Chapter 25

Capacitance

25.2: Capacitance:

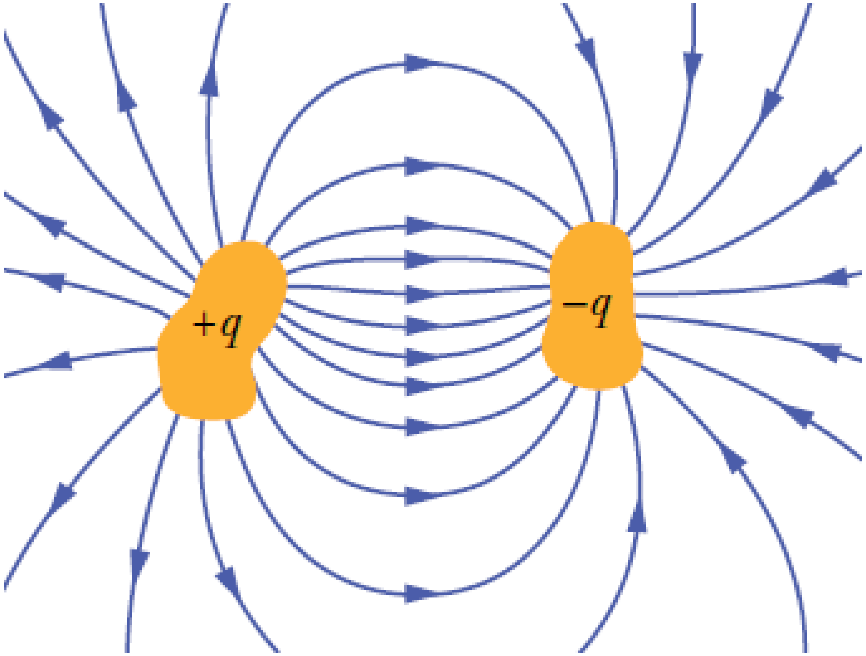


Fig. 25-2 Two conductors, isolated electrically from each other and from their surroundings, form a *capacitor*. When the capacitor is charged, the charges on the conductors, or *plates* as they are called, have the same magnitude q but opposite signs. *(Paul Silvermann/Fundamental Photographs)*

25.2: Capacitance:

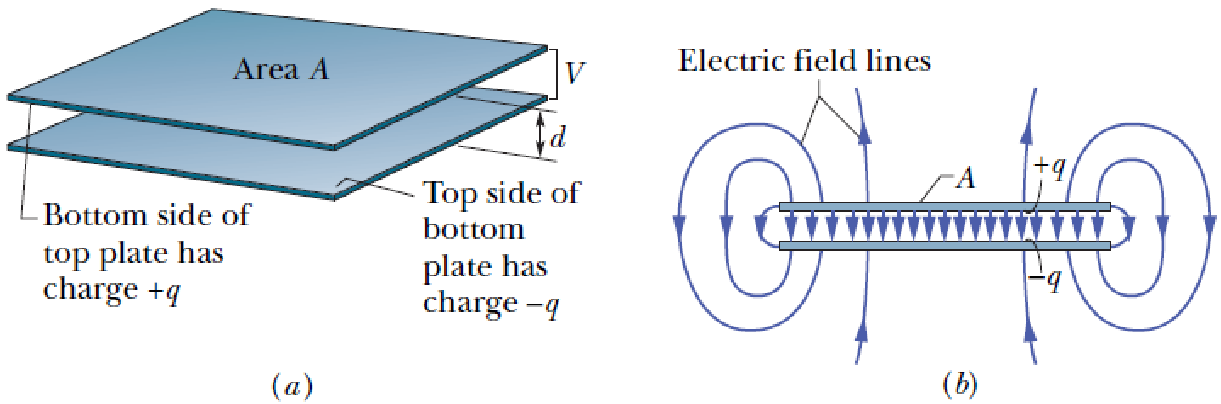


Fig. 25-3 (a) A parallel-plate capacitor, made up of two plates of area A separated by a distance d. The charges on the facing plate surfaces have the same magnitude q but opposite signs. (b) As the field lines show, the electric field due to the charged plates is uniform in the central region between the plates. The field is not uniform at the edges of the plates, as indicated by the "fringing" of the field lines there.

When a capacitor is charged, its plates have charges of equal magnitudes but opposite signs: q+ and q-. However, we refer to the charge of a capacitor as being q, the absolute value of these charges on the plates.

The charge q and the potential difference V for a capacitor are proportional to each other:

$$q = CV.$$

The proportionality constant *C is called the* ***capacitance*** of the capacitor. Its value depends only on the geometry of the plates and not on their charge or potential difference.

The SI unit is called the *farad (F):* **1 farad (1 F)= 1 coulomb per volt =1 C/V.**

25.2: Capacitance: Charging a Capacitor:

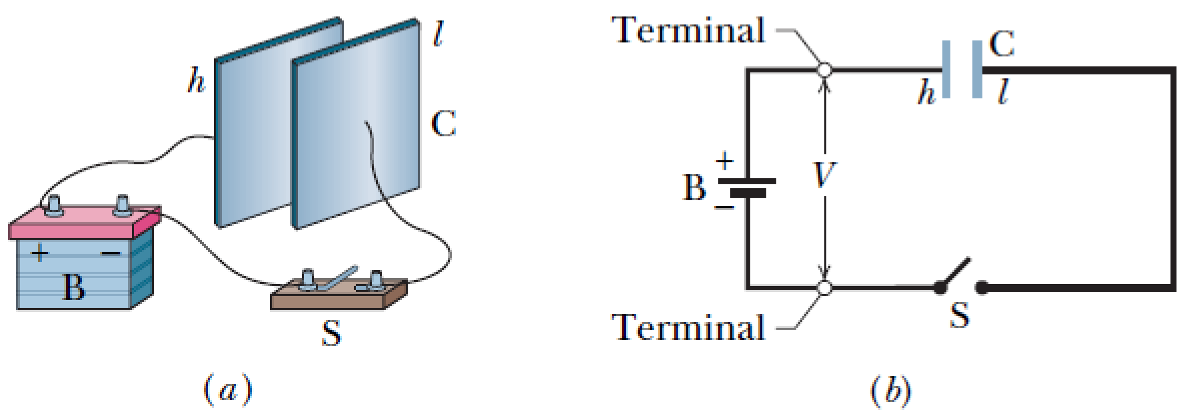


Fig. 25-4 (*a*) Battery B, switch S, and plates *h* and *l* of capacitor C, connected in a circuit. (*b*) A schematic diagram with the *circuit elements* represented by their symbols.

The circuit shown is incomplete because switch S is open; that is, the switch does not electrically connect the wires attached to it. When the switch is closed, electrically connecting those wires, the circuit is complete and charge can then flow through the switch and the wires.

As the plates become oppositely charged, that potential difference increases until it equals the potential difference V between the terminals of the battery. With the electric field zero, there is no further drive of electrons. The capacitor is then said to be fully charged, with a potential difference V and charge q.

25.3: Calculating the Capacitance:

To relate the electric field E between the plates of a capacitor to the charge *q* on either plate, we use Gauss' law:

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q.$$

Here *q* is the charge enclosed by a Gaussian surface and $\oint \vec{E} \cdot d\vec{A}$ is the net electric flux through that surface. In our special case in the figure,

$$q = \varepsilon_0 EA$$

in which *A* is the area of that part of the Gaussian surface through which there is a flux.

the potential difference between the plates of a capacitor is related to the field ***E*** by

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

If *V* is the difference V_f -V_i ,

$$V = \int_-^+ E\,ds = E\int_0^d ds = Ed.$$

$$C = \frac{\varepsilon_0 A}{d} \qquad \text{(parallel-plate capacitor).}$$

Here, $\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}.$

We use Gauss' law to relate *q* and *E*. Then we integrate the *E* to get the potential difference.

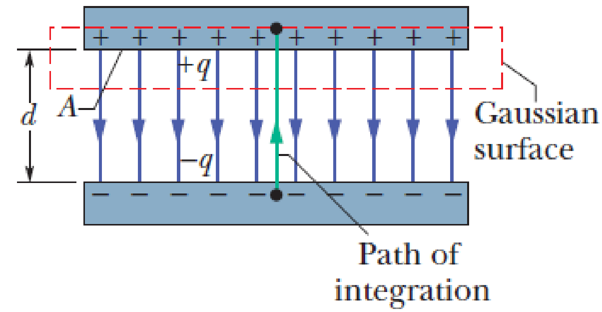


Fig. 25-5 A charged parallel-plate capacitor. A Gaussian surface encloses the charge on the positive plate. The integration of Eq. 25-6 is taken along a path extending directly from the negative plate to the positive plate.

25.3: Calculating the Capacitance, A Cylindrical Capacitor :

As a Gaussian surface, we choose a cylinder of length L and radius r, closed by end caps and placed as is shown. It is coaxial with the cylinders and encloses the central cylinder and thus also the charge q on that cylinder.

$$q = \varepsilon_0 EA = \varepsilon_0 E(2\pi rL),$$

$$E = \frac{q}{2\pi\varepsilon_0 Lr}.$$

$$V = \int_-^+ E\,ds = -\frac{q}{2\pi\varepsilon_0 L}\int_b^a \frac{dr}{r} = \frac{q}{2\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right),$$

$$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)} \quad \text{(cylindrical capacitor).}$$

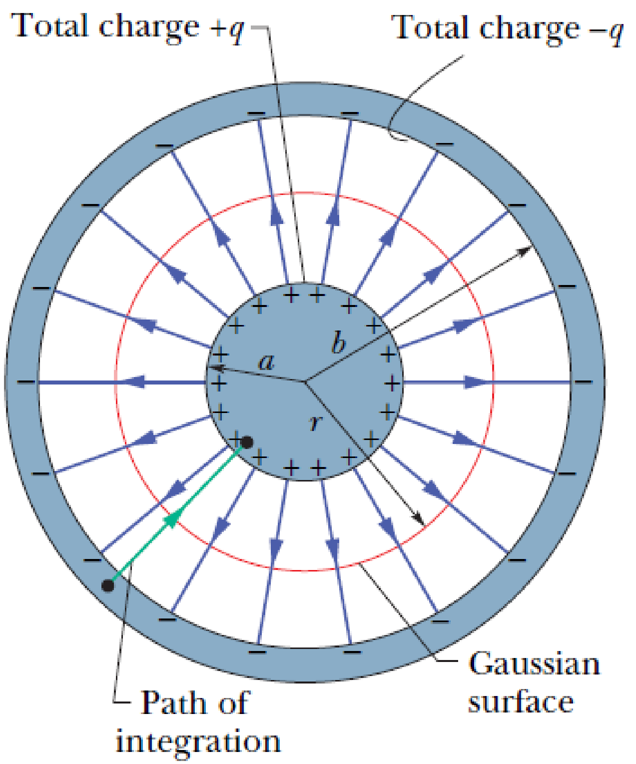


Fig. 25-6 A cross section of a long cylindrical capacitor, showing a cylindrical Gaussian surface of radius r (that encloses the positive plate) and the radial path of integration along which Eq. 25-6 is to be applied. This figure also serves to illustrate a spherical capacitor in a cross section through its center.

25.3: Calculating the Capacitance, A Spherical Capacitor:

$$q = \varepsilon_0 EA = \varepsilon_0 E(4\pi r^2),$$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2},$$

$$V = \int_{-}^{+} E\,ds = -\frac{q}{4\pi\varepsilon_0}\int_b^a \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{q}{4\pi\varepsilon_0}\frac{b-a}{ab},$$

$$\Rightarrow \quad C = 4\pi\varepsilon_0 \frac{ab}{b-a} \qquad \text{(spherical capacitor).}$$

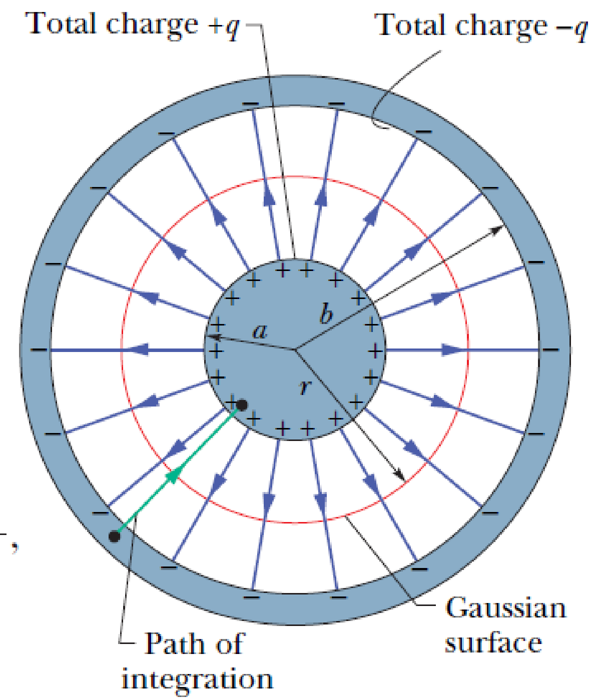


Fig. 25-6 A cross section of a long cylindrical capacitor, showing a cylindrical Gaussian surface of radius r (that encloses the positive plate) and the radial path of integration along which Eq. 25-6 is to be applied. This figure also serves to illustrate a spherical capacitor in a cross section through its center.

25.3: Calculating the Capacitance, An Isolated Sphere:

We can assign a capacitance to a single isolated spherical conductor of radius *R* by assuming that the "missing plate" is a conducting sphere of infinite radius.

The field lines that leave the surface of a positively charged isolated conductor must end somewhere; the walls of the room in which the conductor is housed can serve effectively as our sphere of infinite radius.

To find the capacitance of the conductor, we first rewrite the capacitance as:

$$C = 4\pi\varepsilon_0 \frac{a}{1 - a/b}.$$

Now letting $b \to \infty$, and substituting *R* for *a*,

$$C = 4\pi\varepsilon_0 R \qquad \text{(isolated sphere).}$$

Example, Charging the Plates in a Parallel-Plate Capacitor:

In Fig. 25-7*a*, switch S is closed to connect the uncharged capacitor of capacitance $C = 0.25\ \mu F$ to the battery of potential difference $V = 12$ V. The lower capacitor plate has thickness $L = 0.50$ cm and face area $A = 2.0 \times 10^{-4}\ m^2$, and it consists of copper, in which the density of conduction electrons is $n = 8.49 \times 10^{28}$ electrons/m^3. From what depth d within the plate (Fig. 25-7*b*) must electrons move to the plate face as the capacitor becomes charged?

KEY IDEA

The charge collected on the plate is related to the capacitance and the potential difference across the capacitor by Eq. 25-1 ($q = CV$).

Calculations: Because the lower plate is connected to the negative terminal of the battery, conduction electrons move up to the face of the plate. From Eq. 25-1, the total charge magnitude that collects there is

$$q = CV = (0.25 \times 10^{-6}\ F)(12\ V) = 3.0 \times 10^{-6}\ C.$$

Dividing this result by e gives us the number N of conduction electrons that come up to the face:

$$N = \frac{q}{e} = \frac{3.0 \times 10^{-6}\ C}{1.602 \times 10^{-19}\ C} = 1.873 \times 10^{13} \text{ electrons.}$$

These electrons come from a volume that is the product of the face area A and the depth d we seek. Thus, from the density of conduction electrons (number per volume), we can write

$$n = \frac{N}{Ad},$$

or

$$d = \frac{N}{An} = \frac{1.873 \times 10^{13} \text{ electrons}}{(2.0 \times 10^{-4}\ m^2)(8.49 \times 10^{28} \text{ electrons/m}^3)} = 1.1 \times 10^{-12}\ m = 1.1\ pm. \quad \text{(Answer)}$$

In common speech, we would say that the battery charges the capacitor by supplying the charged particles. But what the battery really does is set up an electric field in the wires and plate such that electrons very close to the plate face move up to the negative face.

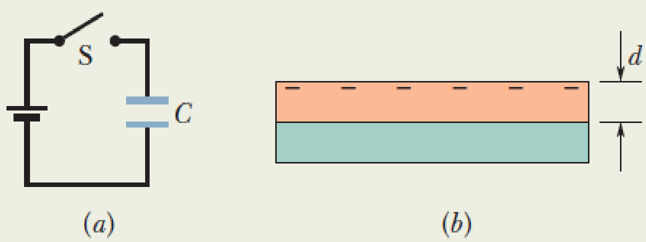


Fig. 25-7 (*a*) A battery and capacitor circuit. (*b*) The lower capacitor plate.

25.4: Capacitors in Parallel and in Series:

❖When a potential difference V is applied across several capacitors connected in parallel, that potential difference V is applied across each capacitor. The total charge q stored on the capacitors is the sum of the charges stored on all the capacitors.

❖Capacitors connected in parallel can be replaced with an equivalent capacitor that has the same total charge q and the same potential difference V as the actual capacitors.

$$q_1 = C_1V, \quad q_2 = C_2V, \quad \text{and} \quad q_3 = C_3V.$$

⇓

$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V.$$

⇓

$$C_{eq} = \frac{q}{V} = C_1 + C_2 + C_3,$$

⇓

$$C_{eq} = \sum_{j=1}^{n} C_j \qquad (n \text{ capacitors in parallel}).$$

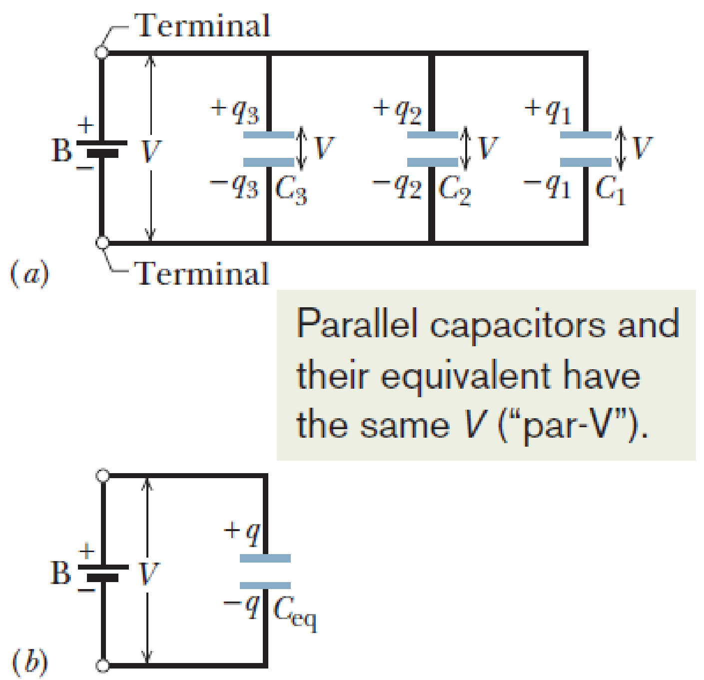


Fig. 25-8 (*a*) Three capacitors connected in parallel to battery B. The battery maintains potential difference V across its terminals and thus across *each* capacitor. (*b*) The equivalent capacitor, with capacitance C_{eq}, replaces the parallel combination.

25.4: Capacitors in Parallel and in Series:

❖When a potential difference V is applied across several capacitors connected in series, the capacitors have identical charge q. The sum of the potential differences across all the capacitors is equal to the applied potential difference V.

❖Capacitors that are connected in series can be replaced with an equivalent capacitor that has the same charge q and the same total potential difference V as the actual series capacitors.

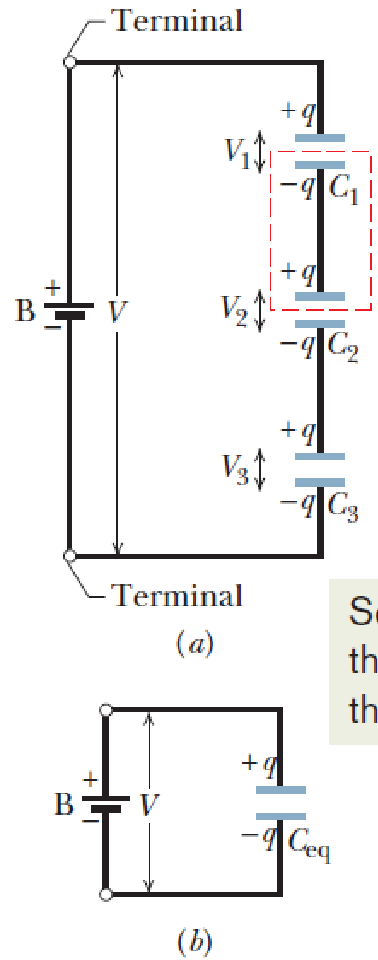
$$V_1 = \frac{q}{C_1}, \quad V_2 = \frac{q}{C_2}, \quad \text{and} \quad V_3 = \frac{q}{C_3}.$$

$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right).$$

$$C_{eq} = \frac{q}{V} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3},$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j} \quad (n \text{ capacitors in series}).$$



Series capacitors and their equivalent have the same q (“seri-q”).

Fig. 25-9 (*a*) Three capacitors connected in series to battery B. The battery maintains potential difference V between the top and bottom plates of the series combination. (*b*) The equivalent capacitor, with capacitance C_{eq}, replaces the series combination.

Example, Capacitors in Parallel and in Series:

(a) Find the equivalent capacitance for the combination of capacitances shown in Fig. 25-10*a*, across which potential difference V is applied. Assume

$$C_1 = 12.0\ \mu F, \quad C_2 = 5.30\ \mu F, \quad \text{and} \quad C_3 = 4.50\ \mu F.$$

⇨

$$C_{12} = C_1 + C_2 = 12.0\ \mu F + 5.30\ \mu F = 17.3\ \mu F.$$

$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3}$$

$$= \frac{1}{17.3\ \mu F} + \frac{1}{4.50\ \mu F} = 0.280\ \mu F^{-1},$$

$$C_{123} = \frac{1}{0.280\ \mu F^{-1}} = 3.57\ \mu F. \qquad \text{(Answer)}$$

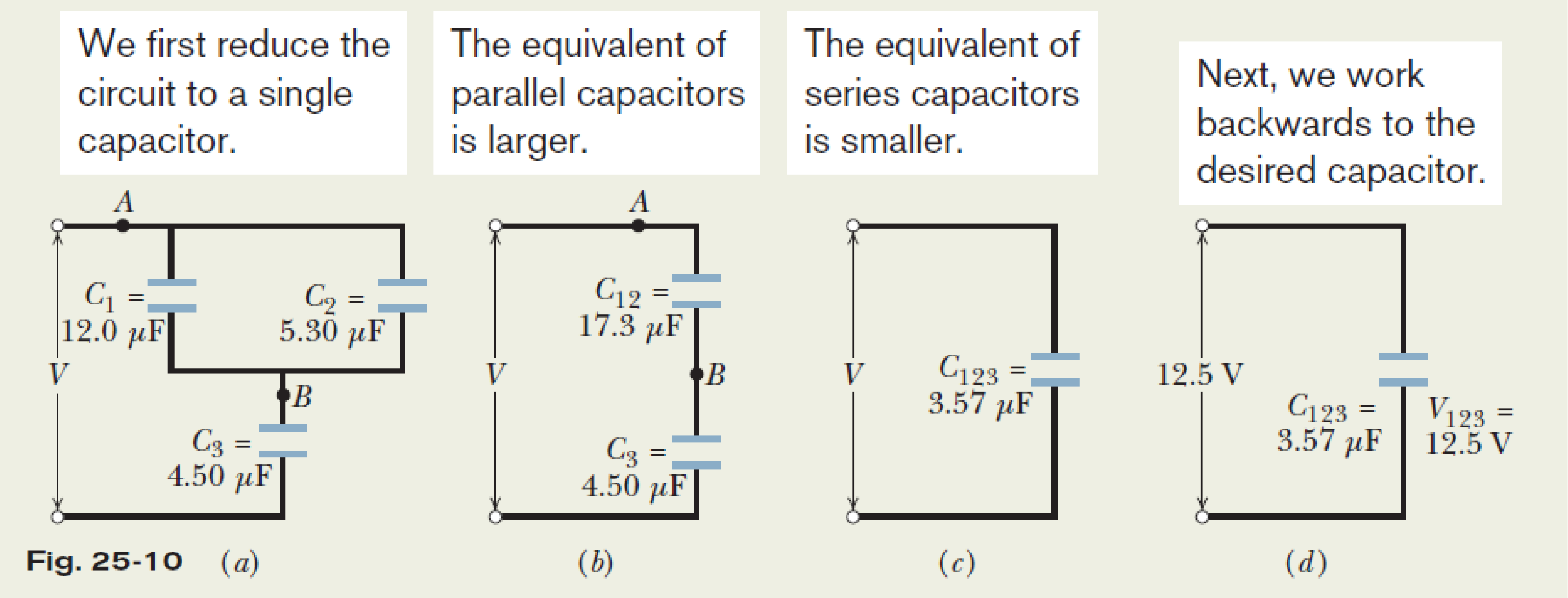


Fig. 25-10

Example, Capacitors in Parallel and in Series:

(b) The potential difference applied to the input terminals in Fig. 25-10a is $V = 12.5$ V. What is the charge on C_1?

We first reduce the circuit to a single capacitor.

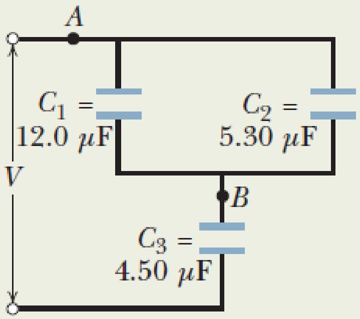
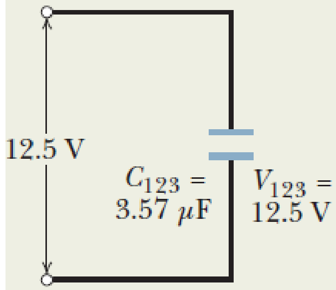


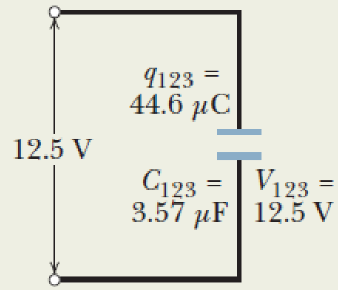
Fig. 25-10 (a)

Next, we work backwards to the desired capacitor.



(d)

Applying $q = CV$ yields the charge.



(e)

$$q_{123} = C_{123}V = (3.57\ \mu\text{F})(12.5\ \text{V}) = 44.6\ \mu\text{C}.$$

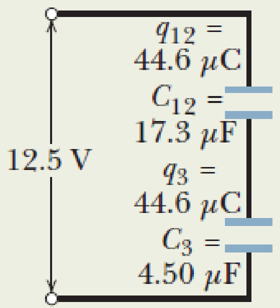
$$q_{12} = q_{123} = 44.6\ \mu\text{C}.$$

$$V_{12} = \frac{q_{12}}{C_{12}} = \frac{44.6\ \mu\text{C}}{17.3\ \mu\text{F}} = 2.58\ \text{V}.$$

$$V_1 = V_{12} = 2.58\ \text{V},$$

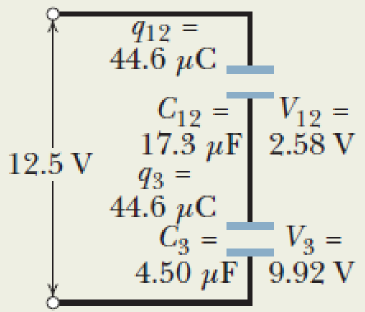
$$q_1 = C_1V_1 = (12.0\ \mu\text{F})(2.58\ \text{V}) = 31.0\ \mu\text{C}.$$

Series capacitors and their equivalent have the same q ("seri-q").



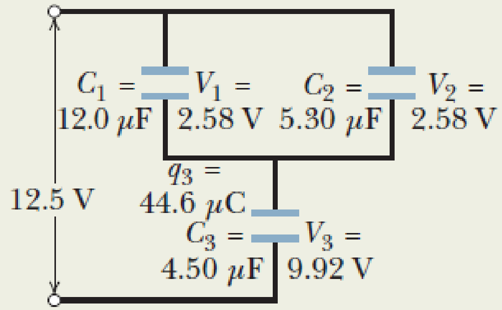
(f)

Applying $V = q/C$ yields the potential difference.



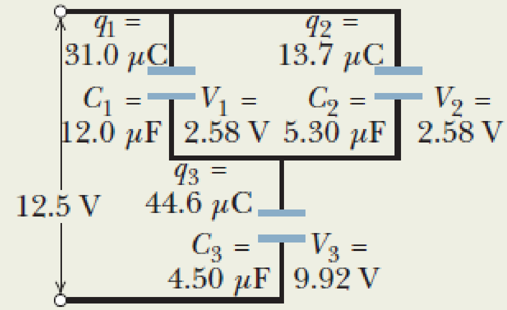
(g)

Parallel capacitors and their equivalent have the same V ("par-V").



(h)

Applying $q = CV$ yields the charge.



(i)

Example, One Capacitor Charging up Another Capacitor:

Capacitor 1, with $C_1 = 3.55\ \mu F$, is charged to a potential difference $V_0 = 6.30$ V, using a 6.30 V battery. The battery is then removed, and the capacitor is connected as in Fig. 25-11 to an uncharged capacitor 2, with $C_2 = 8.95\ \mu F$. When switch S is closed, charge flows between the capacitors. Find the charge on each capacitor when equilibrium is reached.

After the switch is closed, charge is transferred until the potential differences match.

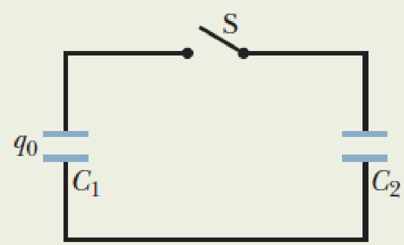


Fig. 25-11 A potential difference V_0 is applied to capacitor 1 and the charging battery is removed. Switch S is then closed so that the charge on capacitor 1 is shared with capacitor 2.

Calculations: Initially, when capacitor 1 is connected to the battery, the charge it acquires is, from Eq. 25-1,

$$q_0 = C_1V_0 = (3.55 \times 10^{-6}\ \text{F})\ (6.30\ \text{V}) = 22.365 \times 10^{-6}\ \text{C}.$$

When switch S in Fig. 25-11 is closed and capacitor 1 begins to charge capacitor 2, the electric potential and charge on capacitor 1 decrease and those on capacitor 2 increase until

$$V_1 = V_2 \qquad \text{(equilibrium).}$$

From Eq. 25-1, we can rewrite this as

$$\frac{q_1}{C_1} = \frac{q_2}{C_2} \qquad \text{(equilibrium).}$$

Because the total charge cannot magically change, the total after the transfer must be

$$q_1 + q_2 = q_0 \qquad \text{(charge conservation);}$$

thus

$$q_2 = q_0 - q_1.$$

We can now rewrite the second equilibrium equation as

$$\frac{q_1}{C_1} = \frac{q_0 - q_1}{C_2}.$$

Solving this for q_1 and substituting given data, we find

$$q_1 = 6.35\ \mu C. \qquad \text{(Answer)}$$

The rest of the initial charge ($q_0 = 22.365\ \mu C$) must be on capacitor 2:

$$q_2 = 16.0\ \mu C. \qquad \text{(Answer)}$$

25.5: Energy Stored in an Electric Field:

The potential energy of a charged capacitor may be viewed as being stored in the electric field between its plates.

Suppose that, at a given instant, a charge *q'* has been transferred from one plate of a capacitor to the other. The potential difference *V'* between the plates at that instant will be *q'/C*. If an extra increment of charge *dq'* is then transferred, the increment of work required will be,

$$dW = V'\,dq' = \frac{q'}{C}\,dq'.$$

The work required to bring the total capacitor charge up to a final value *q* is

$$W = \int dW = \frac{1}{C}\int_0^q q'\,dq' = \frac{q^2}{2C}.$$

This work is stored as potential energy *U* in the capacitor, so that,

$$U = \frac{q^2}{2C} \qquad \text{(potential energy).}$$

This can also be expressed as:

$$U = \tfrac{1}{2}CV^2 \qquad \text{(potential energy).}$$

25.5: Energy Stored in an Electric Field: Energy Density

In a parallel-plate capacitor, neglecting fringing, the electric field has the same value at all points between the plates. Thus, the **energy density *u***—that is, the potential energy per unit volume between the plates—should also be uniform.

We can find *u* by dividing the total potential energy by the volume *Ad of the* space between the plates.

$$u = \frac{U}{Ad} = \frac{CV^2}{2Ad}.$$

But since($C = \varepsilon_0 A/d$), this result becomes $$u = \tfrac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2.$$

However, ($E=-\Delta V/\Delta s$), *V/d* equals the electric field magnitude *E*. Therefore.

$$u = \tfrac{1}{2}\varepsilon_0 E^2 \qquad \text{(energy density).}$$

Example, Potential Energy and Energy Density of an Electric Field:

An isolated conducting sphere whose radius R is 6.85 cm has a charge $q = 1.25$ nC.

(a) How much potential energy is stored in the electric field of this charged conductor?

KEY IDEAS

(1) An isolated sphere has capacitance given by Eq. 25-18 ($C = 4\pi\varepsilon_0 R$). (2) The energy U stored in a capacitor depends on the capacitor's charge q and capacitance C according to Eq. 25-21 ($U = q^2/2C$).

Calculation: Substituting $C = 4\pi\varepsilon_0 R$ into Eq. 25-21 gives us

$$U = \frac{q^2}{2C} = \frac{q^2}{8\pi\varepsilon_0 R}$$

$$= \frac{(1.25 \times 10^{-9}\ \mathrm{C})^2}{(8\pi)(8.85 \times 10^{-12}\ \mathrm{F/m})(0.0685\ \mathrm{m})}$$

$$= 1.03 \times 10^{-7}\ \mathrm{J} = 103\ \mathrm{nJ}. \qquad \text{(Answer)}$$

(b) What is the energy density at the surface of the sphere?

KEY IDEA

The density u of the energy stored in an electric field depends on the magnitude E of the field, according to Eq. 25-25 ($u = \frac{1}{2}\varepsilon_0 E^2$).

Calculations: Here we must first find E at the surface of the sphere, as given by Eq. 23-15:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}.$$

The energy density is then

$$u = \tfrac{1}{2}\varepsilon_0 E^2 = \frac{q^2}{32\pi^2\varepsilon_0 R^4}$$

$$= \frac{(1.25 \times 10^{-9}\ \mathrm{C})^2}{(32\pi^2)(8.85 \times 10^{-12}\ \mathrm{C^2/N\cdot m^2})(0.0685\ \mathrm{m})^4}$$

$$= 2.54 \times 10^{-5}\ \mathrm{J/m^3} = 25.4\ \mu\mathrm{J/m^3}. \qquad \text{(Answer)}$$

25.6: Capacitor with a Dielectric:

In a region completely filled by a dielectric material of dielectric constant κ, all electrostatic equations containing the permittivity constant ε_0 are to be modified by replacing ε_0 with $\kappa\varepsilon_0$.

A *dielectric*, is an insulating material such as mineral oil or plastic, and is characterized by a numerical factor κ, called the *dielectric constant of the material.*

Some dielectrics, such as strontium titanate, can increase the capacitance by more than two orders of magnitude.

The introduction of a dielectric also limits the potential difference that can be applied between the plates to a certain value *Vmax*, called the breakdown potential. Every dielectric material has a characteristic *dielectric strength*, which is the maximum value of the electric field that it can tolerate without breakdown.

Table 25-1

Some Properties of Dielectrics[a]

Material	Dielectric Constant κ	Dielectric Strength (kV/mm)
Air (1 atm)	1.00054	3
Polystyrene	2.6	24
Paper	3.5	16
Transformer oil	4.5	
Pyrex	4.7	14
Ruby mica	5.4	
Porcelain	6.5	
Silicon	12	
Germanium	16	
Ethanol	25	
Water (20°C)	80.4	
Water (25°C)	78.5	
Titania ceramic	130	
Strontium titanate	310	8

For a vacuum, κ = unity.

[a]Measured at room temperature, except for the water.

Example, Work and Energy when a Dielectric is inserted inside a Capacitor:

A parallel-plate capacitor whose capacitance C is 13.5 pF is charged by a battery to a potential difference $V = 12.5$ V between its plates. The charging battery is now disconnected, and a porcelain slab ($\kappa = 6.50$) is slipped between the plates.

(a) What is the potential energy of the capacitor before the slab is inserted?

KEY IDEA

We can relate the potential energy U_i of the capacitor to the capacitance C and either the potential V (with Eq. 25-22) or the charge q (with Eq. 25-21):

$$U_i = \tfrac{1}{2}CV^2 = \frac{q^2}{2C}.$$

Calculation: Because we are given the initial potential V (= 12.5 V), we use Eq. 25-22 to find the initial stored energy:

$$U_i = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(13.5 \times 10^{-12}\ \text{F})(12.5\ \text{V})^2$$
$$= 1.055 \times 10^{-9}\ \text{J} = 1055\ \text{pJ} \approx 1100\ \text{pJ}. \quad \text{(Answer)}$$

(b) What is the potential energy of the capacitor–slab device after the slab is inserted?

KEY IDEA

Because the battery has been disconnected, the charge on the capacitor cannot change when the dielectric is inserted. However, the potential *does* change.

Calculations: Thus, we must now use Eq. 25-21 to write the final potential energy U_f, but now that the slab is within the capacitor, the capacitance is κC. We then have

$$U_f = \frac{q^2}{2\kappa C} = \frac{U_i}{\kappa} = \frac{1055\ \text{pJ}}{6.50}$$
$$= 162\ \text{pJ} \approx 160\ \text{pJ}. \quad \text{(Answer)}$$

When the slab is introduced, the potential energy decreases by a factor of κ.

The "missing" energy, in principle, would be apparent to the person who introduced the slab. The capacitor would exert a tiny tug on the slab and would do work on it, in amount

$$W = U_i - U_f = (1055 - 162)\ \text{pJ} = 893\ \text{pJ}.$$

If the slab were allowed to slide between the plates with no restraint and if there were no friction, the slab would oscillate back and forth between the plates with a (constant) mechanical energy of 893 pJ, and this system energy would transfer back and forth between kinetic energy of the moving slab and potential energy stored in the electric field.

25.7: Dielectrics, an Atomic View:

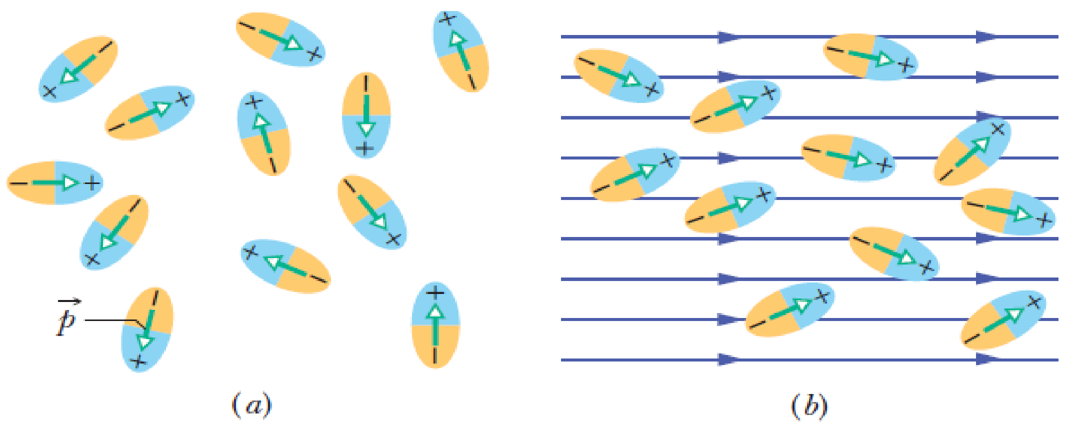


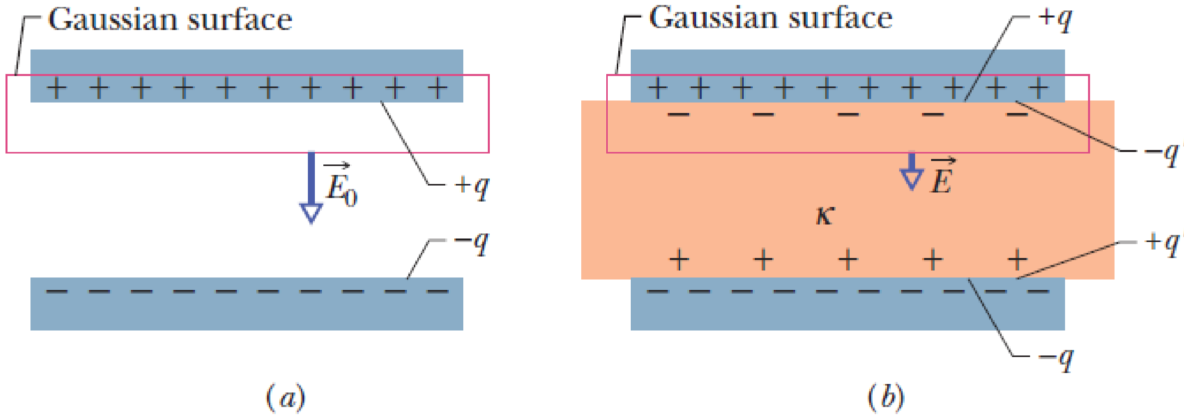
Fig. 25-14 (*a*) Molecules with a permanent electric dipole moment, showing their random orientation in the absence of an external electric field. (*b*) An electric field is applied, producing partial alignment of the dipoles. Thermal agitation prevents complete alignment.

1. ***Polar dielectrics.*** The molecules of some dielectrics, like water, have permanent electric dipole moments. In such materials (called *polar dielectrics)*, the electric dipoles tend to line up with an external electric field as in Fig. 25-14. Since the molecules are continuously jostling each other as a result of their random thermal motion, this alignment is not complete, but it becomes more complete as the magnitude of the applied field is increased (or as the temperature, and thus the jostling, are decreased).The alignment of the electric dipoles produces an electric field that is directed opposite the applied field and is smaller in magnitude.

2. ***Nonpolar dielectrics.*** Regardless of whether they have permanent electric dipole moments, molecules acquire dipole moments by induction when placed in an external electric field. This occurs because the external field tends to "stretch" the molecules, slightly separating the centers of negative and positive charge.

25.8: Dielectrics and Gauss' Law:

Fig. 25-16 A parallel-plate capacitor (*a*) without and (*b*) with a dielectric slab inserted. The charge *q* on the plates is assumed to be the same in both cases.



For the situation of Fig. 25-16*a*, without a dielectric, the electric field between the plates can be found using Gauss's Law. We enclose the charge *q* on the top plate with a Gaussian surface and then apply Gauss' law. If E_0 represents the magnitude of the field, we have

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 EA = q, \quad \Longrightarrow \quad E_0 = \frac{q}{\varepsilon_0 A}.$$

In Fig. 25-16*b*, with the dielectric in place, we can find the electric field between the plates (and within the dielectric) by using the same Gaussian surface. Now the surface encloses two types of charge: It still encloses charge $+q$ on the top plate, but it now also encloses the induced charge $-q'$ *on* the top face of the dielectric. The charge on the conducting plate is said to be *free charge because it can move if we change the electric potential of the plate; the* induced charge on the surface of the dielectric is not free charge because it cannot move from that surface.

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 EA = q - q', \quad \Longrightarrow \quad E = \frac{q - q'}{\varepsilon_0 A}.$$

The effect of the dielectric is to weaken the original field E_0 by a factor of κ: $E = \dfrac{E_0}{\kappa} = \dfrac{q}{\kappa \varepsilon_0 A}.$

Since $q - q' = \dfrac{q}{\kappa}.$ $\Longrightarrow$

$$\varepsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q \qquad \text{(Gauss' law with dielectric).}$$

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1. The flux integral now involves ***κE*,** not just ***E*.** (The vector is sometimes called the electric displacement, ***D***. The above equation can be written as: $\oint \vec{D} \cdot d\vec{A} = q.$
2. The charge *q* enclosed by the Gaussian surface is now taken to be the free charge only. The induced surface charge is deliberately ignored on the right side of the above equation, having been taken fully into account by introducing the dielectric constant κ on the left side.
3. ε_0 gets replaced by $\kappa\varepsilon_0$. We keep κ inside the integral of the above equation to allow for cases in which κ is not constant over the entire Gaussian surface.

Example, Dielectric Partially Filling a Gap in a Capacitor:

Figure 25-17 shows a parallel-plate capacitor of plate area A and plate separation d. A potential difference V_0 is applied between the plates by connecting a battery between them. The battery is then disconnected, and a dielectric slab of thickness b and dielectric constant κ is placed between the plates as shown. Assume $A = 115\ \text{cm}^2$, $d = 1.24$ cm, $V_0 = 85.5$ V, $b = 0.780$ cm, and $\kappa = 2.61$.

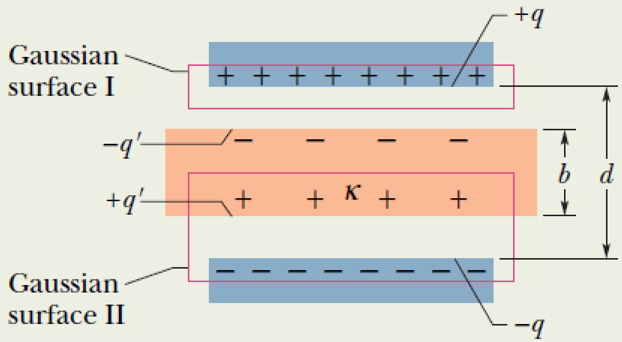

Fig. 25-17 A parallel-plate capacitor containing a dielectric slab that only partially fills the space between the plates.

(a) What is the capacitance C_0 before the dielectric slab is inserted?

Calculation: From Eq. 25-9 we have

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}\ \text{F/m})(115 \times 10^{-4}\ \text{m}^2)}{1.24 \times 10^{-2}\ \text{m}}$$
$$= 8.21 \times 10^{-12}\ \text{F} = 8.21\ \text{pF}. \quad \text{(Answer)}$$

(b) What free charge appears on the plates?

Calculation: From Eq. 25-1,

$$q = C_0 V_0 = (8.21 \times 10^{-12}\ \text{F})(85.5\ \text{V})$$
$$= 7.02 \times 10^{-10}\ \text{C} = 702\ \text{pC}. \quad \text{(Answer)}$$

(c) What is the electric field E_0 in the gaps between the plates and the dielectric slab?

Calculations: That surface passes through the gap, and so it encloses *only* the free charge on the upper capacitor plate. Electric field pierces only the bottom of the Gaussian surface. Because there the area vector $d\vec{A}$ and the field vector $\vec{E}_0$ are both directed downward, the dot product in Eq. 25-36 becomes

$$\vec{E}_0 \cdot d\vec{A} = E_0\, dA \cos 0° = E_0\, dA.$$

Equation 25-36 then becomes

$$\varepsilon_0 \kappa E_0 \oint dA = q.$$

The integration now simply gives the surface area A of the plate. Thus, we obtain

$$\varepsilon_0 \kappa E_0 A = q,$$

or

$$E_0 = \frac{q}{\varepsilon_0 \kappa A}.$$

We must put $\kappa = 1$ here because Gaussian surface I does not pass through the dielectric. Thus, we have

$$E_0 = \frac{q}{\varepsilon_0 \kappa A} = \frac{7.02 \times 10^{-10}\ \text{C}}{(8.85 \times 10^{-12}\ \text{F/m})(1)(115 \times 10^{-4}\ \text{m}^2)}$$
$$= 6900\ \text{V/m} = 6.90\ \text{kV/m}. \quad \text{(Answer)}$$

Note that the value of E_0 does not change when the slab is introduced because the amount of charge enclosed by Gaussian surface I in Fig. 25-17 does not change.

Example, Dielectric Partially Filling a Gap in a Capacitor, cont.:

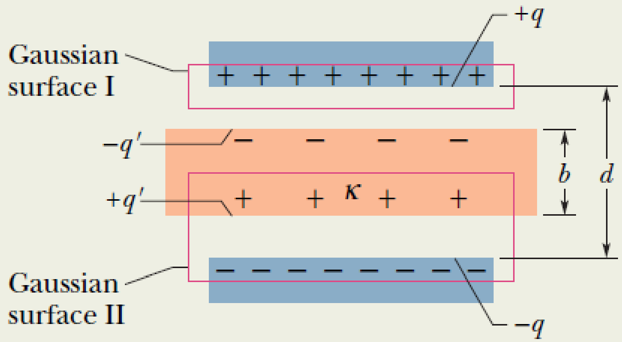

Fig. 25-17 A parallel-plate capacitor containing a dielectric slab that only partially fills the space between the plates.

(d) What is the electric field E_1 in the dielectric slab?

Calculations: That surface encloses free charge $-q$ and induced charge $+q'$, but we ignore the latter when we use Eq. 25-36. We find

$$\varepsilon_0 \oint \kappa \vec{E}_1 \cdot d\vec{A} = -\varepsilon_0 \kappa E_1 A = -q. \qquad (25\text{-}37)$$

The first minus sign in this equation comes from the dot product $\vec{E}_1 \cdot d\vec{A}$ along the top of the Gaussian surface because now the field vector $\vec{E}_1$ is directed downward and the area vector $d\vec{A}$ (which, as always, points outward from the interior of a closed Gaussian surface) is directed upward. With 180° between the vectors, the dot product is negative. Now $\kappa = 2.61$. Thus, Eq. 25-37 gives us

$$E_1 = \frac{q}{\varepsilon_0 \kappa A} = \frac{E_0}{\kappa} = \frac{6.90 \text{ kV/m}}{2.61}$$

$$= 2.64 \text{ kV/m}. \qquad \text{(Answer)}$$

(e) What is the potential difference V between the plates after the slab has been introduced?

Calculation: Within the dielectric, the path length is b and the electric field is E_1. Within the two gaps above and below the dielectric, the total path length is $d - b$ and the electric field is E_0. Equation 25-6 then yields

$$V = \int_-^+ E\,ds = E_0(d-b) + E_1 b$$

$$= (6900 \text{ V/m})(0.0124 \text{ m} - 0.00780 \text{ m})$$

$$+ (2640 \text{ V/m})(0.00780 \text{ m})$$

$$= 52.3 \text{ V}. \qquad \text{(Answer)}$$

This is less than the original potential difference of 85.5 V.

(f) What is the capacitance with the slab in place between the plates of the capacitor?

Calculation: Taking q from (b) and V from (e), we have

$$C = \frac{q}{V} = \frac{7.02 \times 10^{-10} \text{ C}}{52.3 \text{ V}}$$

$$= 1.34 \times 10^{-11} \text{ F} = 13.4 \text{ pF}. \qquad \text{(Answer)}$$

This is greater than the original capacitance of 8.21 pF.