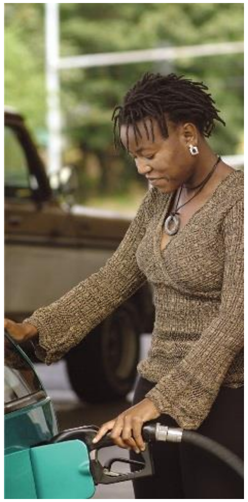
Chapter 24

Electric Potential



24.1 What is Physics?:

Experimentally, physicists and engineers discovered that the electric force is conservative and thus has an associated electric potential energy.

The motivation for associating a potential energy with a force is that we can then apply the principle of the conservation of mechanical energy to closed systems involving the force.

24.2: Electric Potential Energy

When an electrostatic force acts between two or more charged particles within a system of particles, we can assign an **electric potential energy *U*** to the system**.**

If the system changes its configuration from an initial state *i* to a different final state *f*, the electrostatic force does work *W* on the particles. If the resulting change is ΔU, then $\Delta U = U_f - U_i = -W.$

As with other conservative forces, the work done by the electrostatic force is *path independent.*

Usually the reference configuration of a system of charged particles is taken to be that in which the particles are all infinitely separated from one another. The corresponding reference potential energy is usually set be zero. Therefore,

$$U = -W_\infty.$$

Example, Work and potential energy in an electric field:

Electrons are continually being knocked out of air molecules in the atmosphere by cosmic-ray particles coming in from space. Once released, each electron experiences an electrostatic force $\vec{F}$ due to the electric field $\vec{E}$ that is produced in the atmosphere by charged particles already on Earth. Near Earth's surface the electric field has the magnitude $E = 150$ N/C and is directed downward. What is the change ΔU in the electric potential energy of a released electron when the electrostatic force causes it to move vertically upward through a distance $d = 520$ m (Fig. 24-1)?

KEY IDEAS

(1) The change ΔU in the electric potential energy of the electron is related to the work W done on the electron by the electric field. Equation 24-1 ($\Delta U = -W$) gives the relation.

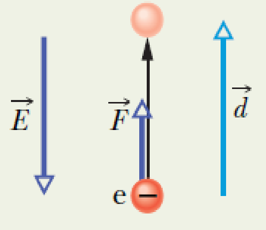


Fig. 24-1 An electron in the atmosphere is moved upward through displacement $\vec{d}$ by an electrostatic force $\vec{F}$ due to an electric field $\vec{E}$.

(2) The work done by a constant force $\vec{F}$ on a particle undergoing a displacement $\vec{d}$ is

$$W = \vec{F} \cdot \vec{d}. \tag{24-3}$$

(3) The electrostatic force and the electric field are related by the force equation $\vec{F} = q\vec{E}$, where here q is the charge of an electron ($= -1.6 \times 10^{-19}$ C).

Calculations: Substituting for $\vec{F}$ in Eq. 24-3 and taking the dot product yield

$$W = q\vec{E} \cdot \vec{d} = qEd \cos \theta, \tag{24-4}$$

where θ is the angle between the directions of $\vec{E}$ and $\vec{d}$. The field $\vec{E}$ is directed downward and the displacement $\vec{d}$ is directed upward; so $\theta = 180°$. Substituting this and other data into Eq. 24-4, we find

$$W = (-1.6 \times 10^{-19}\ \text{C})(150\ \text{N/C})(520\ \text{m}) \cos 180°$$
$$= 1.2 \times 10^{-14}\ \text{J}.$$

Equation 24-1 then yields

$$\Delta U = -W = -1.2 \times 10^{-14}\ \text{J}. \qquad \text{(Answer)}$$

This result tells us that during the 520 m ascent, the electric potential energy of the electron *decreases* by 1.2×10^{-14} J.

24.3 Electric Potential:

The potential energy per unit charge at a point in an electric field is called the **electric potential *V (or simply the potential)*** at that point. This is a scalar quantity. Thus, $V = \frac{U}{q}.$

The *electric potential difference V* between any two points *i* and *f* in an electric field is equal to the difference in potential energy per unit charge between the two points. Thus,

$$\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q}. = -\frac{W}{q} \quad \text{(potential difference defined).}$$

The potential difference between two points is thus the negative of the work done by the electrostatic force to move a unit charge from one point to the other.

If we set $U_i = 0$ at infinity as our reference potential energy, then the electric potential V must also be zero there. Therefore, the electric potential at any point in an electric field can be defined to be

$$V = -\frac{W_\infty}{q} \quad \text{(potential defined)}$$

Here W_∞ is the work done by the electric field on a charged particle as that particle moves in from infinity to point *f*.

The SI unit for potential is the joule per coulomb. This combination is called the *volt (abbreviated V).*

$$1 \text{ volt} = 1 \text{ joule per coulomb.}$$

24.3 Electric Potential: Units:

This unit of volt allows us to adopt a more conventional unit for the electric field, E, which is expressed in newtons per coulomb.

$$1\ \mathrm{N/C} = \left(1\frac{\mathrm{N}}{\mathrm{C}}\right)\left(\frac{1\ \mathrm{V\cdot C}}{1\ \mathrm{J}}\right)\left(\frac{1\ \mathrm{J}}{1\ \mathrm{N\cdot m}}\right) = 1\ \mathrm{V/m}.$$

We can now define an energy unit that is a convenient one for energy measurements in the atomic/subatomic domain: One *electron-volt (eV)* is the energy equal to the work required to move a single elementary charge *e*, such as that of the electron or the proton, through a potential difference of exactly one volt. The magnitude of this work is $q\Delta V$, and

$$1\ \mathrm{eV} = e(1\ \mathrm{V}) = (1.60 \times 10^{-19}\ \mathrm{C})(1\ \mathrm{J/C}) = 1.60 \times 10^{-19}\ \mathrm{J}.$$

24.3 Electric Potential: Work done by an Applied Force:

If a particle of charge q is moved from point i to point f in an electric field by applying a force to it, the applied force does work W_{app} on the charge while the electric field does work W on it. The change K in the kinetic energy of the particle is

$$\Delta K = K_f - K_i = W_{\text{app}} + W.$$

If the particle is stationary before and after the move, Then K_f and K_i are both zero.

$$W_{\text{app}} = -W.$$

Relating the work done by our applied force to the change in the potential energy of the particle during the move, one has

$$\Delta U = U_f - U_i = W_{\text{app}}.$$

We can also relate W_{app} to the electric potential difference ΔV between the initial and final locations of the particle:

$$W_{\text{app}} = q\,\Delta V.$$

24.4 Equipotential Surfaces:

Adjacent points that have the same electric potential form an equipotential surface, which can be either an imaginary surface or a real, physical surface.
No net work *W* is done on a charged particle by an electric field when the particle moves between two points *i* and *f* on the same equipotential surface.

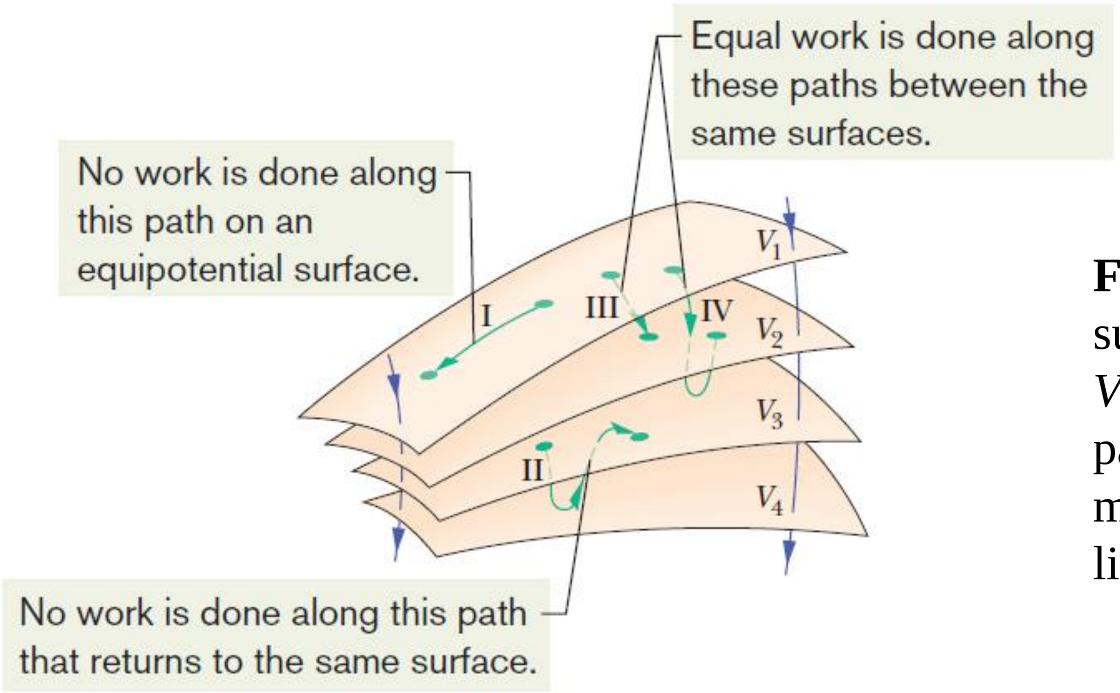


Fig. 24-2 Portions of four equipotential surfaces at electric potentials V_1=*100 V*, V_2=80 *V*,V_3 *=60 V, and* V_4 *=40 V.* Four paths along which a test charge may move are shown. Two electric field lines are also indicated.

24.4 Equipotential Surfaces:

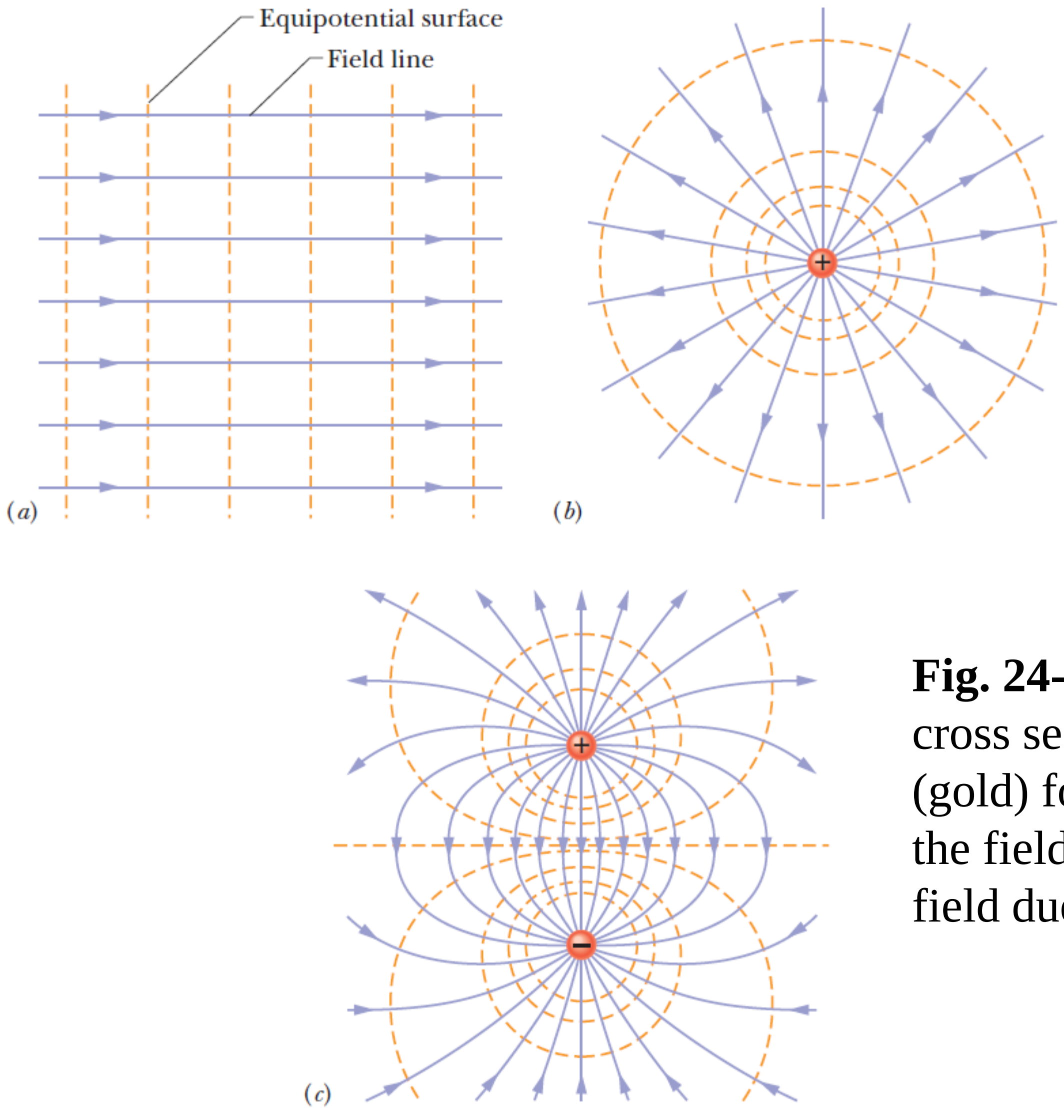


Fig. 24-3 Electric field lines (purple) and cross sections of equipotential surfaces (gold) for *(a)* a uniform electric field, *(b)* the field due to a point charge, and *(c)* the field due to an electric dipole.

24.5 Calculating the Potential from the Field:

$$dW = \vec{F} \cdot d\vec{s}.$$

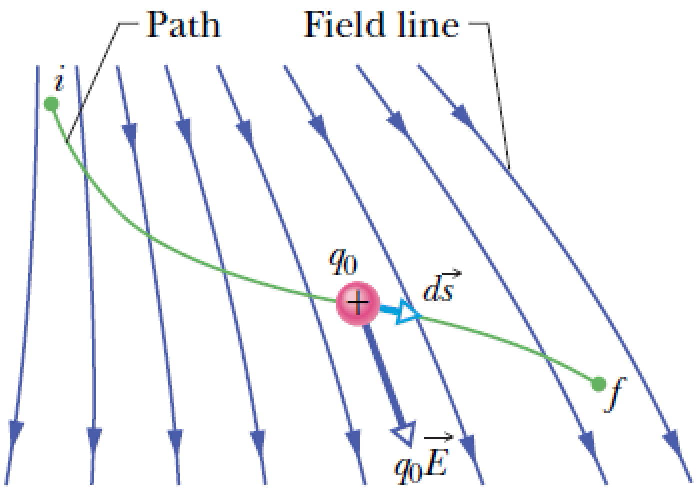


Fig. 24-4 A test charge q_0 moves from point i to point f along the path shown in a nonuniform electric field. During a displacement $d\vec{s}$, an electrostatic force $q_0\vec{E}$ acts on the test charge. This force points in the direction of the field line at the location of the test charge.

For the situation of Fig. 24-4, $dW = q_0\vec{E} \cdot d\vec{s}.$

Total work: $$W = q_0 \int_i^f \vec{E} \cdot d\vec{s}.$$

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}.$$

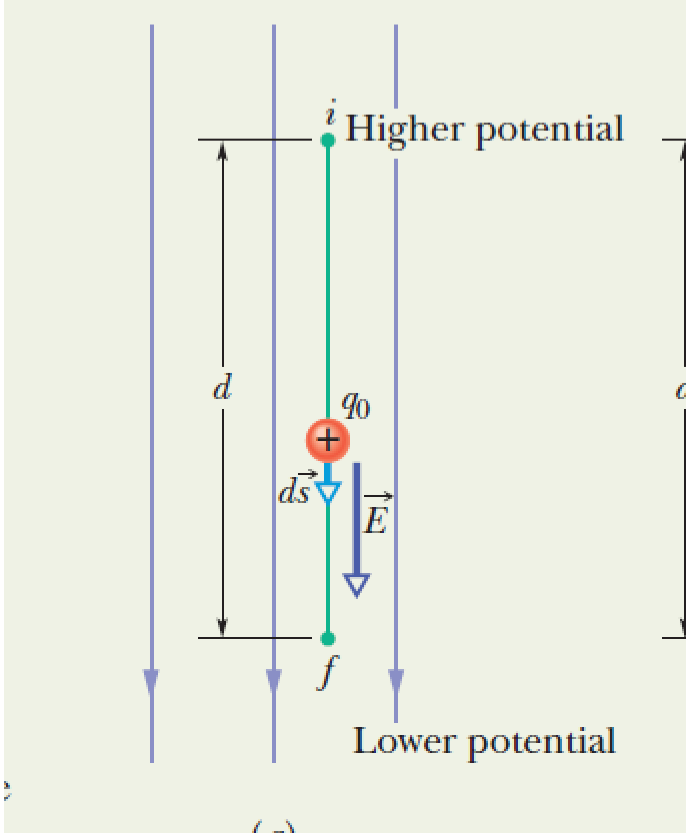
Thus, the potential difference $V_f - V_i$ between any two points i and f in an electric field is equal to the negative of the line integral from i to f. Since the electrostatic force is conservative, all paths yield the same result.
If we set potential $V_i = 0$, then

$$V = -\int_i^f \vec{E} \cdot d\vec{s},$$

This is the potential V at any point f in the electric field relative to the zero potential at point i. If point i is at infinity, then this is the potential V at any point f relative to the zero potential at infinity.

Example, Finding the Potential change from the Electric Field:

(a) Figure 24-5*a* shows two points *i* and *f* in a uniform electric field $\vec{E}$. The points lie on the same electric field line (not shown) and are separated by a distance *d*. Find the potential difference $V_f - V_i$ by moving a positive test charge q_0 from *i* to *f* along the path shown, which is parallel to the field direction.



Calculations: We begin by mentally moving a test charge q_0 along that path, from initial point *i* to final point *f*. As we move such a test charge along the path in Fig. 24-5*a*, its differential displacement $d\vec{s}$ always has the same direction as $\vec{E}$. Thus, the angle θ between $\vec{E}$ and $d\vec{s}$ is zero and the dot product in Eq. 24-18 is

$$\vec{E} \cdot d\vec{s} = E\, ds \cos\theta = E\, ds. \tag{24-20}$$

Equations 24-18 and 24-20 then give us

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s} = -\int_i^f E\, ds. \tag{24-21}$$

Since the field is uniform, *E* is constant over the path and can be moved outside the integral, giving us

$$V_f - V_i = -E \int_i^f ds = -Ed, \qquad \text{(Answer)}$$

Example, Finding the Potential change from the Electric Field:

(b) Now find the potential difference $V_f - V_i$ by moving the positive test charge q_0 from i to f along the path icf shown in Fig. 24-5b.

Calculations: The Key Idea of (a) applies here too, except now we move the test charge along a path that consists of two lines: ic and cf. At all points along line ic, the displacement $d\vec{s}$ of the test charge is perpendicular to $\vec{E}$. Thus, the angle θ between $\vec{E}$ and $d\vec{s}$ is 90°, and the dot product $\vec{E} \cdot d\vec{s}$ is 0. Equation 24-18 then tells us that points i and c are at the same potential: $V_c - V_i = 0$.

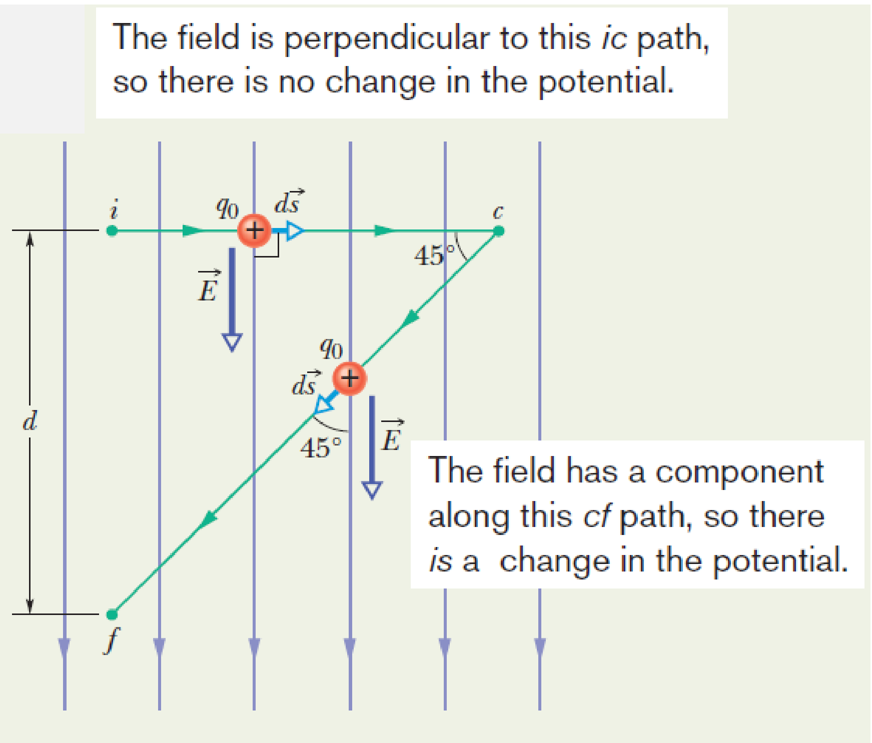
For line cf we have $\theta = 45°$ and, from Eq. 24-18,

$$V_f - V_i = -\int_c^f \vec{E} \cdot d\vec{s} = -\int_c^f E(\cos 45°)\, ds$$

$$= -E(\cos 45°) \int_c^f ds.$$

The integral in this equation is just the length of line cf; from Fig. 24-5b, that length is $d/\cos 45°$. Thus,

$$V_f - V_i = -E(\cos 45°)\frac{d}{\cos 45°} = -Ed. \quad \text{(Answer)}$$



(b)

24.6 Potential Due to a Point Charge:

A positively charged particle produces a positive electric potential. A negatively charged particle produces a negative electric potential.

Consider a point *P* at distance *R* from a fixed particle of positive charge *q*. Imagine that we move a positive test charge q_0 from point *P* to infinity. The path chosen can be the simplest one— a line that extends radially from the fixed particle through *P* to infinity.

$$V_f - V_i = -\int_R^\infty E\,dr.$$

If $V_f = 0$ (at ∞) and $V_i = V$ (at R). Then, for the magnitude of the electric field at the site of the test charge,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}.$$

That gives:
$$0 - V = -\frac{q}{4\pi\varepsilon_0}\int_R^\infty \frac{1}{r^2}\,dr = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_R^\infty$$
$$= -\frac{1}{4\pi\varepsilon_0}\frac{q}{R}.$$

Switching *R* to *r*,
$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

To find the potential of the charged particle, we move this test charge out to infinity.

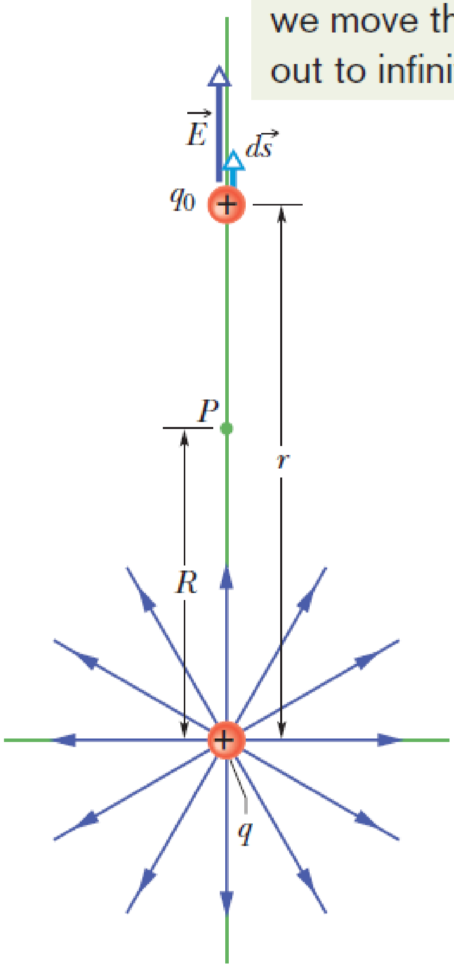


Fig. 24-6 The positive point charge *q* produces an electric field $\vec{E}$ and an electric potential *V* at point *P*. We find the potential by moving a test charge q_0 from *P* to infinity. The test charge is shown at distance *r* from the point charge, during differential displacement $d\vec{s}$.

24.7 Potential Due to a Group of Point Charges:

The net potential at a point due to a group of point charges can be found with the help of the superposition principle. First the individual potential resulting from each charge is considered at the given point. Then we sum the potentials.

For *n* charges, the net potential is

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i} \qquad (n \text{ point charges}).$$

Example, Net Potential of Several Charged Particles:

What is the electric potential at point P, located at the center of the square of point charges shown in Fig. 24-8a? The distance d is 1.3 m, and the charges are

$$q_1 = +12\text{ nC}, \qquad q_3 = +31\text{ nC},$$
$$q_2 = -24\text{ nC}, \qquad q_4 = +17\text{ nC}.$$

KEY IDEA

The electric potential V at point P is the algebraic sum of the electric potentials contributed by the four point charges.

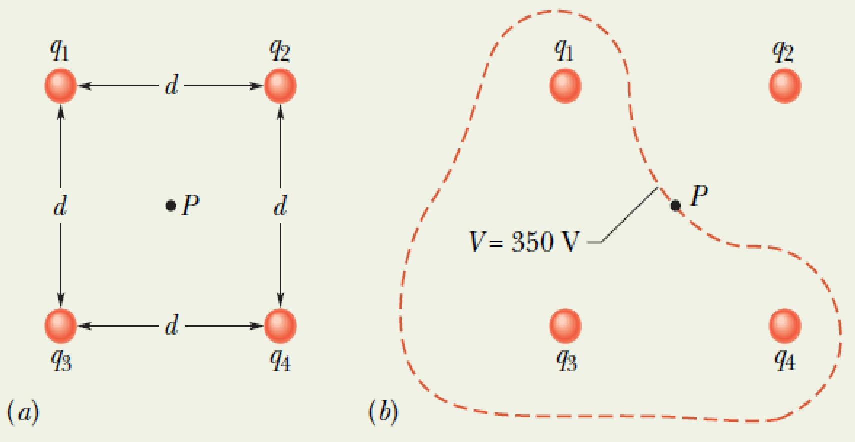


Fig. 24-8 (a) Four point charges are held fixed at the corners of a square. (b) The closed curve is a cross section, in the plane of the figure, of the equipotential surface that contains point P. (The curve is drawn only roughly.)

(Because electric potential is a scalar, the orientations of the point charges do not matter.)

Calculations: From Eq. 24-27, we have

$$V = \sum_{i=1}^{4} V_i = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r} + \frac{q_2}{r} + \frac{q_3}{r} + \frac{q_4}{r}\right).$$

The distance r is $d/\sqrt{2}$, which is 0.919 m, and the sum of the charges is

$$q_1 + q_2 + q_3 + q_4 = (12 - 24 + 31 + 17) \times 10^{-9}\text{ C}$$
$$= 36 \times 10^{-9}\text{ C}.$$

Thus, $$V = \frac{(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(36 \times 10^{-9}\text{ C})}{0.919\text{ m}}$$

$$\approx 350\text{ V}. \qquad \text{(Answer)}$$

Close to any of the three positive charges in Fig. 24-8a, the potential has very large positive values. Close to the single negative charge, the potential has very large negative values. Therefore, there must be points within the square that have the same intermediate potential as that at point P. The curve in Fig. 24-8b shows the intersection of the plane of the figure with the equipotential surface that contains point P. Any point along that curve has the same potential as point P.

Example, Potential is not a Vector:

(a) In Fig. 24-9*a*, 12 electrons (of charge $-e$) are equally spaced and fixed around a circle of radius R. Relative to $V = 0$ at infinity, what are the electric potential and electric field at the center C of the circle due to these electrons?

KEY IDEAS

(1) The electric potential V at C is the algebraic sum of the electric potentials contributed by all the electrons. (Because electric potential is a scalar, the orientations of the electrons do not matter.) (2) The electric field at C is a vector quantity and thus the orientation of the electrons *is* important.

Calculations: Because the electrons all have the same negative charge $-e$ and are all the same distance R from C, Eq. 24-27 gives us

$$V = -12 \frac{1}{4\pi\varepsilon_0} \frac{e}{R}. \qquad \text{(Answer)} \quad (24\text{-}28)$$

Because of the symmetry of the arrangement in Fig. 24-9*a*, the electric field vector at C due to any given electron is canceled by the field vector due to the electron that is diametrically opposite it. Thus, at C,

$$\vec{E} = 0. \qquad \text{(Answer)}$$

Potential is a scalar and orientation is irrelevant.

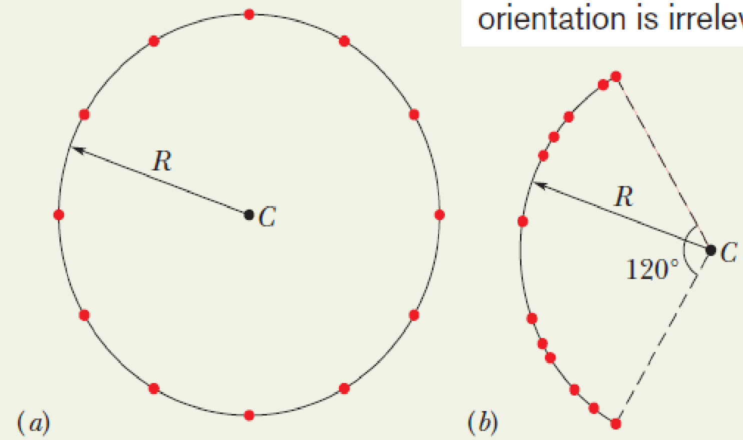


Fig. 24-9 (*a*) Twelve electrons uniformly spaced around a circle. (*b*) The electrons nonuniformly spaced along an arc of the original circle.

(b) If the electrons are moved along the circle until they are nonuniformly spaced over a 120° arc (Fig. 24-9*b*), what then is the potential at C? How does the electric field at C change (if at all)?

Reasoning: The potential is still given by Eq. 24-28, because the distance between C and each electron is unchanged and orientation is irrelevant. The electric field is no longer zero, however, because the arrangement is no longer symmetric. A net field is now directed toward the charge distribution.

24.8 Potential Due to an Electric Dipole:

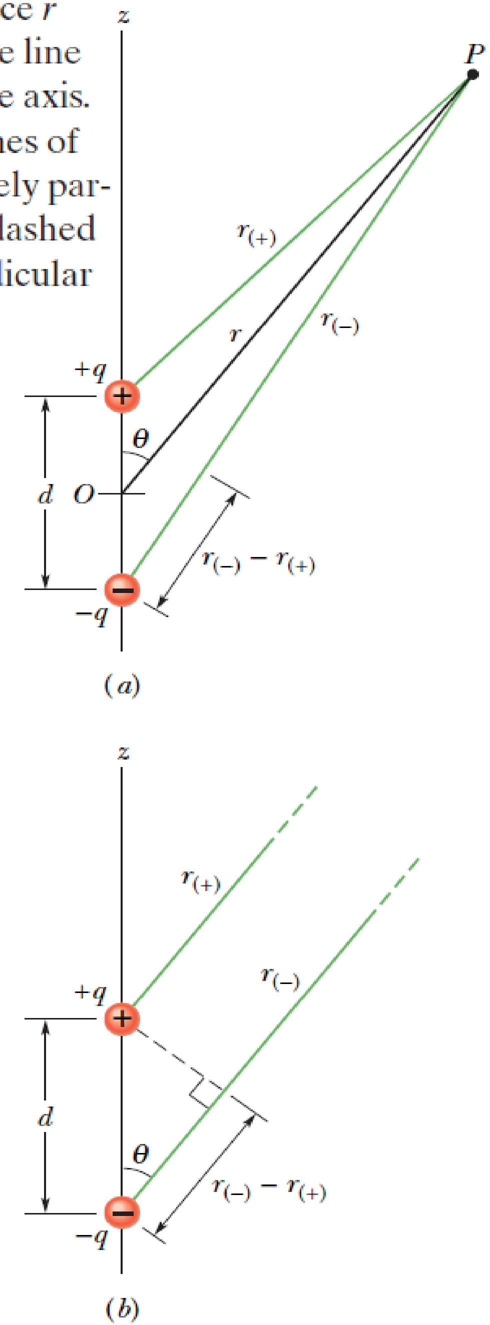


Fig. 24-10 (*a*) Point P is a distance r from the midpoint O of a dipole. The line OP makes an angle θ with the dipole axis. (*b*) If P is far from the dipole, the lines of lengths $r_{(+)}$ and $r_{(-)}$ are approximately parallel to the line of length r, and the dashed black line is approximately perpendicular to the line of length $r_{(-)}$.

At *P*, the positive point charge (at distance $r_{(+)}$) sets up potential $V_{(+)}$ and the negative point charge (at distance $r_{(-)}$) sets up potential $V_{(-)}$. Then the net potential at P is:

$$V = \sum_{i=1}^{2} V_i = V_{(+)} + V_{(-)} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_{(+)}} + \frac{-q}{r_{(-)}}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\,\frac{r_{(-)} - r_{(+)}}{r_{(-)}r_{(+)}}.$$

Naturally occurring dipoles are quite small; so we are usually interested only in points that are relatively far from the dipole, such that d«r, where *d* is the distance between the charges. If $p = qd$,

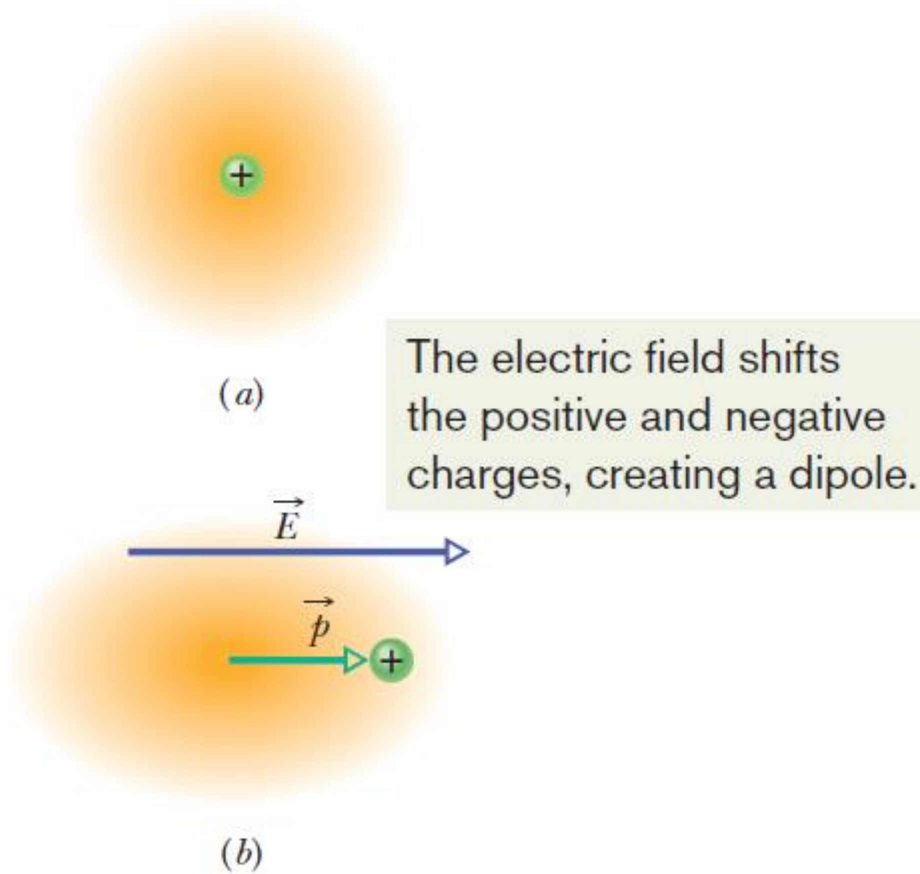
$$r_{(-)} - r_{(+)} \approx d\cos\theta \quad \text{and} \quad r_{(-)}r_{(+)} \approx r^2.$$

$$V = \frac{q}{4\pi\varepsilon_0}\,\frac{d\cos\theta}{r^2}$$

$$V = \frac{1}{4\pi\varepsilon_0}\,\frac{p\cos\theta}{r^2} \qquad \text{(electric dipole)},$$

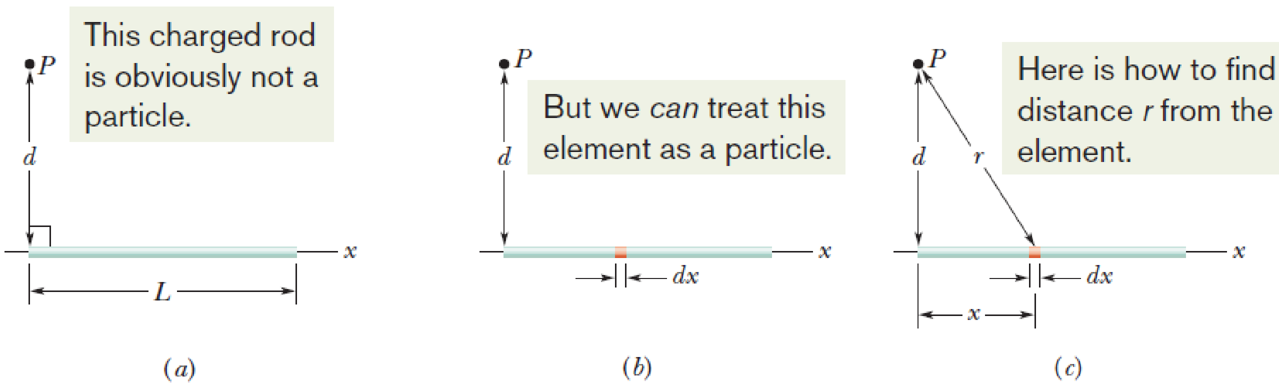
24.8 Induced Dipole Moment:

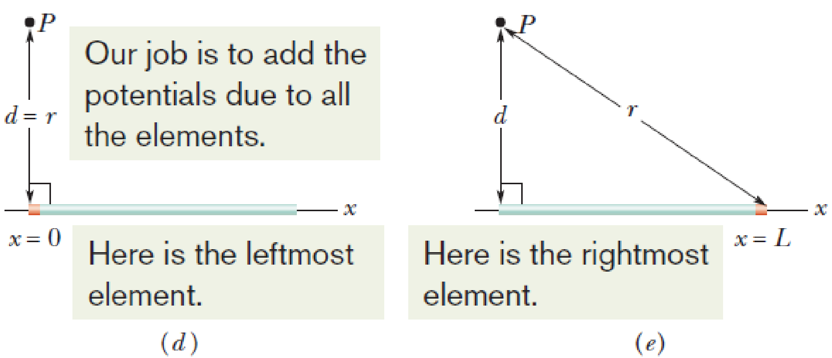
Fig. 24-11 (a) An atom, showing the positively charged nucleus (green) and the negatively charged electrons (gold shading).The centers of positive and negative charge coincide. *(b)* If the atom is placed in an external electric field ***E***, the electron orbits are distorted so that the centers of positive and negative charge no longer coincide. An induced dipole moment ***p*** appears. The distortion is greatly exaggerated here.



24.9 Potential Due to a Continuous Charge Distribution: Line of Charge:

Fig. 24-12 ***(a)*** A thin, uniformly charged rod produces an electric potential *V* at point *P*. *(b) An* element can be treated as a particle. (*c*) The potential at *P* due to the element depends on the distance *r*. *We* need to sum the potentials due to all the elements, from the left side (*d*) to the right side (*e*).





If λ is the charge per unit length, then the charge on length *dx* is:

$$dq = \lambda\, dx.$$

$$r = (x^2 + d^2)^{1/2}$$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{(x^2 + d^2)^{1/2}}.$$

$$V = \int dV = \int_0^L \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{(x^2 + d^2)^{1/2}}\, dx = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{(x^2 + d^2)^{1/2}} = \frac{\lambda}{4\pi\varepsilon_0}\Big[\ln\Big(x + (x^2 + d^2)^{1/2}\Big)\Big]_0^L$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\Big[\ln\Big(L + (L^2 + d^2)^{1/2}\Big) - \ln d\Big]. = \frac{\lambda}{4\pi\varepsilon_0}\ln\left[\frac{L + (L^2 + d^2)^{1/2}}{d}\right].$$

24.9 Potential Due to a Continuous Charge Distribution: Charged Disk:

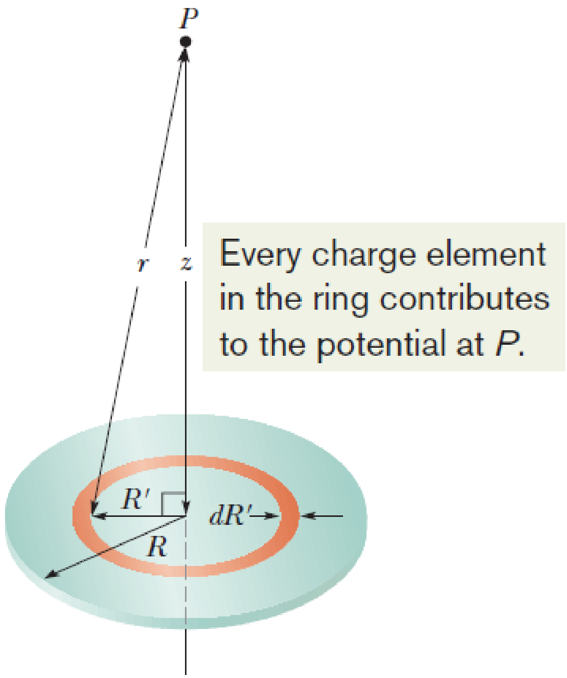


Fig. 24-13 A plastic disk of radius R, charged on its top surface to a uniform surface charge density σ. We wish to find the potential V at point P on the central axis of the disk.

In Fig. 24-13, consider a differential element consisting of a flat ring of radius R' and radial width dR'. Its charge has magnitude

$$dq = \sigma(2\pi R')(dR')$$

The contribution of this ring to the electric potential at P is:

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma(2\pi R')(dR')}{\sqrt{z^2 + R'^2}}$$

The net potential at P can be found by adding (via integration) the contributions of all the rings from $R'=0$ to $R'=R$:

$$V = \int dV = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{R'\,dR'}{\sqrt{z^2 + R'^2}} = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2 + R^2} - z\right)$$

24.10 Calculating the Field from the Potential:

Suppose that a positive test charge q_0 moves through a displacement from one equipotential surface to the adjacent surface. The work the electric field does on the test charge during the move is $-q_0\, dV$.

The work done by the electric field may also be written as the scalar product or $(q_0\vec{E})\cdot d\vec{s} = q_0 E(\cos\theta)\, ds$.

Therefore, $$-q_0\, dV = q_0 E(\cos\theta)\, ds,$$

That is, $$E\cos\theta = -\frac{dV}{ds}$$

Since $E\cos\theta$ is the component of **E** in the direction of **ds**,

$$E_s = -\frac{\partial V}{\partial s}$$

If we take the *s* axis to be, in turn, the x, y, and z axes, the x, y, and z components of **E** at any point are

$$E_x = -\frac{\partial V}{\partial x}; \qquad E_y = -\frac{\partial V}{\partial y}; \qquad E_z = -\frac{\partial V}{\partial z}.$$

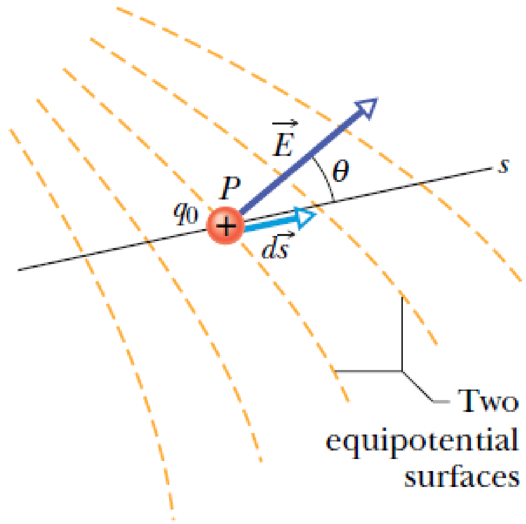


Fig. 24-14 A test charge q_0 moves a distance $d\vec{s}$ from one equipotential surface to another. (The separation between the surfaces has been exaggerated for clarity.) The displacement $d\vec{s}$ makes an angle θ with the direction of the electric field $\vec{E}$.

Therefore, the component of E in any direction is the negative of the rate at which the electric potential changes with distance in that direction.

Example, Finding the Field from the Potential:

The electric potential at any point on the central axis of a uniformly charged disk is given by Eq. 24-37,

$$V = \frac{\sigma}{2\varepsilon_0}(\sqrt{z^2 + R^2} - z).$$

Starting with this expression, derive an expression for the electric field at any point on the axis of the disk.

KEY IDEAS

We want the electric field $\vec{E}$ as a function of distance z along the axis of the disk. For any value of z, the direction of $\vec{E}$ must be along that axis because the disk has circular symmetry about that axis. Thus, we want the component E_z of $\vec{E}$ in the direction of z. This component is the negative of the rate at which the electric potential changes with distance z.

Calculation: Thus, from the last of Eqs. 24-41, we can write

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\sigma}{2\varepsilon_0}\frac{d}{dz}(\sqrt{z^2 + R^2} - z)$$

$$= \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right). \qquad \text{(Answer)}$$

24.11 Electric Potential Energy of a System of Point Charges:

The electric potential energy of a system of fixed point charges is equal to the work that must be done by an external agent to assemble the system, bringing each charge in from an infinite distance.

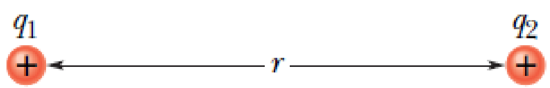


Fig. 24-15 Two charges held a fixed distance r apart.

Figure 24-15 shows two point charges q_1 *and* q_2, separated by a distance *r*. When we bring q_1 in from infinity and put it in place, we do no work because no electrostatic force acts on q_1. However, when we next bring q_2 in from infinity and put it in place, we must do work because q_1 exerts an electrostatic force on q_2 during the move.

The work done is q_2V, where V is the potential that has been set up by q_1 at the point where we put q_2.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}$$

$$U = W = q_2V = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$$

Example, Potential Energy of a System of Three Charged Particles:

Figure 24-16 shows three point charges held in fixed positions by forces that are not shown. What is the electric potential energy U of this system of charges? Assume that $d = 12$ cm and that

$$q_1 = +q, \quad q_2 = -4q, \quad \text{and} \quad q_3 = +2q,$$

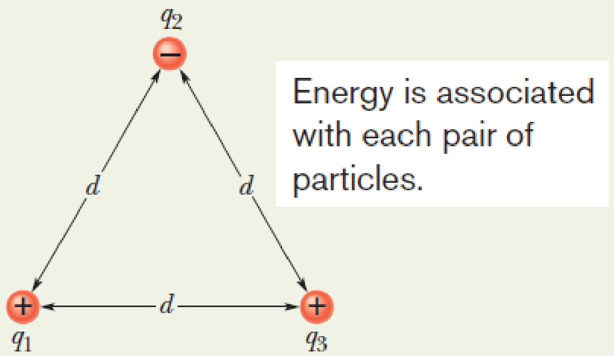


Fig. 24-16 Three charges are fixed at the vertices of an equilateral triangle. What is the electric potential energy of the system?

Calculations: Let's mentally build the system of Fig. 24-16, starting with one of the point charges, say q_1, in place and the others at infinity. Then we bring another one, say q_2, in from infinity and put it in place. From Eq. 24-43 with d substituted for r, the potential energy U_{12} associated with the pair of point charges q_1 and q_2 is

$$U_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d}.$$

We then bring the last point charge q_3 in from infinity and put it in place. The work that we must do in this last step is equal to the sum of the work we must do to bring q_3 near q_1 and the work we must do to bring it near q_2. From Eq. 24-43, with d substituted for r, that sum is

charges. This sum (which is actually independent of the order in which the charges are brought together) is

$$U = U_{12} + U_{13} + U_{23}$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{(+q)(-4q)}{d} + \frac{(+q)(+2q)}{d} + \frac{(-4q)(+2q)}{d}\right)$$

$$= -\frac{10q^2}{4\pi\varepsilon_0 d}$$

$$= -\frac{(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(10)(150 \times 10^{-9}\ \text{C})^2}{0.12\ \text{m}}$$

$$= -1.7 \times 10^{-2}\ \text{J} = -17\ \text{mJ}. \qquad \text{(Answer)}$$

Example, Conservation of Mechanical Energy with Electric Potential Energy:

An alpha particle (two protons, two neutrons) moves into a stationary gold atom (79 protons, 118 neutrons), passing through the electron region that surrounds the gold nucleus like a shell and headed directly toward the nucleus (Fig. 24-17). The alpha particle slows until it momentarily stops when its center is at radial distance $r = 9.23$ fm from the nuclear center. Then it moves back along its incoming path. (Because the gold nucleus is much more massive than the alpha particle, we can assume the gold nucleus does not move.) What was the kinetic energy K_i of the alpha particle when it was initially far away (hence external to the gold atom)? Assume that the only force acting between the alpha particle and the gold nucleus is the (electrostatic) Coulomb force.

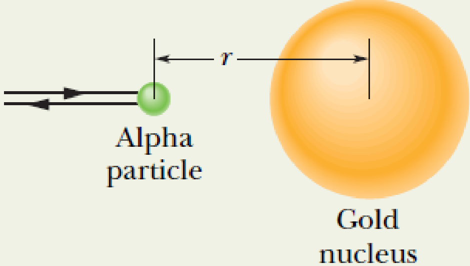


Fig. 24-17 An alpha particle, traveling head-on toward the center of a gold nucleus, comes to a momentary stop (at which time all its kinetic energy has been transferred to electric potential energy) and then reverses its path.

Reasoning: When the alpha particle is outside the atom, the system's initial electric potential energy U_i is zero because the atom has an equal number of electrons and protons, which produce a *net* electric field of zero. However, once the alpha particle passes through the electron region surrounding the nucleus on its way to the nucleus, the electric field due to the electrons goes to zero. The reason is that the electrons act like a closed spherical shell of uniform negative charge and, as discussed in Section 23-9, such a shell produces zero electric field in the space it encloses. The alpha particle still experiences the electric field of the protons in the nucleus, which produces a repulsive force on the protons within the alpha particle.

As the incoming alpha particle is slowed by this repulsive force, its kinetic energy is transferred to electric potential energy of the system. The transfer is complete when the alpha particle momentarily stops and the kinetic energy is $K_f = 0$.

Calculations: The principle of conservation of mechanical energy tells us that

$$K_i + U_i = K_f + U_f. \quad (24\text{-}44)$$

We know two values: $U_i = 0$ and $K_f = 0$. We also know that the potential energy U_f at the stopping point is given by the right side of Eq. 24-43, with $q_1 = 2e$, $q_2 = 79e$ (in which e is the elementary charge, 1.60×10^{-19} C), and $r = 9.23$ fm. Thus, we can rewrite Eq. 24-44 as

$$K_i = \frac{1}{4\pi\varepsilon_0}\frac{(2e)(79e)}{9.23\text{ fm}}$$

$$= \frac{(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(158)(1.60 \times 10^{-19}\ \text{C})^2}{9.23 \times 10^{-15}\ \text{m}}$$

$$= 3.94 \times 10^{-12}\ \text{J} = 24.6\ \text{MeV}. \quad \text{(Answer)}$$

24.12 Potential of a Charged Isolated Conductor:

An excess charge placed on an isolated conductor will distribute itself on the surface of that conductor so that all points of the conductor—whether on the surface or inside—come to the same potential. This is true even if the conductor has an internal cavity and even if that cavity contains a net charge.

We know that $$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}.$$

Since for all points $\boldsymbol{E} = 0$ within a conductor, it follows directly that $V_f = V_i$ for all possible pairs of points i and f in the conductor.

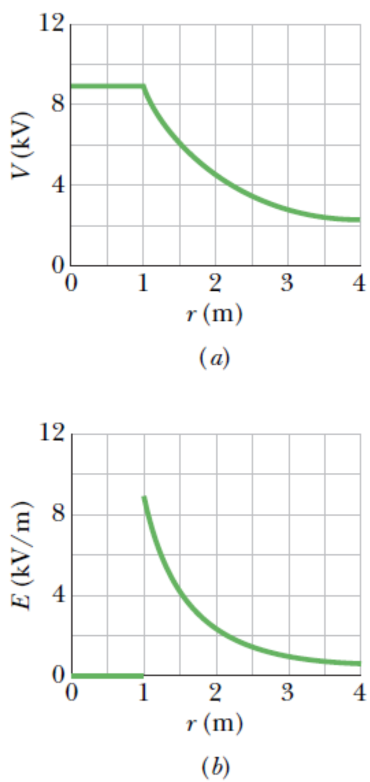


Fig. 24-18 (*a*) A plot of $V(r)$ both inside and outside a charged spherical shell of radius 1.0 m. (*b*) A plot of $E(r)$ for the same shell.

24.12: Spark Discharge from a Charge Conductor:

Fig. 24-19 A large spark jumps to a car's body and then exits by moving across the insulating left front tire (note the flash there), leaving the person inside unharmed. *(Courtesy Westinghouse Electric Corporation)*

On nonspherical conductors, a surface charge does not distribute itself uniformly over the surface of the conductor. At sharp points or edges, the surface charge density—and thus the external electric field, —may reach very high values. The air around such sharp points or edges may become ionized, producing the corona discharge that golfers and mountaineers see on the tips of bushes, golf clubs, and rock hammers when thunderstorms threaten. Such corona discharges are often the precursors of lightning strikes. In such circumstances, it is wise to enclose yourself in a cavity inside a conducting shell, where the electric field is guaranteed to be zero. A car (unless it is a convertible or made with a plastic body) is almost ideal

24.12 Isolated Conductor in an Isolated Electric Field:

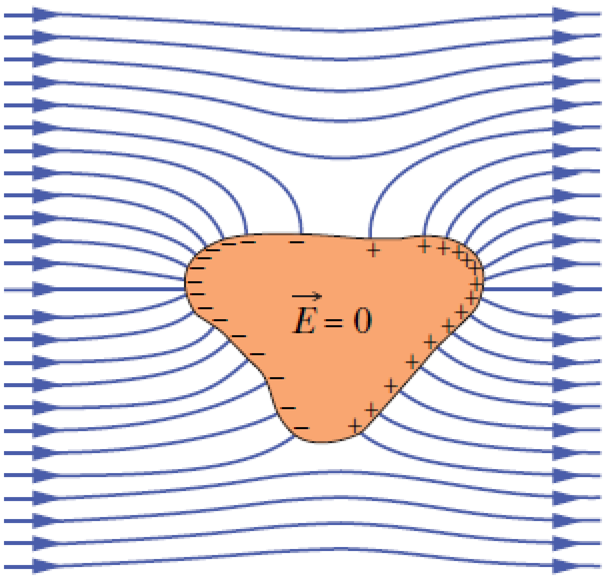


Fig. 24-20 An uncharged conductor is suspended in an external electric field. The free electrons in the conductor distribute themselves on the surface as shown, so as to reduce the net electric field inside the conductor to zero and make the net field at the surface perpendicular to the surface.

If an isolated conductor is placed in an external electric field, all points of the conductor still come to a single potential regardless of whether the conductor has an excess charge.

The free conduction electrons distribute themselves on the surface in such a way that the electric field they produce at interior points cancels the external electric field that would otherwise be there.

Furthermore, the electron distribution causes the net electric field at all points on the surface to be perpendicular to the surface. If the conductor in Fig. 24-20 could be somehow removed, leaving the surface charges frozen in place, the internal and external electric field would remain absolutely unchanged.