i$$ *Boyle's law*

Constant P, constant n: $$\frac{V_f}{T_f} = \frac{V_i}{T_i}$$ *Charles' law*

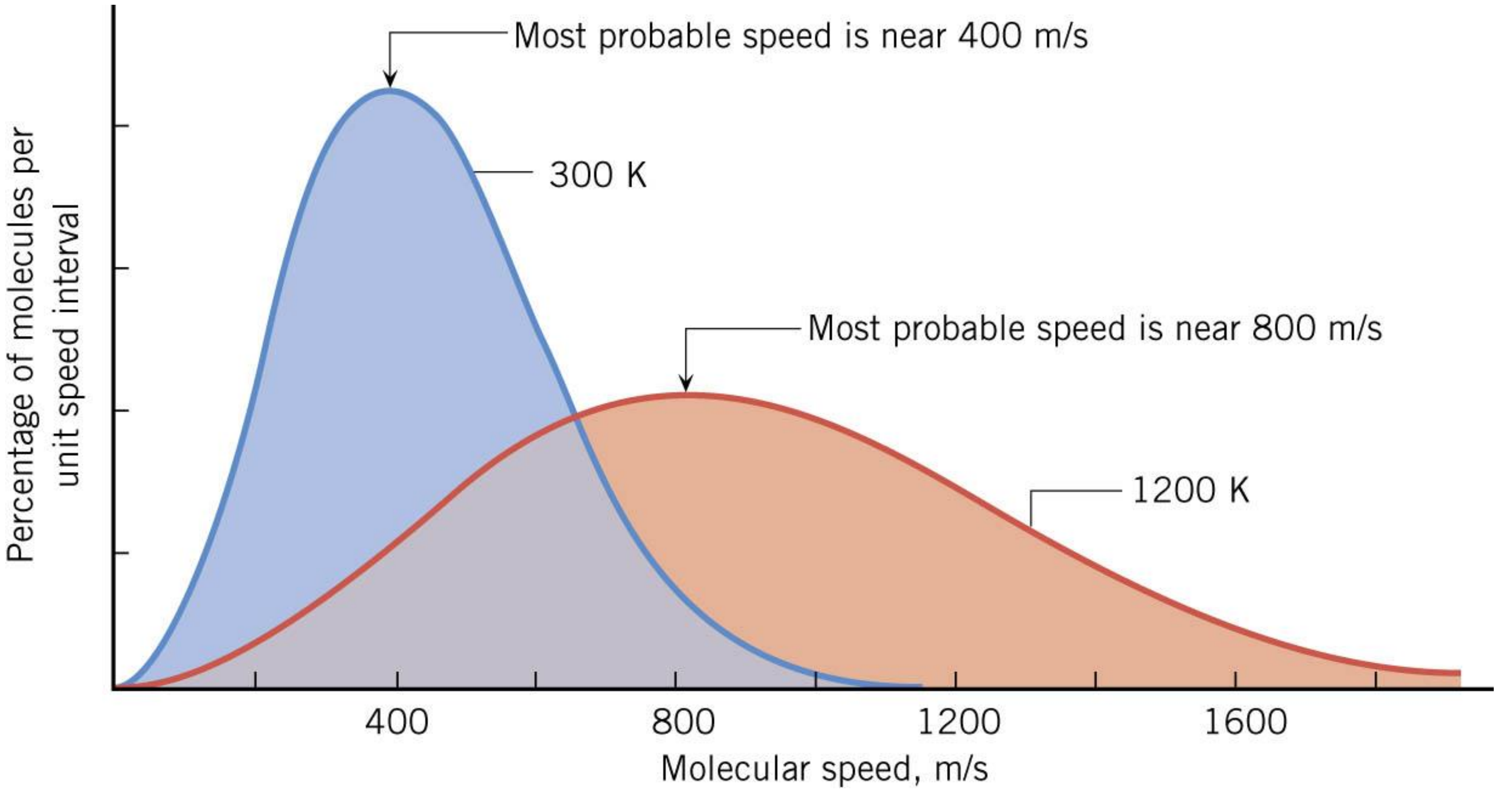
The particles are in constant, random motion, colliding with each other and with the walls of the container.

Each collision changes the particle's speed.

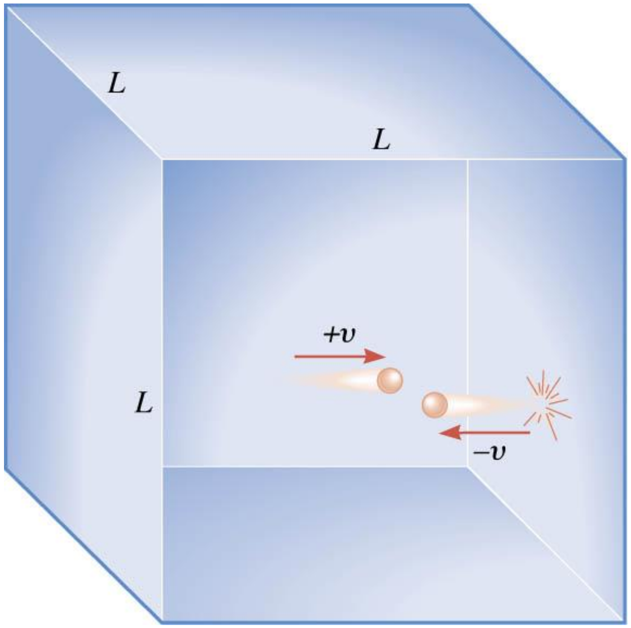
As a result, the atoms and molecules have different speeds.



THE DISTRIBUTION OF MOLECULAR SPEEDS



KINETIC THEORY

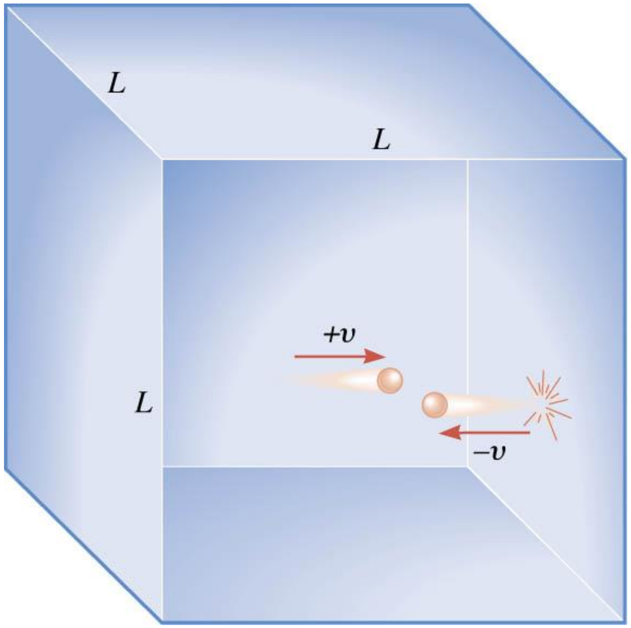


$$\sum F = ma = m\frac{\Delta v}{\Delta t} = \frac{\Delta(mv)}{\Delta t}$$

$$\text{Average force} = \frac{\text{Final momentum - Initial momentum}}{\text{Time between successive collisions}}$$

$$= \frac{(-mv)-(+mv)}{2L/v} = \frac{-mv^2}{L}$$

14.3 *Kinetic Theory of Gases*



For a single molecule, the average force is:

$$F = \frac{mv^2}{L}$$

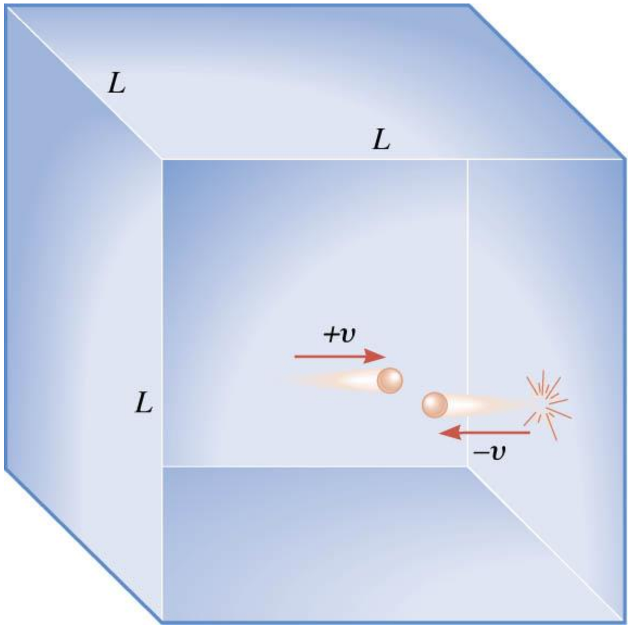
For N molecules, the average force is:

$$F = \left(\frac{N}{3}\right)\left(\frac{m\overline{v^2}}{L}\right)$$

root-mean-square speed

$$P = \frac{F}{A} = \frac{F}{L^2} = \left(\frac{N}{3}\right)\left(\frac{m\overline{v^2}}{L^3}\right)$$

volume



$$P = \left(\frac{N}{3}\right)\left(\frac{m\overline{v^2}}{V}\right)$$

⇩

NkT $\qquad$ $\overline{\text{KE}}$

$$PV = \tfrac{1}{3}N\left(mv_{rms}^2\right) = \tfrac{2}{3}N\left(\tfrac{1}{2}mv_{rms}^2\right)$$

$$\overline{\text{KE}} = \tfrac{1}{2}mv_{rms}^2 = \tfrac{3}{2}kT$$

Conceptual Example 5 **Does a Single Particle Have a Temperature?**

Each particle in a gas has kinetic energy. On the previous page, we have established the relationship between the average kinetic energy per particle and the temperature of an ideal gas.

Is it valid, then, to conclude that a single particle has a temperature?

Example 6 The Speed of Molecules in Air

Air is primarily a mixture of nitrogen N_2 molecules (molecular mass 28.0u) and oxygen O_2 molecules (molecular mass 32.0u). Assume that each behaves as an ideal gas and determine the rms speeds of the nitrogen and oxygen molecules when the temperature of the air is 293K.

$$\tfrac{1}{2} m v_{rms}^2 = \tfrac{3}{2} kT$$

$$\Downarrow$$

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

For nitrogen…

$$m = \frac{28.0\,\mathrm{g/mol}}{6.022\times 10^{23}\,\mathrm{mol}^{-1}} = 4.65\times 10^{-23}\,\mathrm{g} = 4.65\times 10^{-26}\,\mathrm{kg}$$

$$v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\left(1.38\times 10^{-23}\,\mathrm{J/K}\right)\left(293\,\mathrm{K}\right)}{4.65\times 10^{-26}\,\mathrm{kg}}} = 511\,\mathrm{m/s}$$

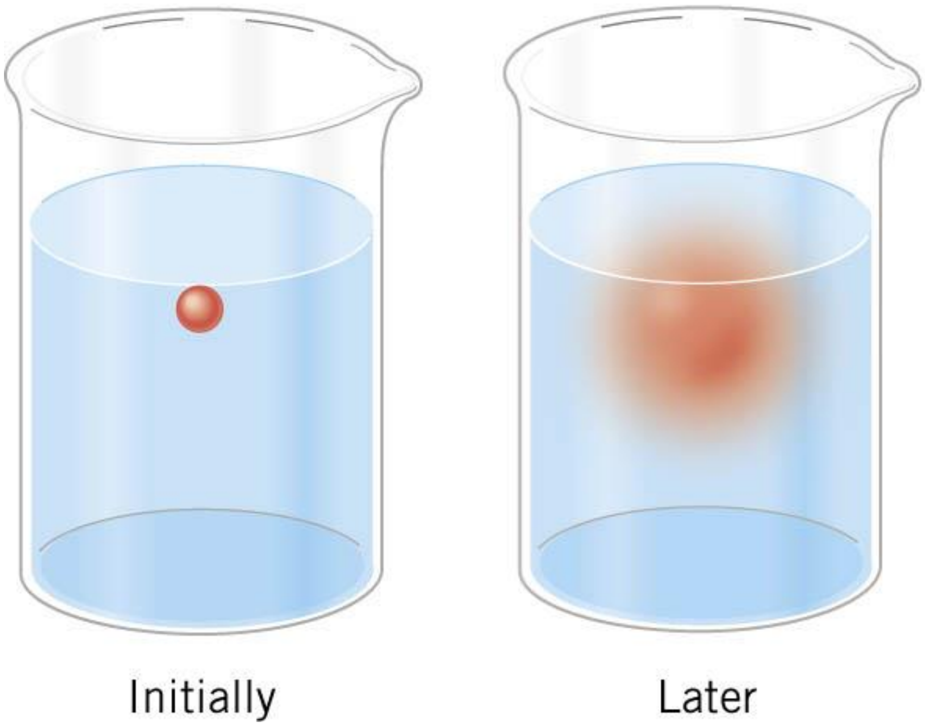
THE INTERNAL ENERGY OF A MONATOMIC IDEAL GAS

$$\overline{\mathrm{KE}} = \tfrac{1}{2} m v_{rms}^2 = \tfrac{3}{2} kT$$

$$U = N\,\tfrac{3}{2} kT = \tfrac{3}{2} nRT$$

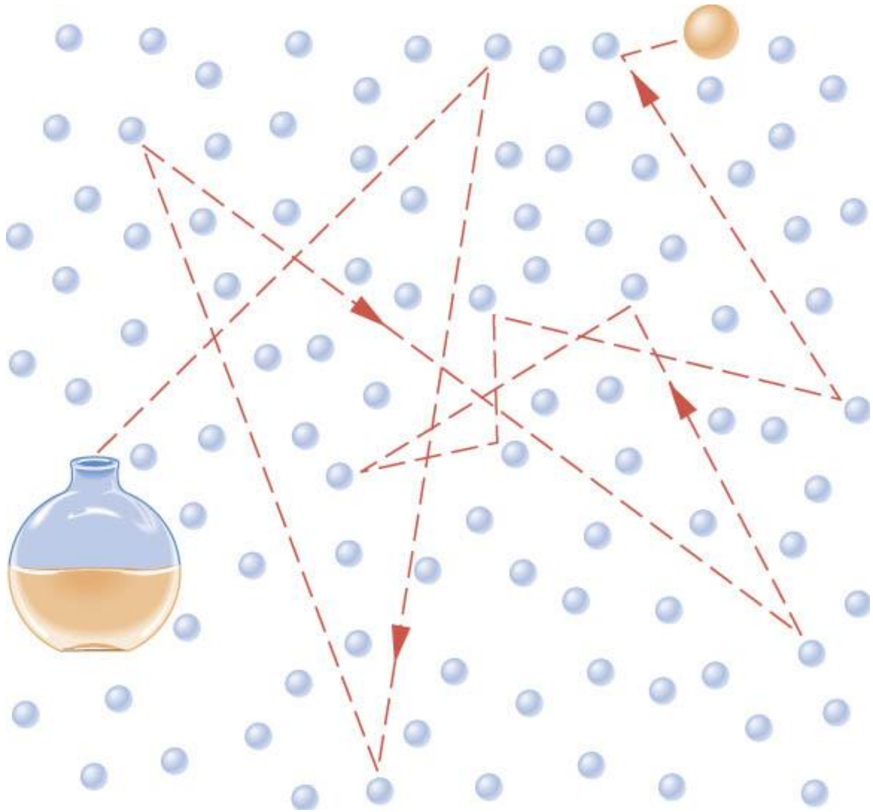
14.4 Diffusion

The process in which molecules move from a region of higher concentration to one of lower concentration is called ***diffusion***.

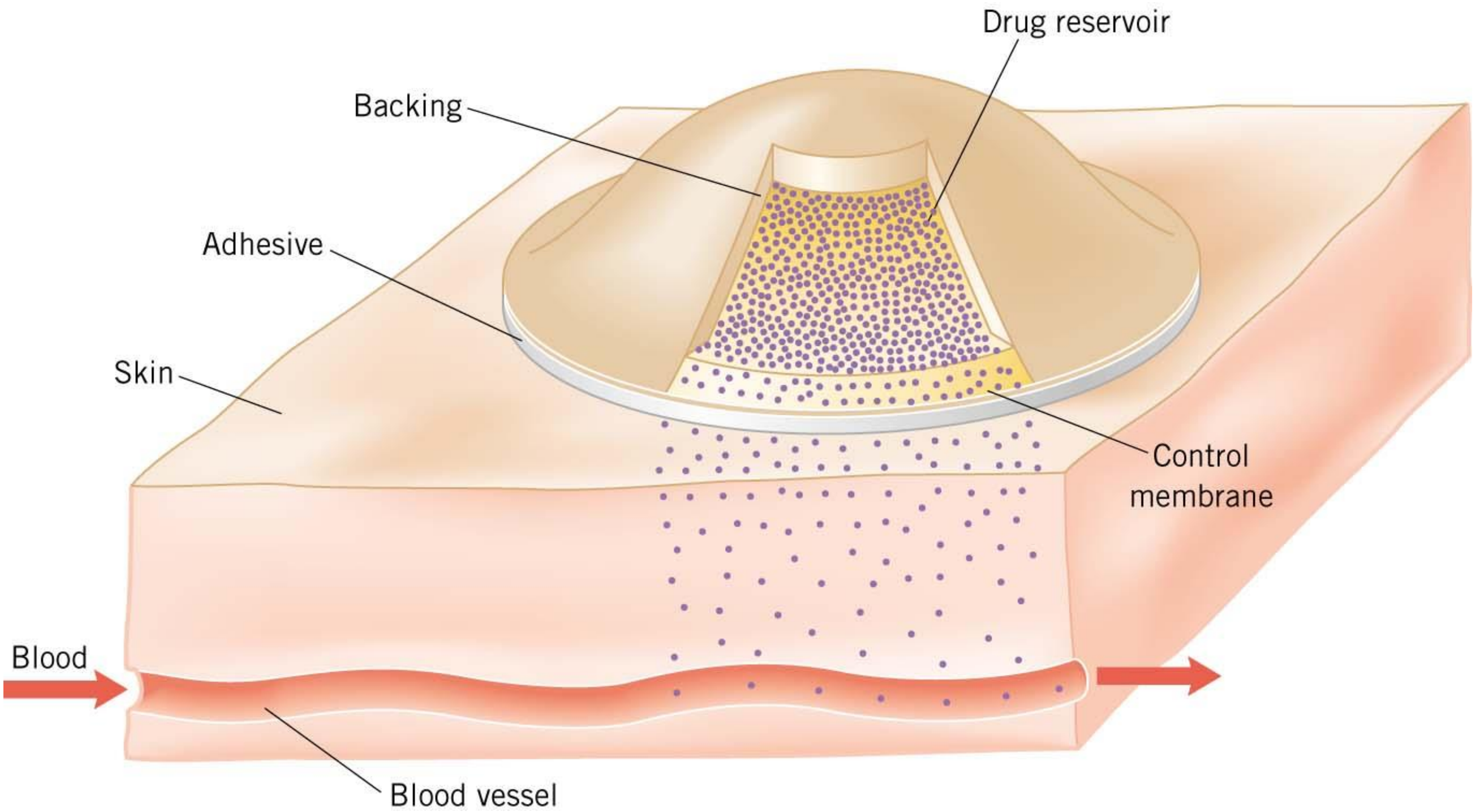


Conceptual Example 7 Why Diffusion is Relatively Slow

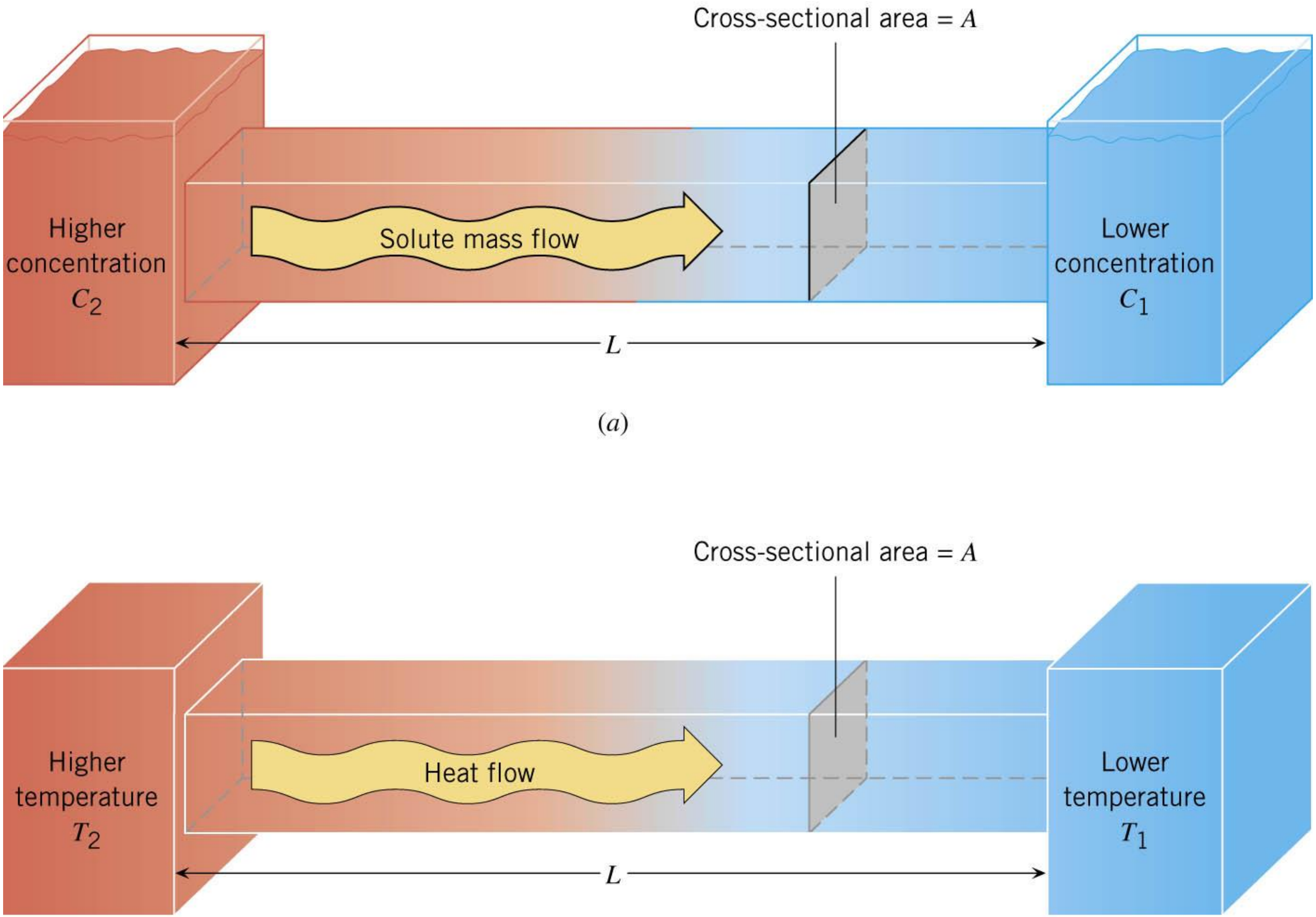
A gas molecule has a translational rms speed of hundreds of meters per second at room temperature. At such speed, a molecule could travel across an ordinary room in just a fraction of a second. Yet, it often takes several seconds, and sometimes minutes, for the fragrance of a perfume to reach the other side of the room. Why does it take so long?



A Transdermal Patch



14.4 Diffusion



(a)

(b)

FICK'S LAW OF DIFFUSION

The mass *m* of solute that diffuses in a time *t* through a solvent contained in a channel of length *L* and cross sectional area *A* is

$$m = \frac{(DA\Delta C)t}{L}$$

diffusion constant

concentration gradient between ends

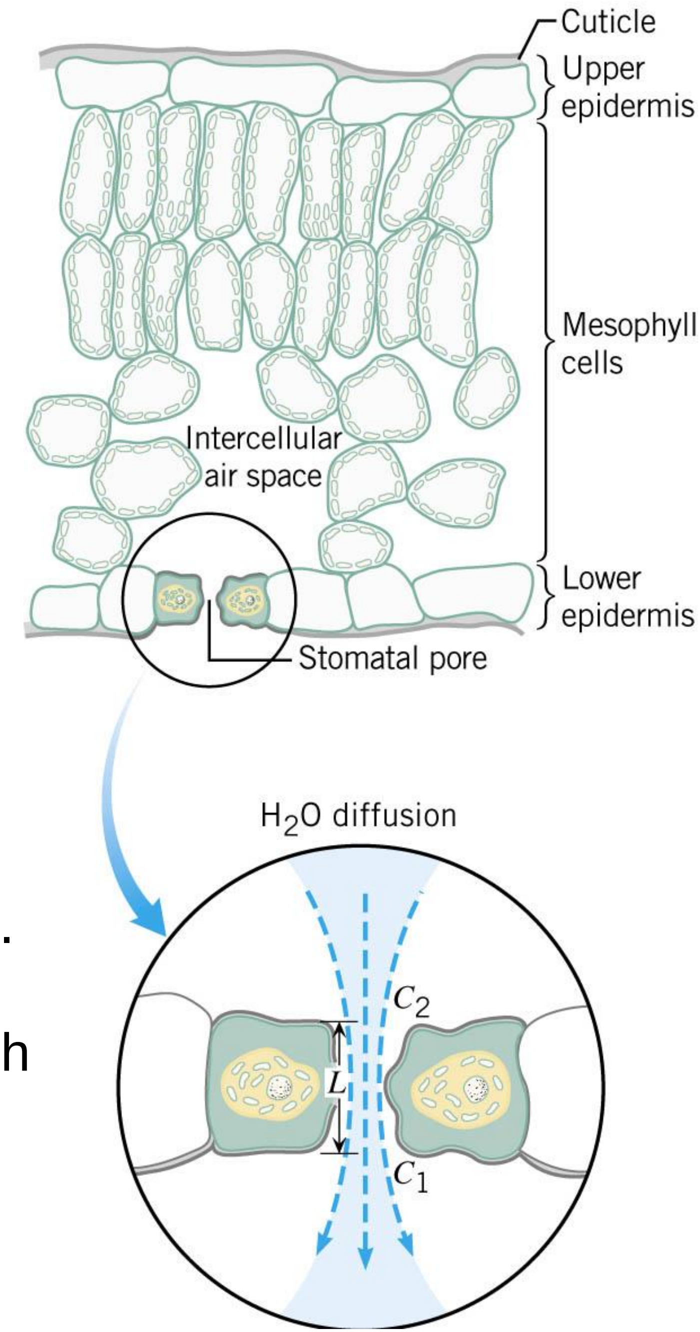
SI Units for the Diffusion Constant: m^2/s

Example 8 Water Given Off by Plant Leaves

Large amounts of water can be given off by plants. Inside the leaf, water passes from the liquid phase to the vapor phase at the walls of the mesophyll cells.

The diffusion constant for water is $2.4 \times 10^{-5} m^2/s$. A stomatal pore has a cross sectional area of about $8.0 \times 10^{-11} m^2$ and a length of about $2.5 \times 10^{-5} m$. The concentration on the interior side of the pore is roughly 0.022 kg/m^3, while that on the outside is approximately 0.011 kg/m^3.

Determine the mass of water that passes through the stomatal pore in one hour.



14.4 Diffusion

$$m = \frac{(DA\Delta C)t}{L}$$

$$= \frac{(2.4\times10^{-5}\ \mathrm{m^2/s})(8.0\times10^{-11}\mathrm{m^2})(0.022\,\mathrm{kg/m^3} - 0.011\,\mathrm{kg/m^3})(3600\,\mathrm{s})}{2.5\times10^{-5}\,\mathrm{m}}$$

$$= 3.0\times10^{-9}\,\mathrm{kg}$$