

# Chapter 26

## Current and Resistance



## 26.2: Electric Current:

Although an electric current is a stream of moving charges, not all moving charges constitute an electric current. If there is to be an electric current through a given surface, there must be a net flow of charge through that surface. Two examples are given.

**1.** The free electrons (conduction electrons) in an isolated length of copper wire are in random motion at speeds of the order of  $10^6$  m/s. If you pass a hypothetical plane through such a wire, conduction electrons pass through it in both directions at the rate of many billions per second—but there is no *net* transport of charge and thus *no current through the wire*. However, if you connect the ends of the wire to a battery, you slightly bias the flow in one direction, with the result that there now is a *net* transport of charge and thus an electric current through the wire.

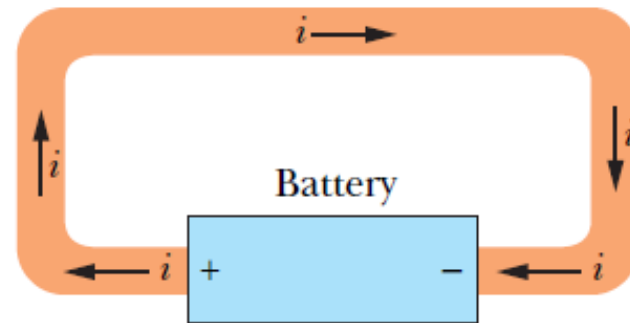
**2.** The flow of water through a garden hose represents the directed flow of positive charge (the protons in the water molecules) at a rate of perhaps several million coulombs per second. There is no net transport of charge, because there is a parallel flow of negative charge (the electrons in the water molecules) of exactly the same amount moving in exactly the same direction.

## 26.2: Electric Current:

**Fig. 26-1** (a) A loop of copper in electrostatic equilibrium. The entire loop is at a single potential, and the electric field is zero at all points inside the copper. (b) Adding a battery imposes an electric potential difference between the ends of the loop that are connected to the terminals of the battery. The battery thus produces an electric field within the loop, from terminal to terminal, and the field causes charges to move around the loop. This movement of charges is a current  $i$ .



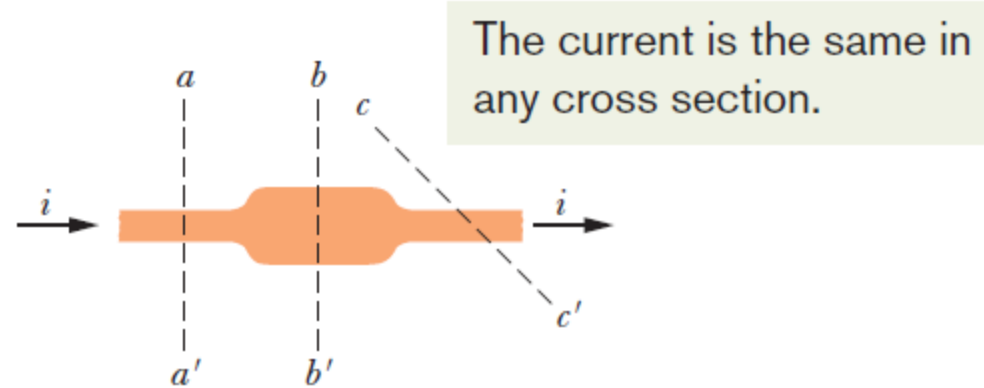
(a)



(b)

## 26.2: Electric Current:

**Fig. 26-2** The current  $i$  through the conductor has the same value at planes  $aa'$ ,  $bb'$ , and  $cc'$ .



The figure shows a section of a conductor, part of a conducting loop in which current has been established. If charge  $dq$  passes through a hypothetical plane (such as  $aa'$ ) in time  $dt$ , then the current  $i$  through that plane is defined as:

$$i = \frac{dq}{dt} \quad (\text{definition of current}).$$

The charge that passes through the plane in a time interval extending from 0 to  $t$  is:

$$q = \int dq = \int_0^t i dt$$

Under steady-state conditions, the current is the same for planes  $aa'$ ,  $bb'$ , and  $cc'$  and for all planes that pass completely through the conductor, no matter what their location or orientation.

The SI unit for current is the coulomb per second, or the ampere (A):

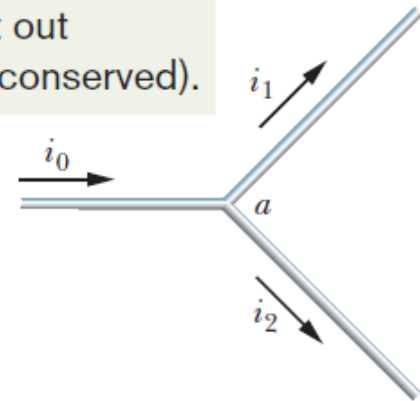
$$1 \text{ ampere} = 1 \text{ A} = 1 \text{ coulomb per second} = 1 \text{ C/s}.$$

## 26.2: Electric Current, Conservation of Charge, and Direction of Current:



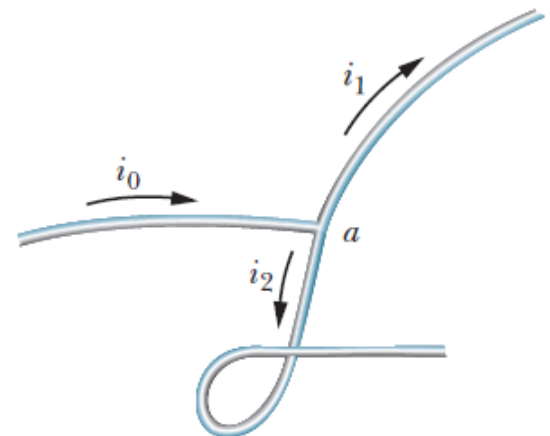
A current arrow is drawn in the direction in which positive charge carriers would move, even if the actual charge carriers are negative and move in the opposite direction.

The current into the junction must equal the current out (charge is conserved).



(a)

**Fig. 26-3** The relation  $i_0 = i_1 + i_2$  is true at junction  $a$  no matter what the orientation in space of the three wires. Currents are scalars, not vectors.



(b)

## Example, Current is the Rate at which charge passes through a point:

Water flows through a garden hose at a volume flow rate  $dV/dt$  of  $450 \text{ cm}^3/\text{s}$ . What is the current of negative charge?

**Calculations:** We can write the current in terms of the number of molecules that pass through such a plane per second as

$$i = \left( \begin{array}{c} \text{charge} \\ \text{per} \\ \text{electron} \end{array} \right) \left( \begin{array}{c} \text{electrons} \\ \text{per} \\ \text{molecule} \end{array} \right) \left( \begin{array}{c} \text{molecules} \\ \text{per} \\ \text{second} \end{array} \right)$$

or

$$i = (e)(10) \frac{dN}{dt}.$$

We substitute 10 electrons per molecule because a water ( $\text{H}_2\text{O}$ ) molecule contains 8 electrons in the single oxygen atom and 1 electron in each of the two hydrogen atoms.

We can express the rate  $dN/dt$  in terms of the given volume flow rate  $dV/dt$  by first writing

$$\left( \begin{array}{c} \text{molecules} \\ \text{per} \\ \text{second} \end{array} \right) = \left( \begin{array}{c} \text{molecules} \\ \text{per} \\ \text{mole} \end{array} \right) \left( \begin{array}{c} \text{moles} \\ \text{per unit} \\ \text{mass} \end{array} \right) \\ \times \left( \begin{array}{c} \text{mass} \\ \text{per unit} \\ \text{volume} \end{array} \right) \left( \begin{array}{c} \text{volume} \\ \text{per} \\ \text{second} \end{array} \right).$$

$$\frac{dN}{dt} = N_A \left( \frac{1}{M} \right) \rho_{\text{mass}} \left( \frac{dV}{dt} \right) = \frac{N_A \rho_{\text{mass}}}{M} \frac{dV}{dt}.$$

Substituting this into the equation for  $i$ , we find

$$i = 10eN_A M^{-1} \rho_{\text{mass}} \frac{dV}{dt}.$$

We know that Avogadro's number  $N_A$  is  $6.02 \times 10^{23}$  molecules/mol, or  $6.02 \times 10^{23} \text{ mol}^{-1}$ , and from Table 15-1 we know that the density of water  $\rho_{\text{mass}}$  under normal conditions is  $1000 \text{ kg/m}^3$ . We can get the molar mass of water from the molar masses listed in Appendix F (in grams per mole): We add the molar mass of oxygen (16 g/mol) to twice the molar mass of hydrogen (1 g/mol), obtaining  $18 \text{ g/mol} = 0.018 \text{ kg/mol}$ . So, the current of negative charge due to the electrons in the water is

$$\begin{aligned} i &= (10)(1.6 \times 10^{-19} \text{ C})(6.02 \times 10^{23} \text{ mol}^{-1}) \\ &\quad \times (0.018 \text{ kg/mol})^{-1}(1000 \text{ kg/m}^3)(450 \times 10^{-6} \text{ m}^3/\text{s}) \\ &= 2.41 \times 10^7 \text{ C/s} = 2.41 \times 10^7 \text{ A} \\ &= 24.1 \text{ MA.} \end{aligned} \quad (\text{Answer})$$

## 26.3: Current Density:

The magnitude of **current density**,  $\mathbf{J}$ , is equal to the current per unit area through any element of cross section. It has the same direction as the velocity of the moving charges if they are positive and the opposite direction if they are negative.

$$i = \int \vec{J} \cdot d\vec{A}.$$

If the current is uniform across the surface and parallel to  $d\mathbf{A}$ , then  $\mathbf{J}$  is also uniform and parallel to  $d\mathbf{A}$ .

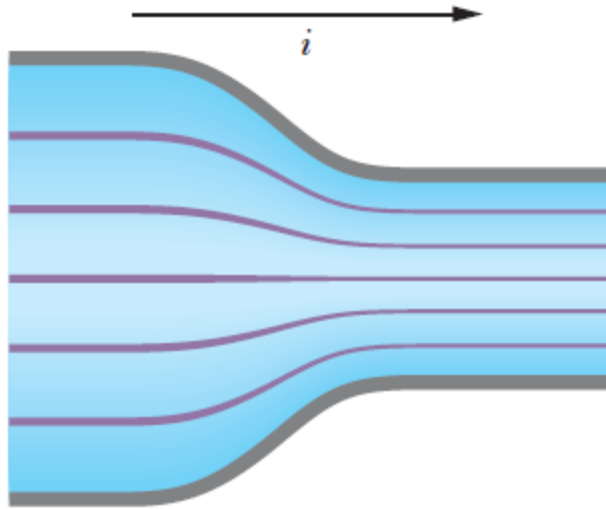
$$i = \int J dA = J \int dA = JA$$

$$J = \frac{i}{A},$$

Here,  $A$  is the total area of the surface.

The SI unit for current density is the ampere per square meter ( $\text{A/m}^2$ ).

## 26.3: Current Density:



**Fig. 26-4** Streamlines representing current density in the flow of charge through a constricted conductor.

Figure 26-4 shows how current density can be represented with a similar set of lines, which we can call *streamlines*.

The current, which is toward the right, makes a transition from the wider conductor at the left to the narrower conductor at the right. Since charge is conserved during the transition, the amount of charge and thus the amount of current cannot change.

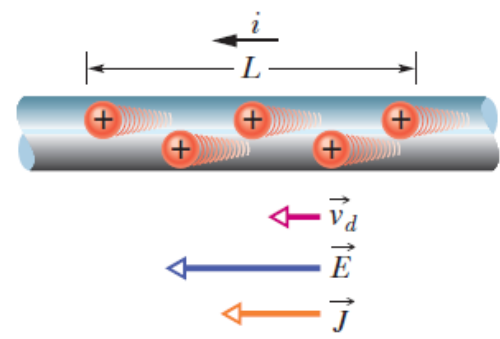
However, the current density changes—it is greater in the narrower conductor.



### 26.3: Current Density, Drift Speed:

Current is said to be due to positive charges that are propelled by the electric field.

**Fig. 26-5** Positive charge carriers drift at speed  $v_d$  in the direction of the applied electric field  $\vec{E}$ . By convention, the direction of the current density  $\vec{J}$  and the sense of the current arrow are drawn in that same direction.



When a conductor has a current passing through it, the electrons move randomly, but they tend to *drift* with a **drift speed**  $v_d$  in the direction opposite that of the applied electric field that causes the current. The drift speed is tiny compared with the speeds in the random motion.

In the figure, the equivalent drift of positive charge carriers is in the direction of the applied electric field,  $\vec{E}$ . If we assume that these charge carriers all move with the same drift speed  $v_d$  and that the current density  $\vec{J}$  is uniform across the wire's cross-sectional area  $A$ , then the number of charge carriers in a length  $L$  of the wire is  $nAL$ . Here  $n$  is the number of carriers per unit volume.

The total charge of the carriers in the length  $L$ , each with charge  $e$ , is then  $q = (nAL)e$ .

The total charge moves through any cross section of the wire in the time interval  $t = \frac{L}{v_d}$ .

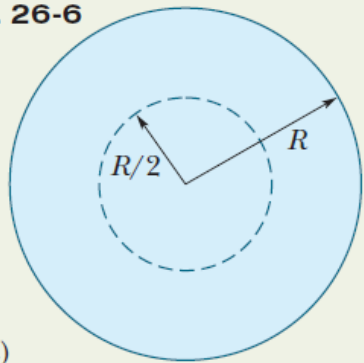
$$\Rightarrow i = \frac{q}{t} = \frac{nALe}{L/v_d} = nAev_d \quad \Rightarrow v_d = \frac{i}{nAe} = \frac{J}{ne}$$

$$\Rightarrow \boxed{\vec{J} = (ne)\vec{v}_d}$$

## Example, Current Density, Uniform and Nonuniform:

(a) The current density in a cylindrical wire of radius  $R = 2.0 \text{ mm}$  is uniform across a cross section of the wire and is  $J = 2.0 \times 10^5 \text{ A/m}^2$ . What is the current through the outer portion of the wire between radial distances  $R/2$  and  $R$  (Fig. 26-6a)?

Fig. 26-6



(a)

**Calculations:** We want only the current through a reduced cross-sectional area  $A'$  of the wire (rather than the entire area), where

$$\begin{aligned} A' &= \pi R^2 - \pi \left( \frac{R}{2} \right)^2 = \pi \left( \frac{3R^2}{4} \right) \\ &= \frac{3\pi}{4} (0.0020 \text{ m})^2 = 9.424 \times 10^{-6} \text{ m}^2. \end{aligned}$$

So, we rewrite Eq. 26-5 as

$$i = JA'$$

and then substitute the data to find

$$\begin{aligned} i &= (2.0 \times 10^5 \text{ A/m}^2)(9.424 \times 10^{-6} \text{ m}^2) \\ &= 1.9 \text{ A.} \end{aligned} \quad \text{(Answer)}$$

## Example, Current Density, Uniform and Nonuniform, cont.:

(b) Suppose, instead, that the current density through a cross section varies with radial distance  $r$  as  $J = ar^2$ , in which  $a = 3.0 \times 10^{11} \text{ A/m}^4$  and  $r$  is in meters. What now is the current through the same outer portion of the wire?

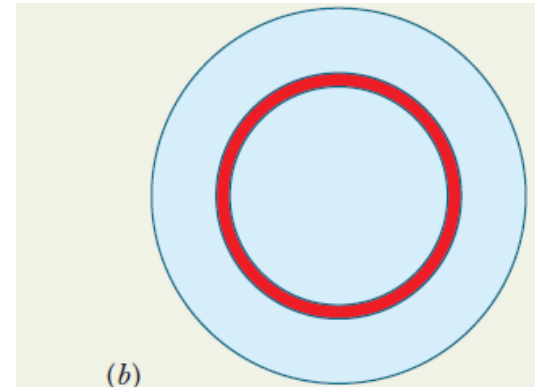
**Calculations:** The current density vector  $\vec{J}$  (along the wire's length) and the differential area vector  $d\vec{A}$  (perpendicular to a cross section of the wire) have the same direction. Thus,

$$\vec{J} \cdot d\vec{A} = J dA \cos 0 = J dA.$$

**Calculations:** The current density vector  $\vec{J}$  (along the wire's length) and the differential area vector  $d\vec{A}$  (perpendicular to a cross section of the wire) have the same direction. Thus,

$$\vec{J} \cdot d\vec{A} = J dA \cos 0 = J dA.$$

We need to replace the differential area  $dA$  with something we can actually integrate between the limits  $r = R/2$  and  $r = R$ . The simplest replacement (because  $J$  is given as a function of  $r$ ) is the area  $2\pi r dr$  of a thin ring of circumference  $2\pi r$  and width  $dr$  (Fig. 26-6b). We can then integrate



$$\begin{aligned} i &= \int \vec{J} \cdot d\vec{A} = \int J dA \\ &= \int_{R/2}^R ar^2 2\pi r dr = 2\pi a \int_{R/2}^R r^3 dr \\ &= 2\pi a \left[ \frac{r^4}{4} \right]_{R/2}^R = \frac{\pi a}{2} \left[ R^4 - \frac{R^4}{16} \right] = \frac{15}{32} \pi a R^4 \\ &= \frac{15}{32} \pi (3.0 \times 10^{11} \text{ A/m}^4) (0.0020 \text{ m})^4 = 7.1 \text{ A}. \end{aligned}$$

(Answer)

## Example, In a current, the conduction electrons move very slowly.:

What is the drift speed of the conduction electrons in a copper wire with radius  $r = 900 \mu\text{m}$  when it has a uniform current  $i = 17 \text{ mA}$ ? Assume that each copper atom contributes one conduction electron to the current and that the current density is uniform across the wire's cross section.

**Calculations:** Let us start with the third idea by writing

$$n = \left( \frac{\text{atoms}}{\text{per unit volume}} \right) = \left( \frac{\text{atoms}}{\text{per mole}} \right) \left( \frac{\text{moles}}{\text{per unit mass}} \right) \left( \frac{\text{mass}}{\text{per unit volume}} \right).$$

The number of atoms per mole is just Avogadro's number  $N_A (= 6.02 \times 10^{23} \text{ mol}^{-1})$ . Moles per unit mass is the inverse of the mass per mole, which here is the molar mass  $M$  of copper. The mass per unit volume is the (mass) density  $\rho_{\text{mass}}$  of copper. Thus,

$$n = N_A \left( \frac{1}{M} \right) \rho_{\text{mass}} = \frac{N_A \rho_{\text{mass}}}{M}.$$

Taking copper's molar mass  $M$  and density  $\rho_{\text{mass}}$  from Appendix F, we then have (with some conversions of units)

$$\begin{aligned} n &= \frac{(6.02 \times 10^{23} \text{ mol}^{-1})(8.96 \times 10^3 \text{ kg/m}^3)}{63.54 \times 10^{-3} \text{ kg/mol}} \\ &= 8.49 \times 10^{28} \text{ electrons/m}^3 \end{aligned}$$

or 
$$n = 8.49 \times 10^{28} \text{ m}^{-3}.$$

Next let us combine the first two key ideas by writing

$$\frac{i}{A} = nev_d.$$

Substituting for  $A$  with  $\pi r^2 (= 2.54 \times 10^{-6} \text{ m}^2)$  and solving for  $v_d$ , we then find

$$\begin{aligned} v_d &= \frac{i}{ne(\pi r^2)} \\ &= \frac{17 \times 10^{-3} \text{ A}}{(8.49 \times 10^{28} \text{ m}^{-3})(1.6 \times 10^{-19} \text{ C})(2.54 \times 10^{-6} \text{ m}^2)} \\ &= 4.9 \times 10^{-7} \text{ m/s}, \end{aligned} \quad \text{(Answer)}$$

which is only 1.8 mm/h, slower than a sluggish snail.

## 26.4: Resistance and Resistivity:

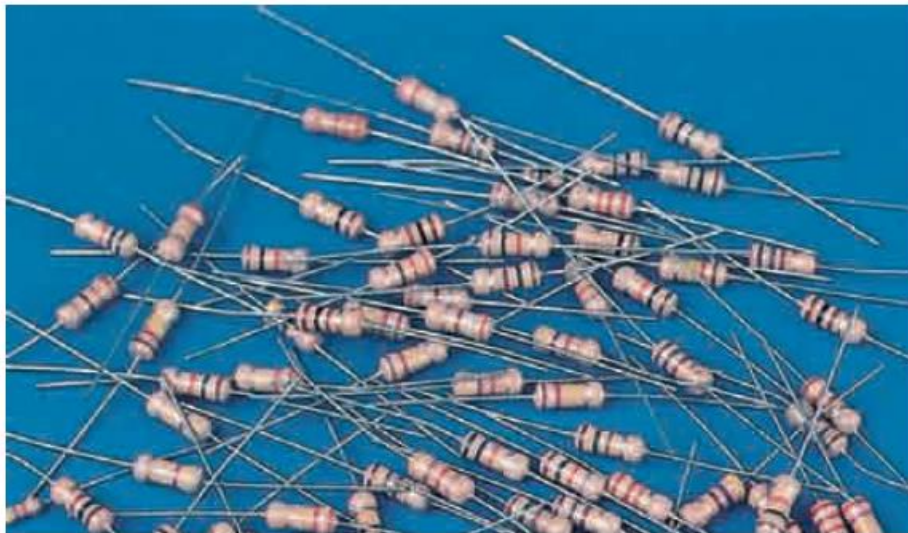
We determine the resistance between any two points of a conductor by applying a potential difference  $V$  between those points and measuring the current  $i$  that results. The resistance  $R$  is then

$$R = \frac{V}{i} \quad (\text{definition of } R).$$

The SI unit for resistance that follows from Eq. 26-8 is the volt per ampere. This has a special name, the **ohm** (symbol  $\Omega$ ):

$$\begin{aligned} 1 \text{ ohm} &= 1 \Omega = 1 \text{ volt per ampere} \\ &= 1 \text{ V/A}. \end{aligned}$$

In a circuit diagram, we represent a resistor and a resistance with the symbol  $\sim\sim\sim$ .



**Fig. 26-7** An assortment of resistors. The circular bands are color-coding marks that identify the value of the resistance. (*The Image Works*)

# 26.4: Resistance and Resistivity:

The resistivity,  $\rho$ , of a resistor is defined as:

$$\rho = \frac{E}{J} \quad \rightarrow \quad \vec{E} = \rho \vec{J}.$$

The SI unit for  $\rho$  is  $\Omega \cdot m$ .

The conductivity  $\sigma$  of a material is the reciprocal of its resistivity:

$$\sigma = \frac{1}{\rho} \quad \rightarrow \quad \vec{J} = \sigma \vec{E}.$$

**Table 26-1**

Resistivities of Some Materials at Room Temperature (20°C)

Material	Resistivity, $\rho$ ( $\Omega \cdot m$ )	Temperature Coefficient of Resistivity, $\alpha$ ( $K^{-1}$ )
<i>Typical Metals</i>		
Silver	$1.62 \times 10^{-8}$	$4.1 \times 10^{-3}$
Copper	$1.69 \times 10^{-8}$	$4.3 \times 10^{-3}$
Gold	$2.35 \times 10^{-8}$	$4.0 \times 10^{-3}$
Aluminum	$2.75 \times 10^{-8}$	$4.4 \times 10^{-3}$
Manganin <sup>a</sup>	$4.82 \times 10^{-8}$	$0.002 \times 10^{-3}$
Tungsten	$5.25 \times 10^{-8}$	$4.5 \times 10^{-3}$
Iron	$9.68 \times 10^{-8}$	$6.5 \times 10^{-3}$
Platinum	$10.6 \times 10^{-8}$	$3.9 \times 10^{-3}$
<i>Typical Semiconductors</i>		
Silicon, pure	$2.5 \times 10^3$	$-70 \times 10^{-3}$
Silicon, n-type <sup>b</sup>	$8.7 \times 10^{-4}$	
Silicon, p-type <sup>c</sup>	$2.8 \times 10^{-3}$	
<i>Typical Insulators</i>		
Glass	$10^{10} - 10^{14}$	
Fused quartz	$\sim 10^{16}$	

## 26.4: Resistance and Resistivity, Calculating Resistance from Resistivity:

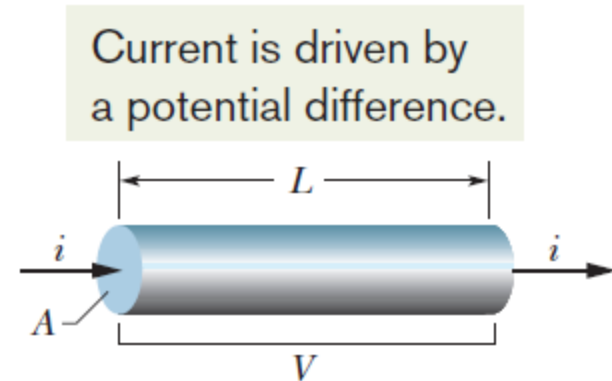


Resistance is a property of an object. Resistivity is a property of a material.

$$E = V/L \quad \text{and} \quad J = i/A.$$

$$\rho = \frac{E}{J} = \frac{V/L}{i/A}.$$

$$R = \rho \frac{L}{A}.$$

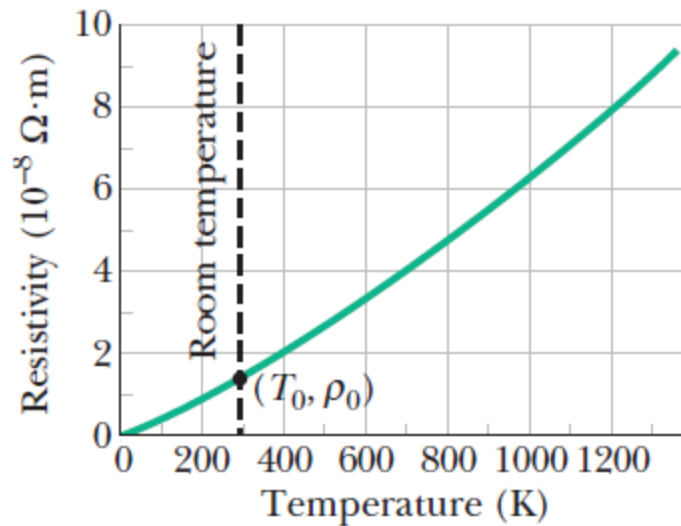


**Fig. 26-9** A potential difference  $V$  is applied between the ends of a wire of length  $L$  and cross section  $A$ , establishing a current  $i$ .

If the streamlines representing the current density are uniform throughout the wire, the electric field,  $E$ , and the current density,  $J$ , will be constant for all points within the wire.

## 26.4: Resistance and Resistivity, Variation with Temperature:

**Fig. 26-10** The resistivity of copper as a function of temperature. The dot on the curve marks a convenient reference point at temperature  $T_0 = 293$  K and resistivity  $\rho_0 = 1.69 \times 10^{-8} \Omega \cdot \text{m}$ .



Resistivity can depend on temperature.

The relation between temperature and resistivity for copper—and for metals in general—is fairly linear over a rather broad temperature range. For such linear relations we can write an empirical approximation that is good enough for most engineering purposes:

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0).$$



## Example, A material has resistivity, a block of the material has a resistance.:

A rectangular block of iron has dimensions  $1.2 \text{ cm} \times 1.2 \text{ cm} \times 15 \text{ cm}$ . A potential difference is to be applied to the block between parallel sides and in such a way that those sides are equipotential surfaces (as in Fig. 26-8*b*). What is the resistance of the block if the two parallel sides are (1) the square ends (with dimensions  $1.2 \text{ cm} \times 1.2 \text{ cm}$ ) and (2) two rectangular sides (with dimensions  $1.2 \text{ cm} \times 15 \text{ cm}$ )?

### KEY IDEA

The resistance  $R$  of an object depends on how the electric potential is applied to the object. In particular, it depends on the ratio  $L/A$ , according to Eq. 26-16 ( $R = \rho L/A$ ), where  $A$  is the area of the surfaces to which the potential difference is applied and  $L$  is the distance between those surfaces.

**Calculations:** For arrangement 1, we have  $L = 15 \text{ cm} = 0.15 \text{ m}$  and

$$A = (1.2 \text{ cm})^2 = 1.44 \times 10^{-4} \text{ m}^2.$$

Substituting into Eq. 26-16 with the resistivity  $\rho$  from Table 26-1, we then find that for arrangement 1,

$$\begin{aligned} R &= \frac{\rho L}{A} = \frac{(9.68 \times 10^{-8} \Omega \cdot \text{m})(0.15 \text{ m})}{1.44 \times 10^{-4} \text{ m}^2} \\ &= 1.0 \times 10^{-4} \Omega = 100 \mu\Omega. \end{aligned} \quad (\text{Answer})$$

Similarly, for arrangement 2, with distance  $L = 1.2 \text{ cm}$  and area  $A = (1.2 \text{ cm})(15 \text{ cm})$ , we obtain

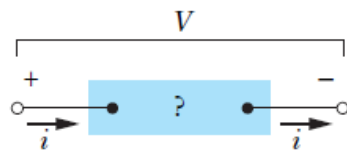
$$\begin{aligned} R &= \frac{\rho L}{A} = \frac{(9.68 \times 10^{-8} \Omega \cdot \text{m})(1.2 \times 10^{-2} \text{ m})}{1.80 \times 10^{-3} \text{ m}^2} \\ &= 6.5 \times 10^{-7} \Omega = 0.65 \mu\Omega. \end{aligned} \quad (\text{Answer})$$

# 26.5: Ohm's Law:

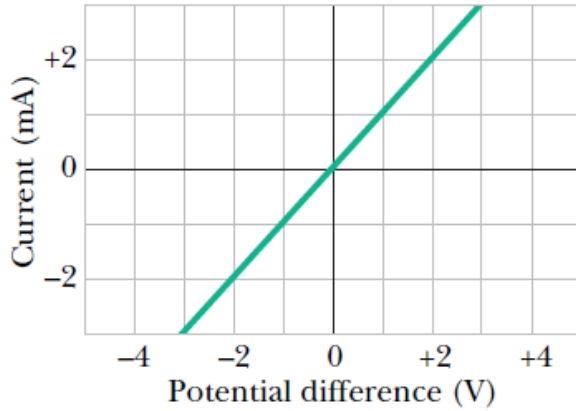
➔ **Ohm's law** is an assertion that the current through a device is *always* directly proportional to the potential difference applied to the device.

➔ A conducting device obeys Ohm's law when the resistance of the device is independent of the magnitude and polarity of the applied potential difference.

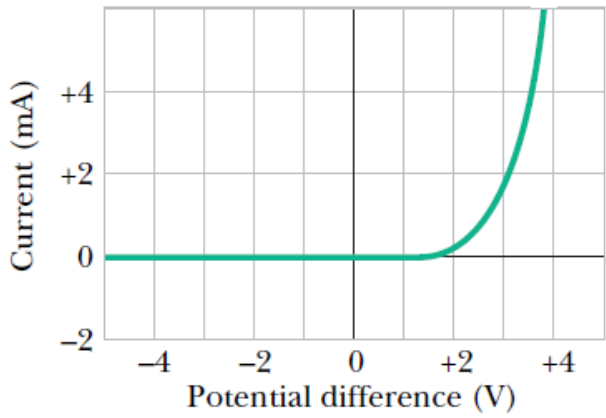
➔ A conducting material obeys Ohm's law when the resistivity of the material is independent of the magnitude and direction of the applied electric field.



(a)



(b)



(c)

**Fig. 26-11** (a) A potential difference  $V$  is applied to the terminals of a device, establishing a current  $i$ . (b) A plot of current  $i$  versus applied potential difference  $V$  when the device is a  $1000\ \Omega$  resistor. (c) A plot when the device is a semiconducting  $pn$  junction diode.

## 26.6: A Macroscopic View of Ohm's Law:

It is often assumed that the conduction electrons in a metal move with a single effective speed  $v_{eff}$ , and this speed is essentially independent of the temperature. For copper,  $v_{eff} = 1.6 \times 10^6 \text{ m/s}$ .

When we apply an electric field to a metal sample, the electrons modify their random motions slightly and drift very slowly—in a direction opposite that of the field—with an average drift speed  $v_d$ . The drift speed in a typical metallic conductor is about  $5 \times 10^{-7} \text{ m/s}$ , less than the effective speed ( $1.6 \times 10^6 \text{ m/s}$ ) by many orders of magnitude.

The motion of conduction electrons in an electric field is a combination of the motion due to random collisions and that due to  $\mathbf{E}$ .

If an electron of mass  $m$  is placed in an electric field of magnitude  $E$ , the electron will experience an acceleration:

$$a = \frac{F}{m} = \frac{eE}{m}.$$

In the average time  $\tau$  between collisions, the average electron will acquire a drift speed of  $v_d = a\tau$ .

$$\Rightarrow v_d = a\tau = \frac{eE\tau}{m}.$$

$$\vec{J} = ne\vec{v}_d \Rightarrow v_d = \frac{J}{ne} = \frac{eE\tau}{m} \Rightarrow E = \left(\frac{m}{e^2n\tau}\right)J. \Rightarrow \rho = \frac{m}{e^2n\tau}.$$

## Example, Mean Free Time and Mean Free Distance:

(a) What is the mean free time  $\tau$  between collisions for the conduction electrons in copper?

### KEY IDEAS

The mean free time  $\tau$  of copper is approximately constant, and in particular does not depend on any electric field that might be applied to a sample of the copper. Thus, we need not consider any particular value of applied electric field. However, because the resistivity  $\rho$  displayed by copper under an electric field depends on  $\tau$ , we can find the mean free time  $\tau$  from Eq. 26-22 ( $\rho = m/e^2n\tau$ ).

**Calculations:** That equation gives us

$$\tau = \frac{m}{ne^2\rho}. \quad (26-23)$$

The number of conduction electrons per unit volume in copper is  $8.49 \times 10^{28} \text{ m}^{-3}$ . We take the value of  $\rho$  from Table 26-1. The denominator then becomes

$$\begin{aligned} (8.49 \times 10^{28} \text{ m}^{-3})(1.6 \times 10^{-19} \text{ C})^2(1.69 \times 10^{-8} \Omega \cdot \text{m}) \\ = 3.67 \times 10^{-17} \text{ C}^2 \cdot \Omega / \text{m}^2 = 3.67 \times 10^{-17} \text{ kg/s}, \end{aligned}$$

where we converted units as

$$\frac{\text{C}^2 \cdot \Omega}{\text{m}^2} = \frac{\text{C}^2 \cdot \text{V}}{\text{m}^2 \cdot \text{A}} = \frac{\text{C}^2 \cdot \text{J/C}}{\text{m}^2 \cdot \text{C/s}} = \frac{\text{kg} \cdot \text{m}^2/\text{s}^2}{\text{m}^2/\text{s}} = \frac{\text{kg}}{\text{s}}.$$

Using these results and substituting for the electron mass  $m$ , we then have

$$\tau = \frac{9.1 \times 10^{-31} \text{ kg}}{3.67 \times 10^{-17} \text{ kg/s}} = 2.5 \times 10^{-14} \text{ s}. \quad (\text{Answer})$$

(b) The mean free path  $\lambda$  of the conduction electrons in a conductor is the average distance traveled by an electron between collisions. (This definition parallels that in Section 19-6 for the mean free path of molecules in a gas.) What is  $\lambda$  for the conduction electrons in copper, assuming that their effective speed  $v_{\text{eff}}$  is  $1.6 \times 10^6 \text{ m/s}$ ?

### KEY IDEA

The distance  $d$  any particle travels in a certain time  $t$  at a constant speed  $v$  is  $d = vt$ .

**Calculation:** For the electrons in copper, this gives us

$$\begin{aligned} \lambda &= v_{\text{eff}}\tau & (26-24) \\ &= (1.6 \times 10^6 \text{ m/s})(2.5 \times 10^{-14} \text{ s}) \\ &= 4.0 \times 10^{-8} \text{ m} = 40 \text{ nm}. & (\text{Answer}) \end{aligned}$$

This is about 150 times the distance between nearest-neighbor atoms in a copper lattice. Thus, on the average, each conduction electron passes many copper atoms before finally hitting one.

# 26.7: Power in Electric Circuits:

In the figure, there is an external conducting path between the two terminals of the battery. A steady current  $i$  is produced in the circuit, directed from terminal  $a$  to terminal  $b$ . The amount of charge  $dq$  that moves between those terminals in time interval  $dt$  is equal to  $i dt$ .

This charge  $dq$  moves through a decrease in potential of magnitude  $V$ , and thus its electric potential energy decreases in magnitude by the amount

$$dU = dq V = i dt V.$$

The power  $P$  associated with that transfer is the rate of transfer  $dU/dt$ , given by

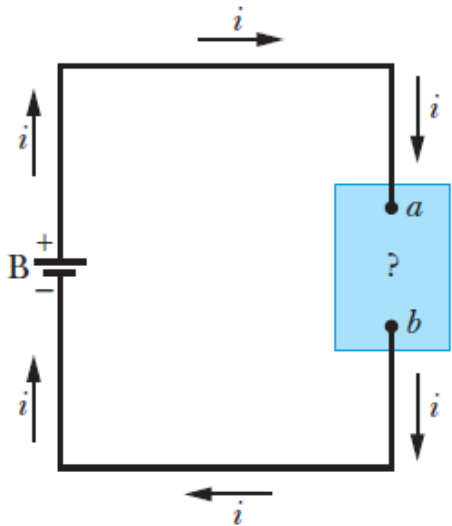
$$P = iV \quad (\text{rate of electrical energy transfer}).$$

$$P = i^2R \quad (\text{resistive dissipation})$$

$$\rightarrow P = \frac{V^2}{R} \quad (\text{resistive dissipation}).$$

The unit of power is the volt-ampere (V A).  $\rightarrow 1 \text{ V} \cdot \text{A} = \left(1 \frac{\text{J}}{\text{C}}\right) \left(1 \frac{\text{C}}{\text{s}}\right) = 1 \frac{\text{J}}{\text{s}} = 1 \text{ W}.$

The battery at the left supplies energy to the conduction electrons that form the current.



**Fig. 26-13** A battery B sets up a current  $i$  in a circuit containing an unspecified conducting device.

## Example, Rate of Energy Dissipation in a Wire Carrying Current:

You are given a length of uniform heating wire made of a nickel–chromium–iron alloy called Nichrome; it has a resistance  $R$  of  $72\ \Omega$ . At what rate is energy dissipated in each of the following situations? (1) A potential difference of  $120\ \text{V}$  is applied across the full length of the wire. (2) The wire is cut in half, and a potential difference of  $120\ \text{V}$  is applied across the length of each half.

### KEY IDEA

Current in a resistive material produces a transfer of mechanical energy to thermal energy; the rate of transfer (dissipation) is given by Eqs. 26-26 to 26-28.

**Calculations:** Because we know the potential  $V$  and resistance  $R$ , we use Eq. 26-28, which yields, for situation 1,

$$P = \frac{V^2}{R} = \frac{(120\ \text{V})^2}{72\ \Omega} = 200\ \text{W}. \quad (\text{Answer})$$

In situation 2, the resistance of each half of the wire is  $(72\ \Omega)/2$ , or  $36\ \Omega$ . Thus, the dissipation rate for each half is

$$P' = \frac{(120\ \text{V})^2}{36\ \Omega} = 400\ \text{W},$$

and that for the two halves is

$$P = 2P' = 800\ \text{W}. \quad (\text{Answer})$$

This is four times the dissipation rate of the full length of wire. Thus, you might conclude that you could buy a heating coil, cut it in half, and reconnect it to obtain four times the heat output. Why is this unwise? (What would happen to the amount of current in the coil?)

## 26.8: Semiconductors:

Table 26-2

Some Electrical Properties of Copper and Silicon

Property	Copper	Silicon
Type of material	Metal	Semiconductor
Charge carrier density, $m^{-3}$	$8.49 \times 10^{28}$	$1 \times 10^{16}$
Resistivity, $\Omega \cdot m$	$1.69 \times 10^{-8}$	$2.5 \times 10^3$
Temperature coefficient of resistivity, $K^{-1}$	$+4.3 \times 10^{-3}$	$-70 \times 10^{-3}$

Pure silicon has a high resistivity and it is effectively an insulator. However, its resistivity can be greatly reduced in a controlled way by adding minute amounts of specific “impurity” atoms in a process called *doping*.

A semiconductor is like an insulator except that the energy required to free some electrons is not quite so great. The process of doping can supply electrons or positive charge carriers that are very loosely held within the material and thus are easy to get moving. Also, by controlling the doping of a semiconductor, one can control the density of charge carriers that are responsible for a current.

The resistivity in a conductor is given by: 
$$\rho = \frac{m}{e^2 n \tau},$$

In a semiconductor,  $n$  is small but increases very rapidly with temperature as the increased thermal agitation makes more charge carriers available. This causes a decrease of resistivity with increasing temperature. The same increase in collision rate that is noted for metals also occurs for semiconductors, but its effect is swamped by the rapid increase in the number of charge carriers.

## 26.9: Superconductors:



A disk-shaped magnet is levitated above a superconducting material that has been cooled by liquid nitrogen. The goldfish is along for the ride. (Courtesy Shoji Tonaka/International Superconductivity Technology Center, Tokyo, Japan)

In 1911, Dutch physicist Kamerlingh Onnes discovered that the resistivity of mercury absolutely disappears at temperatures below about 4 K. This phenomenon is called **superconductivity**, and it means that charge can flow through a superconducting conductor without losing its energy to thermal energy.

One explanation for superconductivity is that the electrons that make up the current move in coordinated pairs. One of the electrons in a pair may electrically distort the molecular structure of the superconducting material as it moves through, creating nearby a short-lived concentration of positive charge. The other electron in the pair may then be attracted toward this positive charge. Such coordination between electrons would prevent them from colliding with the molecules of the material and thus would eliminate electrical resistance. New theories appear to be needed for the newer, higher temperature superconductors.